large k, say $k>2k_F$. If this idea turns out to be right, some of the approximations that we have used may become unreliable. For example, the approximation of neglecting the dependence of κ_{lm} on the momenta of states l and m will certainly become much worse. The numerical work will then become more difficult. Making the single-particle potential attractive for low-lying unoccupied states will also increase[26] the value of κ and therefore worsen the convergence oi the energy expansion.

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Be^8 States at 16.6 and 16.9 MeV and Final-State Coulomb Interactions*

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The shape, width, and location of the 16-MeV resonances in the α-particle energy spectrum from the $Li^6(Li^6, \alpha)Be^8$ reaction were found to be a function of the angle taken by the breakup fragments in the subsequent decay of the Be^8. This was caused by the long-range electrostatic interaction between the α particle and the breakup particles.

I. INTRODUCTION

THE effects of final-state Coulomb interactions were studied with the reaction $Li^6(Li^6, \alpha_0)Be^8$, where the Be^8 was left in the 16.6- and 16.9-MeV states and where the Li^6 beam energy was 6 MeV. Preliminary results from this work have been described under the title, "Distortion of Nuclear Spectra by Final-State Coulomb Interactions".[1] It was shown that effects of this kind can be important in many nuclear reactions and are particularly important when precision measurements are made on unbound nuclear levels. If corrections for these effects were applied to the values in the literature, some of the energy levels would need to be lowered slightly and made more narrow.

The lifetimes of the two Be^8 states are such that, on the average, the α_0 particles are about 100 F (10^{-11} cm) away from the Be^8 before it disintegrates. At this distance the electrostatic potential energy of the Be^8-α_0 system is 115 keV. When the Be^8 breaks up in a direction perpendicular to the direction of α_0, the α_0 does not get its full share of this energy. The energy lost is about 30 keV.

An effect that is five times larger occurs when the Be^8 breaks up in a direction roughly parallel to the direction taken by α_0. In this case α_0 gains energy because the breakup particle follows along behind α_0, allowing the electrostatic repulsion to occur over a very long time.

At all angles, calculations using classical mechanics were found to agree moderately well with the experimental energy shifts. The calculation for the change in the widths as a function of angle were uncertain by as much as a factor of 2 at some angles because of the difficulties of using classical mechanics for what is really a quantum-mechanical problem. The experiment was not designed to make precise measurements of the widths.

II. EXPERIMENTAL TECHNIQUES

The 6-MeV Li^6 beam from the University of Iowa Van de Graaff was used to bombard thin Li^6F targets which had been evaporated onto 190-nm nickel foil. The α_0 particles were detected with a 50-mm², 150-μ surface-barrier silicon detector placed at an angle of 15° with respect to the beam direction. An aperture, placed in front of the detector, limited the acceptance angle from the center of the target chamber to 3°. All of the detectors were covered by 5.2-mg/cm² nickel foil to stop scattered beam. The detectors were calibrated with a 10.6-h Th B preparation that emitted 6.05- and

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[1] E. Norbeck and F. D. Ingram, Phys. Rev. Letters 20, 1178 (1968).

8.78-MeV α particles. The over-all error in the energy calibration was less than 40 keV.

The Be^8-breakup fragments of interest were the α particles from the 16.6- and 16.9-MeV levels. It was necessary to detect only one of the breakup particles because the direction and energy of the other was completely determined by conservation of momentum.

The object of the experiment was to measure small shifts in α_0 energy as a function of the angle of the breakup-fragment detector. To avoid problems associated with an absolute energy calibration, we always looked for breakup fragments in several directions at once. This was accomplished by using a position detector,[2] which produced two signals—an energy signal and a position signal. The position was the point of incidence of the particle measured parallel to the long axis of the 10×50-mm face of the detector, which was located so that it spanned an angle of 35°. The signals were digitized and recorded on magnetic tape in their original time order. After the experiment, the data were divided up into sets corresponding to a number of smaller angular ranges.

Coincidence timing signals were generated by inductive time pick-off units (ORTEC 260). With the α_0 detector, the pickoff unit was used in the conventional manner, between the detector and the preamp. With the position detector, which had a particularly slow rise time that varied across the face of the detector, the time pick-off unit would not fire reliably unless it was placed after the preamp. The time interval between the two timing signals was converted into a voltage amplitude with a TAC (time-to-amplitude converter). The three detector signals and the output from the TAC were sent to a set of four ADC's (analog-to-digital converters). The timing windows were set after the experiment. The over-all timing resolution was 60 nsec but could be reduced to 20 or 30 nsec by making the location of the timing window a function of position.

By replacing the position detector with two detectors, similar to the α_0 detector, we could look simultaneously at two breakup angles that were widely separated. The timing resolution was excellent, about 3 nsec, but there was the possibility that some sort of cross talk between the signals from one of the breakup detectors and the α_0 signal might cause a shift in the α_0 energy spectrum. Interchanging the electronics of the two breakup detectors did reveal some cross talk between the ADC's, but this was eliminated by a suitable rearrangement of the electronics.

A CDC 160-A digital computer read the 10-bit outputs of the four ADC's and wrote them onto magnetic tape for later data reduction. It also used these numbers to generate oscilloscope displays for monitoring the progress of the experiment. The most valuable display was a 64×128 two-parameter contour map, where the α_0 energy was plotted horizontally and the energy of the coincident breakup fragment was plotted vertically. Suitable manipulation of a bank of lever switches selected the various contour levels (the number of counts at each location in the energy-energy matrix). Conservation of energy and momentum required all events from the $2Li^6\rightarrow3\ \alpha$ reaction for a given pair of angles to appear along one curve in the energy-energy plane. This requirement was used to identify the two particles as α particles. Since the location of the curve was a function of the angle of the detected breakup fragment, it was not necessary to use a separate contour display for each breakup-fragment angle. The α_0 energy was displayed as a 1024-channel, single-parameter spectrum. When two separate breakup detectors were used, two α_0-energy spectra were displayed, each corresponding to one of the breakup detectors. Space was available in the 16 384 word memory of the computer for more displays, but one two-parameter and two one-parameter displays were found to be sufficient. During the process of setting up the electronics, we made minor modifications to the programs in order to use the same display routines for other combinations of parameters. The program was written so that such minor modifications could be readily made by the experimenter in the laboratory. The short time required to learn how to write computer programs was soon repaid by the time saved because of the increased control of the physicist over his experiment.

Data reduction involved processing the magnetic tape to generate energy spectra free of as many contaminations as possible. When simple data-reduction operations were sufficient, modified data-acquisition programs were used that read the pulse-height parameters from magnetic tape instead of from the ADC's. The more complicated data-reduction programs were written in FORTRAN and run in the University Computer Center on an IBM 7044, or more recently a 360/65. These programs, and also the 160-A data acquisition system, have been described previously.[3] A newer development was the tape format used with the 360/65. An entire reel of magnetic tape was made to look like a single data-set to the 360 Operating System by leaving out all file marks. The individual runs were identified by writing a four-character run number at the beginning of each record. With this format, the program that searched for a run could be written in PL/1 or FORTRAN.

III. CLASSICAL CALCULATION OF FINAL-STATE COULOMB INTERACTION

The effects of final-state Coulomb interactions were calculated by integrating the classical equations of motion with Coulomb's law as the force. Numerical

[2] E. Norbeck and R. R. Carlson, *Instrumentation Techniques in Nuclear Pulse Analysis* (U. S. Government Printing Office, National Academy of Sciences—National Research Council, Washington. D. C., 1964), p. 42.

[3] E. Norbeck and M. D. Mancusi, Nucl. Instr. Methods **56**, 293 (1967).

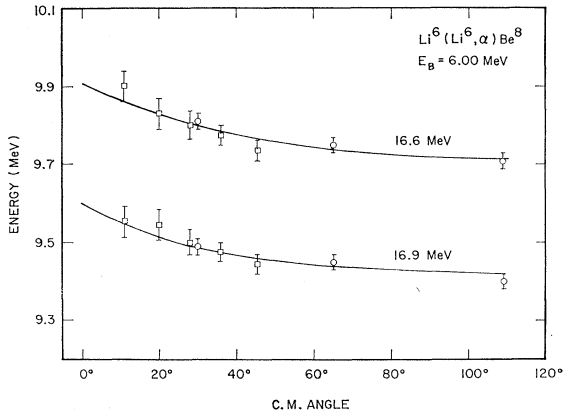

FIG. 1. α-particle energy (lab) at 15° as a function of the angle of emission of coincident particles from the breakup of the indicated Be^8 states.

integration techniques were used to obtain the trajectories of all three α particles. The width of the states were used to calculate the lifetimes. No account was taken of the small effects that would have been caused by angular momentum or quantum-mechanical interference between levels. Both the general outline[1] and some of the specific details[4] of the calculations have been described previously.

The angle between α_0 and one of the breakup fragments (the one with the smallest angle) can be expressed in several different coordinate systems, but in this paper the angle θ will always be measured in the center-of-mass system for the entire reaction. When the Be^8 nucleus breaks up into two α particles in a direction perpendicular to the α_0 direction, the angle θ for both breakup fragments is about 110°. When the angle is small, like 30°, the Be^8 breakups that occur early, when α_0 is close, cause large energy shifts that manifest themselves as a high energy tail. To regulate the amount of this high-energy tail, we assumed, for purposes of calculation, that no Be^8 nuclei decayed until the α_0 particles had reached a radius R. After this time, the Be^8 was assumed to decay exponentially with the lifetime indicated by the width of the state. The measured shape of the curves at 20° and 36° could be fit well with a breakup radius R of 30 F. At larger angles, R also affected the width as measured at half maximum, which means that the widths at most of the larger angles were not well defined. The highest point of a resonance peak was affected relatively little by changes in R, for R near 30 F.

The two solid curves in Fig. 1 show the calculated α_0 energy as a function of θ. By α_0 energy we mean the peak of the resonance curve. At those angles where the energy shift is large, a Breit-Wigner shape gets spread into a much wider asymmetric peak. At 62° the α_0 energy is the same as it would have been in the absence of final-state Coulomb effects.

IV. EXPERIMENTAL RESULTS

Each experimental point which is plotted as a square in Fig. 1 represents the energy at the top of a peak in the α_0 spectrum gated by one small part of the position detector. At small angles, where the energy shift is large, the peaks are conspicuously asymmetric and much broadened. The fitting of these much distorted peaks by the classical theory has been discussed previously.[1] At some angles, the determination of peak

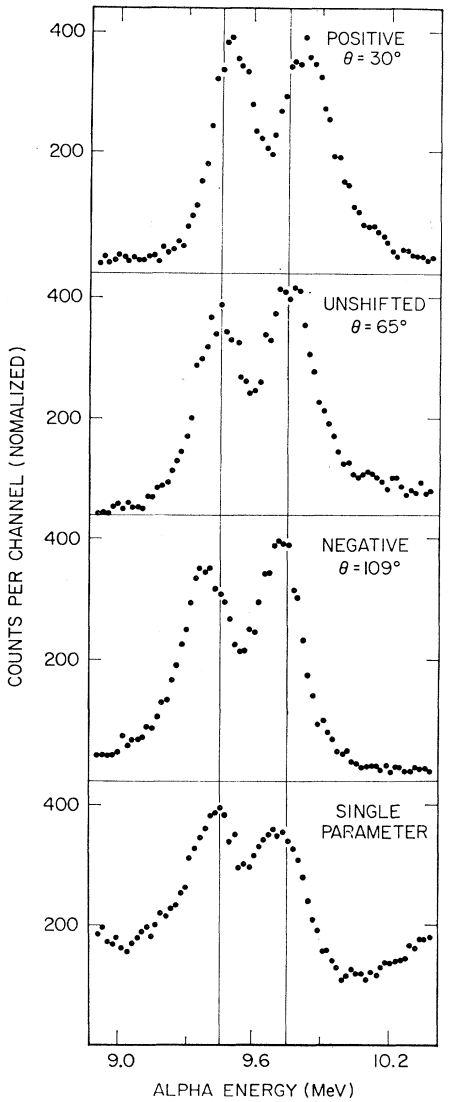

FIG. 2. α-particle spectra from $Li^6(Li^6, \alpha_0)Be^8$. The left peak represents the 16.9-MeV state in Be^8 and the right peak, the 16.6-MeV state. The points are the actual data with the horizontal energy scale corrected for a thin foil in front of the detector (a correction not made in Ref. 1, Fig. 2) and the vertical scale normalized to make the area under the peaks the same for all four curves. The statistical uncertainty may be estimated from the scatter of the points.

[4] F. D. Ingram, University of Iowa Research Report No. 68-34, 1968 (unpublished).

shapes were complicated by a large broad background caused by the capture of Be^8 breakup fragments in the α_0 detector. In almost every case, the background was smooth enough so that it had little effect on the location of the summit of the peak.

For the circled points in Fig. 1, two separate detectors were used to measure two different breakup angles at the same time. This technique automatically eliminated most of the possible systematic errors that could mask a small energy shift. Figure 2 shows the actual spectra for 30°, 65°, and 109°. The angle for the "unshifted" curve is actually 3° away from the 62° angle called for by the theory. The two vertical lines were drawn to make the energy shifts conspicuous. The curve labeled "single parameter" shows the 15° spectrum with no coincidence requirements. This spectrum is essentially the integral of the coincidence data over all angles θ. The spectrum shows an increased width and a small negative energy shift. Because of a $\sin\theta'$ factor (θ' is the angle θ measured in the center of mass of the Be^8 nucleus), most of the contributions to the integral over θ come from large angles where the energy shift is negative. The increased width comes from the contributions at small angles. The large background in the integrated spectrum is due mostly to Be^8 breakup fragments.

An analysis of Fig. 1 gives (310±20) keV for the energy difference between the tops of the two peaks. Correcting this number for quantum-mechanical interference between the two states[5,6] (both 2^+) leads to a value of (295±20) keV for the spacing between the two energy levels. This agrees better with Browne's[5] (289±2) keV than with Marion's[7] (274±3) keV.

The magnetic spectrograph measures an integrated spectrum like the bottom curve in Fig. 2. The shift must, therefore, be integrated over all θ, which would require a knowledge of the angular correlation function. A knowledge of the general character of these angular correlations for the 16.6- and 16.9-MeV states of Be^8, as formed by various reactions,[8,9] allows an estimate of the magnitude of the distortion caused by long-range Coulomb effects.

As the wider of the two states is the lowest in excitation, the shift will be in the direction that makes the peaks appear too close together. The peak locations of the two states as measured with the $Li^6(Li^6,\alpha)Be^8$ reaction would be shifted up in excitation by approximately 10 to 17 keV depending upon the angular correlation function. The two states would also be widened by 20 to 40% because of the Coulomb interactions. Similarly, a noncoincidence experiment measuring the energy distributions of the α's in the $B^{10}(d,\alpha)Be^8$ reaction with a beam energy of 12 MeV should produce peaks shifted up in excitation by about 3 to 8 keV, whereas the $Li^7(He^3,d)Be^8$ reaction with a beam energy of 12 MeV would produce peaks shifted upward 10 to 18 keV. The peaks observed with these two reactions would be broadened by about 10 to 20%. Because of these distortions, a high precision α-α scattering experiment, which would measure the true width and position of the 16.6- and 16.9-MeV states in Be^8, would be most useful at this point.

Our data showed the effects of quantum-mechanical interference, although the energy resolution was not adequate for a quantitative study. The interference manifested itself as a deeper valley between the peaks in Fig. 2 than could be obtained with uncoupled Breit-Wigner shapes.

V. CONCLUSIONS

Corrections for final-state Coulomb interactions can be essential for accurate determination of the location and width of nuclear energy levels. The relevant parameters affecting the magnitude of the correction are the charge and velocity of the interacting particles and the width of the nuclear state. A classical calculation has been shown to be adequate for making corrections for one pair of 100-keV wide levels in Be^8. The errors would undoubtedly be larger in cases where quantum-mechanical effects played a larger role.

[5] C. P. Browne, W. D. Callender, and J. R. Erskine, Phys. Letters **23**, 371 (1966).
[6] F. C. Barker, Australian J. Phys. **20**, 341 (1967).
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[9] C. Moazed, J. E. Etter, H. D. Holmgren, and M. A. Waggoner, Rev. Mod. Phys. **37**, 441 (1965).