where we have set $q_P = k_P$, $p_P = k_N$, and $a_N = r_N$. Note that by taking the limit $a_N \to \infty$ in such a manner that a_P and a_D increase in value at the same rate as a_N, we will find that the ratio b_P/a_N remains constant.

The neutron channel entrance comprises a region for which $\tan\theta < b_P/a_N$. Thus the neutron channel entrance can only contribute to reactions for which $k_N/k_P < b_P/a_N$. Similarly, the proton channel entrance can only contribute to reactions for which $k_P/k_N < b_N/a_P$, and the deuteron channel entrance can only contribute to reactions for which $\rho_N/a_D > k_0/k_D > -\rho_P/a_D$, where

$$k_D = k_N + k_P, \qquad k_0 = \tfrac{1}{2}(k_N - k_P). \tag{12}$$

Using Eq. (12) in the last inequality, one finds $a_N/b_P > k_N/k_P > b_N/a_P$ as the range of three-body reactions to which the deuteron channel may contribute.

It is seen that the three channel entrances each contribute to distinct three-body final states: neutron channel entrance, $k_P/k_N < b_P/a_N$; deuteron channel entrance, $b_P/a_N < k_P/k_N < a_P/b_N$; and proton channel entrance, $a_P/b_N < k_P/k_N$.

The manner in which the three-body final state contributions are distributed among the three types of two-cluster channels is determined by the ratios $a_N : a_P : a_D$.

Clearly, a given three-body final state, one corresponding to a given energy partition (k_P^2, k_N^2), can receive flux from no more than one two-cluster channel entrance. The generalization of this result to more complex many-body systems would appear to be straightforward.

Note added in proof. Finally, it may be noted that although the three-body outgoing part of the asymptotic wave function shown in Eq. (7) has the form of a sum of terms each of which is an outgoing wave for one particle times a standing wave for the other, in the far asymptotic region this wave function becomes the product of outgoing waves for both particles as shown in Eq. (11).

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Remarks on the Regge-Pole Phenomenology in Nuclear Scattering

C. S. Shastry
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803

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Some level parameters of Ne^{20}, tentatively assigned on the basis of Regge-pole phenomenology, are compared with the levels determined in a recent experiment by John, Aldridge, and Davis. The connection between some of the phenomenological models for the partial-wave amplitude in nuclear scattering, the corresponding interaction potential, and the Regge-pole theory is discussed.

THIS paper contains a few interesting remarks on the work of Shanta and the present author[1] dealing with the Regge-pole analysis of α scattering by spin-zero target nuclei. In Ref. 1 it was shown that the compound nucleus levels, when grouped in terms of Regge trajectories, can be used to calculate differential cross sections through suitable Regge-type representations. However, while grouping the levels of O^{16} and Ne^{20}, it was found that if one assumes a smooth and continuous behavior of Regge trajectories, one could "predict" few possible levels of the compound nucleus. A recent experiment by John *et al.*[2] on α-O^{16} scattering and the corresponding partial-wave analysis has confirmed some of our tentative assignments of level parameters in Ne^{20}. John *et al.* restricted their studies to incident energies between 5 and 10 MeV. In this region the nuclear energy-level parameters tentatively assigned by us, and those determined in Ref. 2, are tabulated in Table I, where we have considered the second set of trajectories listed in Ref. 1.

The data on the level 2^+ at $E(\text{lab}) = 7.635$ MeV were not available in the work of Mehta *et al.*,[3] which were used in our phenomenological analysis of Ref. 1, and this level at $E(\text{lab}) = 7.7$ MeV was assigned from the extrapolation of Regge trajectories. Similarly, the 3^- level at $E(\text{lab}) = 9.58$ MeV was ambiguous, the other possible spins being 0^+ and 1^-. Again, the 3^- was chosen out of these alternatives because it was most suggestive from the Regge-trajectory point of view. Again, from the Regge-pole approach, the 3^- level at $E(\text{lab})\,7.2$ MeV was given a width of 65 keV as compared to the width of 81 keV estimated by John *et al.* However, the width of 2^+ level at $E(\text{lab}) =$

[1] R. Shanta and C. S. Shastry, Phys. Rev. **176**, 1254 (1968).
[2] J. John *et al.*, Phys. Rev. **177**, 1755 (1969).
[3] M. K. Mehta *et al.*, Phys. Rev. **160**, 791 (1967).

TABLE I. Comparison of levels of Ne^{20} tentatively assigned using Regge-pole phenomenology (Ref. 1) and the results of Ref. 2. E_{ex} denotes exitation energy.

	E(lab) (MeV)	E_{ex} (MeV)	J	Γ (keV)
Regge-pole phenomenology	7.7	10.913	2^+	150
John *et al.*	7.635	10.837	2^+	13
Regge-pole phenomenology	9.58	12.39	3^-	100
John *et al.*	9.550	12.368	3^-	40
Regge-pole phenomenology	7.2	10.49	3^-	65
John *et al.*	7.092	10.402	3^-	81

7.635 MeV is much smaller than the corresponding width predicted by Regge theory. This means that width assignments from the Regge-pole approach may not be very reliable in all cases.

The above discussion clearly demonstrates that Regge-pole phenomenology can be used along with the other existing methods while investigating the new levels and their quantum numbers. In this connection it may be noted that the work of Ref. 1 predicts for Ne^{20} two 5^- levels, one 6^+, and one 8^+ level in the incident laboratory energies between 11 and 14 MeV. Similar tentative assignments are made for the O^{16} nucleus. If an experimental confirmation becomes available for these levels, Regge-pole phenomenology can be expected to play an important role in nuclear-physics problems.

A second aspect which we wish to point out is the close connection between the phenomenological models for the partial-wave amplitude in nuclear scattering and the corresponding Regge-type representations. For example, in potential scattering it can be shown that if the scattering potential $V(r)$ is analytic for $\mathrm{Re}\,r>0$ and $|r^2V(r)|<\infty$, the corresponding phase shift $\delta_l(k)$ has the following asymptotic behavior[4]:

$$\delta_l(k) \underset{l\to\infty}{\approx} -(2\lambda^{1/2})^{1/2}\frac{V(\lambda/k)}{2k^2} - \lambda^{-1} \times \int_{\lambda/k+\lambda^{-1/2}/k}^{\infty} \frac{V(r)\,dr}{(k^2r^2-\lambda^2)^{1/2}}. \quad (1)$$

Here $\lambda=l+\frac{1}{2}$, and k is the momentum in the c.m. system. Therefore, for large real l, the partial-wave amplitude $a_l(k)$ will behave as

$$a_l(k)\approx(2\lambda^{1/2})^{1/2}\frac{V(\lambda/k)}{2k^2}. \quad (2)$$

Now one of the usual phenomenological forms suggested to the partial-wave amplitude is[5]

$$a_l(k)=\{1+\exp[(l-l_0)/\Delta]\}^{-1}, \quad (3)$$

where l_0 and Δ are parameters determined for a given energy. However, if the interacting potential has the Woods-Saxon radial form, which is usually the case, the expression for $a_l(k)$ given by Eq. (3) is suggested by Eq. (2). The Regge-type representations constructed in Ref. 1 were based on the asymptotic behavior given by Eq. (2). This close connection between the functional form of asymptotic behavior of $a_l(k)$ and the corresponding interaction potential gives more insight in appreciating such phenomenological models. It is found that[5] the fixed poles of the right-hand side of Eq. (3) can be used to generate forward peaks in differential cross sections; however, in our calculations[1] these were not included because they originate basically in the asymptotic form of $a_l(k)$ and do not have other physical significance.

[4] A. O. Barut and J. Dilly, J. Math. Phys. **4**, 1401 (1963).
[5] W. E. Frahn, in *Fundamentals in Nuclear Theory*, edited by A. de-Shalit and C. Villi (International Atomic Energy Agency, Vienna, 1967), p. 38.