

Vibrational Character of Pd¹⁰⁴ and a Spectroscopic Study of Pd¹⁰⁴ by Pickup Reactions*

D. L. DITTMER† AND W. W. DAENHICK

Nuclear Physics Laboratory, University of Pittsburgh, Pittsburgh, Pennsylvania 15213

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The Pd¹⁰⁵(*d*, *t*)Pd¹⁰⁴ and Ag¹⁰⁷(*p*, *α*)Pd¹⁰⁴ pickup reactions were studied at bombarding energies of 17 and 12 MeV, respectively. Tritons were detected by nuclear emulsions in an Enge split-pole spectrograph with 5–7-keV resolution, and 43 (*d*, *t*) angular distributions were measured for transitions to low-lying levels in Pd¹⁰⁴. Comparison of the (*d*, *t*) data with distorted-wave Born-approximation (DWBA) calculations predominantly led to *l*=0, 2, and 4 assignments and yielded spectroscopic factors and quantitative estimates for single-neutron state occupancies. A previously postulated doublet at 1.33 MeV was resolved as a nearly degenerate triplet, in agreement with the two-phonon description for collective vibrations. Further evidence for the vibrational structure of the low-lying levels of Pd¹⁰⁴ might be furnished by the well-isolated multiplet near 1.80 MeV, which may contain some or all of the members of the three-phonon quintet. In the (*p*, *α*) experiment, cross sections were found to be very small (<20 μb/sr), and thick targets had to be used which limited the resolution to 40 keV for Si detector data and ≲30 keV for magnetic spectrograph analysis. (*p*, *α*) angular distributions are given for the four strongest groups below 2 MeV. Very good agreement between the experimental results and *j*-dependent DWBA calculations based on a triton-cluster transfer mechanism supports the interpretation that direct pickup is the dominant (*p*, *α*) reaction process in this mass and energy region.

I. INTRODUCTION

THE Pd isotopes have been widely discussed as good examples of vibrational nuclei. Earlier investigations of Pd¹⁰⁴ include a number of highly selective β-decay studies^{1–5} and a (*d*, *t*) experiment⁶ of low resolution (75 keV), but extend little beyond 2-MeV excitation. The lack of a well-documented level scheme for this interesting nucleus motivated us to complement earlier work by a study of Pd¹⁰⁵(*d*, *t*)Pd¹⁰⁴ with the high resolution made possible by our Enge split-pole spectrograph⁷ and the tandem Van de Graaff. A quantitative comparison of the data with (DWBA) calculations can yield spectroscopic information regarding neutron single-particle occupancies and supplement similar results from (*d*, *p*) stripping^{8,9} in this mass region.

Our initial study of Pd¹⁰⁴ was made via the Ag¹⁰⁷(*p*, *α*)Pd¹⁰⁴ reaction.¹⁰ Previously, (*p*, *α*) reactions had rarely been used for detailed structure investigations in the heavier nuclei, and most published (*p*, *α*) studies on heavy elements have been performed with

cyclotron resolution, and with natural targets.¹¹ The present (*p*, *α*) work was carried out with a resolution of 40 keV and examines the reaction mechanism and *j* dependence in a mass-energy region where one expects compound-nuclear contributions to the reaction to be very small. The experimental results are in good agreement with DWBA triton-transfer model calculations, which include *j* dependence and indicate the potential usefulness of high-resolution (*p*, *α*) reactions for spectroscopic work.

II. EXPERIMENTAL PROCEDURE

A. Pd¹⁰⁵(*d*, *t*)Pd¹⁰⁴ Reaction

Favorable *Q* values for the Pd¹⁰⁵(*d*, *t*) reactions allowed straightforward magnetic analysis of the tritons with nuclear emulsions in the University of Pittsburgh split-pole spectrograph. Incident 17-MeV deuterons were obtained from our three-stage Van de Graaff and triton angular distributions were measured over an angular range 8°–45°. A detailed description of the scattering chamber and of the spectrograph system is given in Ref. 9.

The beam impinging on the target was collimated by a 0.5-mm-wide×2-mm-high target slit in the scattering chamber and had a horizontal divergence of ≈22 mrad. The antiscattering slit was 2 mm wide×5 mm high. By careful tuning, an excellent

* Work supported by the National Science Foundation.

† Present address: Analytic Service Inc., Falls Church, Va.

¹ R. K. Girgis and R. Van Lieshout, Nucl. Phys. **13**, 509 (1959).

² H. Nutley and J. B. Gerhart, Phys. Rev. **120**, 1815 (1960).

³ M. E. Bunker and J. W. Starnes, Bull. Am. Phys. Soc. **5**, 253 (1960).

⁴ K. Wien, Ann. Physik **10**, 281 (1963).

⁵ P. Fettweis and J. Vervier, Z. Physik **201**, 465 (1967).

⁶ B. L. Cohen and R. E. Price, Phys. Rev. **118**, 1582 (1960).

⁷ J. E. Spencer and H. A. Enge, Nucl. Instr. Methods **49**, 181 (1967).

⁸ B. Cujec, Phys. Rev. **131**, 735 (1963).

⁹ B. L. Cohen, J. B. Moorhead, and R. A. Moyer, Phys. Rev. **161**, 1257 (1967).

¹⁰ D. L. Dittmer and W. W. Daehnick, Bull. Am. Phys. Soc. **12**, 18 (1967).

¹¹ See, for instance, C. B. Fulmer and B. L. Cohen, Phys. Rev. **112**, 1672 (1958); R. Sherr and F. P. Brady, *ibid.* **124**, 1928 (1961); J. Kumabe, H. Ogata, T. Komatuzaki, N. Inoue, S. Tomita, Y. Yamada, T. Yamaki, and S. Matsumoto, Nucl. Phys. **46**, 454 (1963); J. B. Ball, C. B. Fulmer, and C. D. Goodman, Phys. Rev. **130**, 2342 (1963); C. B. Fulmer and J. B. Ball, *ibid.* **140**, B330 (1965); J. A. Nolan, Jr., C. M. Glashauser, and M. E. Rickey, Phys. Letters **21**, 705 (1966).

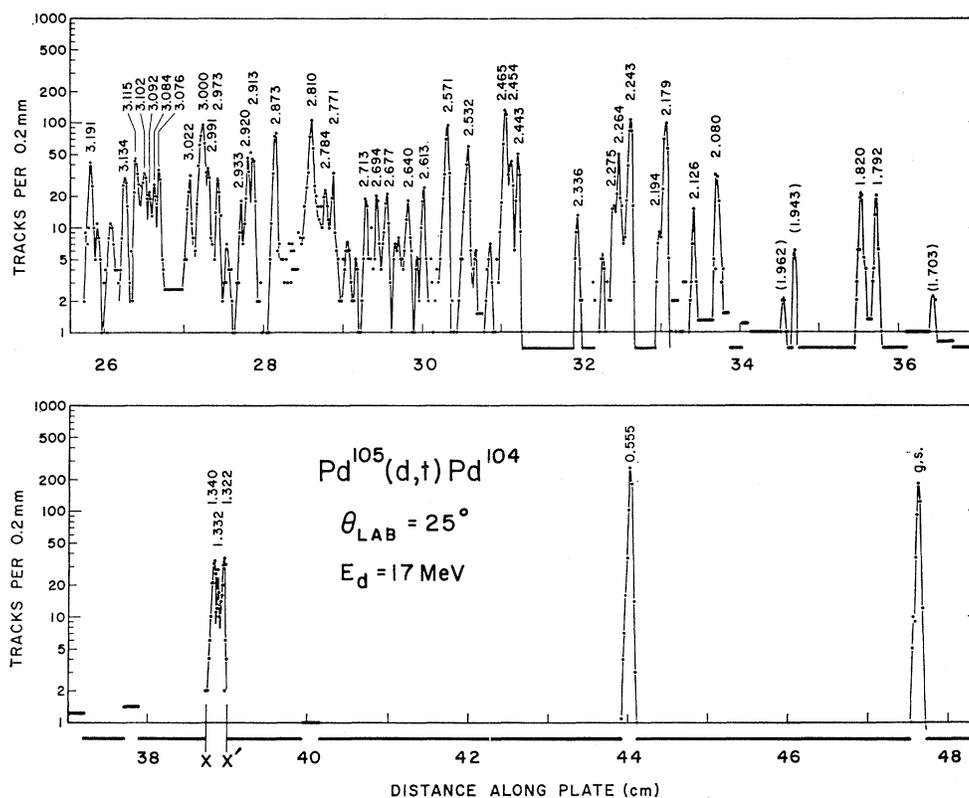


FIG. 1. Energy spectrum for $\text{Pd}^{105}(d,t)\text{Pd}^{104}$ taken at $\theta = 25^\circ$ with the split-pole spectrograph. The excitation energies are indicated (in MeV) above the peaks. Weak, unlabeled groups above 2.45 MeV in excitation probably correspond to isotropic impurities contained in the target. Transitions to single levels exhibit 5–7-keV resolution.

Faraday-cup-slit-current ratio of 35:1 was obtained. For $\theta < 20^\circ$, the target slit was increased to a 1-mm width, which resulted in complete elimination of any slit-current reading. The reaction products entered the spectrograph through the entrance aperture of 1.4 msr, and were detected along the focal plane by 25- μ Kodak NTB photographic plates. Elastic deuterons from Pd^{105} were monitored by a NaI scintillation counter mounted in the scattering chamber at 38° with respect to the beam direction. This counter served as an independent check of the charge integration monitoring that was used in normalizing the differential cross sections.

B. $\text{Ag}^{107}(p,\alpha)\text{Pd}^{104}$ Reaction

The $\text{Pd}^{105}(d,t)\text{Pd}^{104}$ study complemented an earlier $\text{Ag}^{107}(p,\alpha)\text{Pd}^{104}$ experiment.¹⁰ The (p,α) investigations were performed prior to operation of the injector stage of the accelerator and the proton energy limit of 12 MeV, together with the high Coulomb barrier for outgoing α 's, led to differential cross sections on the order of 20 $\mu\text{b}/\text{sr}$ at forward angles for the strongest single transitions. Consequently, resolution had to be sacrificed for the sake of more efficient data accumulation and most of the (p,α) measurements were carried out with a multiple detector setup in our 18-in. scatter-

ing chamber. This chamber was identical to that described by Fulmer and Daehnck¹² except for minor modifications involving freon cooling of the turntable and new beam collimation. Large positive Q values [5.872 MeV for the $\text{Ag}^{107}(p,\alpha)\text{Pd}^{104}$ ground-state transition] permitted simple α -particle identification, and the outgoing α groups were detected in four 300- μ -deep fully depleted Si surface-barrier detectors that were spaced 10° apart and cooled to -30°C . The pulses from each counter were amplified and routed into four separate 1024-channel memory groups of two Nuclear Data 4096 pulse-height analyzers. Owing to the electronics pileup problem, the small-angle runs were taken with photographic plates in the spectrograph.

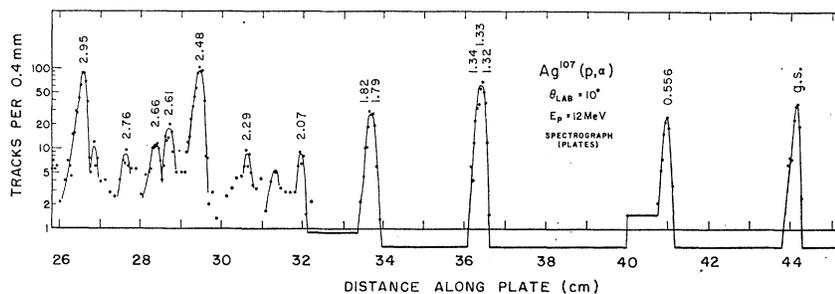
C. Targets and Resolution

Targets were prepared by vapor deposition of metallic Pd^{105} or Ag^{107} on 20–30- $\mu\text{g}/\text{cm}^2$ C foils. The Pd target, 32 $\mu\text{g}/\text{cm}^2$ thick, was only 77.2% isotopically enriched¹³ in Pd^{105} and contained sizable percentages of Pd^{106} and Pd^{104} . However, the (d,t) Q values for these

¹² R. H. Fulmer and W. W. Daehnck, Phys. Rev. **139**, B579 (1965).

¹³ Obtained from Isotopes Development Center, Oak Ridge National Laboratory.

FIG. 2. Spectrum of Ag¹⁰⁷(*p*, α)Pd¹⁰⁴ taken with nuclear emulsions at $\theta_{\text{lab}}=10^\circ$. Resolution is 28 keV for the ground-state α peak.



impurities were such that interference with the spectrum of interest did not occur below 2.45 MeV in excitation. Caution had to be used in assignment of the higher excited levels of Pd¹⁰⁵(*d*, *t*), and weak peaks appearing in this region were not included in the analysis.

Ag¹⁰⁷ foils were produced in the same manner from 99% isotopically pure material and were about 125–150 $\mu\text{g}/\text{cm}^2$ thick. Absolute-thickness determinations were obtained from Rutherford scattering at 5-MeV incident proton energy for the Ag¹⁰⁷ and from deuteron elastic scattering at 11.8-MeV bombarding energy for both the Ag¹⁰⁷ and Pd¹⁰⁵ targets. The latter results were compared with (*d*, *d*) scattering on natural targets.¹⁴

Figures 1 and 2 show typical spectra for Pd¹⁰⁵(*d*, *t*)Pd¹⁰⁴ and Ag¹⁰⁷(*p*, α)Pd¹⁰⁴. The major contributions to the experimental resolution for 16-MeV tritons and 17.5-MeV α 's are listed in Table I. Characteristics of the

TABLE I. Calculated contributions to the experimental spectrograph resolution for 16-MeV tritons and 17.5-MeV α particles from the respective Pd¹⁰⁵(*d*, *t*) and Ag¹⁰⁷(*p*, α) reactions (see Figs. 2 and 3). Energies are in keV.

	Pd ¹⁰⁵ (<i>d</i> , <i>t</i>) $\theta=25^\circ$	Ag ¹⁰⁷ (<i>p</i> , α) $\theta=10^\circ$
(a) Source size	$\lesssim 3.2$	4.5
(b) Kinematic broadening due to horizontal divergence of the incident beam	$\lesssim 7.5$	~ 2.5
(c) Increase in source size due to divergence between slit and target	$\lesssim 2.0$	~ 4.5
(d) Target thickness, straggling, and differential energy loss	1.0	~ 21.5
(e) Spectrograph aberrations	3.2	3.5
(f) Incident beam energy spread	~ 2.0	~ 2.0
(g) Plate scanning resolution	3.2	7.0
(h) Miscellaneous factors including spectrograph and analyzing-magnet field fluctuations and incorrect focal-plane positioning	~ 2.0	> 2.0
Total calculated resolution ($\sum \epsilon_i^2$) ^{1/2}	$\lesssim 10.0$	$\gtrsim 24.0$

¹⁴ G. Mairle and U. Schmidt-Rohr, Max-Planck-Institut für Kernphysik Report No. 1965/V/13 (unpublished).

Enge split-pole spectrograph have been delineated in previous publications,^{7,9,15} and were used in the present estimates. Since the size of many contributions varies with particle energy and angle of detection, the values in Table I were calculated specifically for the spectra displayed in Figs. 1 and 2.

Tritons from the Pd¹⁰⁵(*d*, *t*) reaction typically show measured resolutions of 5–7 keV. For a 0.5-mm horizontal beam-defining slit, the image on the focal plane due to the source size is $(0.5/2)[\cos(\theta-\alpha)/\cos\alpha]$ mm, where θ is the reaction angle and α the target orientation with respect to the incident beam. This led to a source-size contribution of 3.2 keV for $\theta=25^\circ$ and $\alpha=40^\circ$. Other sizable contributions to the triton resolution resulted from the horizontal divergence of the incident beam (7.5 keV, provided the beam completely filled the collimating slits) and from the aberrations of the spectrograph (3.2 keV for a 1.4-msr solid-angle acceptance).

Plates were scanned with 0.2-mm-microscope field-width settings, resulting in a 3.2-keV contribution from effect (g). The 1.33-MeV triplet was later reread with a 0.1-mm scan, which reduced the over-all measured resolution for these peaks by 1 keV and indicated the relatively sizable effect of scanning coarseness in the total linewidth. Since the individual runs were of short duration, fluctuations in the analyzing magnet and the NMR-regulated spectrograph fields were probably minimal.

Adding these terms as the sum of their squares yields a calculated resolution of 10 keV compared to the measured 6.4 keV. Certainly the small ratio of beam intercepted by the 0.5-mm defining slit indicates a sizable overestimate of the dominant calculated term, the incident horizontal divergence. With this assumption the other beam-size-dependent contributions, items (a) and (c), also would be reduced from their values in Table I.

The size of the contributions effecting the resolution for 17.5-MeV α particles from the Ag¹⁰⁷(*p*, α) reaction are listed in the second column of Table I. The target-thickness contribution (d) is the dominant term and could only be significantly reduced at the expense of prohibitively long runs. The 21.5-keV value represents

¹⁵ W. W. Daehnick, Phys. Rev. 177, 1763 (1969).

TABLE II. Optical-model parameters used in the $\text{Pd}^{106}(d, t)\text{Pd}^{104}$ and $\text{Ag}^{107}(p, \alpha)\text{Pd}^{104}$ distorted-wave calculations.

Channel	V_s (MeV)	$r_{0s}=r_{0c}$ (F)	a_s (F)	W_s (MeV)	$4W_D$ (MeV)	r_{0I} (F)	a_I (F)	V_{so} (MeV)
$d+\text{Pd}$	103	1.09	0.85	...	49.6	1.41	0.90	...
$t+\text{Pd}$	153	1.24	0.70	20.8	...	1.42	0.89	...
$p+\text{Ag}$	52.74	1.25	0.65	...	59.2	1.25	0.47	7.5
$\alpha+\text{Pd}$	150	1.49	0.60	20	...	1.49	0.60	...
Neutron well	...	1.25	0.65	$\lambda=25$
Triton well	...	1.38	0.45	$\lambda=25$

a lower limit for these contributions and does not account for additional broadening from possible target nonuniformity. This low-yield reaction necessitated the addition of two neighboring scans for acceptable statistics and introduced a further 0.4-mm or 7-keV width to the resolution. A comparison of the total calculated resolution of 24 keV with the measured value of 28 keV would seem to indicate that item (h) in Table I gives a significant contribution. Since the (p, α) experiment was performed just after installation of the spectrograph and prior to any detailed measurement of its focal characteristics, it is reasonable to assume the fluctuations in the then marginally regulated spectrograph field and errors in focal-plane positioning could account for a sizable broadening.

Resolution for the (p, α) solid-state counter runs, on the order of 40 keV for singlet levels, primarily resulted from target thickness and kinematic broadening. Energy-resolution contributions from detector noise were small due to cooling and magnetic electron suppression between the target and detector.¹⁶

D. Experimental Errors

The dominant source of error in the measured cross sections, particularly the case of (p, α) , was counting statistics. This uncertainty and those due to the background subtraction, particle-group separation, and fluctuations in the charge-to-counter monitor ratio were added as the sum of the squares, and the total random error assigned each point is indicated by the error bars in the figures.

Absolute cross-section uncertainty is due primarily to systematic errors associated with target nonuniformity and thickness determinations, charge collection, and, for the photographic plate runs, microscopist's systematic scanning errors. Based on measurements mentioned earlier, the Ag^{107} and Pd^{105} target thicknesses were assigned an error of $\pm 10\%$. At the initiation of the (p, α) studies, total charge was integrated using an Eldorado C1-110 virtual-earth integrator, which was subject to varying drift. Later an E. J. Rogers Co. model 1000 current integrator was installed and led to much-improved charge monitoring. This latter equipment was used throughout the duration of the (d, t)

work. Charge-to-detector monitor ratios indicated, on the average, a smaller than $\pm 10\%$ uncertainty in monitoring for (p, α) and less than $\pm 5\%$ monitoring uncertainty for the (d, t) experiment. The spectrograph (p, α) data were normalized by comparison with counter data. Rereads of identical (d, t) emulsions by different microscopists indicated occasional discrepancies of as much as $\pm 15\%$. Further systematic errors stem from an uncertainty in the zero scattering angle, primarily in the (p, α) studies, and from uncertainty in the spectrograph entrance aperture setting. In view of this analysis, a scale error of $\pm 15\%$ is assigned to the (d, t) cross sections, and an error of $< \pm 20\%$ to the less precise (p, α) investigations.

The absolute-energy measurements for excited states in Pd^{104} were uncertain to 0.5% of level energy. However, an improved energy calibration for the levels excited by $\text{Pd}^{106}(d, t)$ was obtained by assigning to several low-lying states the excitation energy derived from the γ -ray work of Fettweis and Vervier.⁵ In addition, tritons leading to the 4.433 state of C^{12} from the $\text{C}^{13}(d, t)$ reaction from this target backing were observed out to 20° . They furnished an additional calibra-

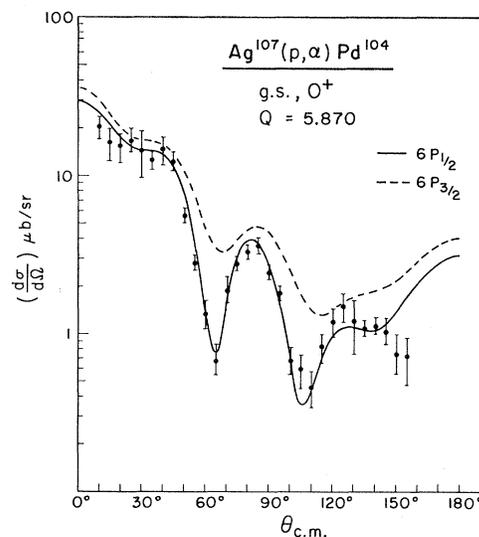


FIG. 3. J dependence for $l=1$ transfer in the $\text{Ag}^{107}(p, \alpha)$ reaction. The distorted-wave calculations (curves) are shown in comparison with the experimental data for the ground-state transition. No attempt was made to optimize the predictions by employing lower integration cutoffs.

¹⁶ G. Andersson-Lindstroem, Nucl. Instr. Methods 56, 309 (1967).

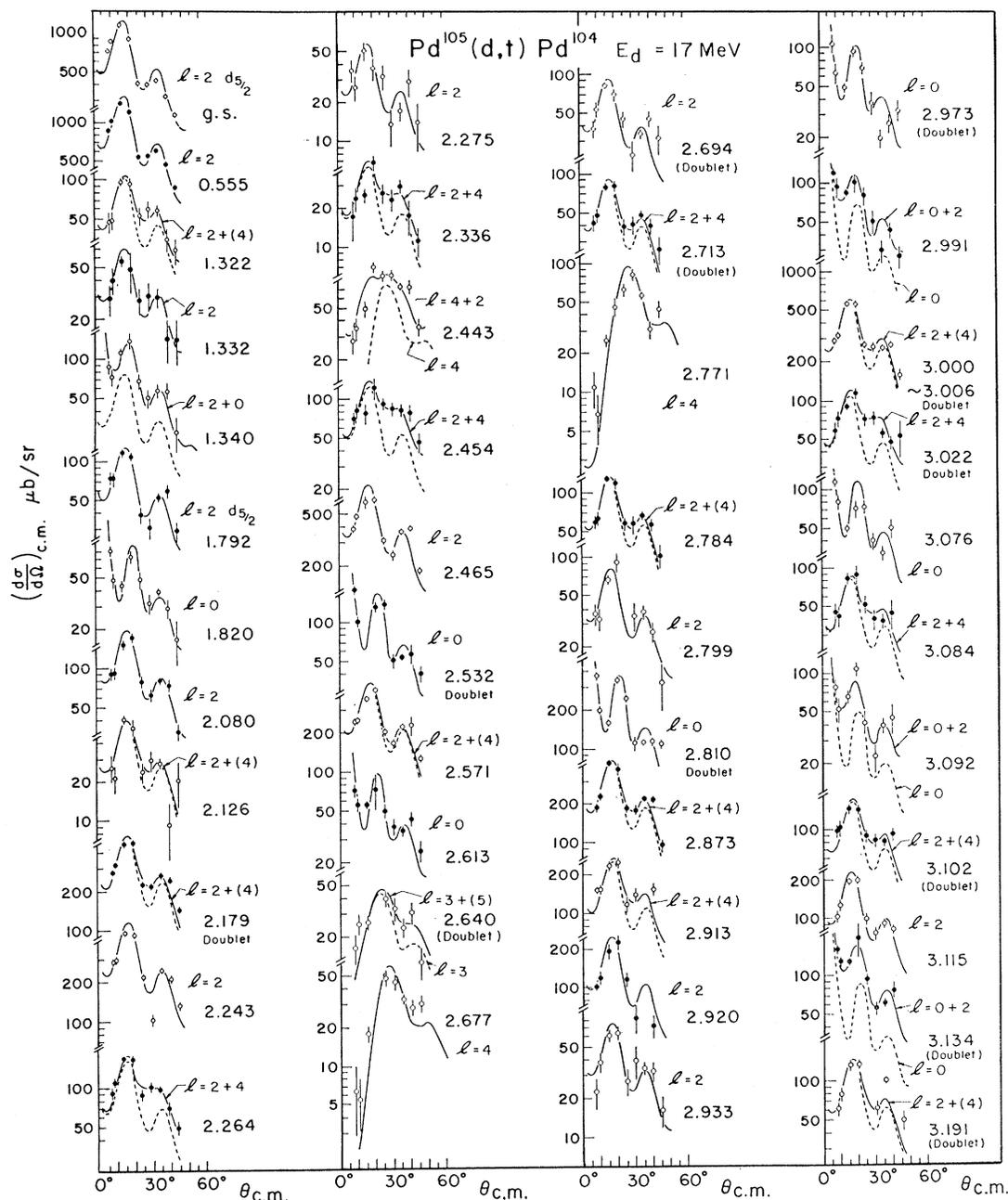


FIG. 4. Triton angular distributions from $\text{Pd}^{105}(d,t)\text{Pd}^{104}$ at $E_d=17$ MeV. Grouping of the data is according to Pd^{104} excitation energy, and error bars indicate the experimental random error. The solid curves represent the DWBA predictions for the indicated l values. For contributions of more than one l value, the dotted trace shows the principal pure- l contribution in the composite predictions.

tion point for the higher excited states of Pd^{104} . Evaluated at each angle, the excitation energies internally agree to ± 3 keV for levels up to 2.5 MeV, with a slightly larger relative discrepancy for the higher excitations. Absolute assignments listed are believed to be better than $\pm 0.2\%$ of excitation energy. Energies obtained from the (p,α) reaction are uncertain to about ± 15 keV.

III. DWBA CALCULATIONS

A. $\text{Pd}^{105}(d,t)\text{Pd}^{104}$

Zero-range DWBA predictions were obtained using code JULIE¹⁷ with the standard options for pickup

¹⁷R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3240, 1962, and supplement, 1966 (unpublished).

TABLE III (Continued)

Excitation energy (MeV)	Present work					Ag ¹⁰⁷ (<i>p</i> , <i>α</i>)Pd ¹⁰⁴			Previous assignments		Best <i>J^π</i> assignment
	<i>l</i>	<i>l_j</i>	<i>J^π</i> limits	<i>dσ/dΩ^c</i> (μb/sr)	<i>S_{l_j}</i>	Excitation energy (MeV)	<i>l_j</i>	<i>J^π</i> limits	Rh ¹⁰⁴ (<i>β⁻</i> , <i>γ</i>) ^a Ag ¹⁰⁴ (<i>β⁺EC</i>) ^b	Excitation energy (MeV) <i>J^π</i>	
2.771	4	<i>g</i> _{7/2,9/2}	1 ⁺ -7 ⁺	96	1.04	2.76					+
2.784	{ 2 (4)	{ (<i>d</i> _{5/2}) (<i>g</i> _{7/2,9/2})	1 ⁺ -5 ⁺	{ 128 15	{ 0.12 0.16						+
2.799	2	(<i>d</i> _{5/2})	0 ⁺ -5 ⁺	58	0.079						+
2.810	0	<i>s</i> _{1/2}	2 ⁺ , 3 ⁺	365	0.14						2 ⁺ , 3 ⁺
2.873	{ 2 (4)	{ (<i>d</i> _{5/2}) (<i>g</i> _{7/2,9/2})	0 ⁺ -5 ⁺	{ 423 (41)	{ 0.41 (0.45)						+
2.913	{ 2 (4)	{ (<i>d</i> _{5/2}) (<i>g</i> _{7/2,9/2})	1 ⁺ -5 ⁺	{ 240 42	{ 0.23 0.48						+
2.920	2	(<i>d</i> _{5/2})	0 ⁺ -5 ⁺	236	0.23						+
2.933	2	(<i>d</i> _{5/2})	0 ⁺ -5 ⁺	79	0.079	2.95	strong				+
2.973(D)	0	<i>s</i> _{1/2}	2 ⁺ , 3 ⁺	104	0.044						2 ⁺ , 3 ⁺
2.991	{ 0 2	{ <i>s</i> _{1/2} (<i>d</i> _{5/2})	2 ⁺ , 3 ⁺	{ 69 58	{ 0.029 0.060						2 ⁺ , 3 ⁺
3.000(D)	2	(<i>d</i> _{5/2})	0 ⁺ -5 ⁺	577	0.60						+
(3.006)	{ 2 (4)	{ (<i>d</i> _{5/2}) (<i>g</i> _{7/2,9/2})	1 ⁺ -7 ⁺	{ 69 0.82							(+)
3.022(D)	{ 2 4	{ (<i>d</i> _{5/2}) <i>g</i> _{7/2,9/2}	0 ⁺ -5 ⁺ 1 ⁺ -7 ⁺	{ 105 38	{ 0.11 0.47						+
3.076	0	<i>s</i> _{1/2}	2 ⁺ , 3 ⁺	112	0.050						2 ⁺ , 3 ⁺
3.084	{ 2 4	{ (<i>d</i> _{5/2}) <i>g</i> _{7/2,9/2}	1 ⁺ -5 ⁺	{ 79 19	{ 0.085 0.25						+
3.092	{ 0 2	{ <i>s</i> _{1/2} (<i>d</i> _{5/2})	2 ⁺ , 3 ⁺	{ 50 44	{ 0.022 0.045						2 ⁺ , 3 ⁺
3.102(D)	{ 2 (4)	{ (<i>d</i> _{5/2}) (<i>g</i> _{7/2,9/2})	0 ⁺ -5 ⁺	{ 150 31	{ 0.16 0.39						+
3.115	2	(<i>d</i> _{5/2})	0 ⁺ -5 ⁺	219	0.25						+
3.134(D)	{ 2 0	{ (<i>d</i> _{5/2}) <i>s</i> _{1/2}	2 ⁺ , 3 ⁺	{ 93 85	{ 0.10 0.13						2 ⁺ , 3 ⁺
3.191(D)	{ 2 (4)	{ (<i>d</i> _{5/2}) (<i>g</i> _{7/2,9/2})	0 ⁺ -5 ⁺ 1 ⁺ -7 ⁺	{ 144 18	{ 0.16 0.23						+

^a See Ref. 5.^b See Ref. 2.^c Measured cross sections near the maximum and listed for $\theta=20^\circ$ for $l=0$, $\theta=15^\circ$ for $l=2$, and $\theta=25^\circ$ for $l=4$.

reactions. This assumes that the transferred particle moves in a Coulomb-plus-Woods-Saxon potential, the real depth (V_0) of which is adjusted to yield the correct binding energy. For the (*d*, *t*) studies, the conventional form-factor well geometry of $r_0=r_e=1.25$ F and $a=0.65$ F was used. Optical parameters for the entrance channel were obtained from the analysis of

Perey and Perey¹⁸ of deuteron scattering from Pd at 15 MeV, and for the exit channel from elastic triton scattering from Sn¹¹⁶ at 20 MeV.¹⁹ The form-factor spin-orbit parameter λ was taken as 25. This corre-

¹⁸ C. M. Perey and F. G. Perey, Phys. Rev. **132**, 755 (1963).¹⁹ J. C. Hafele, F. R. Flynn, and A. G. Blair, Phys. Rev. **155**, 1238 (1967).

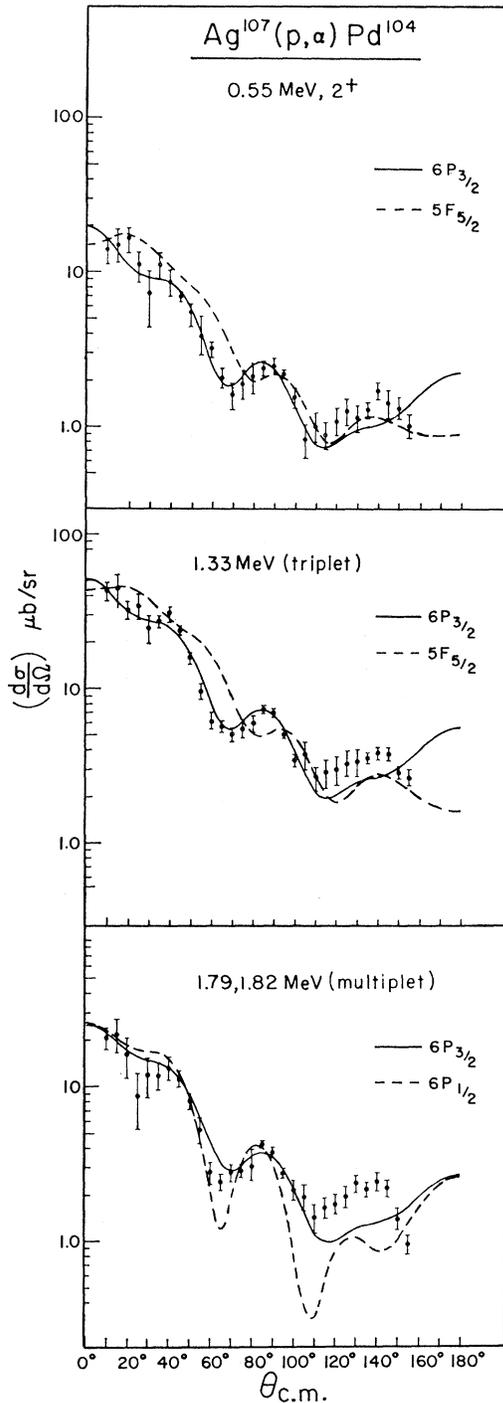


FIG. 5. α -particle angular distributions resulting from the $\text{Ag}^{107}(p, \alpha)\text{Pd}^{104}$ reaction at $E_p = 12$ MeV. The curves indicate DWBA triton-transfer predictions for the indicated j transfers.

sponds to an $1 \cdot \sigma$ potential depth of ~ 7 MeV. Table II lists the parameters used in describing the Woods-Saxon optical-model scattering potentials employed in the calculations.

In addition to l transfers, quantitative spectroscopic

information can be extracted from the (d, t) reaction. We used Bassel's²⁰ normalization for the differential cross section

$$d\sigma/d\omega = 5\frac{2}{3} \sum_{jl} S_{lj} \sigma_{lj}(\theta), \quad (1)$$

where $\sigma_{lj}(\theta)$ is the cross section calculated by code JULIE and the S_{lj} is the spectroscopic factor for a given single-particle pickup. The sum of the spectroscopic factors for all transfers of a given (l, j) is related to the fullness V_{lj}^2 of the quasiparticle state by²¹

$$\sum_i S_{ij}^2 = (2j+1) V_{ij}^2. \quad (2)$$

The extracted values V_{ij}^2 are presented below.

B. $\text{Ag}^{107}(p, \alpha)\text{Pd}^{104}$ Calculations

Our analysis of the (p, α) angular distributions assumes that the transferred quasitriton is adequately represented by a single (triton) bound-state wave function in a Ag^{107} well. The "triton-well" geometry ($r_0 = 1.38$ F, $a = 0.45$ F) was suggested by elastic triton scattering parameters and was fixed in detail by requiring a good reproduction of the unambiguous $p_{1/2}$ ground-state transition, consistent with physically reasonable geometric parameters (see Fig. 3). The inclusion of $1 \cdot \sigma$ terms in the form factor and for the scattered proton waves resulted in a remarkable fit to the data out to $\theta = 145^\circ$ (solid curve). This well geometry was then used throughout all the subsequent distorted-wave calculations. The j dependence predicted by the calculations is best judged by comparing the $p_{1/2}$ prediction with the dotted shallower structured $p_{3/2}$ DWBA curve. It results primarily from the spin-orbit interaction in the entrance channel. The absolute normalization of the two theoretical curves in Fig. 3 is arbitrary. J -dependent effects for $l=3$ transfer were not isolated since the nonzero target spin allowed an admixture of other l contributions to final states populated by f transitions. Distorted waves describing the incoming channel were generated from optical-model parameters of Perey,²² and for the outgoing channel from the deep-well α parameters of Cheston and Glassgold.²³

An objection might be raised to the use of the phenomenological cluster model in computing the radial form factor. A more realistic picture would incorporate the motion of the three transferred nucleons in their finite-well shell-model orbits.²⁴ Such a microscopic method has proved highly successful in the treatment

²⁰ R. H. Bassel, Phys. Rev. **149**, 791 (1966).

²¹ S. Yoshida, Nucl. Phys. **38**, 380 (1962).

²² F. G. Perey, Phys. Rev. **131**, 745 (1963).

²³ W. B. Cheston and A. E. Glassgold, Phys. Rev. **106**, 1215 (1957).

²⁴ B. F. Bayman, Argonne National Laboratory Report No. ANL-6878, 1964, p. 335 (unpublished).

of two-nucleon pickup,²⁵ but is at present not easily extended to the more complex three-particle case. In recent (d, α) calculations²⁶ it was shown that in many instances a microscopic form factor can be well represented by a cluster form factor; the difference is manifest only in the magnitude of the predicted cross sections. It is believed that a similar approximation would also work for the triton-transfer situation. Some previous microscopic (p, α) ²⁷ and (α, p) ²⁸ calculations used simple-harmonic-oscillator orbitals for the transferred nucleons, which produce an incorrect asymptotic-form-factor shape and lead to poor agreement with experiment. However, the triton-cluster approximation²⁸ as used in the present calculations predicts the correct radial form-factor tail and, since (p, α) is primarily a surface reaction, achieves a good qualitative description of the data.

IV. EXPERIMENTAL RESULTS

Angular distributions for 43 levels excited by $\text{Pd}^{105}(d, t)\text{Pd}^{104}$ are shown in Fig. 4. Since the target spin is $\frac{5}{2}^+$, all but 0^+ final states can be populated by more than one l transfer, and more than half of the experimental angular distributions are characterized by l mixtures. Over 80% of the angular distributions displayed some $l=2$ pickup, and are characterized by a primary maximum near 15° and a secondary peak in the vicinity of 35° . The downward slope of the data toward small angles is less extreme than in the calculations [see, for instance, the ground-state (g.s.) transition], but in general there is excellent agreement between experiment and DWBA predictions. Only two pure $l=4$ transfers were observed, although considerable $l=4$ strength enters in the admixtures, indicated by a filling in of the deep 30° minimum in the $l=2$ pattern. 0° peaking and a relatively sharp secondary maximum at 20° are indicative of transitions dominated by $l=0$. Roughly 25% of the distributions showed significant $l=0$ strength. One odd- l transfer was observed, an admixture of f and h pickup. The $h_{11/2}$ state is still fairly empty in this isotope⁹ and the existing strength is distributed among at least six levels. Hence this result is consistent with expectations.

In Table III are listed the excitation energies, l values, parity, spin limits, experimental differential cross sections, and spectroscopic factors for the (d, t) studies in comparison with the limited (p, α) results, and with previous β -decay assignments^{3,5} obtained at other laboratories. On the simplest shell-model picture, the

²⁵ N. K. Glendenning, Phys. Rev. **137**, B102 (1965); R. M. Drisko and F. Rybicki, Phys. Rev. Letters **16**, 275 (1966).

²⁶ W. W. Daehnick and Y. S. Park, Phys. Rev. **180**, 1062 (1969); Y. S. Park, Ph.D. thesis, University of Pittsburgh, 1969 (unpublished).

²⁷ J. A. Nolan, Jr., Ph.D. thesis, Princeton University, 1965 (unpublished).

²⁸ L. L. Lee, A. Marinov, C. Mayer-Boricke, J. P. Schiffer, R. H. Bassel, R. M. Drisko, and G. R. Satchler, Phys. Rev. Letters **14**, 261 (1965).

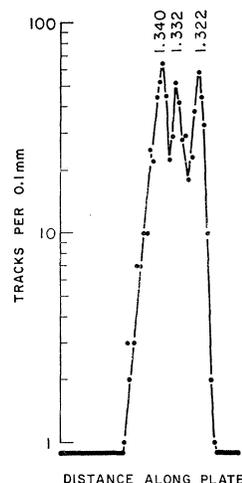


FIG. 6. Magnified portion of the $\text{Pd}^{105}(d, t)\text{Pd}^{104}$ spectrum showing the two-phonon quadrupole triplet. Previous studies (Refs. 1-5) failed to resolve the intermediate 0^+ level at 1.332 MeV and resulted in a doublet assignment for this multiplet.

Pd^{105} target is described as containing nine neutrons in the low-lying single-particle levels of the $N=50-82$ shell. Although the $2d_{5/2}$ neutron level is considered the lowest in this region, the $2d_{3/2}$ probably shows some occupancy as evidenced by a proposed 18% fullness⁹ in Pd^{106} , and pickup from this higher level cannot be ruled out. J dependence affects the magnitude of the DWBA calculations and results in approximately 20% larger cross sections at the stripping peak for the $d_{5/2}$ than the $d_{3/2}$, while a spin-independent form factor predicts σ_{DWBA} intermediate between these two. Hence quantitative use of the DWBA in determining the spectroscopic factor S_{lj} requires a particular l_j assignment to each of the transitions. The S_{lj} listed in Table III were computed from the intermediate σ_{DWBA} value, and thus are about 10% high if one encounters the more likely $d_{5/2}$ transfer, and 10% low for actual $d_{3/2}$ pickup.

Since pickup from both the closed $g_{9/2}$ as well as the filling $g_{7/2}$ shell is likely, the spectroscopic factors for the $l=4$ transitions also were computed by disregarding j dependence in the calculation. The resulting scale error for $S_{l=4}$ is more extreme than for the $l=2$ case and amounts to a $\pm 25\%$ deviation in σ_{DWBA} from the spin-independent $l=4$ determination. s transfers present no problem since all can be interpreted as neutron pickup from the $3s_{1/2}$ single-particle level. The absolute magnitude of the peak cross section for the $s_{1/2}$ calculation was roughly twice that of $l=2$ and 20 times that of the $l=4$ maxima.

Figures 3 and 5 display the (p, α) transitions to levels in Pd^{104} . The excellent fit to the known $\frac{1}{2}^-$ pickup was discussed in Sec. III. With the exception of the $p_{3/2}$ -dominated excitation of the first excited state, the angular distributions shown in Fig. 5 are likely to involve two or more unresolved levels, and all may involve more than one l value. However, no attempt was made to construct composite DWBA curves. Surprisingly, a single l value does quite well; even the multiplet near 1.8 MeV can be well represented by a mixture of

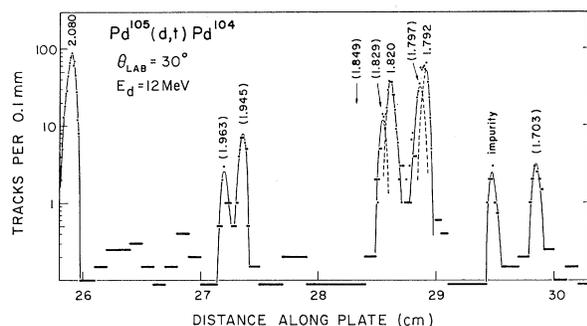


FIG. 7. Expanded plot of the region near the 1.8-MeV multiplet of the 12-MeV $\text{Pd}^{105}(d,t)\text{Pd}^{104}$ spectrum. Horizontal bars indicate averages over several points where statistics are poor.

$p_{1/2}$ and $p_{3/2}$. This fact would seem to indicate that the protons picked up are predominantly p protons.

V. DISCUSSION

The structure of even-even nuclei in this mass region can often be interpreted in terms of collective vibrations about a spherical equilibrium shape. The first excited states are generally one-phonon quadrupole excitations. Their single-particle character is very similar to the ground state.²⁹

The simplest vibrational models predict a (degenerate) 0^+ , 2^+ , 4^+ two-phonon triplet at twice the energy of the one-phonon level. In a systematic investigation of quadrupole vibrational states in even-even nuclei of the mass-100 region, Yoshizawa³⁰ found that the two phonon levels lie at 1.9–2.5 times the single-phonon excitations. Fettweis and Vervier,⁵ who used $\text{Ge}(\text{Li})$ detectors to study the γ -ray spectra resulting from the β decay of the $\text{Rh}^{104}(1^+)$ and $\text{Rh}^{104}(5^+)$ isomers, assigned levels at 1.324 and 1.340 MeV as the 4^+ and 2^+ members, respectively, of this multiplet. They further interpreted the next-higher-lying 0^+ level in their scheme at 1.792-MeV excitation as the 0^+ two-phonon state. However, the 1.792-MeV level in Pd^{104} has considerably more than three times the excitation of the first excited state, and we suggest that it is not part of the two-phonon triplet. Our argument is primarily supported by the fact that the high-resolution (d,t) spectra show an almost degenerate (within 18 keV) but well-resolved triplet near 1.33 MeV (see Figs. 1 and 6). The J^π limits obtained from the measured (d,t) transfers are consistent with the assignment of 4^+ and 2^+ to the 1.322- and 1.340-MeV levels, in agreement with previous γ -ray results (see Table III). The triton angular distribution for the new 1.332-MeV level is well represented (see Fig. 4) by $l=2$. It is therefore quite likely that this new level is the “missing” 0^+ two-phonon excitation. Recent Coulomb excitation of

three other even Pd isotopes³¹ furnishes evidence that a collective triplet occurs in these nuclei at similar energies. It remains to fit this 1.332-MeV state into the energy-level scheme of Fettweis and Vervier. The conclusions from their high-resolution studies are based, for the most part, on γ -ray coincidence relationships established by Wien.⁴ However, a weak 0.778-MeV line appearing in their spectra has been used to tentatively affirm a level at 2.102 excitation independent of any experimental correlation. In view of the present proof for the existence of a level at 1.332 MeV and lack of further substantiation^{1,2,4} of the proposed 2.102-MeV state, it is quite compelling to propose that this weak γ ray arises from $E2$ decay of the 1.332-MeV level to the 2^+ level at 0.555 MeV. The reinterpretation of the γ -decay scheme achieves consistency between level structures deduced from our (d,t) and previous γ -decay studies.

The (p,α) group leading to one or more of the states of the 1.33-MeV triplet is very well explained by the $p_{3/2}$ DWBA curve (see Fig. 5). This might indicate that it predominantly excites the 2^+ state at 1.340 MeV by removal of a neutron pair plus $p_{3/2}$ proton from $\text{Ag}^{107}(1/2^-)$. However, a noticeable broadening of this strong group suggests that at least one of the other triplet states is excited as well. This seemingly presents a contradiction, since a $\Delta J = \frac{3}{2}$ transfer can lead to neither the 0^+ nor the 4^+ level. We observe that a superposition of $f_{5/2}$ and $p_{1/2}$ would yield an angular distribution similar to $p_{3/2}$. While the latter explanation is acceptable and would imply further evidence for a 0^+ level at this energy, the problem under discussion illustrates the fact that low-resolution work (here $\Delta E_{p,\alpha} = 40$ keV) can lead to misinterpretations of levels as “safe” as the second excited state in even-even nuclei, and must be interpreted with extreme caution.

The above-mentioned β^- investigations as well as those of Bunker and Starnes³ document a 0^+ state at

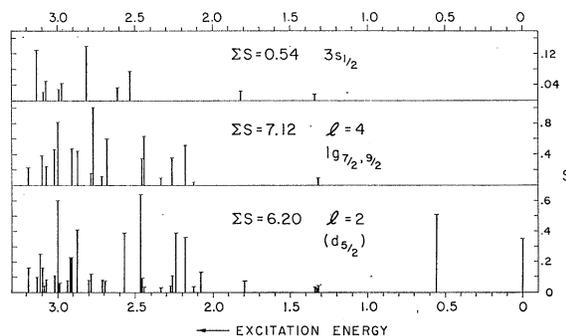


FIG. 8. Graphical display of the spectroscopic strengths as a function of excitation energy for the $\text{Pd}^{105}(d,t)\text{Pd}^{104}$ experiment. ΣS yields a lower limit for the fullness of the single-particle neutron orbits and is listed for the observed l transfers. The $l=2$ pickup is probably mostly $d_{5/2}$.

²⁹ A. Bohr and B. R. Mottelson, *Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd.* **27**, No. 16 (1953).

³⁰ Y. Yoshizawa, *Phys. Letters* **2**, 261 (1962).

³¹ R. L. Robinson, F. K. McGowan, P. H. Stelson, W. T. Milner, and R. O. Sayer, *Bull. Am. Phys. Soc.* **13**, 1467 (1968).

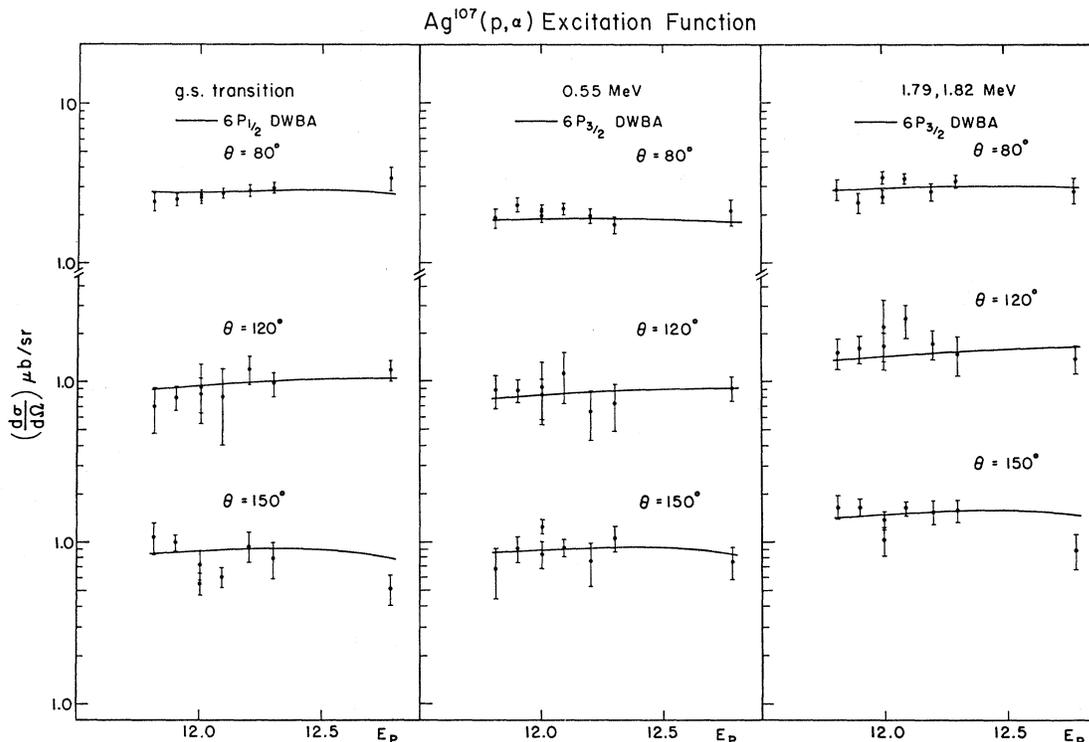


FIG. 9. Variation of the differential (p, α) cross section with bombarding energy at three detection angles. The separate columns refer to the ground state and two excited (p, α) transitions. Solid curves are DWBA $\sigma(E)$ predictions and are normalized to best reproduce the data.

1.792 MeV, while the selective β^+ decay of Ag¹⁰⁴ populates a different level,^{1,2} a 2^+ state at 1.812 MeV. Excitation of both levels is observed in (p, α) and in the high-resolution (d, t) data. Experimental triton distributions exhibit pure $l=2$, for the lower level, and $l=0$ for the higher one and corroborate the 0^+ and 2^+ assignments suggested by the decay studies. The vibrational model predicts a three-phonon (0^+ , 2^+ , 3^+ , 4^+ , 6^+) quintet at 2.8 to 3.9 times³⁰ the excitation of the first 2^+ level, and possibly some or all of the observed levels near 1.8 MeV are members of this collective multiplet. A state at 1.849 MeV was postulated from inferred β^- decay in conjunction with an observed γ line,⁵ but was not detected in the present (d, t) work. However, we find some indications for other levels near 1.8 MeV not previously reported. Figure 7 shows an expanded section of the region 1.65–2.1 MeV in excitation energy. The spectrum was taken at $E_d=12$ MeV with better statistics and resolution close to that of the 17-MeV runs and clearly shows the strong well-established levels at 1.792, 1.820, and 2.080 MeV. It also shows weak but statistically significant groups near 1.703, 1.943, and 1.962 MeV. Traces of these groups are seen in almost all Pd¹⁰⁵ (d, t) spectra 8° to 45° ; however, poor statistics do not permit excitation-energy assignments to better than ± 10 keV, so that we

cannot rule out the possibility that these three weak groups stem from heavy target impurities with masses near the Pd mass.

In most of the 17-MeV data, the 1.820- and 1.792-MeV peaks show low-energy humps or peak broadening which is not observed in the known singlets. Figure 7 includes our statistically most significant examples. It appears that two weaker satellites at 1.829 and 1.797 MeV are almost resolved from the 1.820- and 1.792-MeV states. The new level at 1.829 MeV finds some independent support from the existence of a newly found, but as yet unassigned 1.273-MeV γ ray.³² This transition would lead to the 0.556-MeV 2^+ state and limit the spin of the 1.829-MeV level from $0 \rightarrow 4$. It is conceivable that the three-phonon quintet could be nearly as degenerate as the two-phonon triplet near 1.33 MeV, since it is well below the single-particle energy gap and separated by 250 keV from the closest established level. Assuming that the three-phonon interpretation is valid, one would expect $J^\pi=3^+$ (4^+) for the (1.829 ± 0.005) -MeV level, since we already have a 0^+ assignment for the 1.792-MeV level and 2^+ for the 1.820-MeV group. The suggested level at 1.797 MeV would then have to be one of the remaining high-spin levels: 4^+ , 6^+ , or (less likely) 3^+ .

³²T. Doron (private communication).

The case for a close three-phonon quintet is weakened by the fact that, as yet, β decay from the 5^+ isomer in Rh^{104} and Ag^{104} to a level close to 1.8 MeV has not been reported. Neither is there any published evidence for the expected *strong* γ transition between the tentatively proposed three-phonon levels near 1.8 MeV and the three two-phonon states at 1.33 MeV. It is conceivable that these γ rays of about 500 keV are masked by the extremely strong 556-keV ground-state transition, and another look at low-energy γ transitions in coincidence with the 556-keV γ ray should clarify the issue.

The remaining higher levels proposed by previous studies (Table III), with the exception of the 2.102-MeV state, are in general agreement with the (d, t) Q values and parity and spin limits, although the less selective neutron pickup reaction excites, as expected, many additional levels in this region. l assignments for Pd^{104} have been extended through 3.191 MeV by the present (d, t) experiment and indicate a complex level structure for this even-even nucleus. Jolly *et al.*³³ postulated six negative-parity states up to 3.2-MeV excitation by applying the Blair phase rule to inelastic deuteron scattering. The resolution of their experiment (~ 75 keV) was not adequate to select out individual states; direct comparison with our (d, t) level scheme is difficult. However, an assignment of negative parity to the two-phonon multiplet near 1.33 MeV contradicts our present data and the nature of the excitation, and shows the weakness of the Blair phase rule in (d, d') . No firm evidence was found in (d, t) for odd- l transfer to levels near the postulated³³ negative-parity states at 2.20, 2.80, 3.10, and 3.19 MeV, although the 2.64-MeV excitation agrees well with the 2.640-MeV negative-parity transition, $l=3+(5)$, observed in the present (d, t) studies.

Figure 8 displays the single-particle pickup strength versus excitation energy in Pd^{104} for the three orbital angular momentum transfers observed in $\text{Pd}^{105}(d, t)$. Since $\sum_i S_{1j}^i$ is indicative of the number of neutrons contained in the shell-model state j in the target nucleus, the fullness of the $s_{1/2}$ and $d_{5/2}$ single-particle states in Pd^{105} can be determined and compared with previous results for other nuclei in this region. For the $l=2$ levels, $\sum S \simeq 6.2$ (compared with 5.8 if one assumes all the pickup occurs from the $2d_{5/2}$ orbit), whereas stripping⁹ predicts 5.1 neutrons in the d states (4.4 in $d_{5/2}$, 0.7 in $d_{3/2}$) for the neighboring Pd^{106} . The $3s_{1/2}$ strength is about a factor of 2 ($\sum S_{1/2}^i = 0.54$, i.e., $V_{1/2}^2 = 0.27$) weaker than implied by Cohen's $\text{Pd}^{106}(d, p)$, which predicts $V_{1/2}^2 = 0.43$. Since a good deal of the (d, t) $s_{1/2}$ sum is contributed by levels in the vicinity of 3 MeV, it is likely that significant strength has been missed in this and possibly also in the (d, p) experiment by not extending the analysis to high enough excitation energies. We conclude that $0.27 < V_{1/2}^2 < 0.43$. Repro-

duction of the (d, p) angular distributions by zero-range DWBA was, however, less successful than for the present (d, t) studies and could indicate additional difficulties in extracting (d, p) spectroscopic factors. The $l=4$ strength seen is due to $g_{9/2}$ and $g_{7/2}$ transitions, and only a lower limit $V_{l=4}^2 \lesssim 0.40$ can be given for the fullness of the $1g$ states.

In the analysis of the (p, α) reaction (Sec. III), a direct triton-cluster transfer was assumed. Owing to the very large number of open decay channels in the compound nucleus, little (p, α) compound contribution to low-lying states in this mass and energy region is expected. In order to check this assumption, excitation functions for three $\text{Ag}^{107}(p, \alpha)$ transitions were measured at 80° , 120° , and 150° over an incident energy range 11.8–12.8 MeV and are presented in Fig. 9. The smooth variation of cross-section with bombarding energy, as well as the qualitative agreement with DWBA expectations, lends increased support to the postulated directness of the reaction. In the absence of Ericson fluctuations, compound processes should have symmetry about 90° , but such a trend definitely is not observable in the data of Fig. 3. Confidence in the direct triton-transfer model is strengthened by the excellent agreement of experiment with DWBA theory for the known transitions to the ground and first excited states (Figs. 3 and 5). Other direct processes, such as α -particle knockout with a capture of the incident proton into a bound state of the final nucleus, have been often suggested as competing mechanisms. However, amplitudes for these exchange processes are generally considered quite small relative to nonexchange modes.²⁴ They, possibly, attain increased significance at much higher bombarding energies.³⁴

VI. SUMMARY

Results of the high-resolution $\text{Pd}^{105}(d, t)$ studies allow a consistent description of the low-lying states of the even-even nucleus Pd^{104} in terms of collective vibrational excitations. The nearly degenerate two-phonon levels were resolved, and some evidence for a three-phonon quintet was found. Discrepancies with several of the β -decay⁵ conclusions can be resolved. The level scheme for this nucleus has been extended up to 3.2 MeV, with the assignment of 38 new states. Standard DWBA calculations led to l -value determinations and J^π limits as well as spectroscopic factors for 43 transitions. Subject to the precaution that important pickup strength may arise from excitations above those included in the analysis, sum rules suggest that the $2d_{5/2}$ neutron level is 80–90% full while the $3s_{1/2}$ single-particle state is on the order of $\gtrsim 25\%$ full. The measured $l=4$ strength includes $1g_{7/2}$ and $1g_{9/2}$ transfer.

It was shown in principle that the $\text{Ag}^{107}(p, \alpha)\text{Pd}^{104}$

³³ R. K. Jolly, E. K. Lin, and B. L. Cohen, Phys. Rev. **128**, 2292 (1962).

³⁴ J. Muto, H. Itoh, K. Okano, N. Shiomi, K. Fukuda, Y. Omori, and M. Kihara, Nucl. Phys. **47**, 19 (1963).

reaction can provide much information regarding the spectroscopy of Pd¹⁰⁴, but because of limited resolution the data were primarily of interest as a reaction-mechanism study. Comparison of the experimental angular distributions and energy dependence with DWBA theory furnishes excellent evidence that direct triton pickup is the dominant process. A strong j dependence for $l=1$ transitions was observed in the

data and well reproduced by DWBA triton-cluster transfer calculations.

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Precise Measurement of the Muonic X Rays in the Lead Isotopes*

H. L. ANDERSON

Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637

AND

C. K. HARGROVE, E. P. HINCKS,† J. D. McANDREW,‡ AND R. J. McKEE

National Research Council of Canada, Ottawa, Canada

AND

R. D. BARTON AND D. KESSLER

Carleton University, Ottawa, Canada

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The size and shape of the nuclear charge distribution was determined for the Pb isotopes from precise measurements of their μ -atomic transition energies. Use of the $2p$ - $1s$ transitions was avoided in determining the nuclear parameters because of the perturbation caused by the presence of the muon in the $1s$ orbit. A simple two-parameter Fermi distribution was used in the analysis, but the rms radius determined thereby should be fairly model-independent. We find $\langle r^2 \rangle^{1/2} = 5.4839 \pm 0.0028$ fm for Pb²⁰⁶. This quantity increases by 0.0139 ± 0.0011 fm in going to Pb²⁰⁸. The calculated energy of the $1s$ level is found to be too high by 6.8 ± 2.3 keV, an effect which we have interpreted as being due to nuclear polarization, although the inadequacies in our treatment of the radiative corrections and of the effect of nuclear motion may account for a part of this difference. The measurement of the $4f$ - $3d$ and $5g$ - $4f$ transition energies provides a check of the vacuum-polarization correction, which is just at the limit of the higher-order contributions. The intensity ratios $I(2p_{3/2}-1s_{1/2})/I(2p_{1/2}-1s_{1/2})$ are found to be anomalously low for all three Pb isotopes and by as much as $(12 \pm 3)\%$ in the case of Pb²⁰⁸. In general, the intensities are reasonably well described by a cascade calculation, but the indication is that radiationless transitions do occur which can raise the Pb nucleus to an excited state. We detect *prompt* nuclear γ rays corresponding to this process. Some 15μ -capture γ rays with yields ≥ 0.01 per μ capture are reported for Pb²⁰⁶. One with a yield of 0.18 per μ capture is attributed to the $\frac{3}{2}^+ \rightarrow \frac{1}{2}^+$ g.s. transition in Tl²⁰⁶, following the emission of one neutron.

I. INTRODUCTION

IN this paper, we present the results of a careful study of the muonic x-ray lines of the Pb isotopes, with particular emphasis on Pb²⁰⁶. Our purpose was to perform a high-precision absolute measurement of the energies of the principal transitions of the Pb isotopes and use these energies to determine the parameters of their nuclear charge distribution. Since the same quantities are also determined by electron-scattering measurements, it is of interest to see to what extent such complementary methods give consistent results. When made with sufficient precision, such measurements provide a demanding test of our understanding of

hydrogenlike atoms, including the applicable radiative corrections and the effects of nuclear structure. We were interested in finding out how far the calculation of the energy levels could be relied on to give correct results.

The early measurements of muonic x-ray energies were limited by the resolution of NaI spectrometers. The development of the Li-drifted Ge detector by Tavendale¹ improved the resolution by an order of magnitude and made possible energy measurements correspondingly more precise. In our previous work^{2,3}

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* Research supported by the U. S. National Science Foundation and the National Research Council of Canada.

† Also at Carleton University, Ottawa, Canada.

‡ Present address: Memorial University of Newfoundland, St. Johns, Newfoundland, Canada.