Inelastic Scattering of 14.4-MeV Protons by the Even Isotopes of Titanium*

H. F. Lutz, W. Bartolini, T. H. Curtis, and G. M. Klody†
Lawrence Radiation Laboratory, University of California, Livermore, California 94550
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Elastic and inelastic scattering of 14.4-MeV protons from isotopically enriched targets of ^{46,48,50}Ti were studied. Angular distributions for many groups separated by more than 40 keV were obtained. The data were analyzed using the coupled-channel calculation. The vibrational collective model was used to describe the states of the bombarded nucleus. Several octupole states were found in both ^{46}Ti and ^{48}Ti but only one in ^{50}Ti. These results were compared with the results of α-particle scattering on the same nuclei. It was found that a discrepancy exists between the results of the experiments in the magnitudes of the deformabilities obtained for the octupole states. Spins and parities of some states in ^{48}Ti and ^{50}Ti were suggested. Electromagnetic transition rates were inferred from the deformabilities deduced from inelastic scattering data.

INTRODUCTION

In this paper, we report our experimental data of the scattering of 14.4-MeV protons from isotopically enriched targets of ^{46,48,50}Ti and the analysis of the data using the coupled-channel calculation[1] in the form of Tamura's computer code.[2,3] This work is part of a series of experiments in which we also studied the even-mass isotopes of cadmium[4] and germanium.[5] It was our desire to study the inelastic scattering from all stable even-mass isotopes of titanium in the same experiment and subject the data to an analysis that could handle both single-step and multiple-step excitations within the framework of the collective vibrational model. Several items of particular interest are brought out by the scattering of particles from these isotopes. These are the presence of several octupole states in ^{46}Ti and ^{48}Ti but only one in ^{50}Ti (which closes the $1f_{7/2}$ neutron shell); the possible description of certain states in terms of two-quadrupole-phonon states; and the existence of a large number of states that are excited by inelastic scattering but which do not seem to be readily accountable in terms of direct reaction theory as we apply it.

Bernstein *et al.*[6] have studied ^{48}Ti with α-particle scattering and have summarized the α-particle experiments of the Saclay[7] and Argonne[8] groups on the other even-mass isotopes of titanium. In general, we agree with their location of octupole states, but there is a significant difference in the magnitude of the octupole deformability parameter deduced in α-particle and proton experiments.

Peterson and Perlman[9] have studied proton scattering from ^{46}Ti at 17.5 MeV and Barnard and Jones[10] at 21.2 MeV. Above 3-MeV excitation energy, the energy and spin assignments of Barnard and Jones[10] are at variance with those of Peterson and Perlman[9]; the present experiment agrees with the data of the latter authors. Inelastic scattering of 14.65-MeV protons by ^{48}Ti has been performed by Matsuda.[11]

Table I. Isotopic compositions of Ti targets used in the present experiment.

at. %	Target ^{46}Ti	^{48}Ti	^{50}Ti
46	77.1	0.2	3.1
47	2.3	0.3	2.4
48	17.3	99.0	22.4
49	1.5	0.4	2.4
50	1.8	0.1	69.7
Thickness mg cm^{-2}	0.70	1.10	1.01

The magnitudes of his cross sections are in good agreement with our experimental results. Belote *et al.*[12] have measured the energy levels of ^{48}Ti to within 10 keV with a magnetic spectrograph but did not obtain angular distributions. These energy assignments have been used in the present experiment. The isotope ^{50}Ti has been investigated by Funsten *et al.*[13] with 17.5-MeV protons and by Gray *et al.*[14] with 18.2-MeV protons. The present experiment studies ^{46,48,50}Ti with

* Work performed under the auspices of the U.S. Atomic Energy Commission.
† Summer visitor; permanent address: Physics Department, University of Wisconsin, Madison, Wisc.
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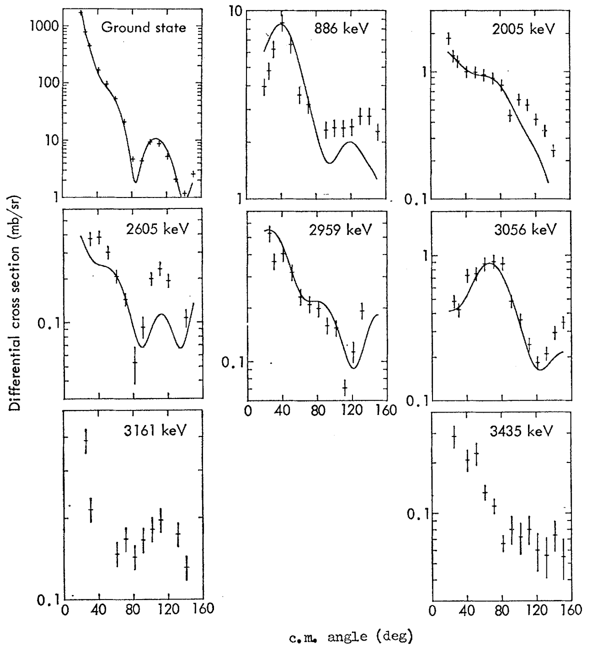

FIG. 1. Angular distributions of 14.4-MeV protons scattered from ^{46}Ti, from ground to 3435 keV.

14.4-MeV protons and was performed with a resolution that equalled or exceeded the resolution in the abovementioned experiments, with the exception of Belote *et al.*[12]

Much useful information on the titanium isotopes has already been obtained by particle-transfer-reaction investigations and by particle–γ-ray angular-correlation studies. Barnes *et al.*[15] have studied the (d, p) reaction while Kashy and Conlon[16] have studied the (p, d) reaction. Hinds and Middleton[17] have studied the (t, p) reactions leading to ^{48}Ti and ^{50}Ti. Hinds *et al.*[18] have noted that, for $N>28$, the two-neutron transfer reaction has its main $L=0$ cross section going to ground states; whereas for $N\leq 28$, strongly excited 0^+ states are always present. These authors speculate that this change in behavior of the $L=0$ (t, p) spectrum is related to a change in the importance of the short-range component of the residual interactions.

The locations of the 0^+ excited states in ^{46,48,50}Ti have been given by Church *et al.*[19] The γ-ray decays in the Ti isotopes have been investigated by Soga *et al.*[20] Spin and parity assignments have been made in ^{46}Ti by Lewis *et al.*,[21] in ^{48}Ti by Monahan *et al.*,[22] and in ^{50}Ti by Chilosi *et al.*[23]

EXPERIMENTAL PROCEDURE

Protons were accelerated to 14.4 MeV by the Livermore variable-energy cyclotron. The beam was momentum-analyzed by a 90° bending magnet with a 76.2-cm radius of curvature and focused to a spot at the center of a 60.9-cm-diam scattering chamber. The beam was monitored by a Faraday-cup–current-integrator combination.

The targets consisted of self-supporting metallic films having areal densities in the neighborhood of 1 mg cm^{-2}. The isotopic composition and exact thicknesses of the various targets are presented in Table I. The ^{48}Ti target was the purest and appeared as the major contaminant in the other two targets.

A single lithium-drifted silicon detector 2000 μm thick, operated with a reverse bias of 200 V, was used to observe the scattered protons. The detector subtended a solid angle of 2.50×10^{-4} sr. It was cooled to $-25°$C by a thermoelectric cooling device to minimize leakage currents.

The spectra were stored in an 800-channel pulse-height analyzer. The punched paper tape output of the pulse-height analyzer was used to generate computer cards that were used as input to the CDC-

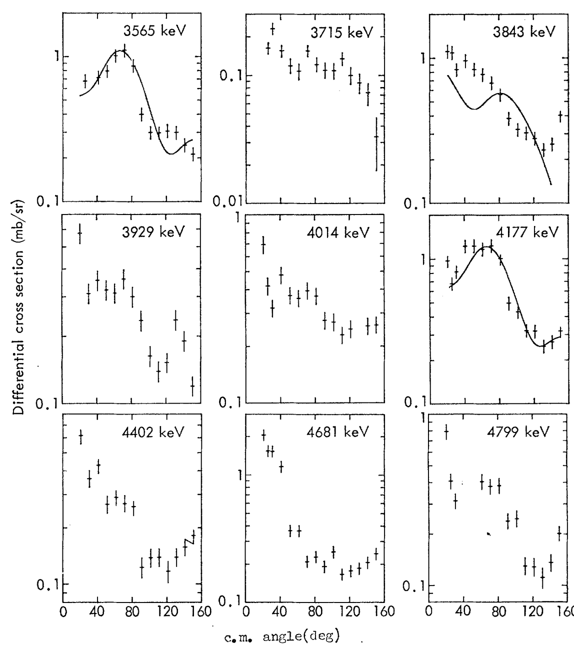

FIG. 2. Angular distributions of 14.4-MeV protons scattered from ^{46}Ti, from 3565 to 4799 keV.

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TABLE II. Levels excited in the present experiment with their spins, parities, and integrated cross sections.

^{46}Ti			^{48}Ti			^{50}Ti		
Ex (keV)	J^{π}	σ (mb)	*Ex* (keV)	J^{π}	σ (mb)	*Ex* (keV)	J^{π}	σ (mb)
886	2^+	42.4	985	2^+	33.3	1550	2^+	16.8
2005	4^+	8.8	2301	4^+	2.7	2686	4^+	4.8
2605	0^+	2.4	2425	2^+	2.6	3208	6^+	1.1
2959	2^+	2.7	3004	0^+	1.9	3770		0.8
3056	3^-	6.5	3230			3880	0^+	1.9
				4^+	5.4			
			3249					
3161	1^+	2.2				4158	4^+	
								8.4
			3365	3^-		4184	2^+	
3435	(5)	1.4			8.8			
			3376	$2^{(+)}$				
3565	3^-	7.0				4322	2^+	3.3
			3520		0.7			
3715		1.4				4420	3^-	5.5
			3625	2^+	1.8			
3843	4^+	6.8				4738		0.8
			3707	1^-	1.2			
3929		3.1				4808	(2^+)	4.9
			3780	1^-	0.7			
4014		4.0				4898	(2^+)	2.9
			3850	3^-	2.1			
4177	3^-	8.6				4989		1.6
			4048					
				1^+	4.0	5206	(4^+)	2.4
4402		2.9	4087					
						5348		1.1
4681		5.2	4203		1.0			
						5417		1.8
4799		3.4	4315		1.1			
			4385	(4^+)	2.8			
			4590	3^-	4.8			
			4806	2^+	2.6			
			4930	2^+	2.9			
			5160		1.8			
			5310		1.7			
			5400		1.2			
			5540	3^-	3.0			
			5630		3.1			
			5800	3^-	1.9			
			5900		1.6			
			6090		1.7			
			6240		1.4			
			6360		1.5			
			6470	(2^+)	2.3			

3600 computer. Several computer programs were then used to plot the spectra, fit the spectra with sums of Gaussian-shaped peaks to obtain peak locations and areas, and deduce and plot angular distributions. In the present experiment, we were able to obtain angular distributions for levels separated by more than 40 keV.

EXPERIMENTAL RESULTS AND COUPLED-CHANNEL ANALYSIS

Experimental Results

The levels seen in the present experiment are summarized in Table II, with their spins and parities (when known) and their integrated cross sections. Since the ^{48}Ti target was the purest target, we were able to extract levels at a higher excitation than in either ^{46}Ti or ^{50}Ti. The values for the integrated cross sections were obtained by fitting the experimental data with a sum of Legendre polynomials. The energy, spin, and parity assignments of ^{46}Ti agree with those made by Peterson and Perlman.[9] The spin assignments up to the 3161-keV level have been determined by a $(p, p'\gamma)$ angular-correlation study performed by Lewis *et al.*[21] Peterson and Perlman[9] also report a level at 3231 keV that we do not see because it was masked by a contaminant peak due to levels in ^{48}Ti at 3230 and 3248 keV. The ^{46}Ti data are grouped in Figs. 1 and 2.

The spin-1 assignments in ^{48}Ti have been made by Monahan *et al.*[22] using $(p, p'\gamma)$ angular-correlation techniques. The even-parity assignments agree with the work of Hinds and Middleton[17] with the exception of the 4591-keV level, which we say is a 3^- and they assign a 0^+. There is the possibility that these are

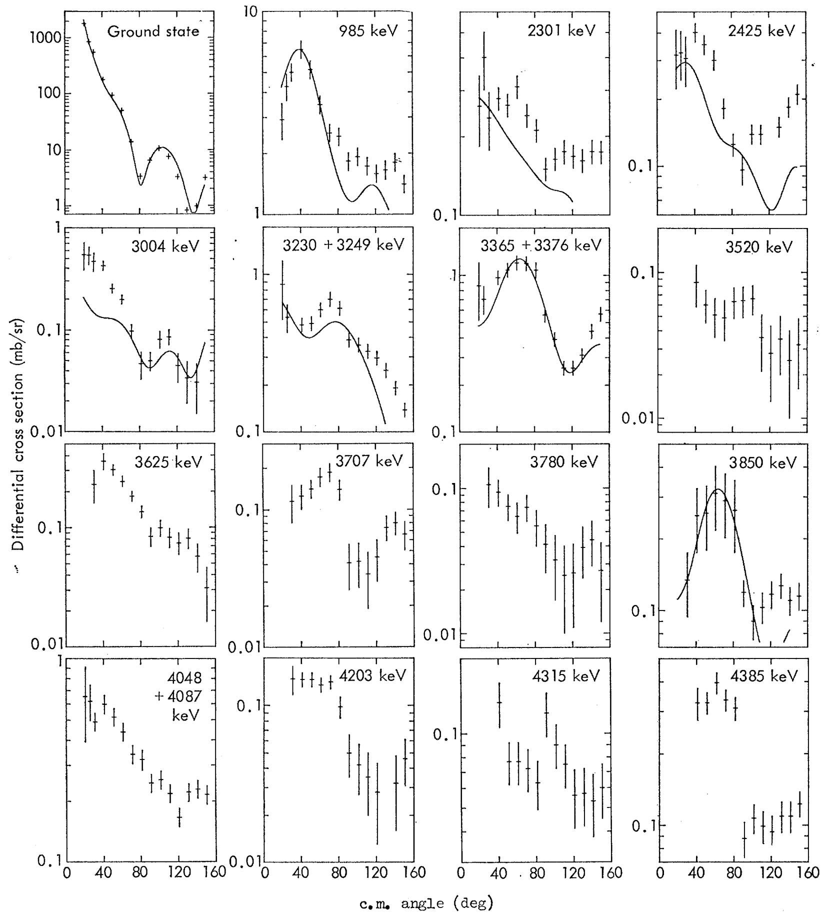

FIG. 3. Angular distributions of 14.4-MeV protons scattered from ^{48}Ti, from ground to 4385 keV.

really two levels, lying close together, each one in turn preferentially excited. The octupole states have been identified by Bernstein *et al.*[6] and our findings agree with their assignments. The graphs for ^{48}Ti are in Figs. 3 and 4.

In ^{50}Ti Chilosi *et al.*[23] have identified the first four levels as being a 0^+-2^+-4^+-6^+ sequence by studying the γ rays following the β decay of ^{50}Sc. The spin and parity assignments up to 4420 keV are summarized by Hinds and Middleton.[17] The 2^+ assignments to the 4808- and 4898-keV levels, and the 4^+ assignment to the 5206-keV level have been made in the present experiment. The graphs of the ^{50}Ti data are in Fig. 5.

The solid curves in Figs. 1–5 are theoretical fits to the experimental data generated by the coupled-channel computer code. This code uses the vibrational collective model[24,25] to describe the states of the bombarded nucleus but permits great flexibility, since the various strength parameters that enter the calculation can be varied from the values assigned by the vibrational model and the states can also be described as mixtures of one-phonon and two-phonon

TABLE III. Parameters used in calculations that included the first 2^+ state.

A	V (MeV)	W_D (MeV)	r_0 (fm)	Q (keV)	β_{02}
46	49.1	9.9	1.25	−886	0.25
48	49.8	10.5	1.25	−985	0.22
50	50.5	11.1	1.25	−1550	0.14

$a=0.64$ fm, $\bar{a}=0.47$ fm, $V_{SO}=6.5$ MeV, $W=0.0$ MeV

TABLE IV. Parameters of the analyses of states with one- and two-quadrupole phonons.

Target	Q (keV)	I^π	β_{02}	Q (keV)	I^π	β_{2I}	β_{0I}
				−2605	0^+	0.25	
^{46}Ti	−886	2^+	0.25	−2959	2^+	0.25	
				−2005	4^+	0.25	0.10
				−3004	0^+	0.22	
^{48}Ti	−985	2^+	0.22	−2425	2^+	0.22	
				−2301	4^+	0.22	
^{50}Ti	−1550	2^+	0.14	−2686	4^+	0.14	0.11

[24] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **26**, No. 14 (1952).
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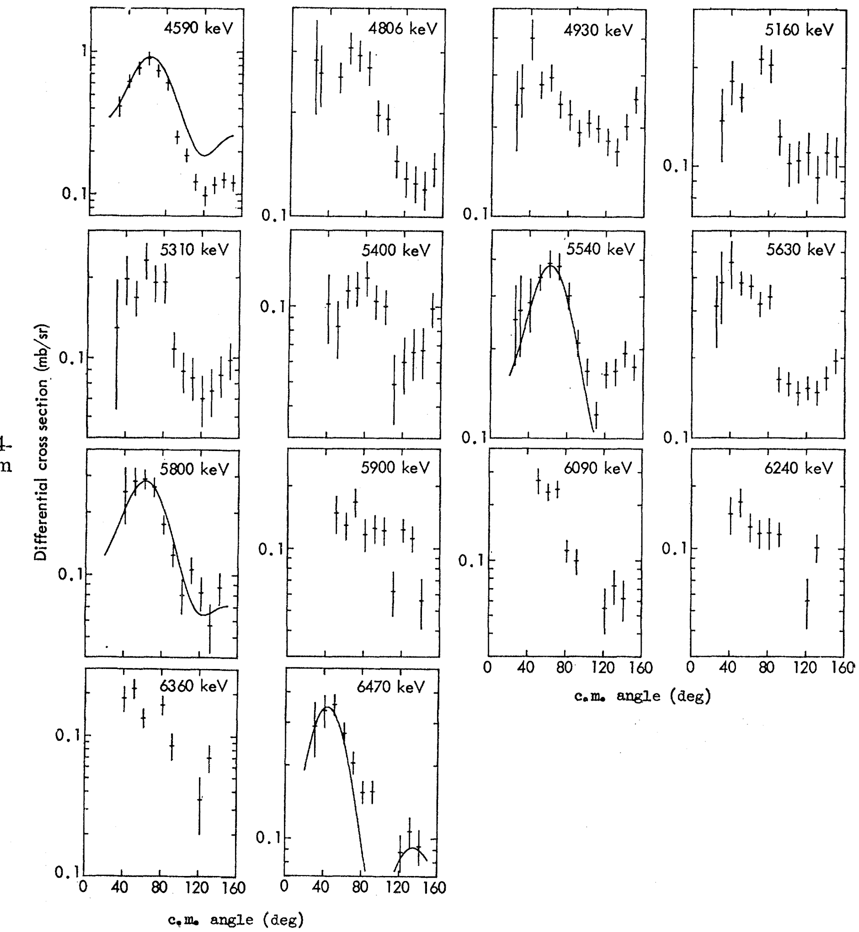

FIG. 4. Angular distributions of 14.4-MeV protons scattered from ^{48}Ti, from 4590 to 6470 keV.

states. All the coupled-channel calculations included spin-orbit distortion and Coulomb excitation, and allowed both real and imaginary parts of the potential to vibrate.

One can see from Table II and the figures that there are a number of levels excited by inelastic proton scattering with non-negligible cross sections that we could not fit with a theoretical curve describing the excitation of a single-phonon state of a given multipolarity. Some of these, no doubt, are due to more than one unresolved level. Others are more complicated excitations, and it is likely that one will have to know more about the structure of these levels and go beyond the harmonic vibrational model to describe their excitation.

TABLE V. Octupole states in ^{46,48,50}Ti.

Target	Q (keV)	Proton results β_{03}	Proton results $\beta_{03}\times 1.25\,A^{1/3}$	α results β_{03}	α results $\beta_{03}\times 1.54\,A^{1/3}$
^{46}Ti	−3056	0.15	0.67	0.067	0.37
^{46}Ti	−3566	0.17	0.76	0.075	0.4[illegible]
^{46}Ti	−4177	0.18	0.81	0.083	0.46
^{48}Ti	−3365	0.18	0.82	0.079	0.44
^{48}Ti	−3850	0.10	0.45	0.056	0.31
^{48}Ti	−4590	0.17	0.77	0.070	0.39
^{48}Ti	−5540	0.11	0.50	0.054	0.30
^{48}Ti	−5800	0.10	0.45	0.054	0.30
^{50}Ti	−4420	0.17	0.78	0.11	0.62

Parameters of the Optical Potential

The optical potentials used in the coupled-channel analyses are given in Table III. These potentials refer to those coupling schemes that included the first quadrupole state. The parameters were chosen as a smoothly varying set, with the known[26] dependence on neutron excess, that would account for the data reasonably well for the isotopic sequence. A least-squares-fitting procedure was not followed. The values for the parameters were chosen by visually inspecting the quality of fits to the data while fitting the scattering to the ground state, one-quadrupole-phonon state and one-octupole-phonon state of each nucleus. The potentials of the Ti isotopic sequence can be represented by the following equations:

$$V=46.3+16.8(N-Z)/A+0.4(Z/A^{1/3}) \qquad (1)$$

and

$$W_D=9.3+14.4(N-Z)/A. \qquad (2)$$

[26] G. R. Satchler, Nucl. Phys. A92, 273 (1967).

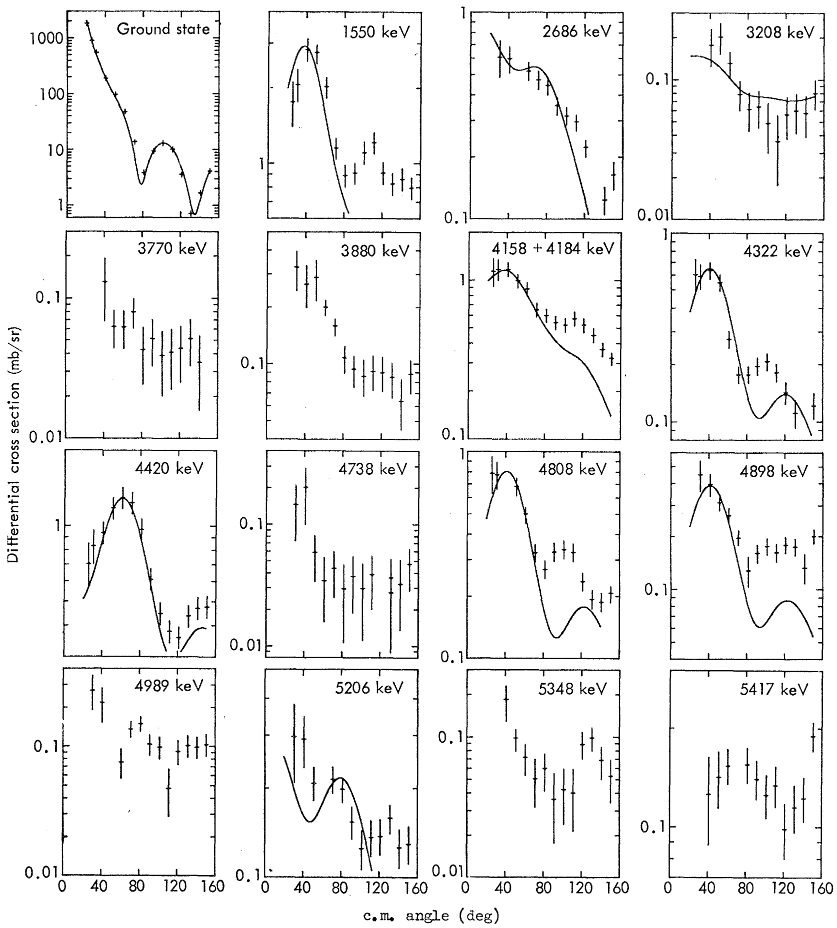

FIG. 5. Angular distributions of 14.4-MeV protons scattered from ^{50}Ti, from ground to 5417 keV.

The values of the constants multiplying $(N-Z)/A$ agree very well with the results of Kossanyi-Demay and de Swiniarski[27] who analyzed elastic scattering in a mass region that included the titanium isotopes.

In the simple calculations that employ the 0^+-I^π coupling scheme [corresponding to the usual distorted-wave Born-approximation (DWBA) analysis)] the imaginary part of the potential must be increased to account for the cross sections for exciting the one-quadrupole-phonon state and the one-octupole-phonon state. In this mass region, we have found that increasing W_D by 0.5 MeV for each 10 mb of integrated cross section approximately accomplishes the above-mentioned task. Hence, W_D for ^{46}Ti, ^{48}Ti, and ^{50}Ti were increased by 2.5, 2.0, and 1.1 MeV, respectively, when used in the 0^+-I^π coupling scheme.

A calculated inelastic cross section depends on the combination of β and W_D, and is somewhat more sensitive to W_D than is the elastic cross section. The interplay between β and W_D is the main factor determining the confidence with which one can deduce values of β from experimental scattering cross sections.

[27] P. Kossanyi-Demay and R. de Swiniarski, Nucl. Phys. **A108**, 557 (1968).

We assign estimated errors of 10% to the β values for the one-quadrupole-phonon and one-octupole-phonon states and errors of at least 15% to other types of states where the quality fit was not as satisfactory as the one-phonon states.

Results for States Included in the 0_0^+-2_1^+-0_2^+-2_2^+-4_2^+ Coupling Scheme

We attempted to analyze the first 2^+ state as a one-quadrupole-phonon state and the next 0^+, 2^+, 4^+ states as members of a two-quadrupole-phonon triplet. We were not very successful in fitting the possible two-quadrupole-phonon states and conclude that a description other than the vibrational collective model should be attempted in addressing the problem of the excitation of these states. This is certainly true in ^{50}Ti which does not have a low-lying second 2^+ state. Certain features are evident just by inspecting the integrated cross sections in Table II. The cross section for exciting the first 2^+ state decreases rapidly reflecting the decrease in the quadrupole moment as the $1f_{7/2}$ neutron shell is filled. The possible two-quadrupole-phonon states, however, do not follow this trend. The excited 0^+ states all have roughly 2-mb cross sections. The cross section of the 4^+ state varies with no discernable pattern as we go from isotope to isotope.

The experimental data for the one-quadrupole-phonon states increase more rapidly than the theoretical curve at small angles; at large angles, the data fall above the theoretical curve. The excited 0^+ states at 2605 keV in ^{46}Ti and 3004 keV in ^{48}Ti have deep minima around 80° and secondary maxima at 110°. These features are, at least qualitatively, reproduced in the two-quadrupole-phonon calculation. The excited 0^+ state at 3880 keV in ^{50}Ti has a more monotonic behavior.

In our analysis it was necessary to add a single-phonon contribution to the description of the first 4^+ state 2005 keV in ^{46}Ti and 2686 keV in ^{50}Ti. It is interesting to note that the (t, p) studies by Hinds and Middleton[17] show that the first 4^+ state is excited in ^{50}Ti but not in ^{48}Ti. Perhaps there is a correlation between the single-phonon admixture and these two-nucleon transfer results. Unfortunately, no such data exist for the production of states in ^{46}Ti by the two-nucleon transfer reaction.

It was not possible to fit the excited 0^+ state of ^{50}Ti which falls at 3880 keV. The theoretical curve was lower than the data by about an order of magnitude, and single-phonon admixtures gave the wrong shape for the angular distribution. The other states were described in terms of two-quadrupole-phonon states excited in a two-step process via the one-quadrupole-phonon state and the calculations employed the same values for β_{2i} as for β_{02}. The symbol β_{2i} denotes the couplings between the one-quadrupole-phonon state and the two-quadrupole-phonon states; β_{02} is the coupling between the ground state and the one-quadrupole-phonon state. Attempts to improve the quality of the fits by adding single-phonon admixtures to states other than the 4^+ states of ^{46}Ti and ^{50}Ti did not succeed. The parameters used are given in Table IV. While the coupled-channel calculations are not entirely satisfactory, they do represent an improvement over the quality of the fits obtained by the usual DWBA analysis.

TABLE VI. Parameters of the analyses of the states of 0^+-I^π coupling schemes.

Target	Q (keV)	I^π	β_{01}
^{46}Ti	−3843	4^+	0.17
^{48}Ti	−3230 −3248	4^+	0.15
^{48}Ti	−6470	2^+	0.09
^{50}Ti	−3208	6^+	0.11
^{50}Ti	−4158	4^+	0.10
^{50}Ti	−4184	2^+	0.09
^{50}Ti	−4322	2^+	0.09
^{50}Ti	+4808	2^+	0.10
^{50}Ti	−4898	2^+	0.07
^{50}Ti	−5206	4^+	0.10

TABLE VII. Inferred electromagnetic transition rates expressed in single-particle units.

Nucleus	E_x (keV)	I^π	β_l	G_l
^{46}Ti	886	2^+	0.25	15.0
	3056	3^-	0.15	6.9
	3566	3^-	0.17	8.9
	3843	4^+	0.17	13.0
	4177	3^-	0.18	10.0
^{48}Ti	985	2^+	0.22	12.0
	3230	4^+	0.15	10.0
	3365	3^-	0.18	10.0
	3850	3^-	0.10	3.1
	4590	3^-	0.17	8.9
	5540	3^-	0.11	3.7
	5800	3^-	0.10	3.1
	6470	2^+	0.09	1.9
^{50}Ti	1550	2^+	0.14	4.7
	4158	4^+	0.10	4.6
	4184	2^+	0.09	1.9
	4322	2^+	0.09	1.9
	4420	3^-	0.17	8.9
	4808	2^+	0.10	2.4
	4898	2^+	0.07	1.2
	5206	4^+	0.10	4.6

Results for Octupole States Calculated by the 0_0^+-2_1^+-3_1^- Coupling Scheme

The number of octupole states is one of the more interesting features of the nuclear spectroscopy of the even-mass Ti isotopes. The fits to the octupole states were made using a 0_0^+-2_1^+-3_1^- coupling scheme that included the ground state and one-quadrupole-phonon state, and considered each octupole state in turn. The quality of the fits to the octupole states are quite good and give one confidence in their assignment. In general, we have confirmed the assignments of the locations of these states made by inelastic scattering of α particles although we were not able to look as high in excitation energy. There is, however, a significant difference in the deformabilities deduced in each type of experiment. The deformabilities are about twice as large in the proton experiments as those in the α experiments. The difference in radius parameters (1.25 for protons and 1.54 for α particles) cannot explain the magnitude of the discrepancy. The relevant data are presented in Table V, in which the α-particle scattering data is taken from the paper of Bernstein *et al.*[6]

The values of β_{03} that Peterson and Perlman[9] deduce for the first three octupole states in ^{46}Ti are 0.16, 0.17, and 0.21 compared to our values of 0.15, 0.17, and 0.18. Barnard and Jones,[10] on the other hand, identify only a single octupole state at 4.05-MeV excitation energy with a deformability of 0.17. This, perhaps, corresponds to our state at 4.177 MeV. Matsuda[11] identified two octupole states in ^{48}Ti at 3.36 and 4.58 MeV, corresponding to the two stronger octupole states that we have seen. His data were not subjected to a DWBA analysis to deduce deformabilities but the magnitude of our experimental data agrees quite well with his results. For the octupole state in ^{50}Ti Funsten *et al.*[13] have deduced a value of 0.17 for the deformability as have Gray *et al.*[14] The latter authors also identify a weak 3^- state at 5.20-MeV excitation energy. We prefer to call this a 4^+ state and note that its formation in (d, p) stripping by a $l_n=1$ neutron transfer implies a positive parity for the state.

We may conclude from the above discussion that the proton experiments are in general agreement as to the magnitudes of the deformabilities and, in the case of the Ti isotopes, a serious discrepancy exists between the results of the proton scattering experiments and the α-particle scattering experiments.

Results for States Included in 0^+-I^π Coupling Schemes

The results for the levels analyzed in terms of the simple 0^+-I^π coupling scheme are summarized in Table VI. The 3843-keV level of ^{46}Ti has been assigned $J^\pi=4^+$ by Peterson and Perlman[9] because its shape is similar to that of the known 4^+ state at 2005 keV. We show a single-step hexadecapole excitation to compare with these authors. It is, however, likely that two-step process such as the one used to generate the theoretical fit to the 2005-keV level would do a better job.

In ^{50}Ti the levels at 4158 and 4184 keV were unresolved. We have fit the resulting group with a sum of two theoretical curves corresponding to a quadrupole and hexadecapole excitation. The levels at 4322, 4808, and 4898 keV have similar shapes. The 4322-keV level is known to be 2^+ [from the (t, p) work of Hinds and Middleton[17]]. Hence all three levels were analyzed as 2^+ states. The 5206-keV level should have positive parity from the stripping results. We have analyzed it as a 4^+ state although the assignment is very tentative.

Electromagnetic Transition Rates Inferred from Deformabilities

From the deformabilities deduced from the inelastic proton data, one may infer electromagnetic transition rates expressed in single-particle units

$$B_{\mathrm{s.p.}}(El, 0\rightarrow l)=[(2l+1)/4\pi][3/(3+l)]^2(R_{\mathrm{EM}})^{2l}e^2\mathrm{fm}^2$$

by the prescription

$$G_L=\frac{K(l)\,(1.25/1.20)\,(3+l)^2}{4\pi(2l+1)}Z^2\beta_l^2,$$

where $K(l)$ corrects[28] for the underestimate made by using a uniformly charged sphere instead of the more realistic Fermi-charge distribution. Following Bernstein *et al.*[6] $K(l)$ equals 1.14, 1.43, and 2.0 for multipolarities of 2, 3, and 4, respectively. These results are presented in Table VII.

SUMMARY

We have measured the elastic and inelastic scattering of 14.4-MeV protons from ^{46,48,50}Ti. A total of 63 angular distributions were obtained, of which 31 were not fit by theoretical curves. The data were analyzed in terms of the coupled-channel computer code employing the vibrational collective model to describe the nuclear states and the couplings between them. A nuclear optical model whose real and imaginary potentials depended on the neutron excess was used in these calculations. Attempts to fit the 0^+, 2^+, 4^+ states that fall above the first quadrupole state as two-quadrupole-phonon states were not very successful, although in ^{46}Ti and ^{48}Ti the magnitudes and certain qualitative features of the angular distributions were accounted for and this is an improvement over the usual DWBA analysis. The 0^+ state at 3880 keV in ^{50}Ti has too large a cross section in relation to the cross section of the first 2^+ state to be a two-quadrupole-phonon state. The numerous octupole states indicated by α-particle scattering experiments were verified, but the deformabilities deduced in the present experiment were about twice as large as the α-particle scattering results. Even after adjusting for the larger radius used in the α-particle scattering experiments, a discrepany remains. On the basis of the results of the present experiment, the 6470-keV level of ^{48}Ti has been assigned $J^\pi=2^+$, as have the 4808- and 4898-keV levels of ^{50}Ti. Electromagnetic transition rates have been inferred from the deformabilities deduced from the inelastic-scattering data.

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[28] L. W. Owen and G. R. Satchler, Nucl. Phys. **51**, 155 (1964).