Projected BCS-Tamm-Dancoff Approximation for Even Tin Isotopes*

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A model for even "superconductor" nuclei is proposed and developed. In this model the ground state is approximated by the component of a Bardeen-Cooper-Schrieffer (BCS) state corresponding to a fixed particle number. The low-lying excited states are then obtained by diagonalizing the nuclear Hamiltonian in the space spanned by the particle-hole elementary excitations and, for $J=0$, by the ground state itself. Expressions for the matrix elements of the electromagnetic operators are also given. In the final section, the results obtained from this model for some even tin isotopes are compared with the experimental results and the corresponding results of an ordinary BCS-Tamm-Dancoff approximation. Thus, we are able to give a direct evaluation of the effects of the particle-number nonconservation in the Tamm-Dancoff approximation.

I. INTRODUCTION

An exact shell-model calculation for tin isotopes is practically impossible, because of the dimensions of the problem. In general, the situation does not change qualitatively if we limit ourselves, as usual, to consider only a few valence nucleons in a few accessible subshells, and we treat the remaining nucleons as an inert core. Consequently, it is necessary to introduce approximate methods for treating the nucleons in the extra-core configurations.

The fact that the character of the nuclear forces contributes to the formation of pairs of nucleons with a resultant angular momentum $J=0$, suggests that the lowest-lying nuclear states must correspond to low values of the seniority quantum number. This guess has proved to be correct for the nickel isotopes described by a shell model within the identical-nucleon configurations $(2p_{3/2}, 1f_{5/2}, 2p_{1/2})^n$.[1-3] Unfortunately, for tin isotopes this seniority truncation leaves the dimensions of the problem still beyond the possibility of using computing techniques. For example, for Sn^{116} considered as an identical-nucleon system wherein neutrons occupy the subshells $2d_{5/2}$, $1g_{7/2}$, $3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$, there are 110 states of seniority zero and over 1000 $J=2^+$ states of seniority 2.[1] Hence, for nuclei such as tin isotopes, the low-seniority states should be approximated. For this purpose the techniques derived from the BCS theory of superconductivity are a very powerful tool.

The BCS theory assumes that, at the lowest order, the ground state of a spherical even-even nucleus can be approximated by the quasiparticle (qp) vacuum. The properties of the low-lying levels are then obtained by conveniently taking into account the residual interaction between the quasiparticles. Satisfactory agreement with the experiment is in general attained by an exact diagonalization of the residual interaction between all two-qp states.[4] This procedure is known as the Tamm-Dancoff approximation (TDA). In a more sophisticated treatment (which we shall refer to as TDA4), the residual Hamiltonian is diagonalized in the space spanned by zero-, two- and four-qp excitations.[3,5,6] Here we do not mention the random-phase approximation[7] (RPA) because it was found that TDA and RPA gave essentially equivalent results.[4,8]

Formally, the Tamm-Dancoff approximations (TDA and TDA4) provide the advantage of reducing a problem of several nucleons into a problem of very few quasiparticles. On the other hand, unfortunately, in the BCS theory the particle number is not conserved. The BCS ground state for an even system results in a sum of components with all possible even numbers of particles from 0 to the maximum occupation number

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of the subshells we consider. Only the expectation value of the number operator is kept equal to the effective-particle number.

For the excited states, the particle-number nonconservation introduces spurious components in the space in which we want to diagonalize the Hamiltonian.[9] Take, for instance, the ordinary TDA. If we operate on the BCS ground state with the particle-number operator and take the part that is orthogonal to the BCS state, we obtain a 0^+ spurious state. This is the only spurious state which is completely contained in the TDA basis. Other spurious components which arise from the successive powers of the particle-number operator are partially inside and partially outside the TDA space. Various methods have been proposed and applied to eliminate the most important spurious states before the diagonalization.[3,5,6,10] However, their complete elimination by these methods is practically impossible.

In order to deal with this difficulty, we suggest a modification of the ordinary BCS-Tamm-Dancoff approximation for the extra-core nucleons which permits particle-number conservation in a simple manner.

We shall refer to the components of the BCS state and the two-qp states, corresponding to a fixed particle number, as projected states (projected BCS state and projected two-qp states, respectively).[1]

We will assume that the ground state can be conveniently described by taking a projected BCS state as a trial function in a variational sense. It is possible then, by acting on the so-defined ground state with the nucleon annihilation and creation operators, to introduce particle-hole excitations. We shall assume that the lowest-lying levels can be suitably obtained by diagonalizing the nuclear Hamiltonian in the space spanned by the ground state and by the particle-hole elementary excitations (for $J=0$, these states are not orthogonal to each other).

This permits us to make direct use of a simple formalism we have described in a previous work.[11] In fact, through the parameters determined by the ground-state energy-minimization procedure, we can introduce (in the sense of the Bogoliubov-Valantin canonical transformation) qp creation and annihilation operators. These operators automatically define a projected BCS state and projected two-qp states. The so-defined projected BCS state coincides exactly with what we have taken as the ground state. Moreover, it can be proved that the space spanned by these projected states is equivalent to the space in which we wish to diagonalize the Hamiltonian. In other words, it is exactly equivalent to "work" with the projected states or with the particle-hole excitations. Therefore, we shall call this model a projected BCS-Tamm-Dancoff approximation (PTDA). It differs from the projected model applied by various authors[1,3] to study nickel isotopes in two respects: First, we determine the ground-state properties by directly minimizing the energy of a projected BCS state. Of course, that is more consistent than solving the BCS equations and then projecting the required eigenstate of the number operator from the obtained BCS state. Second, we make use of a different method for calculating matrix elements between projected states. This method is based on the generating-function technique. In Sec. II we develop the mathematical formalism of the PTDA, and in Sec. III we derive expressions for the matrix elements of the electromagnetic operators in the framework of this formalism. In Sec. IV the model will be applied to the even tin isotopes with A ranging from 116 to 124. In these isotopes, 50 protons and 50 neutrons will be treated as an inert core. We shall limit ourselves to considering the remaining $2p$ extra-core neutrons, distributed among the five subshells: $2d_{5/2}$, $1g_{7/2}$, $3s_{1/2}$, $2d_{3/2}$, and $1h_{11/2}$, as the only active ones.

II. THEORY

We describe an identical-nucleon system by the standard shell-model Hamiltonian

$$H=\sum_{\alpha} E_\alpha{}^0 c_\alpha{}^\dagger c_\alpha+\tfrac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta\,|\,U\,|\,\gamma\delta\rangle c_\alpha{}^\dagger c_\beta{}^\dagger c_\delta c_\gamma, \quad (1)$$

where $U=V(1,\ 2)(1-\hat{P}_{12})$ is the antisymmetrized nucleon-nucleon interaction potential and $E_\alpha{}^0$ the energy corresponding to the single-particle shell-model state φ_α. Here, α designates all the quantum numbers which characterize a single-particle state, namely, $\alpha\equiv n_\alpha$, l_α, j_α, $m_\alpha=a$, $m_\alpha(-\alpha\equiv a,\ -m_\alpha)$. The operators $c_\alpha{}^\dagger$ and c_α are creation and annihilation operators for a nucleon in the state φ_α.

We assume that the ground state of the system should be conveniently approximated by the trial function

$$|\,\psi_{2p}\rangle=\Big(\prod_{\alpha>0} u_a/p!\Big)\Big[\sum_{\alpha>0}(v_a/u_a)s_\alpha c_\alpha{}^\dagger c_{-\alpha}{}^\dagger\Big]^p\,|\,0\rangle, \quad (2)$$

where $s_\alpha=(-)^{j_\alpha-m_\alpha}$. It is a projected BCS state (not normalized) corresponding to $2p$ particles. The parameters u_a and v_a are then to be determined from the requirement that the expectation value

$$E=\langle\psi_{2p}\,|\,H\,|\,\psi_{2p}\rangle/\langle\psi_{2p}\,|\,\psi_{2p}\rangle \quad (3)$$

be stationary.[12] We shall impose the usual condition $u_a{}^2+v_a{}^2=1$. The resulting equations for the u_a and v_a coefficients are given in Sec. IV.

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[9] A complete description of the spurious states introduced by particle-number nonconservation will be found in M. Gmitro, A. Rimini, J. Sawicki, and T. Weber, International Atomic Energy Agency Report No. IC/68/29, (unpublished).

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[12] K. Dietrich, H. J. Mang, and J. H. Pradal, Phys. Rev. **135**, B22 (1964).

Starting from $|\psi_{2p}\rangle$ we introduce particle-hole excitations as

$$|\psi_{\rm ph}(abJM)\rangle=\sum_{m_\alpha m_\beta}(j_a j_b; m_\alpha m_\beta|JM)s_\alpha c_\beta^\dagger c_{-\alpha}|\psi_{2p}\rangle, \quad (4)$$

where $(j_a j_b;m_\alpha m_\beta|JM)$ are the usual Clebsch-Gordan coefficients.

We shall suppose that the energies and the wave functions of the lowest-lying levels can be conveniently obtained by diagonalizing the shell-model Hamiltonian in the space spanned by the vectors of Eq. (4) and by $|\psi_{2p}\rangle$ (for $J=0$, these states are nonorthogonal to each other).

However, instead of start directly from the particle-hole states for constructing the space which we wish to diagonalize the Hamiltonian, we shall follow, for our convenience, a slightly different but equivalent way.

Through the u and v parameters determined by the minimization procedure of Eq. (3), let us define qp creation and annihilation operators by a Bogoliubov-Valantin canonical transformation:

$$a_\alpha^\dagger=u_a c_\alpha^\dagger-s_\alpha v_a c_{-\alpha} \quad \text{and} \quad a_\alpha=u_a c_\alpha-s_\alpha v_a c_{-\alpha}^\dagger. \quad (5)$$

We shall indicate their vacuum by $|\psi_0\rangle$. Starting from $|\psi_0\rangle$, we can also introduce two-qp states, defined as

$$|\psi_{JM}(ab)\rangle=\sum_{m_\alpha m_\beta}(j_a j_b; m_\alpha m_\beta|JM)a_\beta^\dagger a_\alpha^\dagger|\psi_0\rangle. \quad (6)$$

States (6) are always defined with the ordering $a\leq b$, in order to avoid repetitions. It is seen immediately that the $2p$-particle component of $|\psi_0\rangle$ coincides with $|\psi_{2p}\rangle$, the projected BCS state which we have assumed as ground state. We shall indicate the $2p$-particle component of state (6) by $|\psi_{2p,JM}(ab)\rangle$. As we have said in Sec. I, it will be referred to as a projected two-qp state.

Particle-hole states and projected states are not independent. In fact, it can be proved that

$$|\psi_{\rm ph}(abJM)\rangle=-u_b v_a|\psi_{2p,JM}(ab)\rangle+\delta_{J0}\hat{j}_a v_a^2|\psi_{2p}\rangle, \quad (7)$$

where $\hat{j}_a=(2j_a+1)^{1/2}$.

It is then evident that we can make use of the projected states $|\psi_{2p}\rangle$ and $|\psi_{2p,JM}(ab)\rangle$ as starting point for constructing an orthonormal basis in the space in which we wish to diagonalize the Hamiltonian. For $J\neq 0$, this construction does not present any difficulty because the projected two-qp states are already orthogonal to each other (in this case they directly coincide with the particle-hole states). The situation is more complicated for $J=0$. In fact, for $J=0$, projected states are no longer orthogonal to each other. Full details on the orthonormal basis construction will be given in Sec. IV. For now, we merely point out that, starting from the projected states, we can make direct use of a technique which we formulated in a previous work for calculating matrix elements.[11] This technique is based on the use of the generating functions[13] for the projected states defined as

$$|\psi(z)\rangle=\sum_p z^p|\psi_{2p}\rangle \quad (8)$$

and

$$|\psi_{JM}(ab;z)\rangle=\sum_p z^{p+1}|\psi_{2p,JM}(ab)\rangle. \quad (9)$$

The functions (8) and (9) have the property that matrix elements between projected states of any operator $\hat{O}$ preserving the particle number can be derived, apart from an identical constant factor, by using the equations

$$\langle\psi_{2p}|\hat{O}|\psi_{2p}\rangle=\oint dz\, z^{-2p-1}\langle\psi(z)|\hat{O}|\psi(z)\rangle, \quad (10)$$

$$\langle\psi_{2p}|\hat{O}|\psi_{2p,JM}(aa')\rangle=\oint dz\, z^{-2p-2}\times\langle\psi(z)|\hat{O}|\psi_{JM}(aa';z)\rangle, \quad (11)$$

and

$$\langle\psi_{2p,JM}(bb')|\hat{O}|\psi_{2p,J'M'}(aa')\rangle=\oint dz\, z^{-2p-3}\times\langle\psi_{JM}(bb';z)|\hat{O}|\psi_{J'M'}(aa';z)\rangle, \quad (12)$$

where the contour of integration includes the origin.

The generating functions can be expressed in terms of the $c_\alpha^\dagger$ and c_α operators. In fact, we have

$$|\psi(z)\rangle=\prod_{\alpha>0}(u_a+s_\alpha z v_a c_\alpha^\dagger c_{-\alpha}^\dagger)|0\rangle \quad (13)$$

and

$$|\psi_{JM}(aa';z)\rangle=\sum_{m_\alpha m_{\alpha'}}(j_a j_{a'}; m_\alpha m_{\alpha'}|JM)\times(u_{a'}z c_{\alpha'}^\dagger-s_{\alpha'}v_{a'}c_{-\alpha'})(u_a z c_\alpha^\dagger-s_\alpha v_a c_{-\alpha})|\psi(z)\rangle. \quad (14)$$

If we want to use Eqs. (10)–(12) for the matrix elements of the Hamiltonian between projected states, we have to be able to calculate in a simple manner the corresponding matrix elements between generating functions. This can be done if we define a new set of creation and annihilation operators through a z-dependent canonical transformation

$$d_\alpha^\dagger=\frac{u_a c_\alpha^\dagger-s_\alpha z v_a c_{-\alpha}}{(u_a^2+z^2v_a^2)^{1/2}}, \qquad d_\alpha=\frac{u_a c_\alpha-s_\alpha z v_a c_{-\alpha}^\dagger}{(u_a^2+z^2v_a^2)^{1/2}}. \quad (15)$$

The generating function $|\psi(z)\rangle$ is the vacuum for these operators, and $|\psi_{JM}(aa';z)\rangle$ can be rewritten as

$$|\psi_{JM}(aa';z)\rangle=\frac{z^2D_{JM}^\dagger(aa')-\delta_{J0}\hat{j}_a u_a v_a z(z^2-1)}{(u_a^2+z^2v_a^2)^{1/2}(u_{a'}^2+z^2v_{a'}^2)^{1/2}}|\psi(z)\rangle, \quad (16)$$

where

$$D_{JM}^\dagger(aa')=\sum_{m_\alpha m_{\alpha'}}(j_a j_{a'}; m_\alpha m_{\alpha'}|JM)d_{a'}^\dagger d_\alpha^\dagger. \quad (17)$$

Transformation of the shell-model Hamiltonian (1) in terms of the $d_\alpha^\dagger$ and d_α operators does not present any difficulty. All we have to do is ob-

[13] B. F. Bayman, Nucl. Phys. 15, 33 (1960).

serve that the transformation (15) is perfectly equivalent to a Bogoliubov-Valatin canonical transformation if we change u_a into $u_a(u_a^2+z^2v_a^2)^{-1/2}$ and v_a into $zv_a(u_a^2+z^2v_a^2)^{-1/2}$. Obviously, the operators $d_\alpha^\dagger$ and d_α reduce, for $z=1$, to creation and annihilation operators for quasiparticles of the BCS theory [see Eq. (5)]. Hence, the trick consists in simply taking the Hamiltonian that one obtains after the Bogoliubov-Valatin transformation and before the elimination of the dangerous terms (by setting the chemical potential equal to zero), and in changing u_a into $u_a(u_a^2+z_2v_a^2)^{-1/2}$ and v_a into $zv_a(u_a^2+z^2v_a^2)^{-1/2}$.

The final result is that we can write the transformed Hamiltonian as (i.e., see Refs. 5 and 7)

$$H=H_0+H_{11}+H_{02}+H_{20}+H_{40}+H_{31}+H_{22}+H_{13}+H_{04}, \tag{18}$$

where

$$H_0=z^2\left[\sum_a \frac{\hat{j}_a^2 v_a^2 E_a^0}{u_a^2+z^2v_a^2}+\tfrac{1}{8}\sum_{ab}\hat{j}_a\hat{j}_b\frac{2z^2v_a^2v_b^2f_0(aabb)+u_av_au_bv_bg_0(aabb)}{(u_a^2+z^2v_a^2)(u_b^2+z^2v_b^2)}\right], \tag{19}$$

$$H_{11}=-\left[\sum_a \frac{\hat{j}_a(u_a^2-z^2v_a^2)E_a^0}{u_a^2+z^2v_a^2}+\tfrac{1}{2}z^2\sum_{ab}\hat{j}_b\frac{(u_a^2-z^2v_a^2)v_b^2f_0(aabb)-u_av_au_bv_bg_0(aabb)}{(u_a^2+z^2v_a^2)(u_b^2+z^2v_b^2)}\right]\bar{D}_{00}(aa), \tag{20}$$

$$H_{02}=H_{20}^\dagger=-z\left[\sum_a \frac{\hat{j}_a u_a v_a E_a^0}{u_a^2+z^2v_a^2}+\tfrac{1}{8}\sum_{ab}\hat{j}_b\frac{4z^2u_av_av_b^2f_0(aabb)-u_bv_b(z^2v_a^2-u_a^2)g_0(aabb)}{(u_a^2+z^2v_a^2)(u_b^2+z^2v_b^2)}\right]D_{00}^\dagger(aa), \tag{21}$$

$$H_{40}=H_{04}^\dagger=-\tfrac{1}{8}\sum_{JM}(-)^{J+M}\sum_{abcd}\frac{z^2u_au_bv_cv_dg_J(bacd)}{[(u_a^2+z^2v_a^2)(u_b^2+z^2v_b^2)(u_c^2+z^2v_c^2)(u_d^2+z^2v_d^2)]^{1/2}}D_{J-M}^\dagger(ba)D_{JM}(cd), \tag{22}$$

$$H_{31}=H_{13}^\dagger=-\tfrac{1}{4}\sum_{JM}(-)^{J+M}\sum_{abcd}\frac{z(u_au_bu_cv_d-z^2v_av_bv_cu_d)g_J(bacd)}{[(u_a^2+z^2v_a^2)(u_b^2+z^2v_b^2)(u_c^2+z^2v_c^2)(u_d^2+z^2v_d^2)]^{1/2}}D_{J-M}^\dagger(ba)\bar{D}_{JM}(cd), \tag{23}$$

$$H_{22}=\tfrac{1}{8}\sum_{JM}\sum_{abcd}\frac{(u_au_bu_cu_d+z^4v_av_bv_cv_d)g_J(badc)+4z^2u_av_bu_cv_df_J(badc)}{[(u_a^2+z^2v_a^2)(u_b^2+z^2v_b^2)(u_c^2+z^2v_c^2)(u_d^2+z^2v_d^2)]^{1/2}}D_{JM}^\dagger(ba)D_{JM}(dc). \tag{24}$$

The operators $\bar{D}_{JM}(ab)$ and $D_{JM}(ab)$ are defined as

$$D_{JM}(ab)=[D_{JM}^\dagger(ab)]^\dagger \tag{25}$$

and

$$\bar{D}_{JM}(ab)=\sum_{m_\alpha m_\beta}s_\alpha(j_aj_b;-m_\alpha m_\beta\,|\,JM)d_\beta^\dagger d_\alpha. \tag{26}$$

$f_J(abcd)$ and $g_J(abcd)$ are antisymmetrized matrix elements of the interaction and are defined, for example, in Ref. 5.

From Eqs. (10)–(12), (16), (19)–(24) and using the commutation rules for the operators D, D^+, and $\bar{D}$ (see Appendix A of Ref. 5), we can derive matrix elements between projected states of the Hamiltonian (18). We find

$$\langle\psi_{2p}\,|\,H\,|\,\psi_{2p}\rangle=R_0^{2p}, \tag{27}$$

$$\langle\psi_{2p}\,|\,H\,|\,\psi_{2p,JM}(ss')\rangle=\delta_{J0}\{R_{20}^{2p}(ss)-\hat{j}_su_sv_s[R_0^{2p-2}(s)-R_0^{2p}(s)]\}, \tag{28}$$

$$\begin{aligned}\langle\psi_{2p,JM}(ss')\,|\,H\,|\,\psi_{2p,JM}(tt')\rangle=\delta_{J0}\{&-\hat{j}_su_sv_s[R_{20}^{2p-2}(tts)-R_{20}^{2p}(tts)]\\&-\hat{j}_tu_tv_t[R_{20}^{2p-2}(sst)-R_{20}^{2p}(sst)]+\hat{j}_su_sv_s\hat{j}_tu_tv_t[R_0^{2p-4}(st)-2R_0^{2p-2}(st)+R_0^{2p}(st)]\}\\&+\delta_J(ss',tt')[R_0^{2p-2}(ss')+R_{11}^{2p-2}(sss')+R_{11}^{2p-2}(s's's)]+R_{22}^{2p}(ss'tt',J),\end{aligned} \tag{29}$$

where

$$R_0^k(p\cdots t)=\sum_a\hat{j}_a^2v_a^2E_a^0I^{k-2}(ap\cdots t)+\tfrac{1}{8}\sum_{ab}\hat{j}_a\hat{j}_b[2v_a^2v_b^2f_0(aabb)I^{k-4}(abp\cdots t)+u_av_au_bv_bg_0(aabb)I^{k-2}(abp\cdots t)], \tag{30}$$

$$\begin{aligned}R_{20}^k(pp'\cdots t)=&-2\hat{j}_pu_pv_pE_p^0I^{k-2}(pp'\cdots t)\\&-\tfrac{1}{4}\sum_a\hat{j}_a\{4u_pv_pv_a^2f_0(aapp)I^{k-4}(app'\cdots t)-u_av_ag_0(aapp)[v_p^2I^{k-4}(app'\cdots t)-u_p^2I^{k-2}(app'\cdots t)]\}\end{aligned} \tag{31}$$

$$\begin{aligned}R_{11}^k(pp'\cdots t)=&E_p^0[u_p^2I^k(pp'\cdots t)-v_p^2I^{k-2}(pp'\cdots t)]\\&+\tfrac{1}{2}\sum_a\hat{j}_a\hat{j}_p^{-1}\{v_a^2f_0(aapp)[u_p^2I^{k-2}(app'\cdots t)-v_p^2I^{k-4}(app'\cdots t)]-u_av_au_pv_pg_0(aapp)I^{k-2}(app'\cdots t)\}\end{aligned} \tag{32}$$

$$\begin{aligned}R_{22}^k(pp'qq',J)=&\tfrac{1}{8}\hat{P}_J(pp')\hat{P}_J(qq')\{[u_pu_{p'}u_qu_{q'}I^{k-2}(pp'qq')\\&+v_pv_{p'}v_qv_{q'}I^{k-6}(pp'qq')]g_J(pp'qq')+4v_pu_{p'}v_qu_{q'}I^{k-4}(pp'qq')f_J(pp'qq')\}.\end{aligned} \tag{33}$$

Here, $\hat{P}_J(pp')=[1-(-)^{J+j_p+j_{p'}}(p\leftrightarrow p')]$ is an antisymmetrization operator acting to the right, and

$$\delta_J(ss',tt')=P_J(ss')\delta_{st}\delta_{s't'}.$$

The residue integrals $I^k(pq\cdots t)$ are defined as

$$I^k(pq\cdots t)=\frac{1}{2\pi i}\oint dz\, z^{-k-1}\frac{\prod_{\alpha>0}(u_a^2+z^2v_a^2)}{(u_p^2+z^2v_p^2)(u_q^2+z^2v_q^2)\cdots(u_t^2+z^2v_t^2)}, \tag{34}$$

where the contour of integration encircles the origin but not the points $\pm i(u_p/v_p)$, $\pm i(u_q/v_q)$, ..., $\pm i(u_t/v_t)$, where the transformation (15) becomes singular.

Obviously, there are no contributions from H_{40}, H_{04}, H_{31}, and H_{13}. A useful expression for the residuum integrals will be given in Appendix A.

III. ELECTROMAGNETIC-TRANSITION PROBABILITIES

Since we treat the tin isotope protons as an inert core, in considering the electromagnetic-transition probabilities, we are forced to dress the active extra-core neutrons with an effective charge e_{eff}. In particular, we shall be interested in Sec. IV in evaluating the quadrupole moment of the 2_1^+ state and the electric-transition probability from the 2_1^+ to the ground state. In fact, for these quantities there are many experimental results and they supply a simple test of the collective character of the 2_1^+ state.

The electric quadrupole operator we shall need is a spherical harmonic tensor of rank two and a one-body operator. Any one-body tensor operator $\hat{T}_\mu^\lambda$, where λ is the rank of the tensor, can be written in the formalism of the second quantization as

$$\hat{T}_\mu^\lambda=\sum_{\alpha\beta}\langle\alpha\mid T_\mu^\lambda\mid\beta\rangle c_\alpha^\dagger c_\beta. \tag{35}$$

In our case, we shall have to deal with matrix elements of this operator between projected states. For calculating these matrix elements we can follow the same procedure used in Sec. II for the matrix elements of the Hamiltonian and valid for any operator preserving the particle number. We start by writing operator (35) in terms of the creation and annihilation operators defined by Eqs. (15) as follows:

$$\hat{T}_\mu^\lambda=-\hat{\lambda}^{-1}\sum_{aa'}\frac{\langle a'\mid\mid T^\lambda\mid\mid a\rangle}{[(u_a^2+z^2v_a^2)(u_{a'}^2+z^2v_{a'}^2)]^{1/2}}\{[(-)^\lambda u_au_{a'}-z^2v_av_{a'}]\bar{D}_{\lambda\mu}(a'a)$$
$$+\tfrac{1}{2}z[u_{a'}v_a+(-)^\lambda u_av_{a'}][D_{\lambda\mu}^\dagger(a'a)+(-)^\mu D_{\lambda-\mu}(a'a)]\}+\delta_{\lambda0}z^2\sum_a\frac{\hat{j}_av_a^2\langle a\mid\mid T^0\mid\mid a\rangle}{(u_a^2+z^2v_a^2)}, \tag{36}$$

where the reduced matrix element $\langle a'\mid\mid T^\lambda\mid\mid a\rangle$ is defined by

$$\langle\alpha'\mid T_\mu^\lambda\mid\alpha\rangle=\hat{j}_{a'}^{-1}(j_a\lambda;m_\alpha\mu\mid j_{a'}m_{\alpha'})\langle a'\mid\mid T^\lambda\mid\mid a\rangle. \tag{37}$$

Using the expression (36) and Eqs. (10)–(12), we obtain

$$\langle\psi_{2p}\mid T_\mu^\lambda\mid\psi_{2p}\rangle=\delta_{\lambda0}\sum_a\hat{j}_av_a^2\langle a\mid\mid T^0\mid\mid a\rangle I^{2p-2}(a), \tag{38}$$

$$\langle\psi_{2p,JM}(ss')\mid T_\mu^\lambda\mid\psi_{2p}\rangle=-\hat{\lambda}^{-1}\langle s\mid\mid T^\lambda\mid\mid s'\rangle[u_sv_{s'}+(-)^\lambda v_su_{s'}]I^{2p-2}(ss')$$
$$-\delta_{\lambda0}\delta_{J0}\hat{j}_su_sv_s\sum_a\hat{j}_av_a^2\langle a\mid\mid T^0\mid\mid a\rangle[I^{2p-4}(as)-I^{2p-2}(as)], \tag{39}$$

$$\langle\psi_{2p,JM}(ss')\mid T_\mu^\lambda\mid\psi_{2p,J'M'}(tt')\rangle=(J'\lambda;M'\mu\mid JM)$$
$$\times\{\delta_{J'0}\delta_{J\lambda}\delta_{M\mu}\hat{\lambda}^{-1}\hat{j}_tu_tv_t(v_{s'}u_s+(-)^\lambda u_{s'}v_s)[I^{2p-4}(ss't)-I^{2p-2}(ss't)]\langle s\mid\mid T^\lambda\mid\mid s'\rangle$$
$$+\delta_{J0}\delta_{J'\lambda}\delta_{M'-\mu}(-)^\lambda\hat{j}_su_sv_s(v_{t'}u_t+(-)^\lambda u_{t'}v_t)[I^{2p-4}(tt's)-I^{p-2}(tt's)]\langle t\mid\mid T^\lambda\mid\mid t'\rangle$$
$$+\hat{J}'(-)^{J+J'}\hat{P}_J(ss')\hat{P}_{J'}(tt')[\delta_{s't'}W(Jj_{s'}\lambda j_t;j_sJ')(u_su_tI^{2p-2}(ss't)-(-)^\lambda v_sv_tI^{2p-4}(ss't))\langle t\mid\mid T^\lambda\mid\mid s\rangle]\}$$
$$+\delta_{\lambda0}\delta_{JJ'}\sum_a\hat{j}_av_a^2\langle a\mid\mid T^0\mid\mid a\rangle\{\delta_J(ss',tt')I^{2p-4}(ass')-\delta_{J0}\hat{j}_su_sv_s\hat{j}_tu_tv_t[I^{2p-6}(ast)-2I^{2p-4}(ast)+I^{2p-2}(ast)]\}, \tag{40}$$

where $W(abcd; ef)$ is the usual Racah coefficient. In the Sec. IV we shall make direct use of expressions (38)–(40) in calculating the quadrupole moment of the 2_1^+ state and the probability for the electric transitions $2_1^+ \rightarrow 0_1^+$.

IV. NUMERICAL CALCULATIONS AND RESULTS FOR EVEN TIN ISOTOPES

In treating the tin isotopes we shall assume 50 protons and 50 neutrons constitute an inert core. Of course, this is a crude approximation. For example, we know that the low-lying levels of the even tin iostopes radiate, and this means that there is no validity in completely neglecting the proton contribution to the

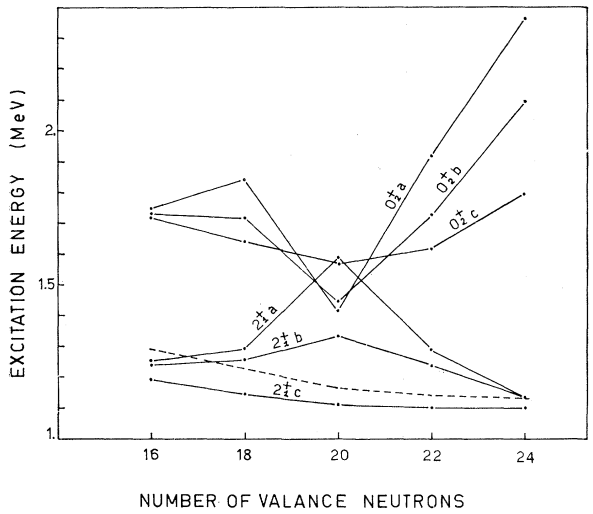

FIG. 1. Energies (in MeV) of the first 2^+ state and of the first excited 0^+ state versus A. The values have been obtained from a PTDA calculation using three different sets of parameters: (a) $E_{5/2^0}=0.0$, $E_{7/2^0}=0.4$, $E_{1/2^0}=1.9$, $E_{3/2^0}=2.2$ and $E_{11/2^0}=2.4$ MeV; $V_0=34$ MeV, $t=0.5$. (b) $E_{5/2^0}=0.0$, $E_{7/2^0}=0.4$, $E_{1/2^0}=1.9$, $E_{3/2^0}=2.2$, $E_{11/2^0}=2.2$ MeV; $V_0=34$ MeV, $t=0.5$. (c) $E_{5/2^0}=0.0$, $E_{7/2^0}=0.4$, $E_{1/2^0}=1.9$, $E_{3/2^0}=2.2$, $E_{11/2^0}=1.9$ MeV; $V_0=34$ MeV, $t=0.5$.

excited states. However, we shall limit ourselves to simulating the core excitations by dressing the extra-core neutrons with an effective charge.

We shall suppose that the extra-core neutrons (from 16 to 24 according to the isotope) are distributed among the following five single-particle levels: $2d_{5/2}$, $1g_{7/2}$, $3s_{1/2}$, $2d_{3/2}$, and $1h_{11/2}$. The single-particle wave functions we take are those of a harmonic oscillator with strength constant $\nu=0.450$ fm^{-1}. For the single-particle energies, we take the values $E_{5/2^0}=0.0$, $E_{7/2^0}=0.20$, $E_{1/2^0}=1.90$, $E_{3/2^0}=2.20$, and $E_{11/2^0}=1.90$ MeV. On adding to these values the self-energy terms evaluated for simplicity from a BCS approximation, we obtain values very similar to those given by Bando or by Nilsson.[14] For example, for ^{116}Sn we obtain $\tilde{E}_{5/2}=0.0$, $\tilde{E}_{7/2}=0.5$, $\tilde{E}_{1/2}=1.7$, $\tilde{E}_{3/2}=2.0$, and $\tilde{E}_{11/2}=2.6$ MeV. For the other isotopes, we find almost identical results. This reflects the fact that we are considering our single-particle energies as solution to the problem of the core neutrons only.

[14] S. G. Nilsson, Kgl. Danske Videnskab. Selskab. Mat.-Fys. Medd. **29**, No. 6 (1955); H. Bando, Progr. Theoret. Phys. (Kyoto) **38**, 1285 (1967).

TABLE I. Tin-isotope ground-state energies (in MeV) calculated from a PTDA for different values of t. The other parameters are fixed as $E_{5/2^0}=0.0$, $E_{7/2^0}=0.20$, $E_{1/2^0}=1.90$, $E_{3/2^0}=2.20$, and $E_{11/2^0}=1.90$ MeV; $V_0=35$ MeV, and $r_0=2$ fm.

t \ A	116	118	120	122	124
−0.555	−3.44	−1.23	1.51	4.78	8.53
−0.3	−17.98	−19.78	−21.51	−23.18	−24.82
0.0	−35.18	−41.73	−48.74	−56.24	−64.23
0.5	−64.11	−78.63	−94.53	−111.78	−130.36

The residual neutron-neutron interaction we have used is a conventional finite-range force of Gaussian form, i.e.,

$$V(r) = -V_0 \exp[-(r/r_0)^2](\hat{P}_s + t\hat{P}_t), \qquad (41)$$

where $r=|\mathbf{r}_1-\mathbf{r}_2|$ and $\hat{P}_s$ and $\hat{P}_t$ are the singlet-even and triplet-odd projection operators. Here V_0, r_0, and t are parameters. In this work the values $r_0=2$ fm, $V_0=35$ MeV, and $t=0.5$ were used. These values, as the single-particle energies, were chosen among various sets of parameters in order to find a good fit to the experimental excitation energies of the low-lying states and to the even-even mass differences.

In Fig. 1 we show the 0_2^+ and 2_1^+ levels of the tin isotopes with A ranging from 116 to 124 obtained from three different sets of single-particle energies. All the remaining parameters are kept fixed. It is evident that we have to take the $1h_{11/2}$ level low enough if we want to reproduce the correct trend of the excitations energies with the mass number. Every time we take the $1h_{11/2}$ above the $2d_{3/2}$ level the most probable configuration for the ground state of ^{120}Sn turns out to be that in which the levels $2d_{5/2}$, $1g_{7/2}$, $3s_{1/2}$, and $2d_{3/2}$ are completely full and the $1h_{11/2}$ is completely empty. This is the reason why the strong inhibition of the first 2^+ state and the lowering of the excitation energy of the 0_2^+ occur. In the ordinary Tamm–Dancoff approximation, this does not happen because the aforesaid configuration is strongly mixed with equally important configurations of the nearest nuclei. For case (a) of Fig. 1, the weight of this configuration is 44% in PTDA and only 0.04% in TDA.

TABLE II. Differences (in MeV) between the tin-isotope ground-state energies calculated and from the projected BCS model from the BCS theory, for different values of t. The other parameters are the same as in Table I.

t \ A	116	118	120	122	124
−0.555	−0.90	−0.87	−0.82	−0.75	−0.65
−0.3	−0.16	−0.13	−0.09	−0.06	−0.04
0.0	0.64	0.66	0.67	0.66	0.61
0.5	1.78	1.75	1.68	1.59	1.44

TABLE III. Occupation probabilities for the various single-particle levels we consider in the BCS state and in the projected BCS state (PBCS). The parameters used are those of Table I with $t=0.5$.

j \ A	116		118		120		122		124	
	BCS	PBCS	BCS	PBCS	BCS	PBCS	BCS	PBCS	BCS	PBCS
1/2	0.45	0.47	0.55	0.63	0.64	0.76	0.74	0.84	0.82	0.89
3/2	0.38	0.33	0.46	0.47	0.56	0.62	0.66	0.75	0.77	0.85
5/2	0.83	0.90	0.87	0.92	0.91	0.94	0.93	0.96	0.95	0.97
7/2	0.77	0.84	0.84	0.89	0.89	0.93	0.93	0.95	0.95	0.97
11/2	0.20	0.13	0.26	0.18	0.32	0.24	0.40	0.33	0.49	0.44

The value of the t parameter was chosen especially from considerations of the ground-state properties. These are very sensible to the value of t used. In Table I, we exhibit the ground-state energy for various values of t.

We can directly compare the even-even mass differences with the experimental neutron-separation energies[15] by using the relation

$$\text{B.E.}(A)-\text{B.E.}(A-2)=S_n(A-1)+S_n(A), \quad (42)$$

where for the binding energy B.E. (A) we take the PTDA ground-state energy with the sign changed. Here $S_n(A)$ is the neutron-separation energy. If we wish the even-even mass difference to decrease with the mass number in order to get the correct saturation properties, then it is evident from Table I that we should take a negative value of t. On the other hand, for negative t the mass differences become excessively small. For $t=0.5$ reasonable values are obtained.

As the starting point of our numerical work, we have minimized expression (3) with regard to the parameters u and v. This leads to the following equation:

$$2\tilde{E}_c^{\,0}u_cv_c-\Delta_c(u_c^2-v_c^2)=0, \quad (43)$$

where the quantities $\tilde{E}_c^{\,0}$ and Δ_c are defined as follows:

$$\Delta_c=-\tfrac{1}{4}\sum_a \hat{j}_a\hat{j}_c^{-1}u_av_ag_0(aacc)I^{2p-2}(ac), \quad (44)$$

$$\begin{aligned}\tilde{E}_c^{\,0}=&E_c^{\,0}I^{2p-2}(c)+\tfrac{1}{2}\sum_a \hat{j}_a\hat{j}_c^{-1}v_a^2f_0(aacc)I^{2p-4}(ac)\\&+\sum_a \hat{j}_a^2\hat{j}_c^{-2}v_a^2E_a^{\,0}\{[I^{2p-4}(ac)-I^{2p-2}(ac)]\nu_c(a)-[I^{2p-2}(c)-I^{2p}(c)](I^{2p-2}(a)/I^{2p})\nu_c\}\\&+\tfrac{1}{2}\sum_{ab}\hat{j}_a\hat{j}_b\hat{j}_c^{-2}\{2v_a^2v_b^2f_0(aabb)[(I^{2p-6}(abc)-I^{2p-4}(abc))\nu_c(ab)-(I^{2p-2}(c)-I^{2p}(c))(I^{2p-4}(ab)/I^{2p})\nu_c]\\&+u_av_au_bv_bg_0(aabb)[(I^{2p-4}(abc)-I^{2p-2}(abc))\nu_c(ab)-(I^{2p-2}(c)-I^{2p}(c))(I^{2p-2}(ab)/I^{2p})\nu_c]\},\end{aligned} \quad (45)$$

with

$$\nu_c(pq\cdots t)=\tfrac{1}{2}\hat{j}_c^2-(\delta_{cp}+\delta_{cq}+\cdots+\delta_{ct}). \quad (46)$$

Equations (43)–(45) were numerically solved by an iteration method. For the calculation of the residuum integrals we used a code based on the expression given in Appendix A.

Using the parameters u and v, we derived the ground-state energy from the expressions (27) and (30). In Table II we show the differences between the ground-state energy from the projected model and the corresponding quantity from a standard BCS approximation for various values of the t parameter.

In the BCS approximation, the occupation and nonoccupation probabilities of a single-particle state φ_α are given by v_a^2 and u_a^2, respectively.

In the projected model, these probabilities become

$$p_{\text{occ}}=v_a^2[I^{2p-2}(a)/I^{2p}], \quad (47)$$

$$p_{\text{nonocc}}=u_a^2[I^{2p}(a)/I^{2p}]. \quad (48)$$

In Table III the occupation probabilities from the two models are compared. It is evident that particle-number conservation tends to make the Fermi surface sharper.

The next point we faced was the construction of an orthonormal basis starting from $|\psi_{2p}\rangle$ and the $|\psi_{2p,JM}(ab)\rangle$'s. We have made use of the following expressions for the scalar products between the projected states:

$$\langle\psi_{2p}\,|\,\psi_{2p}\rangle=I^{2p}, \quad (49)$$

$$\langle\psi_{2p}\,|\,\psi_{2p,JM}(ss')\rangle=-\delta_{J0}\hat{j}_su_sv_s[I^{2p-2}(s)-I^{2p}(s)], \quad (50)$$

$$\begin{aligned}&\langle\psi_{2p,JM}(ss')\,|\,\psi_{2p,JM}(tt')\rangle=\delta_{J0}\hat{j}_su_sv_s\hat{j}_tu_tv_t\\&\times[I^{2p-4}(st)-2I^{2p-2}(st)+I^{2p}(st)]+\delta_J(ss';tt')I^{2p-2}(ss').\end{aligned} \quad (51)$$

We should bear in mind that the projected two-qp states $|\psi_{2p,JM}(ab)\rangle$ are always constructed with the ordering $a\leq b$ in order to avoid repetitions. For $J\neq 0$, the projected states are orthogonal to each other, and all we have to do to normalize them is multiply by a constant factor. For $J=0$, a Schmidt orthonormalization procedure was used. In this procedure, $|\psi_{2p}\rangle$ was taken as the first basis vector. The other basis

[15] R. C. Barber *et al.*, Can. J. Phys. **40**, 1496 (1962); B. L. Cohen, R. Patell, A. Prakash, and E. J. Schneid, Phys. Rev. **135**, B383 (1964).

TABLE IV. In the second column (for any J^π) the results of the PTDA for the lowest-lying levels of tin isotopes with $A=116$, 118, 120, 122, and 124 are exhibited. The third column refers to the corresponding results from an ordinary TDA. The parameters used in both cases are those of Table I with $t=0.5$. In the first column the experimental values are listed (refer to the corresponding reference). All the values are in MeV.

A \ J^π	0^+ Expt	0^+ PTDA	0^+ TDA	2^+ Expt	2^+ PTDA	2^+ TDA	4^+ Expt	4^+ PTDA	4^+ TDA
116	1.762[a]	1.80	2.14	1.291[a]	1.28	1.40	2.391[a]	2.63	2.83
		2.73	3.12	2.108[a]	2.86	3.23	2.531[a]	3.15	3.49
		2.80	3.37	2.224[a]	3.00	3.49	2.803[a]	3.23	3.65
		4.24	4.01		3.07	3.53	3.047[a]	3.53	3.78
118	1.75[b]	1.67	2.08	1.229[a]	1.21	1.38	2.278[a]	2.61	2.74
	2.043[a]	2.67	3.17		2.67	3.12		3.15	3.48
	2.487[a]	3.07	3.30		2.79	3.47		3.47	3.85
		4.84	4.28		3.05	3.55		3.76	3.87
120	1.872[a]	1.58	2.03	1.166[a]	1.15	1.37	2.183[a]	2.40	2.63
	2.16[b]	2.73	3.18		2.57	3.02		3.38	3.55
	2.632[a]	3.59	3.41		2.79	3.37		3.88	3.99
		5.42	4.63		3.25	3.67		4.09	4.09
122		1.64	1.99	1.142[a]	1.14	1.37		2.14	2.50
		2.91	3.12		2.57	2.91		3.78	3.63
		4.28	3.75		2.95	3.29		4.29	4.18
		5.93	5.05		3.63	3.67		4.55	4.28
124		1.84	1.97	1.132[a]	1.13	1.38		1.94	2.37
		3.12	3.09		2.68	2.82		4.23	3.89
		5.07	4.27		3.20	3.26		4.74	4.40
		6.37	5.50		4.13	3.86		5.09	4.58

[a] D. L. Allan, B. H. Armitage, and B. A. Doran, Nucl. Phys. **66**, 481 (1965).

[b] B. L. Cohen and R. E. Price, Phys. Rev. **118**, 1582 (1960); **121**, 1441 (1961).

vectors were constructed by successively utilizing the two-qp projected states $|\psi_{2p,00}(aa)\rangle$ with $j_a=\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$ and $\frac{7}{2}$. $|\psi_{2p,00}(11/2\ 11/2)\rangle$ was not used because only four of the five projected two-qp states corresponding to $J=0$ are independent to each other. This result is evident if we act on the equation

$$\hat{N}\,|\psi_0\rangle=2p\,|\psi_0\rangle-\sum_a u_a v_a \hat{j}_a\,|\psi_{00}(aa)\rangle, \qquad (52)$$

with the operator which projects onto a $2p$-particle subspace. So we obtain the result that

$$\sum_a u_a v_a \hat{j}_a\,|\psi_{2p,00}(aa)\rangle=0. \qquad (53)$$

Another code was used for diagonalizing the Hamiltonian in the so-obtained orthonormal basis. In constructing matrix elements, the expressions (27)–(33) were used. As a result of the diagonalization procedure, the i-th PTDA eigenfunction with angular momentum JM is given by

$$|\Psi_{JM}{}^{(i)}\rangle=\sum_{a\leq b} a_J{}^{(i)}(ab)\,|\tilde{\psi}_{2p,JM}(ab)\rangle \qquad (54)$$

for $J\neq 0$, and

$$|\Psi_{00}{}^{(i)}\rangle=\delta_{i1}a^{(1)}\,|\tilde{\psi}_{2p}\rangle+(1-\delta_{i1})\sum_{n=2}^{s} a_{(n)}{}^{(i)}\,|\tilde{\psi}_{2p(n)}\rangle \qquad (55)$$

for $J=0$, where

$$\begin{aligned}
|\tilde{\psi}_{2p(2)}\rangle&=c_{21}\,|\psi_{2p}\rangle+c_{22}\,|\psi_{2p,00}(\tfrac{1}{2}\tfrac{1}{2})\rangle,\\
&\cdots\\
|\tilde{\psi}_{2p(5)}\rangle&=c_{51}\,|\psi_{2p}\rangle+c_{52}\,|\psi_{2p,00}(\tfrac{1}{2}\tfrac{1}{2})\rangle\\
&\quad+\cdots+c_{55}\,|\psi_{2p,00}(\tfrac{7}{2}\tfrac{7}{2})\rangle.
\end{aligned} \qquad (56)$$

The coefficients $c_{21},\ldots,c_{55}$ were obtained through the Schmidt procedure. The sign over $|\psi_{2p}\rangle$ and $|\psi_{2p,JM}(ab)\rangle$ means that the vectors have been normalized. The notation of Eq. (54) is intended to remind one that the $0_1{}^+$ state (the ground state) coincides exactly with the $|\psi_{2p}\rangle$ state, and the successive 0^+ excited states do not contain the $|\psi_{2p}\rangle$ component. This is a direct consequence of the minimization procedure for the ground-state energy, which makes the correlations between $|\psi_{2p}\rangle$ and the $|\tilde{\psi}_{2p(n)}\rangle$'s vanish. It is equivalent to what happens in the ordinary BCS-Tamm-Dancoff approximation, where the minimization procedure of the BCS-state energy removes the correlations between the BCS state and the two-qp states.

As a check, the calculation was made using, as input for the antisymmetrized matrix elements of the interaction, the following expressions:

$$\begin{aligned}
g_J(abcd)&=4\delta_J(ab,cd),\\
f_J(acdb)&=4(\delta_{J0}\delta_{ac}\delta_{bd}\hat{j}_a\hat{j}_b-\delta_{ad}\delta_{bc}).
\end{aligned} \qquad (57)$$

These expressions correspond to the Hamiltonian

$$H=\hat{N}^2\equiv\sum_a c_\alpha{}^\dagger c_\alpha-\tfrac{1}{2}\sum_{\alpha\beta\gamma\delta}(\delta_{\gamma\beta}\delta_{\alpha\delta}-\delta_{\gamma\alpha}\delta_{\beta\delta})c_\alpha{}^\dagger c_\beta{}^\dagger c_\delta c_\gamma, \qquad (58)$$

and must consequently lead to eigenstates with eigenvalues $\omega=(2p)^2$.

In Table IV, we compare the results for the excitation energies from the projected model with the experimental values and the corresponding results of an ordinary BCS-Tamm-Dancoff approximation. We can

TABLE V. Transition probabilities from the 2_1^+ state to the ground state and quadrupole moments of the 2_1^+ state calculated from the PTDA and from an ordinary BCS-Tamm-Dancoff approximation. The parameters of Table I with $t=0.5$ are used. All the values are referred to $e_{\text{eff}}=1$.

A		116	118	120	122	124
$B(E2)$	PTDA	321.07	299.02	263.54	232.86	207.99
	TDA	339.17	327.61	305.94	274.85	235.29
Q	PTDA	5.62	9.93	12.29	11.03	6.34
	TDA	4.48	6.43	7.47	7.34	5.74

see that a satisfactory agreement with the experimental situation is generally attained, at least, for the lowest levels. The excitation energies obtained from the projected model are generally lower than the corresponding values from the BCS-Tamm-Dancoff model. However, there are several exceptions, particularly for the heaviest isotopes and the highest levels. Of course, in order to evaluate the differences between the energy absolute values in the two models we have to add the ground-state energy shifts given in Table II.

It is interesting to observe that there are several components of comparable size, corresponding to different numbers of particles, which contribute to a BCS-Tamm-Dancoff state. If we analyze, for example, the Tamm-Dancoff states corresponding to the eigenvalues given in Table IV for ^{116}Sn, in terms of components with fixed particle numbers, we obtain that the components corresponding to the particle numbers 114, 116, and 118 are equally important and that the components with particle numbers 112 and 120 are in any case not negligible. This is a typical situation for the BCS-Tamm-Dancoff states and it is quite independent from the parameters used and from the isotope considered.

Finally, we have considered the transition probabilities from the 2_1^+ state to the ground state and the quadrupole moment of the 2_1^+ state. In the PTDA, these quantities are given by

$$(BE2,\ 2_1^+\to 0_1^+)=\tfrac{1}{5}e_{\text{eff}}^2 a^{(1)2}\sum_{\mu}\Big|\sum_{s\le s'} a_2^{(1)}(ss')\times\langle\tilde{\psi}_{2p}\,|\,T_{-\mu}^2\,|\,\tilde{\psi}_{2p,2\mu}(ss')\rangle\Big|^2 \quad (59)$$

and

$$Q_{2_1}{}^+=e_{\text{eff}}(16\pi/5)^{1/2}\sum_{s\le s'}\sum_{t\le t'} a_2^{(1)}(ss')a_2^{(1)}(tt')\times\langle\tilde{\psi}_{2p,22}(ss')\,|\,T_0^2\,|\,\tilde{\psi}_{2p,22}(tt')\rangle. \quad (60)$$

Matrix elements were evaluated by making use of the expressions (39) and (40) with

$$T_0^2=r^2Y_0^2(\omega). \quad (61)$$

In Table V we give our values of $B(E2,\ 2_1^+\to 0_1^+)$ and of the quadrupole moment of the states 2_1^+ computed for $A=116$, 118, 120, 122, and 124. We also give the corresponding BCS-Tamm-Dancoff results. It is interesting to observe that it is impossible to take into account the effects of the particle number nonconservation in the BCS-Tamm-Dancoff approximation by a simple effective charge renormalization. In fact, these effects act in an opposite sense for the $B(E2,\ 2_1^+\to 0_1^+)$ and the $Q_{2_1^+}$. While the particle number conservation increases the quadrupole moments of the 2_1^+ states, it lowers the transition probabilities.

APPENDIX

Here we give an useful expression for the residuum integrals defined by Eq. (34). This expression, which was used in all our numerical codes, is directly obtained by the Cauchy theorem.

$$I^k(pq\cdots t)=\Big(\prod_a u_a{}^{2\nu_a}\Big)\times\sum_{k_1+k_2+\cdots+k_n=k}\prod_a F_a(pq\cdots t;k_a), \quad (A1)$$

where n is the number of subshells involved and k_1, k_2, ..., k_n are zero or positive even numbers. Here

$$\begin{aligned}F_a(pq\cdots t;k_a)&=1, && \text{if } k_a=0\\ &=\nu_a(\nu_a-1)\cdots(\nu_a-\tfrac{1}{2}k_a+1)\\ &\quad\times(v_a/u_a)^{k_a}[(\tfrac{1}{2}k_a)!]^{-1}, && \text{if } k_a\neq 0\end{aligned} \quad (A2)$$

and $\nu_a\equiv\nu_a(pq\cdots t)$ is defined in Eq. (46) of the main text.