where $E_{\rm int}$ is the excitation energy of the intermediate state in the pp diagram and $E_{\rm int}'$ is the excitation energy of the same particles in the ph diagram. We would have (B5a) = (B5b) if $E_{k^{-1}} = -E_k$ and $E_{\rm int}' = E_{\rm int}$. Although these conditions do not hold in general, we will have

$$E_0 - H_0 = (E_0 - H_0)' \tag{B6}$$

under the condition that we have described as *high-lying*, that is, if each of the energy denominators is larger than the variation in E_k or $E_{\rm int}$.

Finally, we find that the ph interaction energy contributed by Fig. 4(b) may be written

$$E_D(jk^{-1};I) = \sum_\lambda (-1)^{\lambda+1} \langle jk^{-1};I \mid \times[T^\lambda \times u^\lambda(k^{-1}k^{-1})]^0 \mid jk^{-1};I\rangle, \tag{B7}$$

which can also be written as a transform of the contribution (B2a) of diagram (a) of Fig. 4 to the pp interaction energy, following (A8):

$$E_D(jk^{-1};I) = -\sum_J [J] W(jkkj;IJ) E_D(jk;J). \tag{B8}$$

This completes the proof of the theorem that *high-lying* diagrams contribute to the *conserving* valence effective interaction $\mathcal{V}_c$ [Eqs. (1.5) and (1.6)].

If the diagram (a) of Fig. 4 has vertices t_0 and t_2 interchanged, then there will be two particle lines between t_2 and t_0. Then the "Pauli-correction" (disconnected) diagram appears for the pp case, and the roles of pp and ph are interchanged in the above proof.

Finally, we note that the restriction that the particle operators A_j and B_j must appear *only* once each is not necessary for the proof, since these operators are contained in T^λ, and do not change in the ph transformation. This implies that our definition of high-lying terms is somewhat more restrictive than necessary. This has no practical effect on the calculation of the *violating* contributions, however, since conserving terms would cancel in (2.15).

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Three-Body Model of Li^6 and Deuteron–Alpha-Particle Scattering*

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The properties of Li^6 and deuteron–α-particle scattering are studied in an exact three-particle model in which the neutron, proton, and α particle interact through separable two-body forces. The α particle is assumed to be structureless, and Coulomb effects are neglected. As a representation of the nucleon–α-particle interaction, a three-term separable potential fit to low-energy neutron–α-particle scattering is introduced in the partial waves $s_{1/2}$, $p_{3/2}$, and $p_{1/2}$. The $s_{1/2}$ interaction is taken to be repulsive, while the other two are attractive. The three-body formalism of Amado is generalized to allow spin-dependent two-body interactions in an arbitrary partial wave. Numerical solution of the resulting three-body equations gives the binding energies of the $T=0$ and $T=1$ states of Li^6 as well as phase shifts, angular distributions, and deuteron polarization in d-α scattering, and also the total cross section for $d+\alpha \to n+p+\alpha$ up to 30 MeV. Most of the calculations have used only an s-wave n-p interaction, but a limited number have been done with the d state of the deuteron included in order to assess its importance. Given the assumptions of the model, the agreement of the calculated quantities with experiment is very good. Some discussion of the results with respect to phenomenological optical-model fits to deuteron-nucleus scattering is also given.

I. INTRODUCTION

In recent years, considerable progress has been made in understanding the three-nucleon system by the use of the separable approximation in equations of the Faddeev type.[1] In this approximation a small number of separable two-body amplitudes are adjusted to fit low-energy nucleon-nucleon scattering and these amplitudes are subsequently used as input in some form of three-body equation. Numerical solution of these equations has yielded results in rather good agreement with experiment but the complexity of the nucleon-nucleon interaction and complications of spin have prevented a really complete calculation from being carried out. What has been learned from this work is that one may successfully employ quite simple two-body interactions if three-body effects are treated exactly. In this paper we carry out a similar analysis[2] for a system consisting of a neutron, proton, and α particle. The spinless nature of the α particle reduces somewhat the complications which hamper a complete treatment of the problem of three nucleons.

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[1] For a review, see *Three Particle Scattering in Quantum Mechanics*, edited by J. Gillespie and J. Nuttall (W. A. Benjamin, Inc., New York, 1968).

[2] A short account of this work appeared in P. E. Shanley, Phys. Rev. Letters 21, 627 (1968).

The model of the six-nucleon system is constructed by assuming that the neutron, proton, and α particle are elementary particles interacting pairwise through separable potentials. The large binding energy of the α particle and its lack of low-energy excited states are taken as evidence that, in its low-energy interaction with nucleons, the α particle may be regarded as an elementary point particle with no internal structure, although some account of the exclusion principle is included in the nucleon-α interaction. The quantities that are calculated from the three-body model are the binding energies of the $T=0$ and $T=1$ states of Li^6 as well as the phase shifts, angular distributions, and deuteron polarization in d-α scattering and also the total cross section for $d+\alpha\rightarrow n+p+\alpha$ up to 30 MeV.

The analysis of d-α scattering carried out here may have some bearing on the optical-model analysis of deuteron-nucleus scattering. Recent measurements of deuteron polarization in deuteron-nucleus elastic scattering[3] have led to the use of various kinds of spin-dependent optical potentials in an attempt to fit the experimental data. The form of the optical potentials is based on general invariance arguments[4] and the origin of these interactions in terms of the more fundamental spin dependence of the nucleon-nucleus or neutron-proton interaction is not clear. The present model allows one to study the deuteron cross sections and polarization that result from various assumptions concerning the spin dependence of the two-body interactions, for example, a neutron-proton tensor force. This is discussed in Sec. IV.

There is considerable previous evidence that the six-nucleon system may be viewed approximately in terms of three particles. Such models have been considered by Wackman and Austern[5] in a variational approach and by Gammel, Hill, and Thaler[6] in a distortionless approximation for d-α scattering. The ground states of He^6 and Li^6 have been studied[7] in separable potential models similar to the present work. In addition, studies have been based on the resonating-group method,[8] the shell model,[9] and an approximate three-body approach[10] based on dispersion theory. Comparison of results with some of this previous work is carried out in Sec. IV.

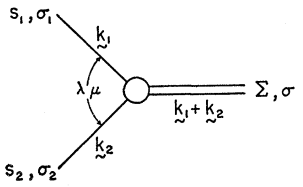

FIG. 1. Basic two-particle vertex employed in the theory.

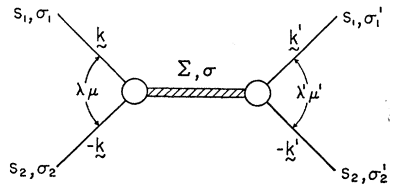

FIG. 2. Two-body elastic scattering graph showing the associated orbital angular momenta and spins.

The three-body formalism used in this paper is one introduced by Amado.[11] The necessary generalization of his method for the problem at hand is given in Sec. II. Section III contains the details of the necessary two-body scattering amplitudes and a discussion of the three-body equations for the six-nucleon problem, their solution, and the results. A summary and discussion is given in Sec. IV. Two appendices deal with the calculation of Born amplitudes and the inclusion of the n-p tensor force.

II. THEORY

In this section we will present an extension of Amado's quasiparticle method that is general enough to permit spin-dependent two-body interactions in an arbitrary partial wave. For the six-nucleon problem this generalization will be required in the N-α subsystem where both s and p waves will be employed and also in the n-p subsystem for the inclusion of the deuteron d state. We will present here a quite general formulation which will be specialized for the six-nucleon problem in Sec. III. In the Amado formalism one assumes that two-body scattering is dominated by a bound state or quasiparticle and proceeds by nonrelativistic Lee-model interactions. This leads to a separable two-body scattering amplitude. Three-particle scattering consists of a bound state or quasiparticle scattering from the remaining third particle. In a second-quantized formalism one requires an interaction Hamiltonian that allows the basic three-point interaction depicted in Fig. 1. Here particle 1 with mass m_1, spin s_1, and projection σ_1 interacts with particle 2 of mass m_2 and spin quantum numbers s_2, σ_2 in a relative orbital angular momentum state λ, μ to form a quasiparticle of mass M and spin Σ, z-component σ, and parity $(-1)^\lambda$. A suitable Hamiltonian is

$$H_I=\sum_{q,Q}\Gamma_{\lambda\bar{s}}{}^{\Sigma}f_{\lambda\bar{s}}{}^{\Sigma}(q)\,Y_\lambda{}^{\mu*}(\hat{q})\,\langle s_1\sigma_1 s_2\sigma_2\mid\bar{s}\bar{\sigma}\rangle\langle\lambda\mu\bar{s}\bar{\sigma}\mid\Sigma\sigma\rangle$$
$$\times\Psi_{\Sigma\sigma}(\mathbf{Q})\Psi_{s_1\sigma_1}{}^{\dagger}[(m_1/M)\mathbf{Q}+\mathbf{q}]\Psi_{s_2\sigma_2}{}^{\dagger}[(m_2/M)\mathbf{Q}-\mathbf{q}]$$
$$+\text{h.c.},\quad(1)$$

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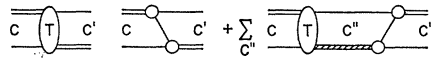

FIG. 3. Pictorial representation of coupled-channel integral equations.

where

$$\mathbf{q}=(m_2\mathbf{k}_1-m_1\mathbf{k}_2)/M, \qquad \mathbf{Q}=\mathbf{k}_1+\mathbf{k}_2,$$
$$M=m_1+m_2. \tag{2}$$

The sum is also over all discrete quantum numbers. The Clebsch-Gordan coefficients indicate the sequence of angular momentum coupling. s_1 and s_2 couple to the total spin $\bar{s}$ which is then coupled with the orbital angular momentum λ to give the quasiparticle spin Σ. The $\Psi^\dagger$ and Ψ create and annihilate particles of given spin and momentum and they commute or anticommute for particles with integral or half-integral spin, respectively. $\Gamma_{\lambda\bar{s}}{}^{\Sigma}$ and $f_{\lambda\bar{s}}{}^{\Sigma}$ are the coupling constant and form factor associated with the vertex and are chosen to fit two-body bound-state and scattering data.

Given an interaction governed by Eq. (1), the most general form of two-body elastic scattering amplitude is shown graphically in Fig. 2. Here we have combined two of the above vertices and the cross-hatched internal line indicates that all bubble insertions have been summed in the intermediate state. The form of this two-body amplitude may be obtained from the Hamiltonian (1) and the calculation is most naturally done in a representation in which the spins of the individual particles are specified. In the center-of-mass system we obtain

$$\langle s_1\sigma_1; s_2\sigma_2; \mathbf{k} \mid t(E) \mid s_1\sigma_1'; s_2\sigma_2'; \mathbf{k}'\rangle$$
$$=\sum \Gamma_{\lambda\bar{s}}{}^{\Sigma} f_{\lambda\bar{s}}{}^{\Sigma}(k)\tau_{\Sigma}(E)\Gamma_{\lambda'\bar{s}'}{}^{\Sigma} f_{\lambda'\bar{s}'}{}^{\Sigma}(k') Y_{\lambda}{}^{\mu*}(\hat{k}) Y_{\lambda'}{}^{\mu'}(\hat{k}')$$
$$\times\langle s_1\sigma_1 s_2\sigma_2 \mid \bar{s}\bar{\sigma}\rangle\langle\lambda\mu\bar{s}\bar{\sigma} \mid \Sigma\sigma\rangle$$
$$\times\langle s_1\sigma_1' s_2\sigma_2' \mid \bar{s}'\bar{\sigma}'\rangle\langle\lambda'\mu'\bar{s}'\bar{\sigma}' \mid \Sigma\sigma\rangle. \tag{3}$$

The sum is over λ, μ, λ', μ', $\bar{s}$, $\bar{\sigma}$, $\bar{s}'$, $\bar{\sigma}'$, Σ, and σ. The quasiparticle spin Σ plays the role of the total angular momentum, since the interaction must proceed through this state. The function τ_{Σ} is the intermediate-state propagator and its precise form will be given later.

It is more convenient to work in a representation in which the total spin or channel spin is specified and the amplitude in this representation is obtained by adding the initial and final spins in Eq. (3). In terms of the initial and final channel spins S and S' we have

$$\langle S, S_z; \mathbf{k} \mid t(E) \mid S', S_z'; \mathbf{k}'\rangle$$
$$=\sum_{\sigma_1\sigma_2\sigma_1'\sigma_2'}\langle s_1\sigma_1 s_2\sigma_2 \mid SS_z\rangle\langle s_1\sigma_1' s_2\sigma_2' \mid S'S_z'\rangle$$
$$\times\langle s_1\sigma_1; s_2\sigma_2; \mathbf{k} \mid t(E) \mid s_1\sigma_1'; s_2\sigma_2'; \mathbf{k}'\rangle \tag{4}$$
$$=\sum_{\lambda\mu\lambda'\mu',\Sigma\sigma}\Gamma_{\lambda S}{}^{\Sigma} f_{\lambda S}{}^{\Sigma}(k)\tau_{\Sigma}(E)\Gamma_{\lambda' S'}{}^{\Sigma} f_{\lambda' S'}{}^{\Sigma}(k')$$
$$\times Y_{\lambda}{}^{\mu*}(\hat{k}) Y_{\lambda'}{}^{\mu'}(\hat{k}')\langle\lambda\mu SS_z \mid \Sigma\sigma\rangle\langle\lambda'\mu' S'S_z' \mid \Sigma\sigma\rangle \tag{5}$$

from which the partial-wave amplitude may be extracted

$$t_{\lambda S,\lambda' S'}{}^{(\Sigma)}(k, k'; E)=\Gamma_{\lambda S}{}^{\Sigma} f_{\lambda S}{}^{\Sigma}(k)\tau_{\Sigma}(E)\Gamma_{\lambda' S'}{}^{\Sigma} f_{\lambda' S'}{}^{\Sigma}(k'). \tag{6}$$

This amplitude has the expected separability in initial and final momenta and is suitable for fitting two-body data as will be discussed further in Sec. III.

We now move on to the three-body sector. In the Amado formalism, the three-body equations have the form of multichannel Lippmann-Schwinger equations for particle-quasiparticle scattering. If we let c represent a partition of the initial three-particle state into a given particle-quasiparticle configuration as well as all other quantum numbers necessary to specify this state, then the transition amplitude between two such channels $T_{cc'}$ satisfies a set of coupled integral equations shown pictorially in Fig. 3. The "potential" for these equations is the single-particle exchange or Born amplitude which is quite complicated if spin-dependent vertices are present. The most general type of Born term required is pictured in Fig. 4 with all spin labels shown. Here we are considering a reaction between distinguishable particles of the form $(1+2)+3\rightarrow 1+(2+3)$ which proceeds in lowest order by exchanging particle 2. If $\mathbf{k}$ and $\mathbf{k}'$ are the initial and final center-of-mass momenta and E is the total energy, then the Born amplitude for this process may be calculated in second-order perturbation theory with the Hamiltonian given in Eq. (1). The result is

$$\langle\Sigma\sigma; s_3\sigma_3; \mathbf{k} \mid B_{cc'}(E) \mid \Sigma'\sigma'; s_1\sigma_1; \mathbf{k}'\rangle$$
$$=\sum\langle s_1\sigma_1 s_2\sigma_2 \mid \bar{s}\bar{\sigma}\rangle\langle\lambda\mu\bar{s}\bar{\sigma} \mid \Sigma\sigma\rangle\langle s_2\sigma_2 s_3\sigma_3 \mid \bar{s}'\bar{\sigma}'\rangle$$
$$\times\langle\lambda'\mu'\bar{s}'\bar{\sigma}' \mid \Sigma'\sigma'\rangle\Gamma_{\lambda\bar{s}}{}^{\Sigma} f_{\lambda\bar{s}}{}^{\Sigma}(q) D^{-1}(k, k'; E)$$
$$\Gamma_{\lambda'\bar{s}'}{}^{\Sigma'} f_{\lambda'\bar{s}'}{}^{\Sigma'}(q') Y_{\lambda}{}^{\mu}(\hat{q}) Y_{\lambda'}{}^{\mu'*}(\hat{q}'), \tag{7}$$

where

$$\mathbf{q}=\frac{m_2\mathbf{k}'-m_1(\mathbf{k}-\mathbf{k}')}{m_1+m_2}, \qquad \mathbf{q}'=\frac{-m_2\mathbf{k}-m_3(\mathbf{k}-\mathbf{k}')}{m_2+m_3},$$
$$D(k, k'; E)=E-k^2/2m_3-k'^2/2m_1-(\mathbf{k}-\mathbf{k}')^2/2m_2, \tag{8}$$

and the sum is over σ_2, $\bar{s}$, $\bar{\sigma}$, $\bar{s}'$, $\bar{\sigma}'$, λ, μ, λ', and μ'. As in the two-body case we want this amplitude in a channel spin representation which we obtain by adding spins in the initial and final states

$$\langle S, S_z; \mathbf{k} \mid B_{cc'}(E) \mid S', S_z'; \mathbf{k}'\rangle$$
$$=\sum_{\sigma\sigma'\sigma_1\sigma_3}\langle\Sigma\sigma s_3\sigma_3 \mid SS_z\rangle\langle\Sigma'\sigma' s_1\sigma_1 \mid S'S_z'\rangle$$
$$\times\langle\Sigma\sigma; s_3\sigma_3; \mathbf{k} \mid B_{cc'}(E) \mid \Sigma'\sigma'; s_1\sigma_1; \mathbf{k}'\rangle. \tag{9}$$

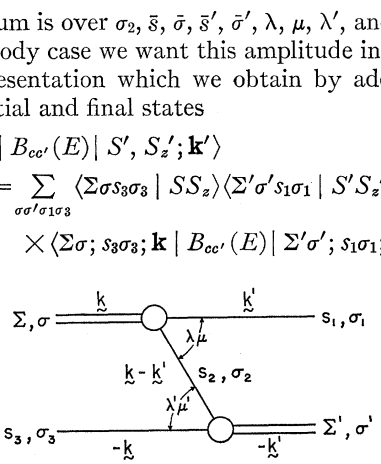

FIG. 4. Graph for three-body Born term involving the exchange of particle 2.

The channel spin amplitude can be decomposed into partial waves as follows:

$$\langle S, S_z; \mathbf{k} \mid B_{cc'}(E) \mid S', S_z'; \mathbf{k}' \rangle = \sum_{lml'm', JM} \langle lmSS_z \mid JM \rangle \langle l'm'S'S_z' \mid JM \rangle \times B_{clS,c'l'S'}{}^{(J)}(k, k'; E) Y_l{}^{m*}(\hat{k}) Y_{l'}{}^{m'}(\hat{k}'), \quad (10)$$

where l and l' are the angular momenta of the quasiparticle relative to the third particle in the initial and final state and J is the total angular momentum. In this partial-wave representation the Born term and also the T-matrix elements have no dependence on magnetic quantum numbers and the three-body integral equations have the form

$$T_{clS,c'l'S'}{}^{(J)}(k, k'; E) = B_{clS,c'l'S'}{}^{(J)}(k, k'; E) + \frac{1}{(2\pi)^3} \times \sum_{c''l''S''} \int_0^\infty n^2 dn T_{clS,c''l''S''}{}^{(J)}(k, n; E) \tau_{c''}(n; E) \times B_{c''l''S'',c'l'S'}{}^{(J)}(n, k'; E). \quad (11)$$

Here, $\tau_{c''}$ is the two-body propagator but now in the three-body space. This function is diagonal in all discrete quantum numbers and a scalar function of $\mathbf{n}$ and therefore requires no partial-wave analysis. In Eq. (11) all intermediate states $c''l''S''$ that conserve total angular momentum J and parity are summed. The calculation of the explicit form of the Born amplitudes is discussed in Appendix A. Although the above integral equation is one dimensional, the limiting factor in its numerical solution is the number of coupled channels involved, which in turn depends on the complexity of the two-body interactions. This equation is applied to Li^6 and deuteron-α scattering in the next section.

III. Li^6 AND d-α SCATTERING

A. Two-Body Subsystems

In this section we give the separable potential parameterization used in the calculation of the bound-state energies of Li^6, both $T=0$ and $T=1$ levels, and the corresponding $T=0$ continuum problem which is deuteron-α scattering. If we assume isospin conservation then the isospin of the three-body system is just that of the n-p pair and the resulting $T=0$ and $T=1$ three-body problems are uncoupled and may be treated separately.

For the isospin zero n-p interaction a complete set of calculations has been performed with an s-wave interaction using the Yamaguchi parameters[12] that fit the deuteron binding energy and the triplet scattering length. In addition to this a few calculations have been done in which the d state of the deuteron has been included in order to assess the importance of this interaction. The n-p parameters used for this purpose are those given by Phillips[13] and are arranged to fit the quadrupole moment and percent d state, in addition to the binding energy and scattering length. For the isospin one n-p system, a single attractive s-wave interaction has been used which fits the singlet scattering length and effective range. We speak of the n-p interaction as proceeding through the d if $T=0$ and through the ϕ if $T=1$.

TABLE I. Parameters for nucleon-α interaction. Units are $2m_N=\hbar^2=1$, ϵ_d (deuteron binding energy) $=\frac{1}{2}$.

λ	Σ	$\Gamma_\lambda{}^\Sigma$	$\beta_\lambda{}^\Sigma$
0	$\frac{1}{2}$	158.1	1.6185
1	$\frac{3}{2}$	500.0	2.4512
1	$\frac{1}{2}$	173.2	1.8364

Experimental studies of low-energy nucleon-α scattering have been quite extensive and the phase shifts in the low partial waves are known[14] and give a basis for fitting effective nucleon-α potentials to the scattering data. At energies below about 10 MeV, the scattering is dominated by s and p waves and the d-wave phase shifts are a few degrees in magnitude and may be safely neglected. Since we have spin-$\frac{1}{2}$–spin-0 scattering, there will generally be two values of J for each l and we must include interactions in the states $s_{1/2}$, $p_{3/2}$, and $p_{1/2}$. Rather than the usual procedure of introducing separate central and spin-orbit potentials, we choose to fit the phase shift in each of these partial waves directly with a separable interaction. In the quasiparticle language one introduces three quasiparticles $\beta_\lambda{}^\Sigma$ with the charge-independent coupling $\beta_\lambda{}^\Sigma \leftrightarrow N+\alpha$. We have three possibilities for (λ, Σ), namely, $(0, \frac{1}{2})$, $(1, \frac{3}{2})$, and $(1, \frac{1}{2})$, and N denotes a neutron or proton. Low-energy neutron-α scattering is dominated by a $p_{3/2}$ resonance at 1.3 MeV and by a broad $p_{1/2}$ level several MeV higher in energy whereas the experimental $s_{1/2}$ phase shift resembles one that would occur in potential scattering for a system with one bound state present. However, the N-α interaction in this partial wave is governed by the Pauli exclusion principle which forbids another nucleon in the closed $1s$ shell of the α particle. Other studies[15] of this problem have shown that the exclusion principle has the effect of introducing a repulsive barrier at small distances. We take a similar view and introduce a simple repulsive separable interaction in this partial wave in an attempt to simulate the exclusion principle. Alternatively, one could fit the $s_{1/2}$ phase shift quite well with an interaction attractive enough to produce one bound state. However, such an interaction would have the unacceptable features of producing too many bound states in Li^6 as well as a spurious stripping

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[14] See, for example, B. Hoop, Jr., and H. H. Barschall, Nucl. Phys. **83**, 65 (1966).

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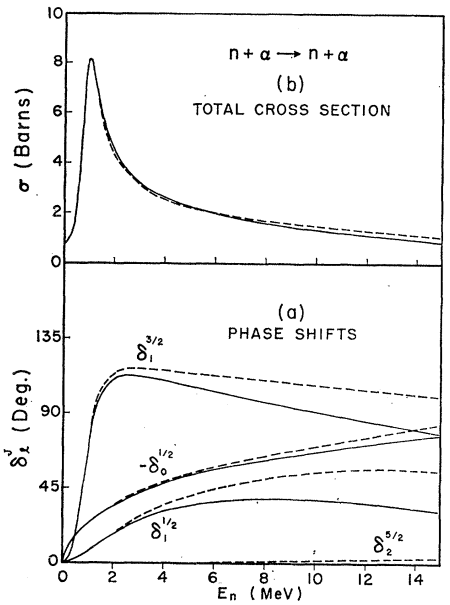

FIG. 5. Comparison of separable potential results (solid curve) and experimental neutron-α scattering (broken curve) for (a) elastic phase shifts and (b) total cross section. The experimental $d_{5/2}$ phase shift is from Ref. 14. All other experimental curves represent the data analysis of Ref. 1b.

channel in d-α scattering. We have chosen the two-body N-α separable form factors to have the functional form

$$f_\lambda^\Sigma(k)=k^\lambda/[k^2+(\beta_\lambda^\Sigma)^2]^{\lambda+1}. \tag{12}$$

We have suppressed spin indices since here the total spin may only be $\frac{1}{2}$. In each partial wave we have two parameters to fix, the coupling constant Γ_λ^Σ and the range parameter β_λ^Σ. On the energy shell, the two-body amplitudes are related to the phase shifts by ($2m_N=1$, $\hbar^2=1$)

$$\begin{aligned} t_{\lambda 1/2}^{(\Sigma)}(k)&=[\Gamma_\lambda^\Sigma f_\lambda^\Sigma(k)]^2\tau_\Sigma(E)\\ &=-[16\pi^2(1+\alpha)/k\alpha]\exp(i\delta_\lambda^\Sigma)\sin\delta_\lambda^\Sigma,\\ E&=[(1+\alpha)/\alpha]k^2, \qquad \alpha=m_\alpha/m_N. \end{aligned} \tag{13}$$

The propagator $\tau_\Sigma(E)$ has the form

$$\tau_\Sigma(E)=\pm\left(1\mp\frac{1}{(2\pi)^3}\int_0^\infty\frac{n^2dn[\Gamma_\lambda^\Sigma f_\lambda^\Sigma(n)]^2}{E-[(1+\alpha)/\alpha]n^2+i\eta}\right)^{-1}. \tag{14}$$

The upper and lower signs are chosen for repulsive and attractive interactions, respectively. The basis used for fitting the experimental data was an effective range analysis of low-energy n-α scattering carried out by Pearce and Swan.[16] The resulting potential parameters are given in Table I and the quality of the resultant fits may be seen in Fig. 5(a) where we compare the separable results with the Pearce and Swan phase shifts which represent the experimental data. In the p waves it is not possible, with the form factors given in Eq. (12), to fit the resonance position and width and still maintain the phase shift at the experimental values at higher energy. In this circumstance, fitting the lower energies was given priority. Also, the $s_{1/2}$ phase shift resulting from the repulsive potential returns to zero at higher energies while the experimental one seems to approach $-\pi$ in agreement with a modified Levinson's theorem given by Swan.[17] These shortcomings of the separable fits do not have much effect on the total cross section which is shown in Fig. 5(b). Here it is apparent that the $p_{3/2}$ resonance completely dominates the low-energy scattering and that the discrepancies in the phase-shift fitting are of little consequence in the total cross section.

B. Three-Body Equations

Having fixed all of the two-body interactions we are now in a position to solve the coupled three-body equations given in Eq. (11). We first indicate the extent of the channel couplings involved. We must evaluate all partial-wave T-matrix elements $T_{clS,c'l'S'}{}^{(J)}$, where c indicates the partition of the initial state into a quasiparticle-particle pair and l and S denote the relative orbital angular momentum and channel spin of that pair, and the primed indices refer to the final state. We let $c=1$ denote the state $d\alpha$ or $\phi\alpha$ for $T=0$ and $T=1$, respectively, and let $c=2$, 3, and 4 stand for the three states $N\beta_\lambda^\Sigma$ involving the quasiparticles β_λ^Σ coupled to nucleon plus α. To study d-α scattering we set $c=1$ for given l and S and we must include all final states $c'l'S'$ that conserve total angular momentum J and parity, as well as sum over all possible intermediate states $c''l''S''$. Table II lists the resulting coupled channels in some low partial waves for the $T=0$ ($d\alpha$) and $T=1$ ($\phi\alpha$) cases. For large J there are nine amplitudes if $l=J$ and 10 if $l=J\pm1$ in the d-α case and there are generally nine amplitudes in the $\phi\alpha$ system. In d-α scattering, only those amplitudes that couple $d\alpha$ to $d\alpha$ ($c=1$ and $c'=1$) are physical amplitudes. The amplitudes for $d+\alpha\rightarrow N+\beta$ ($c=1$, $c'=2$, 3, 4) are included only for the purpose of introducing $N\beta$ intermediate states and do not represent physical amplitudes for $d+\alpha\rightarrow N+\beta$, since β is unstable. To study such processes one would have to calculate the full breakup amplitude for $d+\alpha\rightarrow n+p+\alpha$.

TABLE II. Possible values of clS coupled to a total angular momentum and parity J^π for some low partial waves.

	$c=1$		$c=2$	$c=3$	$c=4$
J^π	$d\alpha(T=0)$	$\phi\alpha$ $(T=1)$	$N\beta_0{}^{1/2}$	$N\beta_1{}^{3/2}$	$N\beta_1{}^{1/2}$
0^+		00	00	11	11
1^+	01, 21		01, 21	11, 12, 32	10, 11
2^+	21	20	20, 21	11, 12, 31, 32	11, 31
3^+	21, 41		21, 41	12, 31, 32, 52	30, 31
0^-	11		11	22	00
1^-	11	10	10, 11	01, 21, 22	01, 21
2^-	11, 31		11, 31	02, 21, 22, 42	20, 21

[16] W. A. Pearce and P. Swan, Nucl. Phys. **78**, 433 (1966).

[17] P. Swan, Proc. Roy. Soc. (London) **A228**, 10 (1955).

In the $T=1$ case, the quasiparticle ϕ is also unstable, but here we are only interested in the Fredholm determinant of the equation and not in any scattering amplitude. A zero of this determinant below the three-body threshold would indicate a $T=1$ bound state in Li^6.

In the present formalism one can see from Eq. (11) that the Born function $B_{c''l''S'',c'l'S'}$ plays the role of a multichannel potential which couples the three-body partial wave channels to one another. In the most complicated case considered above, the Born function is a 10×10 matrix in the discrete indices and since this function involves the coupling of many angular momenta, its evaluation places an upper limit on the feasibility of a given three-body calculation. For the present problem there are two classes of Born terms to consider: one for the stripping process (d or ϕ)$+\alpha\rightarrow N+\beta$ and one representing the α exchange reaction $p+\beta\rightarrow n+\beta$. The evaluation of these amplitudes is discussed in Appendix A.

C. Results

We will first discuss the $T=0$ bound state calculations. These have been performed with n-p forces containing 0, 4, and 7 percent d-state probability. In all calculations the N-α interaction is that given in Table I. For the three n-p interactions we find only one three-body $T=0$ bound state with total angular momentum $J=1^+$ which corresponds to the ground state of Li^6. The resulting binding energies are given in Table III.[18] As is usual in such systems, the inclusion of the d state decreases the binding energy although it is a small effect.[19] We should also make allowance for the Coulomb energy of the ground state which is about 1.0 MeV.[5] Since Coulomb effects have been neglected we should expect too much binding by 1.0 MeV. Instead we find too little binding by $\sim$0.3 MeV. In trying to assess the origin of this discrepancy one would expect that the most uncertain feature of the two-body interactions employed is the repulsive s-wave N-α interaction. To what extent the exclusion principle can be simulated by a repulsive interaction of the form we have chosen is uncertain but the sensitivity of the ground-state binding energy may be seen by noting that a fit to the experimental Li^6 energy, including the Coulomb correction, requires a reduction of the s-wave N-α coupling constant by a factor of 3. Thus the binding energy is not very sensitive to the strength of the s-wave N-α interaction and the ground-state binding comes mainly from the p-wave attraction.

TABLE III. Binding energy of Li for three n-p interactions.

% d state	Binding energy (MeV)
0	3.350
4	3.253
7	3.164
Experimental	3.697

[18] The experimental levels are from T. Lauritsen and F. Ajzenberg-Selove, Nucl. Phys. **78**, 1 (1966).

[19] With a simpler N-α interaction Alessandrini *et al.* (Ref. 7) find that the tensor force increases the Li^6 ground-state binding energy.

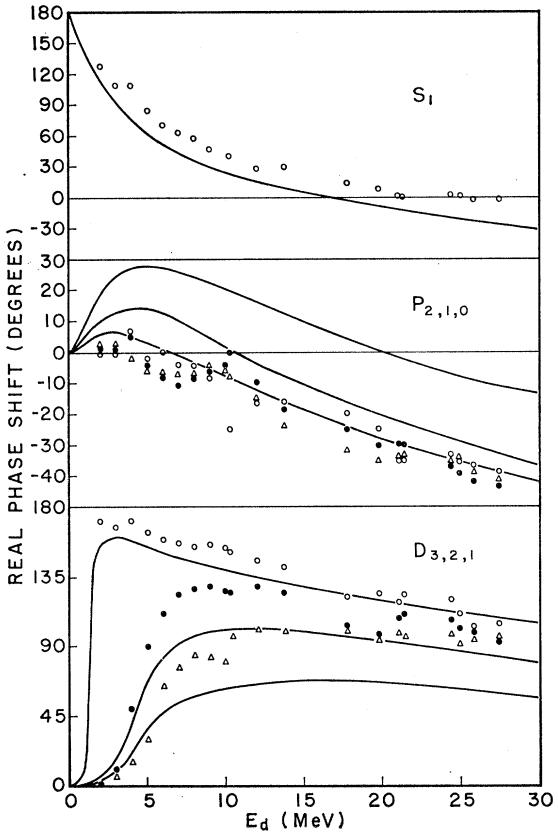

FIG. 6. Theoretical (solid line) and experimental d-α real phase shifts for allowed values of L_J versus energy. For each L, the larger theoretical phase shifts have the larger J. The experimental points are from Ref. 23 below 10 MeV and from Ref. 25 above. The open circle, closed circle, and triangle represent $J=l+1$, l, and $l-1$, respectively.

For the case of $T=1$, two levels are found with the interactions specified above. A $J=0^+$ bound state is obtained at an energy of 3.59 MeV relative to the ground state. In addition to this, a continuum state is found above the three-particle threshold with the quantum numbers $J=2^+$ at an energy of 4.97 MeV. At this energy the real part of the Fredholm determinant vanishes, indicating the presence of a three-particle resonance. Since it is not feasible to search for the position of the pole of the amplitude in the energy plane, no estimate of the width of this state has been obtained. As a check on the interpretation of this zero as a three-particle resonance, it was observed that the level moves down in energy and becomes a bound state if the N-α interaction is artificially made more attractive. In the other $T=1$ partial waves there is little attraction and no other levels are present up to 10-MeV excitation.

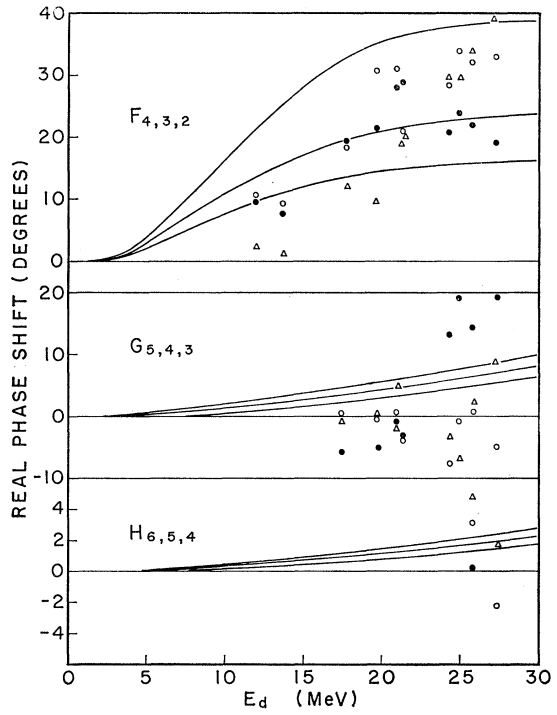

FIG. 7. Theoretical and experimental real d-α phase shifts. See caption to Fig. 6.

Experimentally, $T=1$ levels are seen at 3.562 MeV (0^+) and 5.36 MeV (2^+).[20] The theoretical results are in good agreement with the position of these levels.

The remaining $T=0$ levels of Li^6 are continuum states above the d-α elastic threshold and are studied by extracting the elastic phase shifts from the scattering amplitude. The scattering calculations to be described were performed with a purely s-wave n-p interaction. Some discussion of the effect of the deuteron d state is given in Sec. IV. Above the elastic threshold the integral equations were solved by either matrix inversion or Neumann iteration using the contour rotation method.[21] In some partial waves there are as may as ten coupled equations and computer limitations of both space and time restrict one to the use of a mesh of about 24 points per equation. For complete stability of results against mesh variation one would want a larger number of points but judging from small fluctuations in results, it is estimated that errors in the final phase shifts are less than 10%. The scattering equations were solved at 12 energies between 0- and 30-MeV lab deuteron energy for partial waves through $l=5$. In this energy range, phase shifts extracted from cross section and polarization data have been obtained by several groups.[22–25] The real part of the theoretical phase shifts are compared with the experimental ones of McIntyre and Haeberli[23] from 2–10 MeV and with those of Darriulat *et al.*[25] from 10.3 to 27 MeV in Figs. 6 and 7. The theoretical inelasticities $\eta_l{}^J = \exp(-2\,\mathrm{Im}\delta_l{}^J)$ are given in Fig. 8. Although some of the assumptions of the model are suspect at the higher energies, for example, the $\alpha(d, T)\mathrm{He}^3$ threshold is at 21.5 MeV, it was decided to extend the calculations to this region because the phase-shift analysis of Darriulat *et al.*[25] indicated a sizeable reaction cross section in the higher partial waves, notably in $l=3$ and 4. The calculation reproduces the approximate magnitude of this inelasticity and shows that long-range breakup of the deuteron is significant in these partial waves. At lower energies there is general if not precise agreement between the calculated real phase shifts and the experimental ones. There is a tendency for the odd phase shifts to be too attractive while the even ones are not attractive enough. This effect is probably a result of not treating antisymmetrization correctly and is most pronounced in the P-wave phase shifts which are more positive than the experiments would warrant. In the model, the sign change in these phase shifts arises from the importance of both the repulsive s-wave and the attractive p-wave two-body N-α interactions. At low energy, the attraction dominates but at higher energies

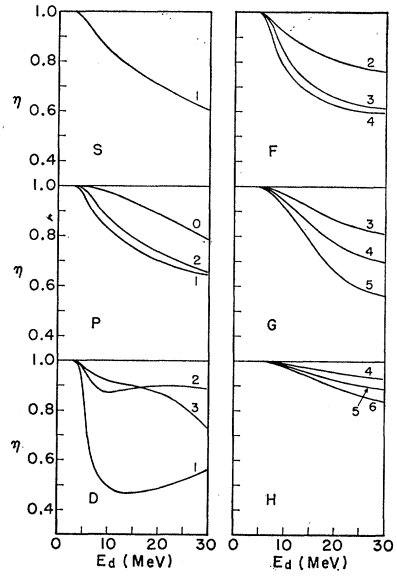

FIG. 8. Theoretical d-α absorption parameters versus deuteron energy through H waves. The value of J for each curve is indicated.

[20] The 5.36-MeV level is most likely $T=1$. See I. Linck, R. Bilwes, L. Kraus, R. Seltz, and D. Magnac-Valette, J. Phys. **30**, 17 (1969).

[21] J. H. Hetherington and L. H. Schick, Phys. Rev. **137**, B935 (1965).

[22] H. Meiner, E. Baumgartner, S. E. Darden, P. Huber, and G. R. Plattner, Helv. Phys. Acta **40**, 483 (1967).

[23] L. C. McIntyre and W. Haeberli, Nucl. Phys. **A91**, 369 (1967); **A91**, 382 (1967).

[24] L. S. Senhouse and T. A. Tombrello, Nucl. Phys. **57**, 624 (1964).

[25] P. Darriulat, D. Garreta, A. Tarrats, and J. Arvieux, Nucl. Phys. **A94**, 653 (1967).

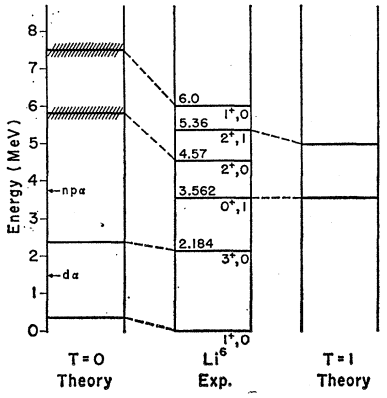

FIG. 9. Comparison of the energy levels of Li^6 (Ref. 18) with those of the three-body calculation. No Coulomb corrections have been applied to the theoretical levels.

the shorter-range repulsion changes the sign of the phase shifts. It should also be noted that there are no P-wave resonances in the energy region studied as had been suggested in an earlier phase-shift analysis.[24] There now seems to be conclusive experimental[23] and theoretical[8] evidence that these levels do not exist.

From Fig. 6 one can see that the triplet of resonances in the partial waves $^3D_3$, $^3D_2$, and $^3D_1$ is fairly well reproduced by the calculation. Compared to experiment, the $^3D_2$ and $^3D_1$ resonances occur at too high an energy and are quite broad and asymmetric. In contrast, the

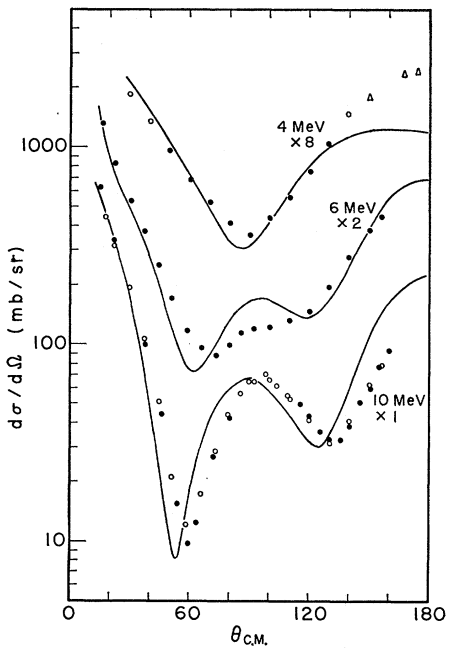

FIG. 10. Experimental and theoretical d-α angular distribution versus angle. At 4 MeV the data are from the following sources: Ref. 24 (dots), G. G. Ohlsen and P. G. Young, Nucl. Phys. **52**, 134 (1964) (circles) and Ref. 26 (triangles); at 6 MeV from Ref. 24; and at 10 MeV from L. Stewart, J. E. Brolley, Jr., and L. Rosen, Phys. Rev. **128**, 707 (1962) (dots) and G. G. Ohlsen and P. G. Young, Nucl. Phys. **52**, 134 (1964) (circles).

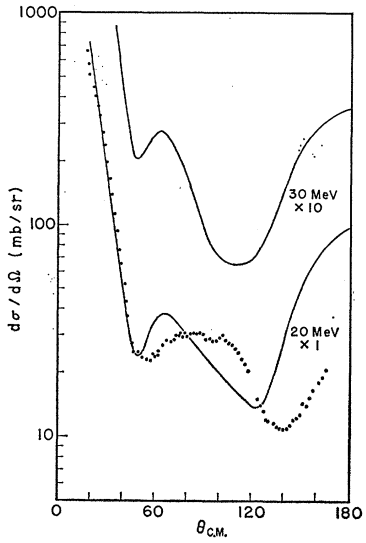

FIG. 11. Theoretical d-α angular distributions versus angle at 20 and 30 MeV. The experimental points are the 21-MeV data of Ref. 28.

lower $^3D_3$ resonance has a position of 1.31 MeV and a width of 0.240 MeV in closer accord with the experimental values[26] of 1.07 and 0.035 MeV. In comparing the theoretical and experimental widths it is more appropriate to do so after Coulomb and centrifugal barrier effects have been removed or in the language of R-matrix theory one should compare reduced widths rather than "experimental" widths. If account is taken of these effects and an adjustment is made for the shift in the position of the theoretical level (a broadening effect), one obtains a theoretical reduced width of about

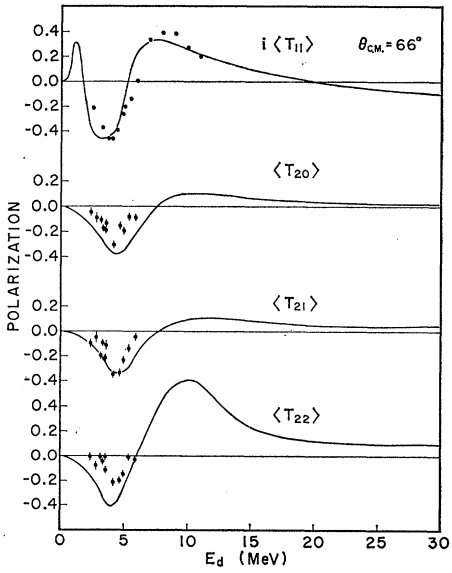

FIG. 12. Theoretical and experimental first- and second-rank deuteron tensor polarization versus energy at a center-of-mass angle of 66°. For references to experimental work, see Ref. 30.

[26] A. Galonsky, R. A. Douglas, W. Haeberli, M. T. McEllistrem, and H. T. Richards, Phys. Rev. **98**, 586 (1955).

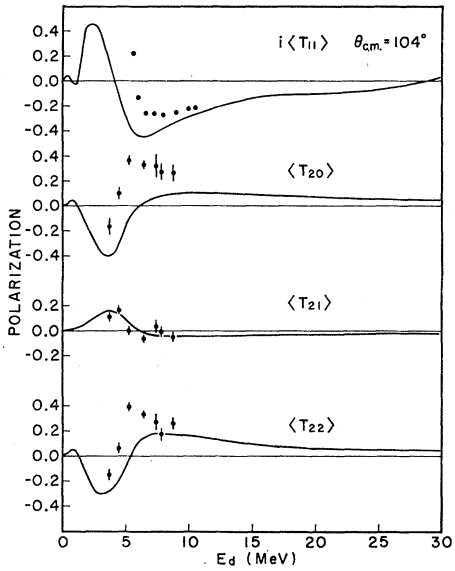

FIG. 13. Theoretical and experimental first- and second-rank deuteron tensor polarization versus energy at a center-of-mass angle of 104°. See Ref. 30.

twice the experimental value[22] of 0.551 MeV. In view of the uncertainties of the R-matrix analysis, this may be considered reasonable agreement. A diagram is shown in Fig. 9 comparing all the theoretical and experimental $T=0$ and $T=1$ low-lying levels of Li^6.

The theoretical results may also be judged by comparing them with experimental angular distributions and polarization. In order to do so Coulomb effects have been included in the usual procedure[27] for combining nuclear and Coulomb amplitudes. Figures 10 and 11 give a comparison of differential cross sections. At the lower energies the agreement is good but there are some deviations at 20 MeV,[28] particularly at large angles. The theoretical results for the first- and second-rank deuteron polarization[29] are compared with experiment[30] as a function of energy in Figs. 12 and 13 for center-of-mass angles of 66° and 104°. As for the differential cross section, the agreement tends to be better at smaller scattering angles. The total deuteron breakup cross section is also available theoretically and is shown in Fig. 14 as a function of energy. The agreement with the available experimental data[31,32] is satisfactory.

Considering the approximations inherent in the model, the agreement with many diverse experimental results is quite satisfying and tends to confirm the experimental phase shift results which are not without uncertainty, particularly at higher energy.

A point of some interest in spin zero-spin one scattering is the amount of l nonconservation present in partial waves with $l=J\pm1$. In our model of d-α scattering there is considerable coupling between the partial waves $^3S_1$ and $^3D_1$ and above the breakup threshold there is inelasticity as well and one would like a general and convenient way of parametrizing the S matrix under these conditions. Arndt[33] has recently given a six-parameter representation which is general, consistent with unitarity, and reduces to the Stapp nuclear-bar phase shifts[34] in the absence of inelasticity. In d-α scattering for $J=1^+$, the S-matrix elements $S_{ll'}{}^{(J)}$ are related to the Arndt parameters by

$$S_{00}{}^{(1)}=\cos\rho_-\cos2\epsilon\exp(2i\delta_-),$$
$$S_{22}{}^{(1)}=\cos\rho_+\cos2\epsilon\exp(2i\delta_+),$$
$$S_{02}{}^{(1)}=i\sin2\epsilon\exp[i(\delta_-+\delta_++\alpha)].$$

For a nonabsorptive situation ρ_-, ρ_+, and α vanish and we are left with the Stapp parameters δ_-, δ_+, and ϵ. In our normalization the connection between the S- and T-matrix elements is

$$S_{ll'}{}^{(J)}=\delta_{ll'}-[ik\alpha/4\pi^2(2+\alpha)]T_{ll'}{}^{(J)}.$$

For the coupled state $^3S_1$-$^3D_1$ we have evaluated these eigenphase parameters and they are plotted versus energy in Fig. 15. We find that the mixing parameter ϵ is quite small and the eigenphases do not exhibit the repulsive behavior[35] noted by McIntyre and Haeberli.[23] The parameters representing the inelasticity are quite

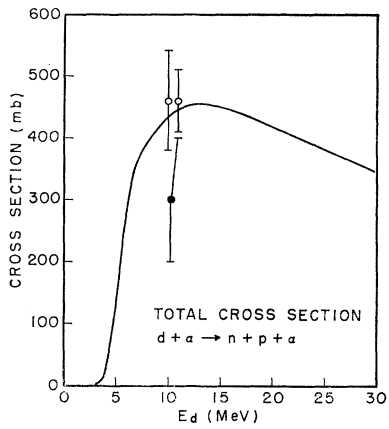

FIG. 14. Theoretical and experimental reaction cross sections versus energy. The open circles are from Ref. 31 and the closed circle is from Ref. 32.

[27] P. E. Hodgson, Advan. Phys. **15**, 329 (1966).
[28] The experimental points are the 21.0-MeV data of H. W. Broek and J. L. Yntema, Phys. Rev. **135**, B678 (1964).
[29] The notation is that of W. Lakin, Phys. Rev. **98**, 139 (1955).
[30] The $\langle iT_{11}\rangle$ data are from A. Trier and W. Haeberli, Phys. Rev. Letters **18**, 915 (1967); the remaining data are from Ref. 23.
[31] G. G. Ohlsen and P. G. Young, Phys. Rev. **136**, B1632 (1964).
[32] J. C. Allred, D. K. Froman, A. M. Hudson, and L. Rosen, Phys. Rev. **82**, 786 (1951).
[33] R. A. Arndt, Rev. Mod. Phys. **39**, 710 (1967).
[34] H. P. Stapp, T. J. Ypsilantis, and N. Metropolis, Phys. Rev. **105**, 302 (1957).
[35] In a simple two-channel system, the eigenvalues of the S matrix, or equivalently, the eigenphases (mod π), cannot be equal in the presence of coupling between channels. See C. J. Goebel and K. W. McVoy, Phys. Rev. **164**, 1932 (1967). If inelasticity is also present, such as in the d-α model above the breakup threshold, the eigenvalues of S no longer have unit modulus and thus the equality of the eigenphases does not imply the equality of the eigenvalues. In the d-α model the eigenphases cross at 6 MeV although the eigenvalues of S are quite different there.

sizeable, particularly ρ_{+-} which reflects the large absorption present in $^3D_1$. In the higher partial waves there is negligible l mixing in the energy region studied and in all cases the mixing parameters are less than 1° below 30 MeV.

IV. DISCUSSION

The results presented in Sec. III show that it is possible to predict many diverse bound-state and scattering properties of the six-nucleon system when viewed in a three-body approximation. For the particular system we are considering, this is quite plausible. In d-α scattering one is dealing with the interaction of a rather diffuse and loosely bound deuteron with an α particle that is more compact and not subject to low-energy excitation. Also, in the bound-state problem, the rms charge radius[36] of Li^6 (2.72 F) is considerably larger than that of the α particle (1.61 F), indicating that the loosely bound valence nucleons are responsible for the large increased spacial extension of Li^6. Thus, given the essential three-particle character of the system, a theoretical model that takes proper account of two-body bound states and low-energy scattering resonances and also treats three-particle states correctly seems to provide a consistent over-all picture of the problem. Of course there is a limit to the extent that one can treat the α particle as an elementary entity and it is of some interest to compare our results with models in which this assumption is unnecessary, such as the resonating-group approach of Thompson and Tang.[8] In their work, antisymmetry of the six nucleons is properly taken into account but deuteron breakup and certain spin-dependent interactions are neglected. It is interesting that those aspects of the respective calculations that overlap give quite similar results and that neither the phenomenological treatment of the Pauli principle used in the present work, nor the neglect of breakup in the resonating-group method very seriously affects the major results of either calculation.

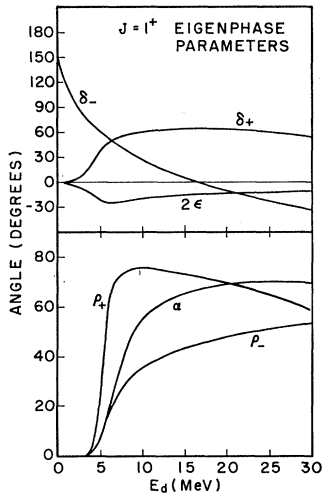

FIG. 15. Calculated d-α eigenphase parameters for $J=1^+$. The phase parameters are defined in the text.

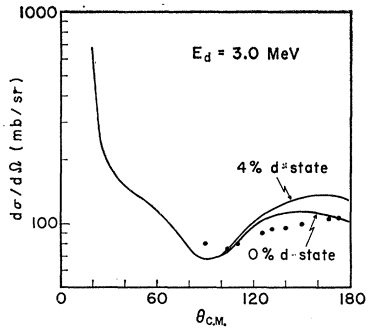

FIG. 16. Comparison of d-α angular distributions at 3 MeV for 0 and 4% deuteron d state. The experimental points are from Ref. 26.

Extensive studies of deuteron-nucleus scattering have been performed in terms of two-body optical potentials and a large body of knowledge exists[37] concerning the shape of the real and imaginary wells as well as more recently acquired information[3] concerning possible spin-dependent terms. It would be useful to extract from the present three-body calculation an equivalent local two-body optical potential and to study its relation to the standard forms. Although this is possible in principle it would be very complicated to carry out and has not been attempted. As a substitute, one may compare some features of the two approaches such as the behavior of absorption and the origin of the spin dependence in d-α scattering in terms of that of the underlying two-body interactions.

As pointed out in Sec. III, the theoretical absorption parameters η_l^J indicate a sizeable reaction cross section in the high partial waves. This points to a long-range component in the breakup process that would support the prevalent view that the large spacial extension of the imaginary optical potential has its origins in deuteron breakup. The η_l^J's also have the characteristic nonmonatomic dependence on l found in the phenomenological studies. It may be seen in Fig. 8 that there is a tendency for more absorption in even waves than odd.

One further point of interest is the origin of the spin dependence in d-α scattering. The scattering results presented in Sec. III arise from a spin-dependent N-α interaction coupled with an s-wave n-p interaction. Thus the d-α spin-dependent effects, such as polarization, arise solely from the N-α spin dependence. One might expect that the next most important spin-dependent effects would stem from the n-p tensor force. To test this idea a few scattering calculations were performed with a separable form of this interaction included (see Appendix B). Figure 16 shows a 3-MeV d-α angular distribution for 0 and 4% d state compared to experiment. The inclusion of the d state produces a small

[36] L. R. B. Elton, *Nuclear Sizes* (Oxford University Press, London, 1961), p. 26.

[37] For a review, see Ref. 27.

change in the angular distribution only at large angles and is in the wrong direction with respect to the experimental points. The effect of the tensor force is to reduce the phase shifts a few percent but to introduce no interesting structure in either the d-α cross sections or polarization. Thus in the present three-body model the spin-dependent effects in deuteron-nucleus scattering arise mainly from the spin dependence of the nucleon-core interaction, or in the usual language from the nucleon-core spin-orbit potential.

The problem of deuteron breakup in d-α collisions is one that could be studied by an extension[38] of the methods outlined here. Interest in this problem is centered on the effect of the final-state N-α resonant interactions on the breakup angular distributions. It is hoped that a unitary three-body model of the breakup process could shed some light on the reaction mechanism by allowing a theoretical separation between direct deuteron breakup, and breakup that proceeds by sequential decay of He^5 and Li^5.

ACKNOWLEDGMENTS

Part of this work was done at the University of California at Davis. I would like to thank the members of the Physics Department for their hospitality. Most of the calculations described here were performed on a CDC 6600 computer at the Lawrence Radiation Laboratory at Berkeley. I wish to thank the Laboratory for making these facilities available. I am also grateful to D. Garreta for supplying a table of phase shifts, and to R. Aaron, J. V. Corbett, L. R. Dodd, G. L. Strobel, and M. T. Vaughn for helpful discussions.

APPENDIX A: BORN AMPLITUDES

In order to solve the three-body integral equations of Sec. II we must calculate the partial-wave projection of the single-particle exchange amplitudes. As an example of the general procedure, the α-particle exchange amplitude for the reaction $p(\mathbf{k})+\beta(-\mathbf{k})\rightarrow n(\mathbf{k}')+\beta(-\mathbf{k}')$ will be calculated. We start by inverting Eq. (10) for the partial-wave amplitude

$$B_{clS,c'l'S'}{}^{(J)}(k,k';E)=(2J+1)^{-1}\sum_{mm'Sz Sz'}\langle lmSS_z\mid JM\rangle \times\langle l'm'S'S_z'\mid JM\rangle \times\int d\Omega_{\mathbf{k}}\int d\Omega_{\mathbf{k}'}Y_l{}^m(\hat{k})\,Y_{l'}{}^{m'*}(\hat{k}') \times\langle S,S_z;\mathbf{k}\mid B_{cc'}(E)\mid S',S_z';\mathbf{k}'\rangle. \quad \text{(A1)}$$

The channel spin amplitude on the right-hand side of (A1) is in turn related to an amplitude in which the spins of the individual particles are specified,

$$\langle S,S_z;\mathbf{k}\mid B_{cc'}(E)\mid S',S_z';\mathbf{k}'\rangle=\sum_{\sigma\sigma'\sigma_n\sigma_p}\langle\Sigma\sigma\tfrac{1}{2}\sigma_p\mid SS_z\rangle \times\langle\Sigma'\sigma'\tfrac{1}{2}\sigma_n\mid S'S_z'\rangle\langle\Sigma\sigma;\tfrac{1}{2}\sigma_p;\mathbf{k}\mid B_{cc'})E)\mid\Sigma'\sigma';\tfrac{1}{2}\sigma_n;\mathbf{k}'\rangle. \quad \text{(A2)}$$

It is this amplitude in the individual spin representation which is most naturally calculated from the interaction Hamiltonian given in Eq. (1). For the process we are considering the result is

$$\langle\Sigma\sigma;\tfrac{1}{2}\sigma_p;\mathbf{k}\mid B_{cc'}(E)\mid\Sigma'\sigma';\tfrac{1}{2}\sigma_n;\mathbf{k}'\rangle = \frac{\Gamma_\lambda{}^\Sigma f_\lambda{}^\Sigma(q)\,\Gamma_{\lambda'}{}^{\Sigma'}f_{\lambda'}{}^{\Sigma'}(q')}{E-k^2-k'^2-(\mathbf{k}+\mathbf{k}')^2/\alpha}\sum_{\mu\mu'}\langle\lambda\mu\tfrac{1}{2}\sigma_n\mid\Sigma\sigma\rangle \times\langle\lambda'\mu'\tfrac{1}{2}\sigma_p\mid\Sigma'\sigma'\rangle\times Y_\lambda{}^\mu(\hat{q})\,Y_{\lambda'}{}^{\mu'*}(\hat{q}'), \quad \text{(A3)}$$

where α is the α-particle-to-nucleon-mass ratio and

$$\mathbf{q}=\mathbf{k}'+\mathbf{k}/(1+\alpha), \qquad \mathbf{q}'=\mathbf{k}+\mathbf{k}'/(1+\alpha). \quad \text{(A4)}$$

Since these are three quasiparticles coupled to $N+\alpha$, Eq. (A3) represents six separate channel amplitudes for c or $c'=2$, 3, 4. The form of the vertex functions $f_\lambda{}^\Sigma$ has been given in Eq. (12).

To evaluate the partial-wave Born term we must insert (A3) into the right-hand side of (A2) and in turn insert (A2) into the right-hand side of (A1). The resulting expression is quite complex and must be reduced to a form suitable for numerical evaluation. Initially we have a product of four spherical harmonics but if we take $\hat{k}$ along the z axis, then we can write the spherical harmonics of argument q and q' in Eq. (A3) as the sums[39]

$$Y_\lambda{}^\mu(\hat{q})=\left[\frac{k}{q(1+\alpha)}\right]^\lambda\sum_{\nu=0}^{\lambda}(2\lambda-2\nu+1)^{1/2}\hat{\nu}^{-1}\binom{2\lambda+1}{2\nu}^{1/2} \times\left(\frac{k}{1+\alpha}\right)^{-\nu}(k')^\nu\langle\lambda-\nu 0\nu\mu\mid\lambda\mu\rangle Y_\nu{}^\mu(\hat{k}'), \quad \text{(A5)}$$

$$Y_{\lambda'}{}^{\mu'*}(\hat{q}')=\left(\frac{k}{q'}\right)^{\lambda'}\sum_{\nu'=0}^{\lambda'}(2\lambda'-2\nu'+1)^{1/2}(\hat{\nu}')^{-1}(k)^{-\nu'} \times\left(\frac{k'}{1+\alpha}\right)^{\nu'}\binom{2\lambda'+1}{2\nu'}^{1/2}\langle\lambda'-\nu'0\nu'\mu'\mid\lambda'\mu'\rangle Y_{\nu'}{}^{\mu'*}(\hat{k}'), \quad \text{(A6)}$$

where we have written $\hat{a}=(2a+1)^{1/2}$. We are then left with sums of a product of three spherical harmonics of argument $\hat{k}'$. This product can now be combined to a

[38] R. Aaron and R. Amado, Phys. Rev. **150**, 857 (1966); J. H. Hetherington and L. H. Schick, *ibid.* **156**, 1647 (1967).

[39] N. Austern, R. M. Drisko, E. C. Halbert, and G. R. Satchler, Phys. Rev. **133**, B3 (1964).

single sum,[40]

$$Y_{l'}^{m'*}(\hat{k}')\,Y_{\nu}^{\mu}(\hat{k}')\,Y_{\nu'}^{\mu'*}(\hat{k}')=(4\pi)^{-1}\hat{\nu}\hat{\nu}'\hat{l}'\times\sum_{LM,\mathfrak{L}\mathfrak{M}}\hat{L}^2(2\mathfrak{L}+1)^{1/2}\begin{pmatrix}\nu' & l' & L\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\nu & L & \mathfrak{L}\\ 0 & 0 & 0\end{pmatrix}\times\begin{pmatrix}\nu' & l' & L\\ \mu' & m' & M\end{pmatrix}\begin{pmatrix}\nu & L & \mathfrak{L}\\ \mu & M & \mathfrak{M}\end{pmatrix}Y_{\mathfrak{L}}^{\mathfrak{M}*}(\hat{k}'). \quad \text{(A7)}$$

The objects with parenthesis are 3-j symbols and the sums are restricted by $|l'-\nu'|\leq L\leq l'+\nu'$ and $|L-\nu|\leq\mathfrak{L}\leq L+\nu$. The remaining task is to perform the 3-j and Clebsch-Gordan sums.[41] The final expression is

$$B_{clS,c'l'S'}^{(J)}(k,k';E)=\hat{l}\hat{l}'\hat{\Sigma}\hat{\Sigma}'\hat{S}\hat{S}'\hat{\lambda}\hat{\lambda}'(-1)^{J+\lambda-\Sigma-\Sigma'}\times\sum_{bd\nu\nu',L\mathfrak{L}}(-1)^{\nu+\nu'+L}\begin{pmatrix}2\lambda+1\\ 2\nu\end{pmatrix}^{1/2}\begin{pmatrix}2\lambda'+1\\ 2\nu'\end{pmatrix}^{1/2}(1+\alpha)^{\nu-\nu'}\times(k'/k)^{\nu+\nu'}\hat{b}^2\hat{d}^2\hat{L}(2\mathfrak{L}+1)^{1/2}\langle\nu'0l'0\,|\,L0\rangle\langle\nu 0L0\,|\,\mathfrak{L}0\rangle\times\langle l0d0\,|\,\lambda'-\nu'0\rangle\langle\mathfrak{L}0d0\,|\,\lambda-\nu 0\rangle W(SS'll';bJ)\times W(\lambda-\nu\nu dL;\lambda\mathfrak{L})\begin{Bmatrix}S' & b & S\\ \frac{1}{2} & \lambda & \Sigma\\ \Sigma' & \lambda' & \frac{1}{2}\end{Bmatrix}\begin{Bmatrix}L & d & \lambda\\ \nu' & \lambda'-\nu' & \lambda'\\ l' & l & b\end{Bmatrix}\times\beta_{\mathfrak{L}}(k,k';E). \quad \text{(A8)}$$

In the sum over b and d, any integer satisfying the triangular conditions of the 9-j symbols is included. The quantity $\beta_{\mathfrak{L}}$ contains the dynamics and may be written as the partial-wave integral

$$\beta_{\mathfrak{L}}(k,k';E)=\frac{1}{2}\int_{-1}^{1}dx P_{\mathfrak{L}}(x)\left[\frac{k}{(1+\alpha)q}\right]^{\lambda}\left(\frac{k}{q'}\right)^{\lambda'}\times\frac{\Gamma_{\lambda}^{\Sigma}f_{\lambda}^{\Sigma}(q)\,\Gamma_{\lambda'}^{\Sigma'}f_{\lambda'}^{\Sigma'}(q')}{E-k^2-k'^2-(\mathbf{k}+\mathbf{k}')^2/\alpha}, \quad \text{(A9)}$$

where $x=\hat{k}\cdot\hat{k}'$. For the functional forms of the f's we have chosen [Eq. (12)], the above integral may be done analytically and the result expressed in terms of sums of Legendre functions of the second kind and their derivatives.

We have outlined the procedure used for calculating the Born term for $p+\beta\rightarrow n+\beta$. Expressions for the other Born terms may be obtained using similar techniques.

[40] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, N. J., 1957), p. 63.

[41] See Appendix II and Appendix III of D. M. Brink and G. R. Satchler, *Angular Momentum* (Oxford University Press, London, 1962).

APPENDIX B: d STATE OF THE DEUTERON

To include the deuteron d state in the quasiparticle formalism we still have a deuteron pair state with total angular momentum 1^+ but we must add a new coupling to a d-wave n-p pair as well as the s-wave coupling. The appropriate interaction Hamiltonian for $d\leftrightarrow n+p$ may be written

$$H_I=\sum_{\mathbf{q},\mathbf{Q}}\Gamma_{\lambda}f_{\lambda}(q)\,\langle\tfrac{1}{2}\sigma_p\tfrac{1}{2}\sigma_n\,|\,1\bar{\sigma}\rangle\langle\lambda\mu 1\bar{\sigma}\,|\,1\sigma\rangle\times Y_{\lambda}^{\mu*}(\hat{q})\,D_{1,\sigma}(\mathbf{Q})\,P_{1/2,\sigma_p}^{\dagger}(\tfrac{1}{2}\mathbf{Q}+\mathbf{q})\,N_{1/2,\sigma_n}^{\dagger}(\tfrac{1}{2}\mathbf{Q}-\mathbf{q})+\text{h.c.}, \quad \text{(B1)}$$

where the sum is also over the discrete indices and D, P, and N are field operators for the deuteron, proton, and neutron, respectively, and the general notation is that given in Eq. (2). λ is the relative angular momentum of the n-p pair and may be 0 or 2. For the vertex functions we choose[42]

$$f_0(q)=(q^2+\beta_0^2)^{-1}, \qquad f_2(q)=q^2(q^2+\beta_2^2)^{-2}. \quad \text{(B2)}$$

The deuteron is taken to have no elementary component so that the bound-state normalization condition is

$$\frac{1}{(2\pi)^3}\int_0^{\infty}\frac{n^2dn[\Gamma_0^2f_0^2(n)+\Gamma_2^2f_2^2(n)]}{(2n^2+\epsilon_d)^2}=1, \quad \text{(B3)}$$

where ϵ_d is the binding energy of the deuteron. Introducing $\epsilon_d=2\alpha_d^2$ we can express Γ_0 and Γ_2 in terms of the percent s and d states P_s and P_d by

$$\Gamma_0^2=128\pi^2\alpha_d\beta_0(\alpha_d+\beta_0)^3P_s,$$

$$\Gamma_2^2=1028\pi^2\beta_2(\alpha_d+\beta_2)^5P_d/(5\alpha_d+\beta_2).$$

Phillips[13] gives values of β_0 and β_2 adjusted to fit the triplet scattering length, effective range, and quadrupole moment for $P_d=4.0$, 5.5, and 7.0%. To obtain the correct sign for the quadrupole moment Γ_2 has been chosen to be negative. The resulting partial-wave two-body amplitude is

$$t_{\lambda\lambda'}^{(1)}(k,k';E)=\Gamma_{\lambda}f_{\lambda}(k)\tau_d(E)\,\Gamma_{\lambda'}f_{\lambda'}(k'), \quad \text{(B4)}$$

where the deuteron propagator $\tau_d(E)$ is given by

$$\tau_d(E)\,\frac{-1}{(2\pi)^3(E+\epsilon_d)}\int_0^{\infty}\frac{n^2dn[\Gamma_0^2f_0^2(n)+\Gamma_2^2f_2^2(n)]}{(E-2n^2+i\eta)(2n^2+\epsilon_d)}. \quad \text{(B5)}$$

In the treatment of the three-body problem with the deuteron d state included there are two changes. First, one must use the above propagator (B5) for intermediate d-α states and second, those Born terms which involve the deuteron (deuteron stripping and pickup amplitudes) become a sum of two amplitudes: one for s-wave and one for d-wave deuteron disassociation. These amplitudes may be evaluated by the methods of Appendix A.

[42] Y. Yamaguchi and Y. Yamaguchi, Phys. Rev. **95**, 1635 (1954).