

## ( $\alpha$ , $\alpha'\gamma$ ) Angular Correlation Studies of Some Excited States in $Mg^{24}$ , $Mg^{26}$ , and $Si^{28}$ †

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A number of states in  $Si^{28}$ ,  $Mg^{26}$ , and  $Mg^{24}$  have been studied using the ( $\alpha$ ,  $\alpha'\gamma$ ) reaction and the angular correlation method-II geometry of Litherland and Ferguson. In  $Si^{28}$ , spin and parity assignments of  $1$ , ( $6^+$ ), and  $1^-$  are made to the 7.38-, 8.51-, and 8.94-MeV states, respectively. The 4.90-MeV state in  $Mg^{26}$  is assigned  $J^\pi = 4^+$ . Earlier  $J^\pi$  assignments to the  $3^-$  6.88-,  $2^+$  7.41-, and  $2^+$  7.93-MeV states in  $Si^{28}$ , and the  $1^-$  8.44-,  $2^+$  9.00-, and  $1^-$  9.15-MeV states in  $Mg^{24}$  are confirmed, as is an earlier value of the multipolarity mixing ratio of the  $2.94 \rightarrow 1.81$   $\gamma$ -ray transition in  $Mg^{26}$ .

### I. INTRODUCTION

IN earlier work at this laboratory, Robinson and Bent<sup>1</sup> measured the lifetimes of several low-lying levels in  $Mg^{24}$ ,  $Mg^{26}$ , and  $Si^{28}$  and compared the results with the predictions of some collective and shell-model descriptions of these nuclei. While there was strong evidence for collective motion in  $Mg^{24}$  and to some extent in  $Mg^{26}$  and  $Si^{28}$ , it was apparent that particle excitations must also be included in order to achieve satisfactory descriptions of the low-lying levels of these nuclei.

More information on higher levels in  $Mg^{24}$ ,  $Mg^{26}$ , and  $Si^{28}$  is needed to test further the validity of the various nuclear models. Lifetime measurements in particular provide very exacting tests of nuclear structure theories; however, before useful nuclear structure information can be extracted from lifetime measurements, the spin and parity assignments to the levels and the multiplicities of the deexcitation  $\gamma$  rays must be determined.

In the present work, the spins of a number of states in  $Mg^{24}$ ,  $Mg^{26}$ , and  $Si^{28}$ , excited through the ( $\alpha$ ,  $\alpha'\gamma$ ) reaction, have been measured in the Litherland-Ferguson method-II angular correlation geometry.<sup>2</sup> The angular distributions measured in this geometry are ideally characterized by only the magnetic substate  $\alpha=0$  being populated. However, because the particle counter does subtend a finite solid angle near  $180^\circ$ , the magnetic substates  $\alpha = \pm 1$  are also populated to a small extent. The procedures followed in fitting these angular distributions are discussed in Sec. III below.

Parity assignments are also made to these states. Litherland<sup>3</sup> has shown that the cross section for exciting unnatural parity states [i.e.,  $\pi = (-)^{J+1}$ ] in even-even nuclei by  $\alpha$ -particle scattering is zero at  $180^\circ$ , and that angular correlations measured in the method-II geometry

are characterized by strong population of the  $\alpha=0$  or the  $\alpha = \pm 1$  magnetic substates for states of natural or unnatural parity, respectively. In this work, known unnatural-parity states were only weakly populated relative to those of known natural parity.<sup>4</sup> Furthermore, nearly all the measured angular distributions were characterized by strong population of the  $\alpha=0$  substate, allowing  $\pi = (-)^J$  assignments to be made.

### II. APPARATUS AND PROCEDURES

The levels studied in this experiment were excited by inelastic scattering of 22-MeV  $\alpha$  particles obtained from the Indiana University cyclotron. The  $Mg^{24}$  and  $Si^{28}$  targets were prepared by evaporating approximately 0.3 mg/cm<sup>2</sup> of natural silicon or magnesium onto gold foils. The  $Mg^{26}$  target was a 0.28 mg/cm<sup>2</sup> self-supporting foil (95%  $Mg^{26}$ ).

The experimental arrangement including the electronic components has been described elsewhere<sup>1</sup> and will be described only briefly here.

The scattered  $\alpha$  particles were detected in an annular silicon solid-state detector through which the beam passed before striking the target. The detector subtended an angle of  $170.6^\circ \pm 2^\circ$  relative to the target center.

The deexcitation  $\gamma$  rays were detected in a  $12.7 \times 12.7$ -cm NaI(Tl) crystal which was mounted on a trolley centered at the target. The front face of the crystal was 20 cm from the center of the target.

Pulses from the particle and  $\gamma$ -ray detectors were routed through single-channel pulse-height analyzers into a fast-slow coincidence unit (resolving time 40 nsec). The single-channel analyzers allowed an energy criterion to be placed upon either the particle spectrum or the  $\gamma$ -ray spectrum. Pulses from the coincidence unit opened a linear gate allowing the desired coincidence spectrum to be stored in one-half of a multi-

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<sup>1</sup> S. W. Robinson and R. D. Bent, Phys. Rev. **168**, 1266 (1968).

<sup>2</sup> A. E. Litherland and A. J. Ferguson, Can. J. Phys. **39**, 788 (1961).

<sup>3</sup> A. E. Litherland, Can. J. Phys. **39**, 1245 (1961).

<sup>4</sup> This is not true at higher energies. At an incident energy of 40 MeV, the differential cross section at back angles for exciting the  $3^+$  state at 5.23 MeV in  $Mg^{24}$  is comparable to that for exciting states of natural parity. See R. E. Malmin, P. P. Singh, D. W. Devins, J. G. Wills, C. R. Bingham, and M. L. Halbert, Bull. Amer. Phys. Soc. **13**, 117 (1968).

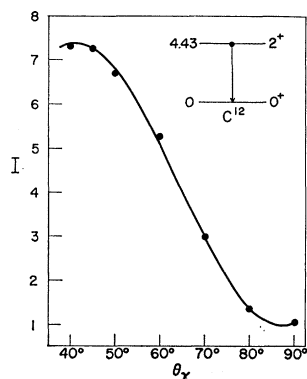


FIG. 1. Angular correlation measured for the decay of the 4.43-MeV state to the ground state in  $C^{12}$ . The solid line is the best theoretical fit for a  $J^\pi=2^+\rightarrow 0^+$  transition.

channel analyzer. A chance coincidence spectrum was stored simultaneously in the second half of the multi-channel analyzer using a circuit logic like that described above except that the  $\alpha$ -particle pulses were delayed by a time equal to that between the cyclotron beam pulses.

The two coincidence spectra were stored simultaneously at a minimum of six angles (from  $40^\circ$  to  $90^\circ$ ) and were normalized to the  $\alpha$ -particle counting rate. The measurements at each angle were performed twice to check reproducibility.

### III. DATA ANALYSIS

The method of analysis followed closely that of Poletti and Warburton.<sup>5</sup> Each measured angular correlation was fitted by the function

$$W(\theta) = \sum_k \rho_k(a) F_k(ab) Q_k P_k(\cos\theta),$$

where  $a$  and  $b$  are the spins of the initial and final states, respectively. The  $\rho_k(a)$  are functions of the population parameters  $P(a)$ , and the  $F_k(ab)$  are functions of the ratio of the amplitudes of the two lowest multipolarities allowed for the  $\gamma$ -ray transition, denoted by  $x$ .  $Q_k$  are attenuation coefficients which correct for the solid angle subtended by the  $\gamma$ -ray detector, and the  $P_k(\cos\theta)$  are Legendre polynomials. For assumed values of  $a$  and  $b$ , fits were made for a discrete set of values of  $x$ . For each value of  $x$ , a best fit was sought by searching<sup>6</sup> for the values of the parameters  $P(0)$  and  $P(1)$  (with the constraint that  $P(a)$  be greater than or equal to zero) which minimized the quantity

$$\chi^2 = (1/n) \sum_i \{ [Y(\theta_i) - W(\theta_i)] / \sigma_i \}^2.$$

<sup>5</sup> A. R. Poletti and E. K. Warburton, Phys. Rev. **137**, B595 (1965).

<sup>6</sup> This calculation was performed using the subroutine STEFIT. The FORTRAN code STEFIT was written by J. P. Chandler of the Indiana University Department of Physics and is distributed by the Quantum Chemistry Program Exchange, Research Computing Center, Indiana University.

$Y(\theta_i)$  is the measured intensity at the angle  $\theta_i$ , the term  $\sigma_i$  is the associated statistical error, and  $n$  is the number of degrees of freedom. A plot of  $\chi^2$  versus  $\arctan x$  was thus generated for each allowed value of the spin  $a$ . The expectation value of  $\chi^2$  is 1.

In the case of a cascade transition of the type  $a \rightarrow b \rightarrow c$ , the analysis consisted of several steps: (1) The angular correlation for the  $a \rightarrow b$  transition was fitted as described above. (2) This transition was refitted for those values of the spin  $a$  for which possible solutions were found with a discrete set of values for the ratio  $P(1)/P(0)$ . That is, for each value of  $x$ , a set of  $\chi_1^2 [P(1)/P(0)]$  were found corresponding to the fits for values of  $P(1)/P(0)$  between zero and 0.20. (3) The angular distribution of the second  $\gamma$  ray ( $b \rightarrow c$ ) was fitted<sup>6</sup> for the same values of the spin  $a$  and in the same manner as described in step (2). (For all cases considered here, the spins  $b$  and  $c$  were known from earlier work and the mixing ratio of the second transition was zero.) Thus a set of  $\chi_2^2 [P(1)/P(0)]$  was generated for each value of  $x$  and  $a$ . A best fit to both angular distributions at a given  $x$  corresponded to a value of  $P(1)/P(0)$  for which the sum  $\chi_1^2 + \chi_2^2$  was a minimum.

Measured angular correlations for transitions of the type  $J \rightarrow 0$  have a multipole mixing ratio  $x=0$ . Furthermore,  $J$  is generally limited to 1, 2, or 3, owing to lifetime considerations. Angular distributions were measured for each of these three possible spin combinations. Figure 1 shows the measured distribution resulting from the decay of the  $C^{12}$  4.43-MeV level to the ground state. The points represent the data and include the statistical error. The solid line represents the best fit to the data [ $\chi^2=1.13$ ,  $P(1)/P(0)=0.056 \pm 0.001$ ] assuming the decay is between states with  $J$  of 2 and 0. The angular correlation obtained for the 6.14-MeV  $\rightarrow 0.0$ -MeV decay in  $O^{16}$  (measured with a thin  $MoO_3$  foil) is shown in Fig. 2(a). The solid line represents the best fit assuming a  $3 \rightarrow 0$  transition [ $\chi^2=1.89$ ,  $P(1)/P(0)=0.33 \pm 0.03$ ]. Although this value for the  $P(1)/P(0)$  ratio appears large, it was found to be reproducible

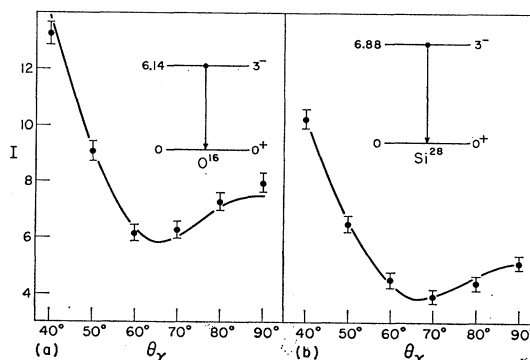


FIG. 2. Angular correlations measured for the decay of (a) the 6.14-MeV state, and (b) the 6.88-MeV state to the ground states in  $O^{16}$  and  $Si^{28}$ , respectively. The solid lines are the best theoretical fits for  $J^\pi=3^-\rightarrow 0^+$  transitions.

(see Sec. IV A 1) indicating that a  $3 \rightarrow 0$  angular distribution was discernible.

A comparison of Figs. 1 and 2(a) with Fig. 11, an example of the angular distribution measured for a  $J=1 \rightarrow 0$  transition, shows that the angular distributions measured for  $J \rightarrow 0$  transitions allow unambiguous spin assignments to be made.

In several cases, two energetically unresolved levels which were both known from earlier work to decay to the ground state were studied. The measured angular distributions did not show one of the characteristic shapes discussed above, indicating that both states were being populated and that they did not have the same spins. In such cases, fits were made with an angular correlation function  $W(\theta) = IW_J(\theta) + I'W_{J'}(\theta)$ , where  $W_J(\theta)$  and  $W_{J'}(\theta)$  are the previously measured angular correlation functions for decays of states with spins  $J$  and  $J'$ , respectively, to a spin-zero state. That is, the  $W_J(\theta)$ 's are the fits shown as solid lines in Figs. 1, 2(a), and 11. The terms  $I$  and  $I'$  are variable parameters representing the intensities of the two transitions. Best fits were sought for each of the three possible combinations of  $J$  and  $J'$ .

#### IV. RESULTS

##### A. $\text{Si}^{28}$ Results

A number of levels above and including 6.88 MeV in  $\text{Si}^{28}$  were studied. A summary of the most probable spin assignments for these levels as obtained from this experiment is given in Fig. 3. The decay of the 8.94-MeV

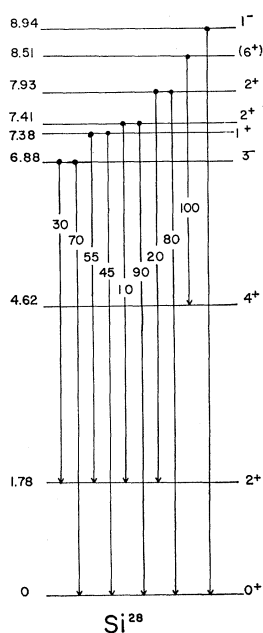


FIG. 3. Energies and decay modes (Refs. 7 and 8) of selected levels in  $\text{Si}^{28}$ . The  $J^\pi$  assignments to levels above and including the 6.88-MeV level were obtained in this experiment.

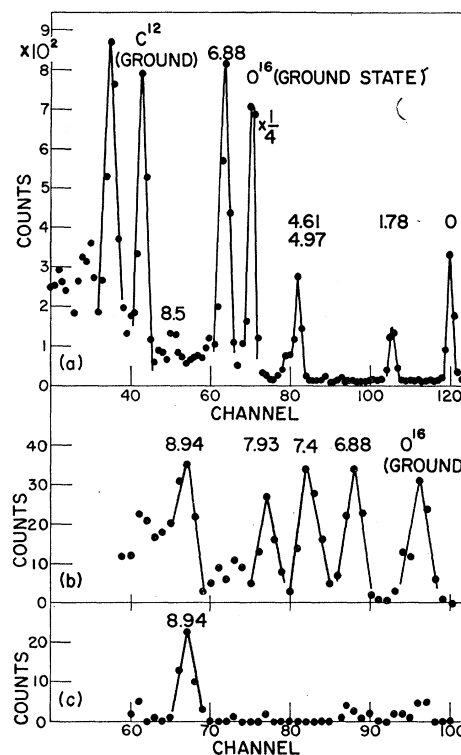


FIG. 4. (a)  $\alpha$ -particle singles spectrum obtained in the  $180^\circ$  annular counter from the  $\text{Si}^{28}(\alpha, \alpha')\text{Si}^{28}$  reaction, and particle spectra obtained in coincidence with  $\gamma$  rays whose energies were in the (b) 6.5–8.5-MeV, and (c) 7.6–9.1-MeV ranges.

level in Fig. 3 is taken from Antoutiev *et al.*<sup>7</sup> The remaining decay modes and branching ratios are taken from Endt and Van der Leun,<sup>8</sup> as are the spin assignments for levels below 6.88 MeV.

A particle spectrum obtained from the  $\text{Si}^{28}(\alpha, \alpha')\text{Si}^{28}$  reaction in the  $180^\circ$  annular counter is shown in Fig. 4(a). The  $\alpha$ -particle peaks are identified by the excitation energies of the levels to which they correspond in  $\text{Si}^{28}$ . Contaminant peaks are also indicated. Figures 4(b) and 4(c) show particle spectra taken in coincidence with  $\gamma$  rays whose energies were in the 6.5–8.5 and 7.6–9.1-MeV ranges, respectively. The peaks at 6.88, 7.4, and 7.93 MeV in Fig. 4(b) and at 8.94 MeV in Fig. 4(c) result from the ground-state  $\gamma$ -ray transitions from each of these states as indicated in Fig. 3.

##### 1. $\text{Si}^{28}$ 6.88-MeV Level

A  $\gamma$ -ray spectrum taken in coincidence with the  $\alpha$ -particle group at 6.88 MeV is shown in Fig. 5. The measured angular distribution of the 6.88-MeV ground-state transition is shown in Fig. 2(b). The solid line is the best fit to the data assuming a  $3 \rightarrow 0$  transition. Note that this distribution is identical to that measured for the  $3 \rightarrow 0$  transition in  $\text{O}^{16}$  [Fig. 2(a)].

<sup>7</sup> Y. P. Antoutiev, D. A. E. Darwish, O. E. Badawy, L. M. El-Nadi, and P. V. Sorokin, *Nucl. Phys.* **56**, 401 (1964).

<sup>8</sup> P. M. Endt and C. Van der Leun, *Nucl. Phys.* **A105**, 1 (1967).

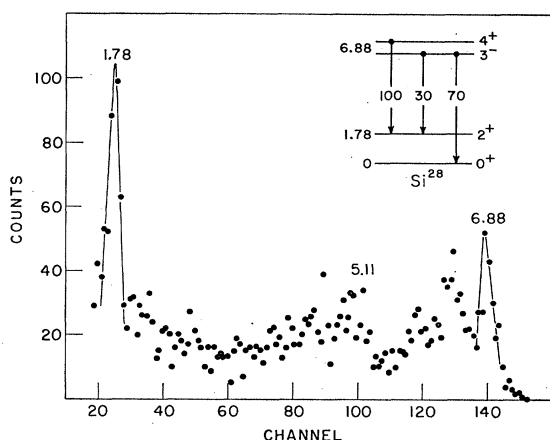


FIG. 5. Spectrum of  $\gamma$  rays in coincidence with the  $\text{Si}^{28}$  6.88-MeV  $\alpha$ -particle group.

The  $J^\pi=3^-$  assignment to this level is consistent with that measured by a number of other groups.<sup>9-11</sup>

### 2. $\text{Si}^{28}$ 7.38- and 7.41-MeV Levels

Figure 6(b) includes a diagram showing that both the 7.38- and 7.41-MeV levels of  $\text{Si}^{28}$  decay to the first excited state and the ground state. This doublet was not resolved either in the particle spectrum or in the coincidence  $\gamma$ -ray spectra. The coincidence  $\gamma$ -ray spectrum shown in Fig. 6(a) is a sum of six spectra taken at six angles from  $40^\circ$  to  $90^\circ$  to the beam direction. From this spectrum and the known branching ratios for these two states,<sup>8</sup> it can be inferred that the 7.41-MeV state was populated with  $0.3 \pm 0.2$  times the intensity of the 7.38-MeV state in the present experiment. The measured correlation of the decay from the doublet to the ground state is therefore a composite of two correlation functions as discussed above.

This correlation was initially fitted assuming that both states have natural parities and consequently that the angular correlations are characterized by strong population of the  $m=0$  substate and weak population of the  $|m|=1$  substates. Visual comparison of the data in Fig. 6(b) with those taken for  $2^+ \rightarrow 0^+$ ,  $3^- \rightarrow 0^+$ , and  $1^- \rightarrow 0^+$  transitions (see Sec. IV A 5) indicates that both states do not have the same spin. Fits were made to the data assuming that the correlation was due to a sum of transitions of the form  $J+J' \rightarrow 0$ , where  $J$  and  $J'$  can take on values of 1, 2, and 3. The best fit to the data assuming a  $1^-+3^- \rightarrow 0^+$  decay yielded a  $\chi^2$  value of 25, thus eliminating this spin combination as a solution. The best theoretical fits assuming a  $1^-+2^+ \rightarrow 0^+$  decay ( $\chi^2=1.68$ ), and a  $3^-+$

$2^+ \rightarrow 0^+$  ( $\chi^2=2.95$ ) were obtained with the population of the  $1^-$  or  $3^-$  state being 0.15 or 0.5 times that of the  $2^+$  state. These fits are consistent with  $J^\pi$  assignments of  $2^+$  and  $1^-$  or  $3^-$  (with  $1^-$  being favored) to the 7.38- and 7.41-MeV states, respectively.

Other data suggest spin assignments of  $J=1$  for the 7.38-MeV state,<sup>12</sup> and  $J^\pi=2^+$  for the 7.41-MeV state.<sup>13</sup> An attempt was therefore made to fit the data shown in Fig. 6(b) assuming that a  $2^+$  state and a  $1^+$  state were populated. The  $2^+ \rightarrow 0^+$  correlation was taken to be like that measured for the  $\text{C}^{12}$ ,  $2^+ \rightarrow 0^+$  transition, with primarily the  $m=0$  substate being populated. The theoretical angular correlation for the  $1^+ \rightarrow 0^+$  transition was calculated assuming that only the  $|m|=1$  substates were populated. A good fit to the data [solid line in Fig. 6(b)] was found assuming  $J^\pi(7.41)=2^+$  and  $J^\pi(7.38)=1^+$  with the population of the 7.41-MeV state being 0.4 times that of the 7.38-MeV state. This result is consistent with the other data<sup>12,13</sup> on these states, but disagrees with the expectation that unnatural-parity states should be populated weakly if the particles are detected near  $180^\circ$ , as was indeed the case in the present experiments for known  $3^+$  states.

### 3. $\text{Si}^{28}$ 7.93-MeV Level

A  $\gamma$ -ray spectrum taken in coincidence with the particle group at 7.9 MeV showed the 7.93-MeV decay

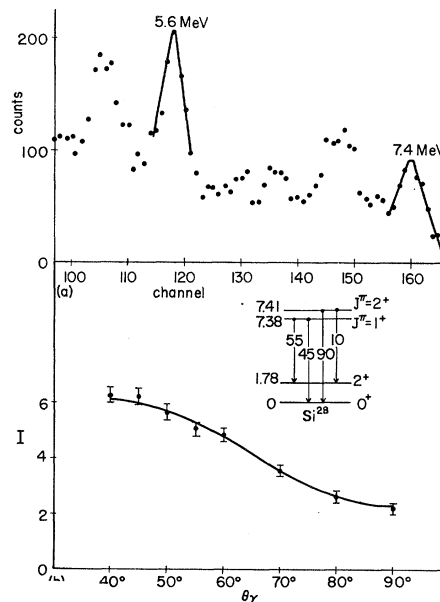


FIG. 6. (a) Partial spectrum of  $\gamma$  rays in coincidence with the  $\text{Si}^{28}$  7.4-MeV  $\alpha$ -particle group. This spectrum is a sum of six spectra taken at six different angles. (b) Angular correlation measured for the decay of the 7.38- and 7.41-MeV states to the ground state in  $\text{Si}^{28}$ . The solid line is the best theoretical fit assuming  $J^\pi+J^{\pi'}=1^++2^+$ .

<sup>9</sup> R. Nordhagen, M. Hoffman, F. Ingebretsen, and A. Tueter, Phys. Letters **163**, (1965).

<sup>10</sup> R. J. Levesque, R. Ollerhead, E. Blackmore, and J. Kuehner, Can. J. Phys. **44**, 1087 (1966).

<sup>11</sup> S. Gorodetzky, J. P. Coffin, G. Frick, A. Gallmann, F. Jundt, and E. Aslanides, Nucl. Phys. **A97**, 475 (1967).

<sup>12</sup> T. K. Alexander, C. Broude, A. J. Ferguson, J. A. Kuehner, A. E. Litherland, R. W. Ollerhead, and P. J. M. Smulders, in *International Nuclear Physics Conference*, edited by R. L. Becker (Academic Press Inc., New York 1967).

<sup>13</sup> A. Heyligers, thesis, Utrecht University, 1964 (unpublished).

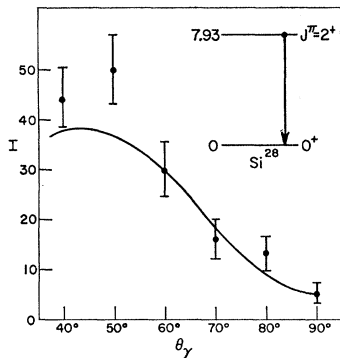


FIG. 7. Angular correlation measured for the decay of the 7.93-MeV state to the ground state in  $\text{Si}^{28}$ . The solid line is the best theoretical fit for a  $J^\pi = 2^+ \rightarrow 0^+$  transition.

to ground reported by Azuma *et al.*<sup>14</sup> A comparison of the angular correlation measured for this transition (Fig. 7) with that measured for the  $\text{C}^{12}$   $4.43(2^+) \rightarrow 0(0^+)$ -MeV transition suggests that the spin of the 7.93-MeV state is 2. The solid line in Fig. 7 is the best fit assuming a  $2 \rightarrow 0$  transition [ $\chi^2 = 1.35$ ,  $P(1)/P(0) = 0.056 \pm 0.042$ ].

Hinds and Middleton<sup>15</sup> measured the angular distributions of several deuteron groups in the  $\text{Al}^{27}(\text{He}^3, d)\text{Si}^{28}$  reaction and assigned an  $l_p$ -value of zero for the transition to the 7.93-MeV level in  $\text{Si}^{28}$ . This limits the  $J^\pi$  possibilities to  $2^+$  and  $3^+$ . Endt and Heyligers<sup>16</sup> combined this result with the observation of a ground-state transition from the 7.93-MeV level to assign  $J^\pi(7.93 \text{ MeV}) = 2^+$ . The direct measurement made in this experiment confirms this assignment.

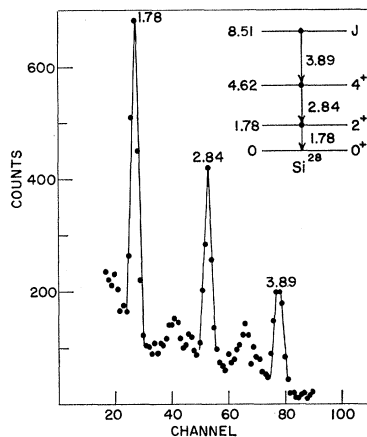


FIG. 8. Spectrum of  $\gamma$  rays in coincidence with the  $\text{Si}^{28}$  8.51-MeV  $\alpha$ -particle group.

<sup>14</sup> R. E. Azuma, A. Charlesworth, K. Jackson, A. Anyas-Weiss, and B. Lalovic, *Can. J. Phys.* **44**, 3075 (1966).

<sup>15</sup> S. Hinds and R. Middleton, *Proc. Phys. Soc. (London)* **76**, 545 (1960).

<sup>16</sup> P. M. Endt and A. Heyligers, *Physica* **26**, 230 (1960).

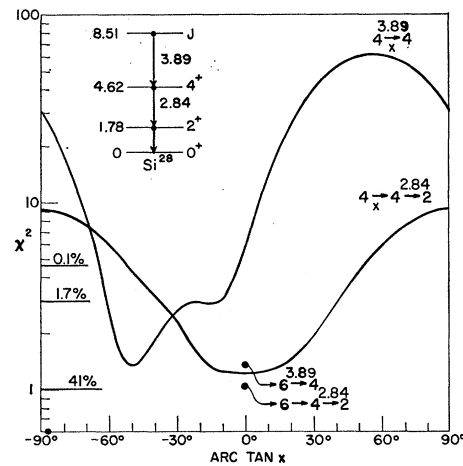


FIG. 9.  $\chi^2$  versus  $\arctan x$  for the decay of the 8.51-MeV state in  $\text{Si}^{28}$ . The 3.89- and 2.84-MeV transitions are fitted simultaneously for assumed values of  $J=4$  and  $J=6$ .

#### 4. $\text{Si}^{28}$ 8.51-MeV Level

A  $\gamma$ -ray spectrum taken in coincidence with the particle group at approximately 8.5 MeV is shown in Fig. 8. The three  $\gamma$  rays are interpreted in terms of a cascade originating from a level at  $8.51 \pm 0.05$  MeV.<sup>17</sup>

The fits to the measured 3.89-MeV correlation for assumed values of  $J$  from 2 to 6 yielded possible solutions for  $J=4$  or 6. In an attempt to remove this ambiguity, the 3.89- and 2.84-MeV correlations were fit simultaneously for assumed values of  $J=4$  and  $J=6$ . The results are shown in Fig. 9, which is a plot of  $\chi^2$  versus  $\arctan x$  for spin assignments of  $4 \rightarrow 4$  and

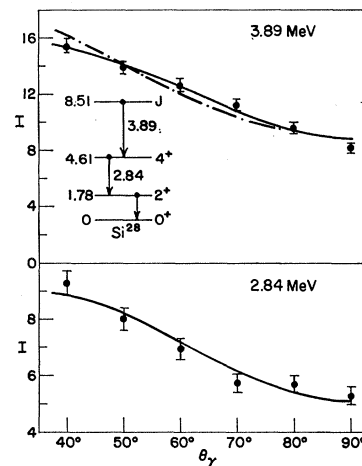


FIG. 10. Angular correlations measured for the decay of the 8.51-MeV state in  $\text{Si}^{28}$ . The solid lines and dashed line are the best fits assuming  $J^\pi = 6^+$  and  $4^+$ , respectively. The fits are so similar for the 2.84-MeV correlation that only one is shown.

<sup>17</sup> Recent  $\text{Ge}(\text{Li})$   $\gamma$ -ray counter studies put this level at 8.543 MeV. See T. K. Alexander, C. Broude, A. J. Ferguson, J. A. Kuehner, A. E. Litherland, R. W. Ollerhead, and P. J. M. Smulders, *Can. J. Phys.* **47**, 651 (1969).

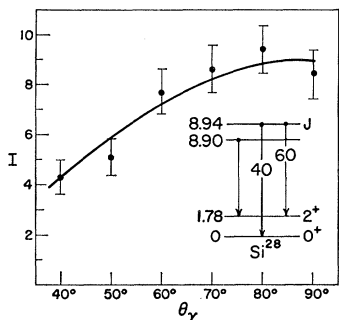


FIG. 11. Angular correlation measured for the decay of the 8.94-MeV state to the ground state in  $\text{Si}^{28}$ . The solid line is the best theoretical fit for a  $J^\pi=1^- \rightarrow 0^+$  transition.

$6 \rightarrow 4$  ( $x=0$ ) for the 3.89-MeV transition, and for  $4 \rightarrow 4 \rightarrow 2$  and  $6 \rightarrow 4 \rightarrow 2$  for the 2.84-MeV transition. This figure shows that the measured distributions are consistent with the  $J^\pi=6^+$  assignment to the 8.5-MeV level [ $P(1)/P(0)=0.00+0.05$ ]. Both  $\chi^2$  distributions have strong minima assuming  $J=4$ ; however, these minima are not obtained at the same values of  $x$ . To find a single set of values for  $P(0)$ ,  $P(1)$ , and  $x$  which represent a best fit to both sets of data for  $J=4$ , a minimum is sought in the sum of the two  $\chi^2$  versus  $\arctan x$  curves shown in Fig. 9. This minimum occurs at  $x=-0.27$ ,  $\arctan x=-15^\circ$ ,  $P(1)/P(0)=0.16$  with  $\chi^2(3.89 \text{ MeV})=2.8$  and  $\chi^2(2.84 \text{ MeV})=1.3$ . The best fits to the data are shown in Fig. 10 for  $J=6$  (solid lines) and  $J=4$  (dashed line). The two fits are so similar for the 2.84-MeV correlation that only one is shown.

These correlation measurements and the absence of other strong-decay modes favor a  $J^\pi$  assignment of  $6^+$  to this level.

### 5. $\text{Si}^{28}$ 8.94-MeV Level

A  $\gamma$  spectrum taken in coincidence with the particle group at 8.9 MeV showed peaks at 8.9, 7.1, and 1.78 MeV, indicating levels at approximately 8.9 MeV decaying to the ground and first excited states. The decay

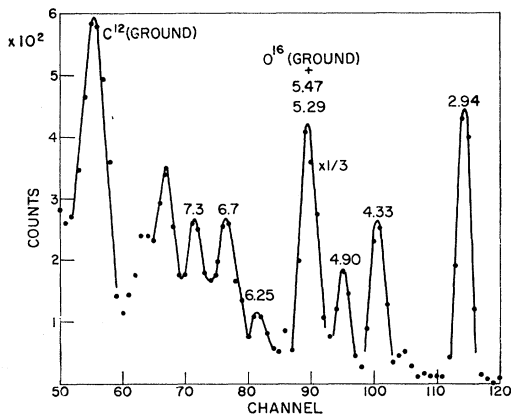


FIG. 12.  $\alpha$ -particle singles spectrum obtained in the  $180^\circ$  annular counter from the  $\text{Mg}^{26}(\alpha, \alpha')\text{Mg}^{26}$  reaction.

modes of the 8.90- and 8.94-MeV levels shown in Fig. 11 were obtained by Nordhagen and Tveter<sup>18</sup> and Antoutiev *et al.*<sup>7</sup>, respectively, by studying the  $\text{Al}^{27}(p, \gamma)\text{Si}^{28}$  reaction. Assuming these to be correct, the ground-state transition originates from the 8.94-MeV state rather than the 8.90-MeV state as was indicated in a preliminary report.<sup>19</sup>

The measured distribution of the ground-state transition is shown in Fig. 11. Comparison of this with Figs. 1 and 2(a) indicates that the spin of this level is not 2 or 3. The solid line represents the best fit assuming a  $1^- \rightarrow 0^+$  transition [ $\chi^2=0.52$ ,  $P(1)/P(0)=0.00+0.05$ ].

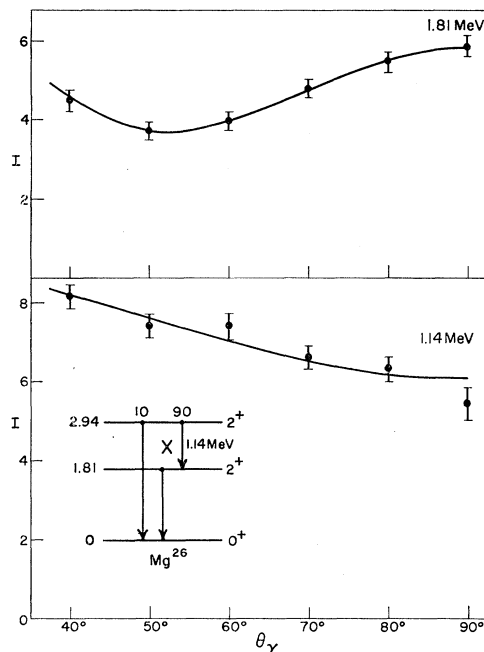


FIG. 13. Angular correlations measured for the decay of the 2.94-MeV state in  $\text{Mg}^{26}$ . The solid lines are the best fits assuming a  $J^\pi=2^+ \rightarrow 2^+ \rightarrow 0^+$  cascade with  $x=0.087 \pm 0.045$ .

## B. $\text{Mg}^{26}$ Results

A particle spectrum obtained from the  $\text{Mg}^{26}(\alpha, \alpha')\text{Mg}^{26}$  reaction is shown in Fig. 12. The  $\alpha$ -particle peaks are identified by the excitation energy to which they correspond in  $\text{Mg}^{26}$ . The  $\alpha$ -particle groups corresponding to ground and first excited state in  $\text{Mg}^{26}$  are not shown. The 2.94- and 4.90-MeV levels were studied in this experiment.

### 1. $\text{Mg}^{26}$ 2.94-MeV Level

The  $2^+$  level at 2.94 MeV in  $\text{Mg}^{26}$  is known to decay predominantly (90%)<sup>8</sup> by a cascade through the  $2^+$ , first excited state at 1.81 MeV to the ground state

<sup>18</sup> R. Nordhagen and A. Tveter, Nucl. Phys. 56, 337 (1964).

<sup>19</sup> T. R. Canada, R. D. Bent, and J. A. Haskett, Bull. Am. Phys. Soc. 12, 570 (1967).

(Fig. 13). An example of a  $\gamma$ -ray spectrum obtained in coincidence with the 2.94-MeV  $\alpha$  group from this laboratory has been published elsewhere.<sup>1</sup> The angular correlations of these two transitions were measured to determine the mixing ratio of the 2.94 ( $2^+$ ) $\rightarrow$ 1.81 ( $2^+$ ) decay.

Figure 13 shows the measured distributions. The solid lines represent the best theoretical fits to the data giving  $x=0.087\pm 0.045$  [ $P(1)/P(0)=0.08\pm 0.05$ ]. This value for the  $E2/M1$  mixing ratio is in agreement with an earlier measurement<sup>20</sup> of  $x=0.12\pm 0.02$ .

### 2. $Mg^{26}$ 4.90-MeV Level

A  $\gamma$ -ray spectrum obtained in coincidence with the particle group at 4.90 MeV is shown in Fig. 14. The

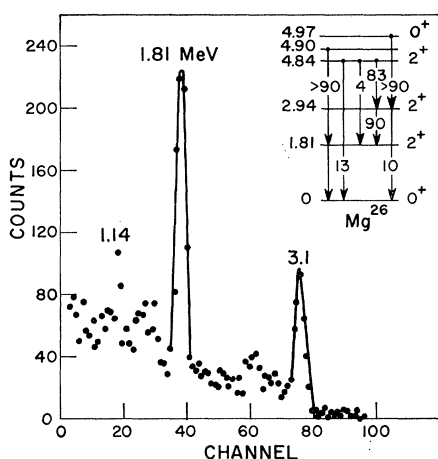


Fig. 14. Spectrum of  $\gamma$  rays in coincidence with the  $Mg^{26}$  4.90-MeV  $\alpha$ -particle group.

accompanying decay scheme is taken from the recent work of Häusser *et al.*,<sup>21</sup> who have studied the  $\gamma$ -ray spectra originating from the  $Mg^{26}(p, p'\gamma)$  reaction using Ge(Li) detectors. The  $J^\pi$  assignments for the 4.84- and 4.97-MeV levels were originally made on the basis of ( $d, p$ )- and ( $t, p$ )-reaction studies by Hinds *et al.*<sup>22</sup> and have been recently confirmed in the angular correlation measurements of Ferguson *et al.*<sup>23</sup> The coincidence spectrum shows that in this experiment, only the 4.90-MeV member of this triplet was strongly excited.

Fits were made to the 3.1-MeV correlation assuming  $J(4.90) = 1, 2, 3,$  and  $4$ . The best theoretical fits assuming  $J$  values of 1 and 3 yielded  $\chi^2$  values of 10.4 and 5.3, respectively, thus ruling out these spin possibilities.

<sup>20</sup> C. Broude and H. E. Gove, *Ann. Phys. (N. Y.)* **23**, 71 (1963).

<sup>21</sup> O. Häusser, T. K. Alexander, and C. Broude, *Can. J. Phys.* **46**, 1035 (1968).

<sup>22</sup> S. Hinds, H. Marchant, and R. Middleton, *Nucl. Phys.* **67**, 257 (1965).

<sup>23</sup> A. J. Ferguson, O. Häusser, C. Broude, and F. Ingebretsen, *Bull. Am. Phys. Soc.* **13**, 86 (1968).

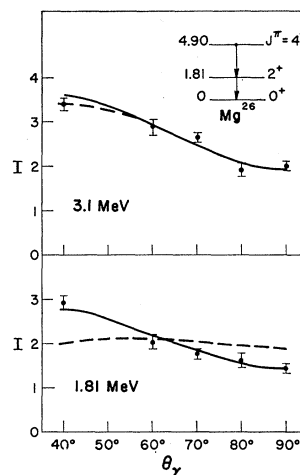


Fig. 15. Angular correlations measured for the decay of the 4.90-MeV state in  $Mg^{26}$ . The solid and dashed lines are the best fits assuming  $J=4$  and  $2(x=-2.7)$ , respectively.

Solutions were found assuming  $J=4$  ( $P(1)/P(0) = 0.10\pm 0.05$ ) and  $J=2$  with an  $E2/M1$  mixing ratio of  $x = -(2.7_{-1.3}^{+0.9})$ ; however, this ambiguity was removed by a simultaneous fit of the 3.1- and 1.81-MeV correlations which allowed only the  $J^\pi=4^+$  assignment (Fig. 15).

The work of Ferguson *et al.*<sup>23</sup> suggested a spin assignment of 3 or 4 to this level. Hinds *et al.*<sup>22</sup> suggested a spin assignment of  $2^+$  or  $3^+$ . The lifetime of the 4.90-MeV level,  $5_{-3}^{+4}\times 10^{-14}$  sec,<sup>21</sup> corresponds to  $12_{-5}^{+18}$  Weisskopf units, assuming a pure  $E2$  transition to the 1.81-MeV level.  $E2$  enhancements this large are not uncommon in this region<sup>1,21</sup> and may indicate that the state has a collective nature.

### C. $Mg^{24}$ Results

The particle groups corresponding to inelastically scattered  $\alpha$  particles which excite the higher levels in

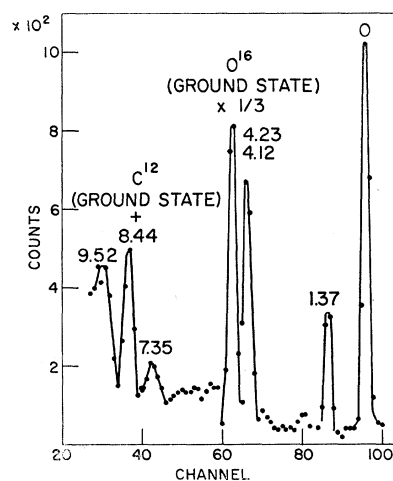


Fig. 16.  $\alpha$ -particle singles spectrum obtained in the  $180^\circ$  annular counter from the  $Mg^{24}(\alpha, \alpha')Mg^{24}$  reaction.

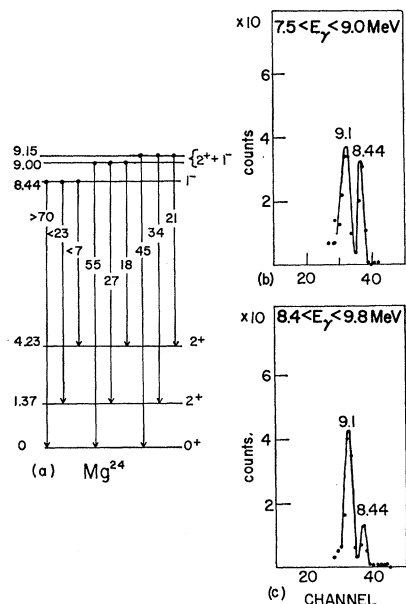


FIG. 17. (a) Energies and decay modes (Refs. 8 and 23) of selected states in  $Mg^{24}$ . The  $J^\pi$  assignments to levels above 4.23 MeV were obtained in this experiment. Particle spectra obtained in coincidence with  $\gamma$  rays whose energies were in the (b) 7.5–9.0-MeV and (c) 8.4–9.8-MeV ranges.

$Mg^{24}$  are obscured in the particle singles spectrum (Fig. 16) by a proton background originating from the  $Mg^{24}(\alpha, p)Al^{27}$  ( $Q = -1.595$  MeV) reaction. These protons were not stopped in the detector.

To enhance these groups and to obtain some information concerning their decay modes, particle spectra were taken in coincidence with  $\gamma$  rays of various energies. These spectra are shown in Fig. 17(b) and 17(c). They indicate possible ground-state transitions from states at approximately 8.44 and 9.10 MeV. This is in agreement with the recent report of Ollerhead *et al.*<sup>24</sup> who have studied a number of levels in  $Mg^{24}$  using the  $C^{12}(O^{16}, \alpha)Mg^{24}$  reaction. The decay modes of levels near 8.44 and 9.1 MeV as measured in Ref. 24 are shown in Fig. 17(a). The  $\gamma$ -ray spectra obtained in coincidence with these particle groups were consistent with this decay scheme but allowed no qualitative analysis, since the various groups were not resolved in the particle spectrum.

### 1. $Mg^{24}$ 8.44-MeV Level

The measured angular distribution of the ground-state transition from an 8.44-MeV state [Fig. 18(a)] has the characteristic  $1^- \rightarrow 0^+$  pattern. The solid line in Fig. 18(a) is the best theoretical fit to the data assuming  $J^\pi = 1^-$ . This assignment confirms that reported earlier.<sup>24</sup>

<sup>24</sup> R. W. Ollerhead, J. A. Kuehner, R. J. A. Levesque, and E. W. Blackmore, *Can. J. Phys.* **46**, 1381 (1968).

### 2. $Mg^{24}$ 9.00-, and 9.15-MeV Levels

Fits to the angular distribution of the ground-state transitions from the levels at approximately 9.1 MeV [Fig. 18(b)] were made assuming two states with spins  $J$  and  $J'$  were contributing. The best theoretical fits assuming spin combinations of  $J+J'=1+3$  and  $2+3$  yielded  $\chi^2$  values of 4.7 and 11.8, respectively, effectively ruling out these possibilities. The best fit assuming spins of  $1^-$  and  $2^+$  ( $\chi^2=0.2$ ) is shown as a solid line in Fig. 18(b). This result is in agreement with that of Ref. 24, where assignments of  $2^+$  and  $1^-$  are made to the levels at 9.00 and 9.15, respectively.

## V. DISCUSSION

There are a number of similarities in the level structures of  $Si^{28}$ ,  $Mg^{26}$ , and  $Mg^{24}$  which are pointed out in Fig. 19. The most obvious of these are the  $K=0$  ground-state rotation bands which may be identified in all three nuclei.<sup>2,5,16,21,24</sup> The assignment of the  $4^+$  member of this band in  $Mg^{26}$  at 4.32 MeV is somewhat arbitrary because of the tentative nature of the spin assignment to this level and because of the existence of two other possible candidates at 4.90 and 5.47 MeV. A possible  $K=2$  rotational band may be identified in both  $Mg^{24}$  and  $Mg^{26}$  (again the placement of the  $4^+$  member of this band at 4.90 MeV is arbitrary) but not in  $Si^{28}$  at a comparable energy. A possible  $\beta$ -vibration,  $0^+$  state in  $Si^{28}$  at 4.97 MeV<sup>25</sup> may be matched with a  $0^+$  state in  $Mg^{26}$  at approximately the same energy. However, a comparable state is not found in  $Mg^{24}$ .

The relationship between energy and  $J(J+1)$  for states within a band (Fig. 20) is very similar for these nuclei. Qualitatively, the moments of inertia appear to

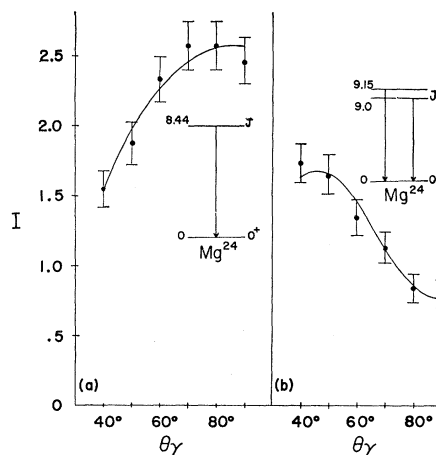


FIG. 18. Angular correlations measured for the decay of the (a) 8.44-MeV and (b) 9.00- and 9.15-MeV states to the ground state in  $Mg^{24}$ . The solid lines are the best theoretical fits assuming (a)  $J^\pi = 1^-$  and (b)  $J^\pi + J'^\pi = 1^- + 2^+$ .

<sup>25</sup> S. Das Gupta and M. Harvey, *Nucl. Phys.* **A94**, 602 (1967).



be almost equal in the  $K=0$  ground-state rotation bands. Similarly, the  $K=2$  rotation bands in  $Mg^{24}$  and  $Mg^{26}$  have approximately the same  $J(J+1)$  dependence.

Figure 20 predicts a  $5^+$  state in  $Mg^{26}$  near 6.5 MeV and two  $6^+$  states between 8 and 9 MeV. The identification of these states and a study of their decay modes would be helpful in assigning the  $4^+$  members of the  $K=0$  and  $K=2$  rotation bands.

In addition, as indicated in Fig. 19,  $Mg^{24}$ ,  $Mg^{26}$ , and  $Si^{28}$  each have a  $0^+$  state near 6.5 MeV and a  $3^-$  state near 7 MeV (6.88 MeV in  $Mg^{26}$  and  $Si^{28}$ , 7.62 MeV in  $Mg^{24}$ ).

These similarities indicate that the same general modes of excitation may be present in all three nuclei.

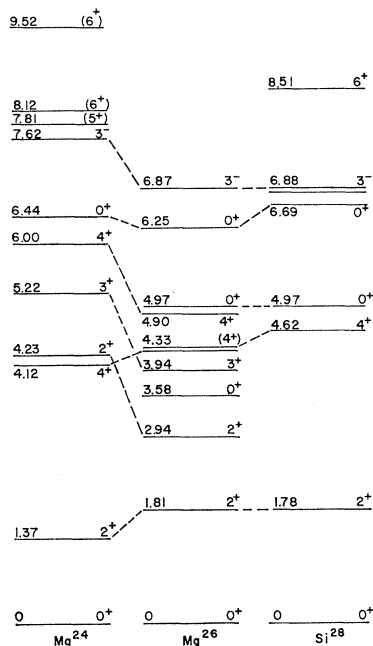


FIG. 19. Energy-level diagrams for  $Mg^{24}$ ,  $Mg^{26}$ , and  $Si^{28}$ . Only selected levels are shown.

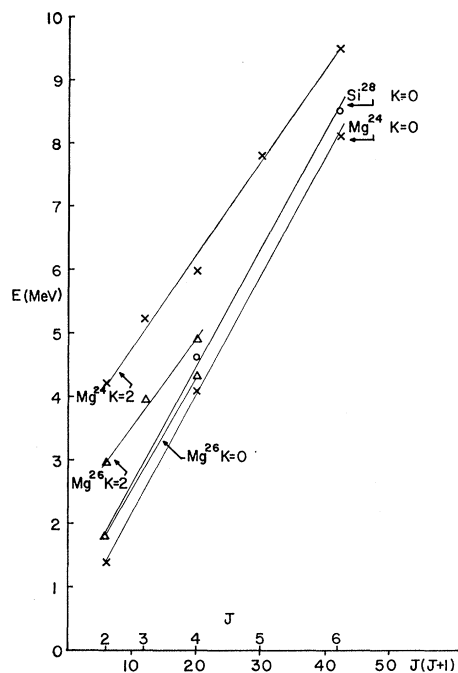


FIG. 20. Energies of selected levels in  $Mg^{24}$  ( $\times$ ),  $Mg^{26}$  ( $\Delta$ ), and  $Si^{28}$  ( $\circ$ ) versus  $J(J+1)$ . The straight lines are approximations to the various groupings.

The degree to which the various collective bands are mixed in a given nucleus will vary. This mixing is reflected in the deviation of the measured lifetimes and branching ratios from the predicted values for pure rotation bands<sup>1,21</sup> and may be especially complex in  $Mg^{26}$ , where the number of excited states below 7 MeV is approximately three times as large as that in  $Mg^{24}$  or  $Si^{28}$ . A number of these states in  $Mg^{26}$  may result from rotation bands based upon particle excitations; for example, there may be a  $K=0$  rotation band associated with the  $0^+$  level at 3.58 MeV which has been suggested to arise from the removal of two neutrons from the  $d_{5/2}$  into the  $2s_{1/2}$  orbital.<sup>22</sup>