Magnetic Transitions in ^{28}Si Excited by 180° Electron Scattering

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States in ^{28}Si have been excited by 180° electron scattering at bombarding energies of 39 and 56 MeV. Three strong $M1$ transitions excited levels at 10.86, 11.41, and 12.27 MeV with ground-state radiation widths of 5.7, 20.8, and 7.3 eV, respectively. Three weaker transitions, probably $M1$, are also treated. The apparent excitation of $M2$ transitions which may possibly be identified with theoretically predicted 2^- spin-isospin giant resonances is discussed. The relation of these results to the theory of Kurath as well as to nuclear-model assumptions is also discussed.

I. INTRODUCTION

MANY examples have now been reported[1–7] of the excitation of analog states in self-conjugate nuclei by means of the magnetic dipole interaction in electron scattering. Since the Morpurgo[8] rule strongly inhibits $\Delta T=0$ magnetic dipole transitions in such nuclei, only $T=1$ states are generally excited. This restriction in the number of states excited is made even more severe as a result of effects discussed by Kurath.[9] He showed that the $M1$ transition strength in $T=0$ nuclei with $A=4N$ (where N is an integer) in the $1p$ and $2s$-$1d$ shells is strongly concentrated in the lowest few levels available for this type of transition.

The beauty of 180° electron scattering is that these effects are highlighted quite strikingly, usually unencumbered by the detracting influence of electric transitions whose probability for excitation is lowest at this angle. Some of the most attractive examples exhibiting these effects have been found in our work[3,10] on ^{24}Mg and ^{20}Ne, and in the ^{28}Si results reported here.

In this paper we discuss the properties of the $M1$ transitions to the states excited, including the identification of these states with $T=1$ analog states in ^{28}Al. In a treatment paralleling that given by Kuehne *et al.*[11] in their photon-scattering work on ^{28}Si, the results on the $M1$ transitions are also used to show how they can affect the choice of an appropriate model for this nucleus. We also include a survey of the giant-resonance region resulting from 56-MeV bombardment, with particular emphasis on the low-excitation portion where it appears that[12] theoretically predicted 2^- $T=1$ spin-isospin resonances may have been excited.

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II. RELATED THEORY

The results reported here are compared with the model-independent theoretical expressions based on the Born approximation [subject to the distorted-wave Born-approximation (DWBA) correction, see Sec. IV] given by Rosen *et al.*[13] Since magnetic transitions usually are predominant at 180°, we use the magnetic scattering differential cross section [Ref. 13, Eq. (5)]

$$\left(\frac{d\sigma}{d\Omega}\right)_{180°}=\frac{\pi\alpha}{[(2L+1)!!]^2}\frac{L+1}{L}\frac{q^{2L}}{k_1^2}B(ML,q),\quad(1)$$

where L is the multipolarity, q is the momentum transfer, k_1 is the incident electron momentum, α is the fine-structure constant, and B is the reduced transition probability. The latter can be expanded in terms of q and the transition radii [Ref. 13, Eq. (13a)]:

$$B(ML,q)=B(ML,0)\left(1-\frac{L+3}{L+1}\frac{(qR_M)^2}{2(2L+3)}+\frac{L+5}{L+1}\frac{(qR_M{}^*)^4}{8(2L+3)(2L+5)}+\cdots\right)^2,\quad(2)$$

where R_M and $R_M{}^*$ are transition radii as defined in Ref. 13.

As explained in some detail in earlier papers,[3,14] by equating the experimental ratio of the cross sections obtained at two bombarding energies to the corresponding ratio of expressions (1), $B(ML,0)$ cancels out and the result is an expression depending on the unknowns L, R_M, and $R_M{}^*$. Since $R_M{}^*\cong R_M$ on theoretical grounds, we set $R_M{}^*=R_M$ in the third term of Eq. (2). If a reasonable tentative value of L is assumed, a value of R_M can be thus found. If it differs too markedly from the ground-state matter radius, or from the average radius for other transitions of the same multipolarity in the same nucleus, generally another value of L must be tried.

The values of L and R_M are then used in Eqs. (1) and (2) with the measured cross section at either energy to determine $B(ML,0)$. The ground-state radiation width can then be determined and is given by

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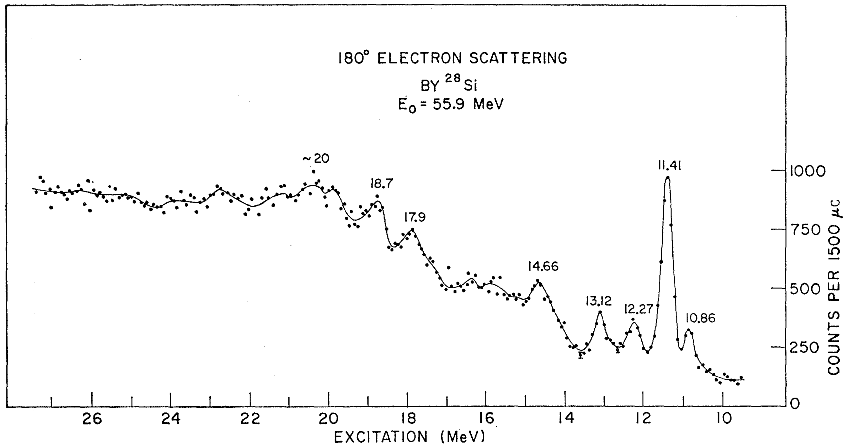

FIG. 1. Spectrum obtained from 180° electron scattering from ^{28}Si at $E_0 = 55.9$ MeV covering the excitation region 10–27 MeV.

[Ref. 13, Eq. (15a)]

$$\Gamma_0 = \frac{8\pi}{[(2L+1)!!]^2} \frac{L+1}{L} \frac{2J_0+1}{2J+1} \omega^{2L+1} B(ML, \omega), \quad (3)$$

where ω is the excitation energy, and J_0 and J are the ground- and excited-state spins,[15] respectively.

Considerable doubt has recently been expressed by Drechsel[16] as to whether R_M has any model-independent physical meaning. In our work thus far[3,14] we have not intended to give it physical meaning as a nuclear radius. Nevertheless, it might be inferred that we did so to the extent that we require the correct value of L to yield a transition radius roughly equal to the nuclear matter radius. It happens that this criterion has yielded consistency in our results, in this region of A. However, even if we relegate R_M to being a convenient parameter, and use the criterion that the transition radii for transitions of the same multipolarity in the same nucleus must be roughly equal, the results obtained from our data would still be the same. Thus it should not be inferred that our results indicate anything definite concerning the physical significance of R_M.

We must also mention that our analysis cannot exclude the possibility of a transverse electric excitation occurring. However, to the best of our knowledge only a few such cases have been reported.[17]

III. EXPERIMENTAL CONSIDERATIONS

In this experiment a ^{28}Si target, isotopically enriched to 99.91% and 90.2 mg/cm^2 thick, was bombarded with 39- and 56-MeV electrons from the NRL 60-MeV Linac. The 180° scattering apparatus (including the magnetic spectrometer, associated detector system, and the beam-measuring system) has been described in detail in Ref. 14. Also discussed there is the use of the elastic peak and the 15.110-MeV inelastic peak from a ^{12}C target as a basis for energy calibration.

Some of the difficulties encountered in the fabrication and use of the ^{28}Si target should be mentioned. Most of our solid targets, including the present one, have been made quite successfully from compressed powders. However, it was found that the silicon was very difficult to compress to the point where it remained in the form of a mechanically stable pellet, because the edges flaked off easily in the target mounting process. However, since the central area of the pellet where the beam struck remained intact, the experiment was carried out. The flaking led to obvious difficulties in calculating the number of target atoms in the beam path by using the weight and area. Therefore, the determination of this number was based primarily on x-ray absorption measurements, which were nevertheless in satisfactory agreement with the value based on the pellet weight, the area, and an estimate of the flaking loss. It has since been learned that compression at about 1000°C in an inert atmosphere might have improved the mechanical rigidity.

IV. RESULTS

A discussion of the treatment of the data (that is, how the quantities L, R_M, and Γ_0 are ultimately extracted from the raw data) is given in an earlier report.[3] Since essentially no structure was found below 10 MeV and since peak analysis usually becomes prohibitively ambiguous beyond about 15 MeV, most attention in terms of quantitative analysis was given to the region within these limits. However, in order to ascertain whether there exist strong magnetic transitions, and to present a qualitative picture of the giant resonance region, data were taken at the 56-MeV electron bombardment over the full available energy range of excitation. This is presented in Fig. 1, which

[15] The ratio involving the nuclear spins in this expression as given in Ref. 3 was erroneously squared. Also, the sign for the summation over L has been omitted here, since only one value of L can contribute for a nucleus with ground-state-spin zero.

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TABLE I. Values of differential cross sections, spin and parity, transition radii, and radiation widths for energy levels excited in ^{28}Si.

Level energy (MeV)	$d\sigma/d\Omega)_{56}$ (10^{-34} cm^2/sr)	$(d\sigma/d\Omega)_{39}$ (10^{-34} cm^2/sr)	J^π	PWBA R_M (fm)	PWBA Γ_0 (eV)	DWBA corrected R_M (fm)	DWBA corrected Γ_0 (eV)
10.48 ± 0.06	10 ± 3	39 ± 9	1^+	4.1	3.3	3.9 ± 0.4	$2.4_{-0.9}^{+1.2}$
10.86 ± 0.04	60 ± 5	117 ± 10	1^+	3.25	8.0	$2.98_{-0.27}^{+0.23}$	$5.7_{-1.1}^{+1.3}$
10.90 ± 0.06[a]					4.8 ± 1.3[a]		
11.41 ± 0.03	250 ± 14	408 ± 28	1^+	2.93	29.2	$2.58_{-0.25}^{+0.21}$	$20.8_{-3.7}^{+4.3}$
11.42 ± 0.02[a]				3.2 ± 0.3[a]	32.4 ± 4.5[a]	3.0 ± 0.3[b]	25.7 ± 3.6[b]
11.42 ± 0.02[c]					47 ± 12[d] 33[e]		22.9 ± 4[c]
12.27 ± 0.04	55 ± 6	103 ± 12	1^+	3.22	10.1	$2.93_{-0.39}^{+0.32}$	$7.3_{-1.8}^{+2.0}$
12.32 ± 0.06[a]					7.5 ± 1.9[a]		
12.79 ± 0.10	17 ± 5	37 ± 12	1^+	3.5	4.5	$3.2_{-1.2}^{+0.7}$	$3.3_{-1.7}^{+2.3}$
13.12 ± 0.05	53 ± 10	55 ± 18	(2^-)	5.0	0.20	$4.6_{-1.3}^{+0.9}$	$0.14_{-0.07}^{+0.10}$
14.01 ± 1.10	20 ± 10	62 ± 24	(1^+)	4.0	12.2	$3.8_{-1.1}^{+0.8}$	$8.9_{-5.0}^{+7.3}$
14.66 ± 0.08	110 ± 35	62 ± 26	(2^-)	3.3	0.28	$2.7_{-2.7}^{+1.8}$	$0.20_{-0.10}^{+0.20}$

[a] Reference 5. In this reference the value of R_M found for the 11.4-MeV level was also used to find Γ_0 for other levels.
[b] Reference 18 using the data of Ref. 5.
[c] Reference 11. Photon scattering result using $\Gamma_0/\Gamma=1$, where Γ is the total level width.
[d] Reference 6.
[e] Reference 7.

covers the entire range from 10 to 27 MeV. Above 15 MeV, peaks are apparent at 17.9, 18.7, and ~20 MeV.

In order to obtain values of Γ_0, R_M, and L for transitions in the 10–15-MeV region, this range was covered at both the 55.9- and 38.9-MeV bombarding energies with greater statistical accuracy. The spectra obtained in terms of the differential cross section and based on 2500- and 1500-μC accumulations, are presented in Figs. 2 and 3, respectively.

The energy, inelastic cross sections (integrated over the energy range of each peak), spin and parity, transition radii, and ground-state radiation widths of each of the levels excited are presented in Table I. Results of other workers are included in the table for comparison. The peak intensities, upon which these results are based, were subjected to the Schwinger, bremsstrahlung, and ionization corrections. The uncertainties given for the cross sections are relative values and represent counting statistics and base-line uncertainties. The uncertainties in the values of the radiation widths are due to uncertainties in the relative cross sections, in the radiation corrections, and in the counter

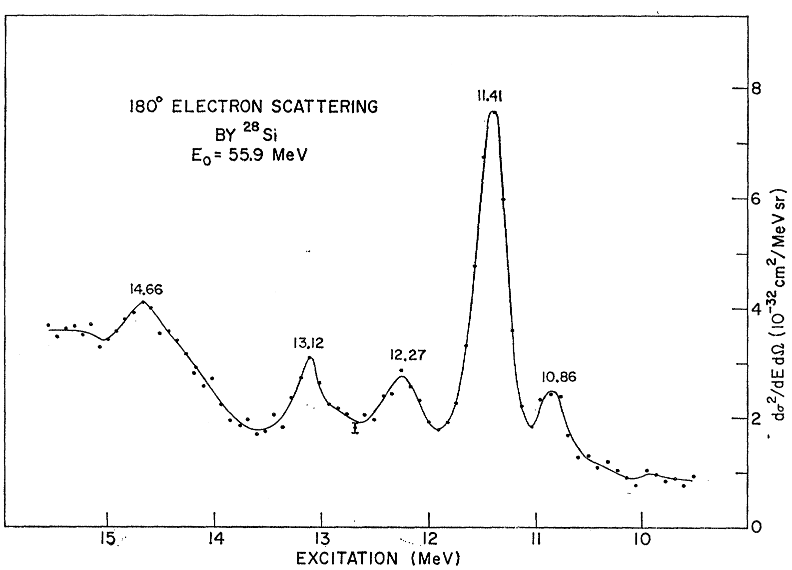

FIG. 2. Differential cross section for 180° electron scattering from ^{28}Si at $E_0=55.9$ MeV covering the excitation energy range 10–15 MeV.

efficiency. Since all but the first of these uncertainties essentially cancel out in the calculations of R_M, the uncertainties given for this quantity are calculated from those shown for the cross sections.

It should be emphasized that our results are compared with a theory based on the plane-wave Born approximation (PWBA) but subjected to a correction based on the DWBA. The plane-wave results for Γ_0 and R_M, uncorrected and with the DWBA correction, are shown in Table I. Our corrected values are based on the calculations of Chertok and Johnson[18] on the 11.4-MeV transition in ^{28}Si. Accordingly, the cross section at 56 MeV was divided by 1.1038, and that at 39 MeV by 1.2798. It was the corrected cross sections that were employed in Eqs. (1)–(3) to obtain the DWBA values of L, R_M, and Γ_0. The same correction was used in all transitions treated, since it was found to be rather insensitive to changes in excitation energy. Because no $M2$ corrections are available, the $M1$ corrections were used for this case.

However, it should be pointed out that the calculations of Chertok and Johnson are based on the theoretical work of Tuan *et al.*,[19] who only take into account nuclear currents and not magnetization density. The work of Drechsel[16] includes both the currents and the magnetization density, but a computer program for calculating the correction ratios is not yet available. Nevertheless, estimates of the error in the correction resulting from the neglect of the magnetization density[20] indicate that it can be no greater than 25% in our case.

V. DISCUSSION

As was mentioned earlier, since the rule of Morpurgo[8] strongly inhibits $\Delta T=0$ magnetic dipole transitions in self-conjugate nuclei, only $T=1$ states are usually excited. This has been true experimentally in all our work with self-conjugate nuclei thus far and is exemplified here by the fact that we could not detect the 1^+ $T=0$ state at 7.38 MeV. The number of measurable $M1$ transitions is further restricted by effects discussed by Kurath,[9] who showed that the $M1$ transition strength in $T=0$ nuclei with $A=4N$ (where N is an integer) in the $1p$ and $2s$-$1d$ shells is strongly concentrated in the lowest few levels available for this type of transition.

In fact, Kurath predicted that ^{28}Si should be a striking example of this effect in the $2s$-$1d$ shell. This is indeed the case experimentally, where it can be seen that most of the strength is concentrated in the 11.41-MeV transition with considerably smaller amounts devoted to the 10.86- and 12.27-MeV transitions. However, a qualitative feature that seems to distinguish the cases of ^{24}Mg and ^{28}Si from similar examples in the $1p$ shell is that it is not the lowest 1^+ $T=1$ level but the second or third lowest which is clearly the most intense in these two nuclei. On the other hand, an example of a $2s$-$1d$ nucleus conforming more precisely to the behavior found in the $1p$ shell has recently been found in ^{20}Ne. Our refrigerated-gas work[10] with this

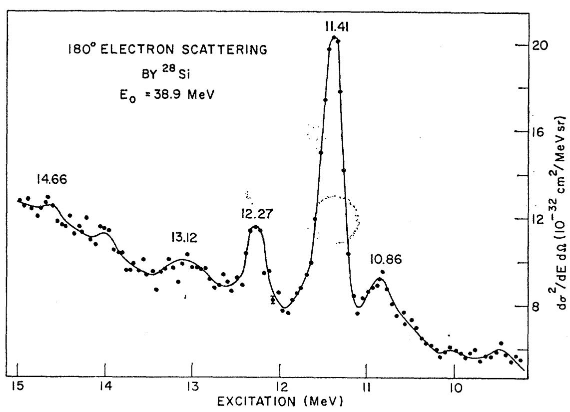

FIG. 3. Differential cross section for 180° electron scattering from ^{28}Si at $E_0=38.9$ MeV covering the excitation energy range 9–15 MeV.

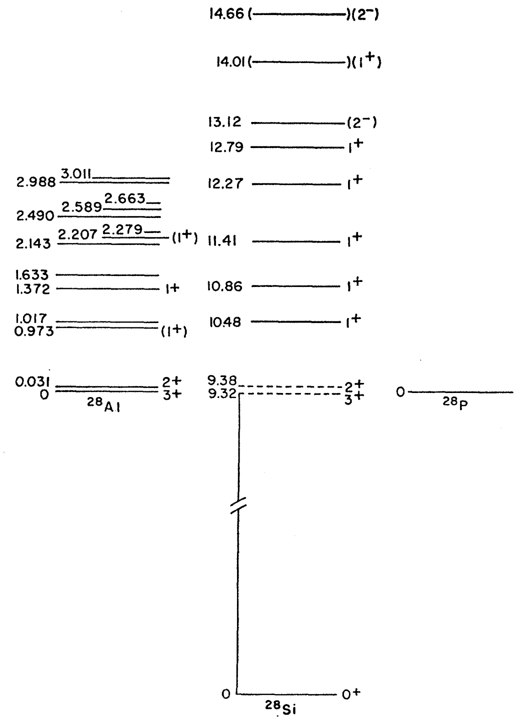

FIG. 4. Energy-level diagram adjusted for the Coulomb displacement so that the ^{28}Si and ^{28}Al analog states are approximately aligned. Only the ^{28}Si states denoted with a solid line were excited in this experiment. The energies given in the ^{28}Al-level scheme are those quoted from Endt and Van der Leun (Ref. 22). The parentheses around the states at 14.66 and 14.01 MeV indicate a considerable degree of uncertainty as to their existence and position.

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nucleus indicates from preliminary data that essentially only one transition at 11.21 MeV is excited, with no smaller partners yet observable. The results from all three nuclei thus show an interesting trend which may or may not be of physical significance: the $M1$ strength being first concentrated in the lowest 1^+ $T=1$ level in ^{20}Ne, essentially split between the lowest two in ^{24}Mg, and mostly in the third lowest in ^{28}Si.

The identification of the three strongest 1^+ levels excited in ^{28}Si with analogs in ^{28}Al may present some difficulties in that the energy differences involved are as large as 0.17 MeV. However, it is reasonable to believe that the 10.86-, 11.41-, and 12.27-MeV levels can be identified with the 1.372-, 2.207-, and 2.988- or 3.011-MeV levels, respectively, in ^{28}Al. This is pictured in Fig. 4, which presents the energy-level diagrams of the two nuclei corrected for the Coulomb displacement so that the analog states are approximately aligned.

There seems to be relatively little question in the case of the identification of the 10.86-MeV level with the 1.372-MeV level. The 11.41-MeV level cannot be identified with the 2.143-MeV level, since this is reached through a neutron orbital of $l_n=0$ in d, p experiments[21] and is given an assignment $(2, 3)^+$ by Endt and Van der Leun.[22] However, both the 2.207- and 2.279-MeV levels are acceptable in this respect, since they are reached as a result of $l_n=2$ neutrons. The 2.207-MeV level is suggested, since it is the closest in energy. However, for the identification of the 12.27-MeV level, the situation is more ambiguous. The 3.011-MeV level is a clear possibility with respect to the above arguments, since it corresponds to a d, p neutron orbital $l_n=2$. There is no information on the assignment of the 2.988-MeV level which, however, is more closely aligned in energy. Thus both are still possible candidates for this identification at this time.

It may be seen from Table I, the plane-wave value of Γ_0 for the 11.41-MeV transition from the present work agrees with the other electron-scattering values[5,7] except for that of Barber *et al.*[6] When our value is DWBA corrected it also agrees with the photon-scattering value given by Kuehne *et al.*[11] The only comparison available for the Γ_0's of the 10.86- and 12.27-MeV transitions is with the values of Liesem.[5] There is satisfactory agreement for the 12.27-MeV transition, but not for that at 10.86 MeV. We have no explanation for the discrepancy.

The value of the transition radius R_M for the 1^+ level at 11.41 MeV agrees within the uncertainties with that found by Liesem. He does not, however, determine values for the 10.86- and 12.27-MeV levels. The corresponding energies agree with those given by other experimenters[5,6,11] to within 0.4%.

Our analysis shows that additional $M1$ transitions occur at 10.48, 12.79, and possibly at 14.01 MeV. The percentage uncertainties in Γ_0 and R_M are considerably greater, as is apparent from Table I. The existence of the 14.01-MeV transition, as well as its assignment as $M1$, is particularly uncertain in view of the assumptions, discussed below, which had to be made in order to extract values of Γ_0 and R_M for the $M2$ strength in this excitation energy region. The 10.48-MeV level may tentatively be identified as the analog of the 0.973-MeV level in ^{28}Al. Similar identifications for the 12.79- and 14.01-MeV levels are virtually impossible at this time in view of the higher level density of this region.

The values of Γ_0 given for all the 1^+ levels can be used in conjunction with Kurath's theory to extract direct nuclear-structure information through the calculation of the ls coupling matrix element. This was first attempted by Kuehne *et al.*,[11] and a comparison between the results of their photon scattering and our electron scattering in the case of ^{24}Mg has since been reported.[3] The ls coupling matrix elements can be calculated using an approximate sum-rule expression[9] relating the matrix element to reduced transition probabilities corresponding to the ground-state $M1$ transitions. This expression was developed by Kurath in extrapolating the behavior of the $4N$ nuclei in the $1p$ shell to the $2s$-$1d$ shell. A somewhat more convenient form of this expression in terms of the Γ_0's instead of the reduced transition probabilities is given by Kuehne *et al.*[11] as

$$\sum_j [\Gamma_{0j}(M1)/3.395 \text{ eV}](10 \text{ MeV}/E_j)^2 = (-a/2 \text{ MeV}) \langle g \mid \sum_i \mathbf{l}_i \cdot \mathbf{s}_i \mid g \rangle, \quad (4)$$

where Γ_{0j} is the radiative width for the $M1$ transition from the excited state of energy E_j to the ground state g, and a is the coefficient of the $\mathbf{l}_i \mathbf{s}_i$ term in the potential of the ith nucleon. For the d shell, $a=-2$ MeV.[11]

Using only the Γ_0 for the 11.4-MeV state, Kuehne *et al.* calculate a value of 5.2 for the ls coupling matrix element. Our value using the Γ_0's from the lowest five of the $M1$ transitions is 8.8. We give this value separately because of the uncertainty, discussed below, of the existence of the 14.01-MeV transition, which, if included into the sum, leads to a value of 10.0. In either case the inclusion of the $M1$ strength from these additional levels leads to a value almost twice that of Keuhne *et al.*

Kurath has pointed out that the question of whether the oblate or prolate shape is more consistent with a deformed axially symmetric model might be clarified by a study of the magnetic dipole strengths in ^{28}Si.

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[22] P. M. Endt and C. Van der Leun, Nucl. Phys. **A105**, 1 (1967).

The value of 5.2 falls between the oblate prediction by Nilsson[23] of $\langle \sum_i \mathbf{l}_i \cdot \mathbf{s}_i \rangle = 7.6$ and his prolate prediction of 4.3 (using $\eta = -4$). Our value of 8.8 clearly is more consistent with the oblate prediction. This is in agreement with recent ground-state quadrupole measurements[24] on this nucleus. However, it must be pointed out that the theoretical result for $\langle \sum_i \mathbf{l}_i \cdot \mathbf{s}_i \rangle$ is quite sensitive to calculational details[11] (e.g., another value obtained for the oblate axially symmetric case[25] is 4.3 instead of 7.6). Furthermore, the results of these theoretical calculations are in considerable doubt because more recent work[26] on ^{24}Mg has considerably altered many of their predictions. Also, the results of Ref. 25 are in disagreement with recent work of Banerjee,[27] who shows the ^{24}Mg nucleus to be triaxial. Thus a conclusive statement on this matter will have to be withheld until more reliable theoretical calculations are available.

The survey of the giant-resonance region, the results of which are presented in Fig. 1, was primarily conducted to see qualitatively what magnetic effects exist. In addition to the peaks already discussed, peaks at 13.12, 14.66, 17.9, 18.7, and ~20 MeV are observed. Measurements with 150- to 225-MeV bombarding energies and angles of 25° to 65° conducted by Gulkarov *et al.*[28] reveal relatively strong peaks at the latter three of these energies for about the same momentum transfer. This may mean we are exciting electric transitions in this region. However, because of the progressive decrease in confidence of the exact location of the base line as the excitation energy increases, quantitative measurements were only attempted on the peaks at 13.12 and 14.66 MeV. Liesem,[5] as well as other workers, has not reported these peaks. The data reported in Table I for the corresponding Γ_0's and R_M's are most consistent with an assumption of $L=2$, indicating 2^- levels at these energies. In the case of the 13.12-MeV peak, this conclusion is somewhat marginal (note the relatively large transition radius in Table I) but analysis still clearly favors an $M2$ assignment.

It should be emphasized that these results are based to a significant extent on our judgment of what the curve of the elastic radiation tail and general background should be in the region of these peaks. This is especially true in the case of the 14.66-MeV peak, where, in addition, it has been very difficult to conclusively analyze all of the contributions to this broad peak. The analysis was conducted under the assumption that the peak widths of all contributors were approximately equal. This reasonable but uncertain assumption was made and analysis attempted because the qualitative behavior of this structure, under variation of the bombarding energy, strongly indicated that higher multipoles were playing a role. Thus the values given for the $M2$ transition at 14.66 MeV and the $M1$ transition at 14.01 MeV should be regarded as tentative estimates of some of the behavior in this region. The analysis on this basis further indicates the presence of a peak at about 14.3 MeV which qualitatively also behaves like a magnetic transition with $L>1$. It was not treated quantitatively and included in Table I because of insufficient measurable intensity at the 39-MeV bombardment.

If there are 2^- levels at 13.12 and 14.66 MeV (and possibly 14.3 MeV), they may provide some confirmatory evidence for the existence of the 2^- $T=1$ spin-isospin giant-resonance states predicted by Hill[12] at 14.3 and 14.8 MeV. If there were nuclear deformation, these states would each split into two states whose energy separation is proportional to the deformation.[29,30] Also, the higher-energy state would be expected to be more intense if the deformation were prolate, and the reverse if oblate. Thus when more refined measurements are available in this region, it may be possible to complement the sum-rule results with the $M1$ transitions in determining something about the nuclear shape. Additional evidence, preliminary in nature, for the existence of $M2$ transitions in this region has been reported by Drake *et al.*[31]

It is interesting to observe that two peaks at 12.9 and 13.4 MeV also appear in roughly the same energy region in our ^{24}Mg work[3] which behave qualitatively the same at the different bombarding energies as the $M2$ transitions discussed here.

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