In the limit $\omega\tau \gg 1$, which we are considering, the frequencies of the longitudinal modes are the roots of the real part of ϵ_T. The imaginary part of ϵ_T leads to absorption, and the absorption coefficient γ is

$$\gamma = (\omega/c)K. \tag{9}$$

The relaxation time τ is determined by the collision of the carriers with phonon and screened impurities in the region of interest here. In treating the impurities, one should take into account the fact that carriers in the two valleys screen differently because of the difference in their masses.[3] Recently, this optical reflection method has been used to measure plasmon-LO-phonon interaction in GaAs.[8]

The same effects can be measured by scattering a laser beam from the plasmon-LO-phonon system and observing the Raman spectrum of the scattered light. This was done for GaAs[9] where the mixed modes were observed, although this measurement had nothing to do with intervalley transfer.

It is clear that the effects we predict here can be used to study quantitatively intervalley transfer, a subject of much current interest. Furthermore, this is not restricted to GaSb but can be used in many other semiconductors as well.[4]

ACKNOWLEDGMENT

I would like to express my thanks and appreciation to Professor Masataka Mizushima for his interest and hospitality during the time when this work was completed.

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Influence of Boundary Conditions on High-Field Domains in Gunn Diodes*

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Using the method of the field of directions, the influence of the boundary conditions on stationary and moving high-field domains in Gunn diodes is analyzed and discussed. A criterion for self-induced instabilities, especially the Gunn oscillations, is given. It is shown that stationary domains must occur preceding the Gunn oscillations, and that such oscillations can only occur for slightly blocking contacts. The analysis given in this paper is similar to the one discussed for field-quenched CdS.

1. INTRODUCTION

It is well known that characteristic high-field domains can occur in homogeneous semiconductors (or photoconductors) exhibiting an N-shaped negative differential conductivity.[1–5] Such a negative differential conductivity is observed if either the free-carrier concentration[1–7] or the mobility[8–11] decreases more than linearly with increasing electric field. For the first case, the qualitative and, to some extent, the quantitative behavior of the high-field domains (first described analytically and discussed by Döhler[12]) could be derived by using the field of directions (introduced in Ref. 3) and a proper boundary condition.[12–14] It is the purpose of this paper to apply the method of the field of directions properly to the case of field-dependent mobility (a first attempt of such an analysis was made earlier by neglecting the spatial dependence of the mobility in the diffusion current).[15,16] Possible stationary high-field domains in Gunn diodes are discussed as a function of the boundary conditions. Necessary conditions are derived for undeformed moving domains. The occurrence of self-induced instabilities, particularly of Gunn domains is analyzed using results similar to those obtained for field-quenched CdS.[5,12–15]

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[13] K. W. Böer and P. Voss, Phys. Rev. **171**, 899 (1968).

[14] K. W. Böer and P. Voss, Phys. Status Solidi **28**, 355 (1968).

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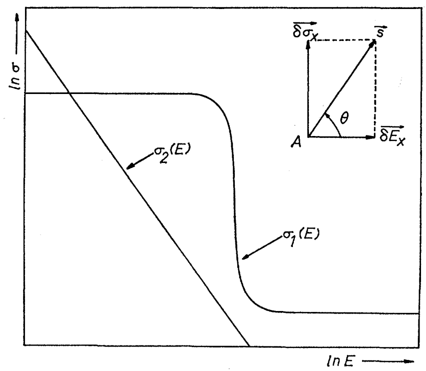


FIG. 1. Neutrality curve $\sigma_1(E)$ and drift-current curve $\sigma_2(E)$ in the σ-E plane.

2. FIELD OF DIRECTIONS

For an n-type Gunn diode, the time-independent Poisson and transport equations can be given by[17]

$$\frac{dE}{dx}=\frac{en_0}{\epsilon\epsilon_0}\frac{\sigma-\sigma_0}{\sigma_0}, \tag{1a}$$

$$\frac{d\sigma}{dx}=\frac{e}{kT}(\sigma E-j), \tag{1b}$$

where the conductivity $\sigma_0=e\mu(E)n_0$ (n_0 being the thermodynamic equilibrium carrier density) possesses an N-shaped negative differential conductivity range.[18] The reason for using the modified diffusion term in the transport equation (1b) may be found in Ref. 17.

The solutions of the autonomous system (1) can be discussed by using the field of directions.[19,20] This field of directions, as given by $d\sigma/dE$ (pointing towards increasing x from cathode towards anode), changes sign at the zeros of Eqs. (1a) and (1b). For a simple analysis of this field of directions, therefore, two auxiliary functions $\sigma_1(E)$ and $\sigma_2(E)$ are conveniently introduced:

(1) The neutrality curve[3] $\sigma_1(E)$ is given by the condition $dE/dx\equiv 0$; from (1a) we obtain $\sigma_1(E)=\sigma_0(E)$.

(2) The drift-current curve $\sigma_2(E)$ is given by the condition $d\sigma/dx\equiv 0$; from (1b) we obtain $\sigma_2(E)=j/E$.

Both the σ_1 and σ_2 curves are given in Fig. 1 (σ-E plane) for a certain current (log-log scale). The drift-current curve shifts parallel to itself toward higher values with increasing current. The singular points of the system (1) [intersections of $\sigma_1(E)$ and $\sigma_2(E)$] represent homogeneous solutions, i.e., spatial-independent conductivity and field. One or three singular points exist,[21] depending on the applied voltage, i.e., on the current.

The curves $\sigma_1(E)$ and $\sigma_2(E)$ divide the σ-E plane into regions of different "quadrants of directions." Consider, for example, the upper right-hand part of Fig. 1 [with $\sigma>\{\sigma_1(E),\sigma_2(E)\}$]. Here dE/dx and $d\sigma/dx$ both have positive values [as can be seen from Eqs. (1a) and (1b)]. At any arbitrary point A in this quadrant, these derivatives can be represented by the vectors $\delta\mathbf{E}_x$ and $\delta\boldsymbol{\sigma}_x$, respectively. The vector $\mathbf{s}=\delta\mathbf{E}_x+\delta\boldsymbol{\sigma}_x$ represents the projection in the σ-E plane of the tangent to the solution curve in the σ-E-x space. (This tangent is oriented because x is considered to increase from cathode to anode.) The value of the angle θ between $\mathbf{s}$ and the E axis ($\tan\theta=d\sigma/dE$) fulfills the relation $0<\theta<90°$ in this quadrant, indicating that both σ and E must increase with increasing x. In Fig. 1, this result is symbolized by an arrow pointing toward the upper right-hand corner. A similar analysis for each quadrant of the σ-E plane yields the other arrows drawn in Figs. 2 for different current values. When the projected solution crosses σ_1 (or σ_2), the angle θ must be 90° or 270° (or $\theta=0°$ or 180°), respectively, since here $dE/dx=0$ (or $d\sigma/dx=0$).

3. STATIONARY HIGH-FIELD DOMAINS

In order to solve system (1), one needs two boundary conditions if the value of the current j is known. Since the current is determined by the applied voltage V, the condition

$$\int_0^L E(x)dx=V \tag{2}$$

can be used to replace j (L is the length of the crystal between the electrodes). One boundary condition can be the electron concentration or the conductivity at the cathode. In the following, we will for simplicity[22] assume the conductivity σ_c "at the cathode" to be known, and, for this section, to be field-independent (a field-dependent σ_c is considered in Sec. 4). As a second boundary condition, the electron density at the anode could be used. However, it is known that in n-type materials, the influence of the anode for the most part can be neglected, since, when forward-biased, it does not markedly influence the current. Therefore, the concentration of electrons there can be rather freely chosen. For a crystal of a length which is long compared to the Debye length, the solution must approach very close to at least one singular point. Therefore, as the second condition, we can assume that the solution must follow the path which enters

[17] B. W. Knight and G. A. Peterson, Phys. Rev. **155**, 393 (1967). For more detailed discussion see Ref. 43.

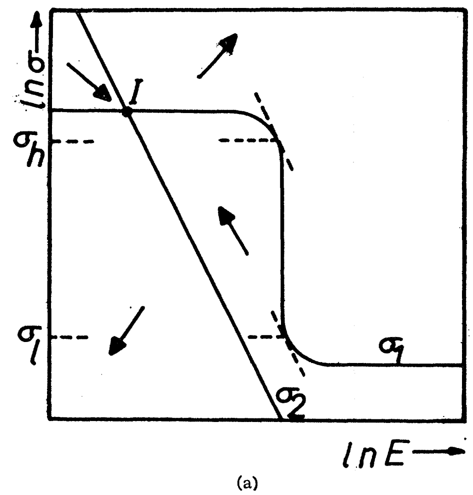
[18] σ_0 is the conductivity which is calculated by assuming the field distribution to be homogeneous.

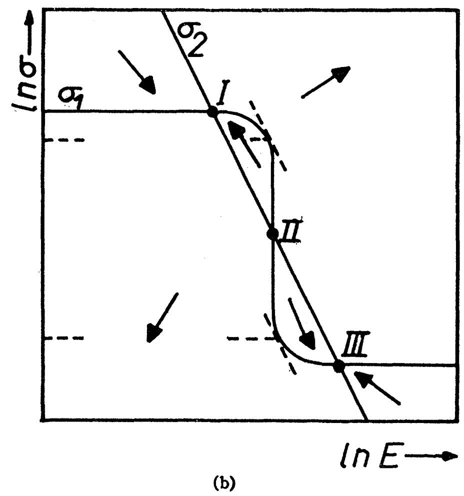
[19] For a more detailed discussion of the field of directions, see Refs. 3, 5, 14–16, and 20.

[20] K. W. Böer, G. Döhler, G. A. Dussel, and P. Voss, Phys. Rev. **169**, 700 (1968).

[21] Tangent points are counted twice.

[22] In fact, only n_c at the cathode is known. μ_c is rather undefined since the field changes rapidly close to the cathode. However, for the main conclusions of the following discussions, that is of no importance.





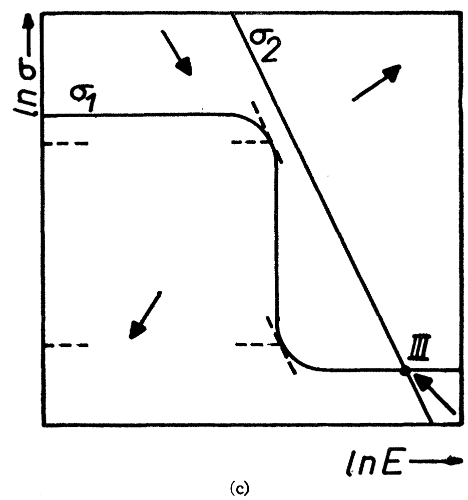


FIG. 2. Field of directions for increasing currents (a)–(c).

the first (or third) singular point, as was done for the case of field-dependent carrier concentration.[12–14,16] The solution curve ends at a point which is determined by the length L of the sample [Eq. (2)]. With these two boundary conditions, the solution has to start at a point $\sigma(E)=\sigma_c$ and to end (or to start) close to a singular point.

We have to distinguish among three possibilities, depending on whether $\sigma_c<\sigma_l$, $\sigma_l<\sigma_c<\sigma_h$, or $\sigma_0(E=0)<\sigma_c$,[23] where σ_l and σ_h are defined by the two possible tangent points between $\sigma_2(E)$ and $\sigma_1(E)$, as shown in Fig. 2. (For a more detailed discussion of the following, see, for example, Refs. 12–14.)

A. $\sigma_c<\sigma_l$: Strongly Blocking Contact

At low currents, the solution, which starts at $\sigma=\sigma_c$, has to reach the only existing singular point I [Fig. 3(a)]. Here the type of the solution $E(x)$ and $\sigma(x)$ is given as curves 1 in Fig. 3.

With increasing voltage the current increases until σ_2 becomes almost tangent to σ_1, thereby creating a "quasisingular point." Then the solution changes from a "Schottky-barrier type" (curves 1) to a type with a high-field domain adjacent to this barrier (curves 2 in Fig. 3). The field strength and the conductivity in the domain stay nearly constant.[24] With increasing applied voltage, the current saturates [Fig. 3(c)] and the width of the high-field domain increases nearly proportionally with the applied voltage. Note that in this domain range the solution has always an additional Schottky-barrier region close to the cathode. With further increase in voltage, the domain width increases

[23] In fact, the intermediate case $\sigma_h<\sigma_c<\sigma_0$ $(E=0)$ should also be discussed, but it is not, in principle, different from the cases mentioned above.

[24] In a Schottky barrier, the field decreases with distance from the cathode with an essentially field-independent characteristic length, the Debye length (screening length for trapped charges). In contrast to this case is that of a high-field domain in which the field-dependent carrier density or mobility forces the field to remain essentially constant over distances long compared to the Debye length.

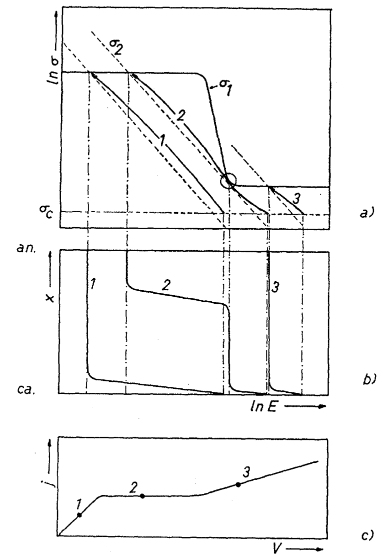


FIG. 3. Highly blocking contact condition. (a) Projection of the solution curves (1–3) into the σ-E plane (for three different currents). (b) Field distribution corresponding to the different solution curves 1–3 in (a). (c) Current-voltage characteristic. Points 1–3 correspond to the solution curves 1–3 in (a) and (b).

until it essentially fills the entire sample. Then the current increases and the solution becomes again a purely Schottky-barrier type (curve 3; here the solution ends at the singular point III).

In summary, a domain near the cathode occurs while the current saturates for the case of a strongly blocking contact. The field strength in the high-field domain does not lie in a range in which the conductivity decreases stronger than linearly with the field. A Schottky barrier always remains close to the cathode.

B. $\sigma_l < \sigma_c < \sigma_h$: Slightly Blocking Contact

At low currents, the solution is represented by curve 1 in Fig. 4, as in Fig. 3(a) (see Sec. 3 A). It also is of a Schottky-barrier type. With increased applied voltage, the current increases until the second singular point II appears and moves towards the intersection point of σ_c with $\sigma_1(E)$. Then the current saturates [Fig. 4(c)] and a cathode-adjacent high-field domain (without Schottky-barrier) appears.[25] With further increased applied voltage, the domain width increases nearly proportionally with the voltage until the entire sample is filled [curves 2 and 3 of Fig. 4(b)]. With still further increase in the applied voltage, an anode-adjacent domain appears while the current stays saturated (curve 4). The width of this anode domain increases again nearly proportionally with the applied voltage, until it almost covers the entire crystal. With further increase in voltage, the solution changes to an "injecting" contact behavior. That is, the saturation ends and the current increases (curve 5).

[25] The curves $\sigma(x)$ and $E(x)$ start with an almost horizontal tangent close to the cathode.

In summary, for the case of slightly blocking contacts and in the current-saturation range, a high-field domain first forms at the cathode. Then, for higher applied voltages, a domain adjacent to the anode is formed while the current remains saturated. The field in the cathode domain, even if it homogeneously fills the entire sample (curves 4 in Fig. 4), lies in the range in which the conductivity decreases more than linearly with the field.

C. $\sigma_1(E=0) < \sigma_c$: Injecting Contact

With increasing applied voltage, the current increases and an injecting contact behavior is observed [curve 1 in Figs. 5(a)–5(c)] until $\sigma_2(E)$ is nearly tangent to $\sigma_1(E)$. Then the current saturates, [Fig. 5(c)] and the solution is able to squeeze between σ_1 and σ_2 ("quasi-singular point"), thus creating a high-field domain at the anode (curves 2 of Fig. 5). The width of this anode domain increases nearly proportionally with the applied voltage. After the anode domain essentially fills the entire sample, the current increases again, thereby returning to an injecting contact behavior (curve 3 of Fig. 5).

In summary, for injecting contacts, the only high-field domain that occurs is adjacent to the anode while the current saturates. The field strength in this domain does *not* lie in the range in which the conductivity decreases stronger than linearly with increasing field. The solution remains always an injection type close to the cathode.

4. BOUNDARY CONDITION

In Sec. 3, we introduced a field-independent conductivity σ_c at the cathode as one boundary condi-

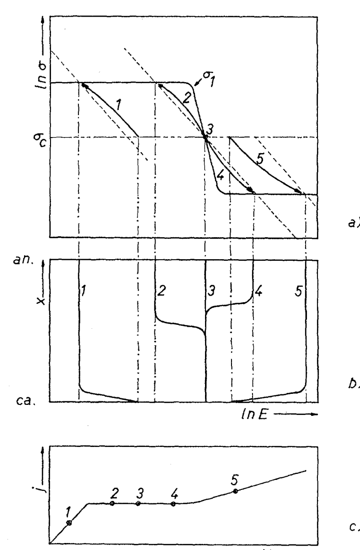


FIG. 4. Slightly blocking contact condition. (a) Projections of the solution curves 1–5 into the σ-E plane (for five different currents). (b) Field distributions corresponding to the solution curves 1–5 in (a). (c) Current-voltage characteristic. Points 1–5 correspond to the curves 1–5 in (a) and (b).

tion.[26,27] This was done mainly to simplify the discussion in the σ-E plane. Similarly, the introduction of a field-independent concentration[12] n_c at the cathode for the case of field-dependent carrier concentration was very fruitful and explained the behavior of stationary domains occurring in CdS[12–14] and Ge:Au.[6]

However, if the conductivity σ_c at the cathode depends on the field and does not decrease stronger than linearly with increasing field,[28,29] the general behavior of the solutions is the same as discussed in Sec. 3. This can easily be seen from the field of directions and from the discussion in Sec. 3. Only the classification of the contact has to be slightly changed:

(1) Strongly blocking contact. The conductivity $\sigma_c(E)$ is always smaller than the conductivity $\sigma_0(E) = \sigma_1(E)$ [or the intersection point $\sigma_c(E)$ with $\sigma_0(E)$ lies below σ_l, as given in Fig. 2].

(2) Injecting contact. The conductivity $\sigma_c(E)$ is larger than the volume conductivity $\sigma_0(E)$.

(3) Slightly blocking contact. The curve $\sigma_c(E)$ intersects the curve $\sigma_0(E)$ in a point σ_x with $\sigma_l < \sigma_x < \sigma_h$.

5. UNDEFORMED MOVING DOMAINS

For moving domains, the displacement current has to be included in the system of Eq. (1). Thus, we obtain

$$\frac{\partial E}{\partial x} = \frac{en_0}{\epsilon\epsilon_0} \frac{\sigma - \sigma_0}{\sigma_0}, \tag{3a}$$

$$j = \sigma E - \frac{kT}{e} \frac{\partial \sigma}{\partial x} + \epsilon\epsilon_0 \frac{\partial E}{\partial t}. \tag{3b}$$

If we consider only undeformed moving domains propagating with a constant velocity c through the sample,[30] then the unknown functions σ and E will only depend on the coordinate $z = x - ct$. Under these conditions, we obtain from (3) the autonomous system

$$\frac{dE}{dz} = \frac{en_0}{\epsilon\epsilon_0} \frac{\sigma - \sigma_0}{\sigma_0}, \tag{4a}$$

$$\frac{kT}{e} \frac{d\sigma}{dz} = \sigma E - j - en_0 c \frac{\sigma - \sigma_0}{\sigma_0}. \tag{4b}$$

Also, since this system fulfills the Lipschitz conditions, the solutions can again be discussed in terms of their projections in σ-E plane. (An attempt to use the field of directions in a more simplified model in order to analyze moving high-field domains was made in Ref.

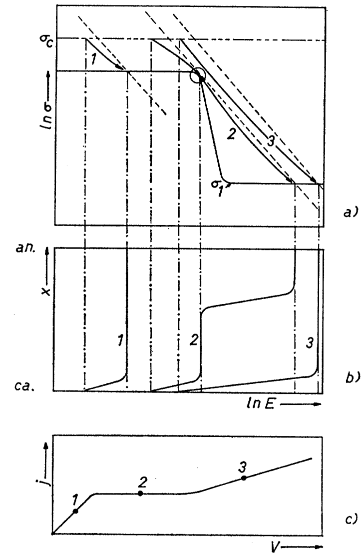


FIG. 5. Injecting contact condition. (a) Projections of the solutions into the σ-E plane (curves 1–3, for three different currents). (b) Field distributions corresponding to the solution curves 1–3 in (a). (c) Current-voltage characteristic. Points 1–3 correspond to curves 1–3 in (a) and (b).

15.) The neutrality curve $\sigma_1(E)$ remains unchanged and is equal to $\sigma_0(E)$, while the drift-current curve σ_2 depends on the parameter c:

$$\sigma_2 = \sigma_0(E) \frac{j - en_0 c}{j_h - en_0 c}, \tag{5}$$

with

$$j_h = e\mu n_0 E.$$

Both functions σ_1 and σ_2 and the field of directions are represented in Fig. 6 for different values of the velocity c.[31] (A current j is chosen so that three singular points I–III occur.) Since a moving domain in this model is represented by a closed curve, one sees immediately that the field of direction in principle allows for such curves even at relatively low velocities. [In fact, the minimum of $\sigma_2(E)$ on the left-hand side of the singular point I in Fig. 6 has to be smaller than the value of $\sigma_1(E)$ taken at the singular point II.]

The singular points of the system of equations (4) lie at the same positions as those of the (stationary) system of equations (1), since the space charge and, therefore, the displacement current vanish at the singular points. From the discussion in Sec. 3 it is known that one or three singular points exist, depending on the applied voltage, i.e., on the current. Only in the latter case, and if all three singular points are well separated from each other, can closed solutions be expected.

The properties of these singular points can be obtained by linearizing the system (4) around a singular

[26] The introduction of the conductivity σ_c at the cathode does not disagree with the boundary condition adopted in Ref. 27.

[27] M. P. Shaw, P. R. Solomon, and H. L. Grubin, United-Aircraft Research Laboratories, East Hartford, Conn., Report No. UAR-H22, 1969 (unpublished).

[28] If the contact conductivity σ_c decreases more than linearly with increasing field, a contact instability may occur. This is a relaxation instability and is different from types of instabilities discussed in this paper (see Ref. 29).

[29] G. Döhler and H. Heckl, Phys. Status Solidi **35**, K77 (1969).

[30] A sample of infinite length is assumed, thus allowing the current to remain time-independent.

[31] Only the positive branches of $\sigma_2(E)$ are relevant.

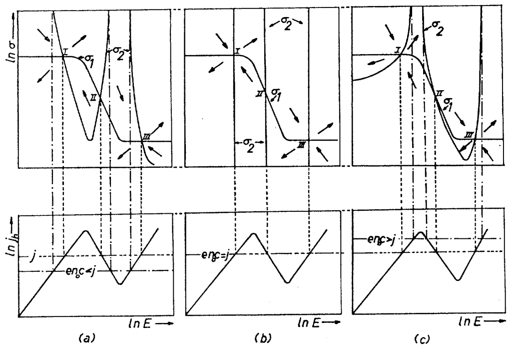


FIG. 6. Field of directions, with the neutrality curve $\sigma_1(E)$ and the drift-current curve $\sigma_2(E)$ for increasing domain velocities c [upper figures, from (a) to (c)]. The lower figures show the current j_h calculated for an assumed homogeneous field distribution, and compare the velocity c with j/en_0 (j being the measured current).

point (σ_s,E_s). Using the ansatz

$$\sigma=\sigma_s+\Delta\sigma\,,\quad E=E_s+\Delta E\,,$$

we obtain from (4), with $\mathbf{u}=(\Delta E,\,\Delta\sigma)$,

$$d\mathbf{u}/dz=(A)\mathbf{u}\,,$$

where the eigenvalues λ_i of the matrix (A) are given by

$$\lambda^2-B\lambda-C=0\,, \tag{6}$$

with

$$B=\frac{e}{kT}\frac{1}{\sigma_s}(\sigma_sE_s-en_0c)-\frac{kT}{e}L_D^{-2}\left(\frac{1}{\sigma_s}\frac{d\sigma_0}{dE}\right)\bigg|_{E=E_s},$$

$$C=\gamma L_D^{-2}\,,\quad \gamma=\frac{d\ln\sigma_0E}{d\ln E}\,,$$

$$L_D^2=\frac{kT}{e}\frac{\epsilon\epsilon_0}{en_0}=(\text{Debye length})^2.$$

For $\gamma>0$, the constant C in Eq. (6) is always positive. Therefore, the eigenvalues $\lambda_{1,2}$ are real and have opposite signs. Thus, the singular points I and III are always saddle points.[33] The property of the singular point II, for which γ is negative, is shown in Fig. 7. The velocities shown in the figure are

$$c_0=v_0\left(1-\frac{kT}{e}\frac{1}{\epsilon\epsilon_0v_0}\frac{d\sigma_0}{dE}\bigg|_{E=E_{\rm II}}\right),$$

$$\delta c=\left(-\frac{kT}{e}\frac{1}{en_0\epsilon\epsilon_0}\gamma\right)^{1/2}\sigma_{\rm II}\,, \tag{7}$$

$$v_0=\mu(E_{\rm I})E_{\rm I}=\mu(E_{\rm II})E_{\rm II}\,.$$

Since with $t\to+\infty$, z tends to $-\infty$, the "mathematical" stability and the "physical" stability do not correspond.[32] The stability indicated in Fig. 7 refers to the physical meaning of the stability.

[32] G. A. Dussel draws our attention to the fact that the stability criterion to be used is the physical rather than the mathematical one (as erroneously used in Ref. 15).

From Poincaré's theory of singular points,[33] it follows that the closed solution can only cycle the singular point II (and cannot include either one or both singular points I and III). From the field of directions, it follows immediately that closed solutions cannot extend below $E_{\rm I}$ or above $E_{\rm III}$.

Since the points I and III are saddle points, two solutions enter and two solutions leave these points. A single moving high- (or low-) field domain in an infinite sample is represented by a curve which leaves and enters the singular point $E_{\rm I}$ (or $E_{\rm III}$) and cycles the singular point $E_{\rm II}$. If this closed curve does not "approach" the singular point III, then the width of the domain is relatively small and the shape of the domain is nearly triangular.[43] If this closed curve "approaches" the singular point III, then the domain becomes "flat-topped."[43]

On the other hand, periodic solutions can only exist if the singular point II is a vortex or if there exists a limit cycle.[33] Let us assume that $c<c_0$ and that a limit cycle exists. Then the singular point II is stable in the physical sense (see Fig. 7). The limit cycle is then stable in the mathematical sense, since the singular point II is a source of solutions (in the mathematical sense). This limit cycle is therefore unstable with respect to t, and this solution has to be excluded. Thus, a stable closed solution can only exist if the velocity c of the domain fulfills the relation

$$c>c_0=v_0\left(1-\frac{kT}{e}\frac{1}{\epsilon\epsilon_0v_0}\frac{d\sigma_0}{dE}\bigg|_{E=E_{\rm II}}\right). \tag{8}$$

This value is higher than the drift velocity at $E_{\rm II}$ and slightly higher than estimated in Refs. 2, 17, 43.

A more detailed analysis of the field of direction is necessary to decide whether a single moving high-field domain (at higher applied voltages this solution is necessarily a flat-top domain with a solution curve approaching close to the first *and* third singular point) or several simultaneous domains (limit-cycle solutions) are actually possible in "long" crystals.

6. SELF-INDUCED INSTABILITIES

We have seen that, in principle, stationary high-field domains may exist in Gunn diodes provided that instabilities are not self-induced. Furthermore, we have seen that the field strength in these domains does not lie in the range in which the conductivity decreases

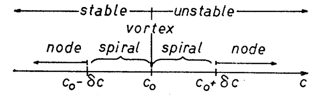


FIG 7. Characteristic behavior of the singular point II as a function of the domain velocity c. The velocities c_0 and δc are defined in Eq. (7).

[33] L. Cesari, *Asymptotic Behavior and Stability Problems in Ordinary Differential Equations* (Springer-Verlag, Berlin, 1959).

more than linearly with increasing field except for domains adjacent to "slightly blocking" cathodes (see Secs. 3 and 4). Since it is well known[2,17,34] that fluctuations can only grow if the field strength lies in a range in which the conductivity decreases stronger than linearly with increasing field, self-induced instabilities can only be expected in samples with such slightly blocking contacts. This behavior, indeed, was observed in field-quenched CdS.[35] This also explains the following:

(1) The classical Gunn effect does not occur in GaAs if the active region is connected with low-field boundaries[27] (acting as quasi-injecting contacts).

(2) The frequent observation of instabilities other than Gunn oscillations is due to "bad" contacts[27] (possibly acting as strongly blocking contacts).

Since the contact has to be slightly blocking, a stationary high-field domain at the cathode and the appearance of current saturation should be observed before instabilities occur. Instabilities can only be self-induced if the width d of the stationary domain at the cathode is larger than a critical width d_c,[34] as experimentally observed in field-quenched CdS.[35] Two different types of self-induced instabilities are described in Ref. 35:

(1) Instabilities due to deformed moving domains which move toward the anode but disappear before reaching this electrode. These instabilities are generally more pronounced if the current is relatively high ("very slightly blocking" contacts). It seems that the oscillations reported in Ref. 27 (Fig. 2, curve A) in GaAs are due to this type of instability.

(2) Instabilities which are due to undeformed moving domains. Generally, these domains were observed when the amplitude of the instabilities of type 1 became large enough in order to flip into a type-2 domain. This was the case when the saturation current was low but not too low[35] ("slightly stronger blocking" contacts).

Let us first discuss type-1 domains, which usually occur first with increasing applied voltage:

The critical width d_c of the high-field domain adjacent to the cathode can be roughly evaluated by studying the growth and decay of small fluctuations.[34] The wavelength of a fluctuation which neither grows nor decays is a measure of the critical width d_c of the domain.[34,36] Such a fluctuation corresponds to the case where the singular point II is a vortex (see Sec. 5). The wavelength of such a traveling wave is given by the inverse of the eigenvalue λ_c of the matrix A. With Eq. (7) one obtains from Eq. (6)

$$1/\lambda_c = L_D/\surd(-\gamma) \simeq d_c \tag{9}$$

as an estimate for the critical domain width.[37] Since $-\gamma$ is usually of the order of 1, the critical domain width d_c is only slightly larger than the Debye length L_D in a trap-free crystal. Consequently, no pronounced current saturation can be observed before oscillations begin. This, however, is different in the trap-controlled case[34,35] where the critical length given by Eq. (9) can be orders of magnitude larger than the screening length.[38] There, pronounced current saturation is observed, and, for low enough free-carrier concentrations, stationary domains can extend over macroscopic dimensions [10^{-1} cm in CdS with $n(E_{\mathrm{II}}) \simeq 10^8$ cm^{-3} [34,35]]. Depending on the ratio of trapped to free electrons, intermediate cases are possible.

It is interesting to note that a detailed analysis of the growth of small fluctuations in CdS leads to alternating stable and unstable ranges for the domain width, i.e., for the applied voltage[34]; that is, with increasing width d of the domain, i.e., of the applied voltage, first stability ($d<d_c$), then instability ($d_{c1}>d>d_c$), and again stability ($d>d_{c1}>d_c$) is predicted (here d_{c1} is another critical width of a high-field domain.[39]) This has been observed in Ref. 35. Similar behavior should be expected for Gunn diodes.

However, with increasing applied voltage, these current oscillations (type 1) grow rather strongly for lower values of σ_c. Consequently, the linearization close to the second singular point becomes invalid. In these cases, termination of the oscillation does not occur, but instead a flipping to type-2 domains is observed in CdS. These domains move undeformed through the sample while the current drops to a well-defined value during the motion of the domain through the sample.

For a discussion of these undeformed moving domains, one can apply a result given by Kröll,[39] namely, that both the current and the applied voltage have to be larger than certain corresponding critical values (j_∞ and V_m) in order to obtain undeformed moving domains.[40] The current j_∞ can be derived by the "equal-area" rule.[41] If the current flowing through the sample is equal to j_∞, then the domain is flat-topped and fills the entire sample.[42] The critical value V_m can be derived by using Kröll's discussion.[39]

Consider, for example, an undeformed moving domain far away from both electrodes [Fig. 8(a)]. The domain potential ΔV (also often referred to as "overvoltage" or Gunn potential) depends on the current j as shown in Fig. 8(b) (curve 1). ΔV increases from zero to infinity while the current decreases from j_m to j_∞.[17,39,43] The total applied voltage is then given by

$$V = \Delta V + E_{\mathrm{I}} L + V_c, \tag{10}$$

[34] G. Döhler, Phys. Status Solidi **30**, 627 (1968).
[35] K. W. Böer and P. Voss, Phys. Status Solidi **30**, 291 (1968).
[36] S. G. Kalashnikov (private communication).
[37] Equation (9) shows also that γ must be smaller than zero for self-induced instabilities to occur (see Sec. 6).
[38] Using the density of trapped rather than free carriers.
[39] K. E. Kröll, Solid State Commun. **6**, 691 (1968).
[40] j_∞ must lie well above a minimum current $j_{\min} = en\mu(E_l)E_l$ (with E_l as defined in Fig. 2) in order to allow for cycling of the solution around the second singular point. The value of j_∞ can be estimated by using the equal-area rule (Ref. 42).
[41] Equal-area law (Refs. 17 and 43).
[42] This occurs at a current value $j_M > j_\infty$ where the voltage drop across the domain increases just linearly with decreasing current.

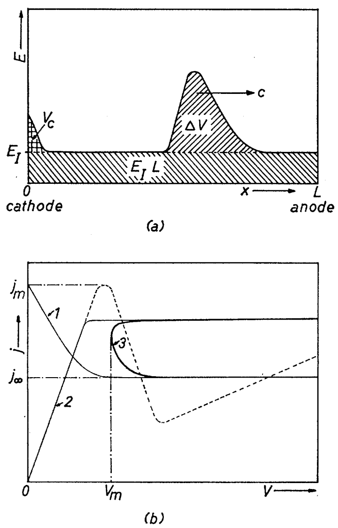


FIG. 8. (a) Undeformed moving high-field domain in a finite sample (voltage drop V_c at the cathode). (b) Curve 1 represents the current-voltage characteristic of the domain potential ΔV [as indicated in (a)]. The dashed curve corresponds to the current-voltage characteristic which would be measured at homogeneous field solutions (j_h, as indicated by Fig. 6, lower part). Curve 2 represents the current-voltage characteristic of a sample with a slightly blocking contact (see Fig. 4), which would be measured if no instability occurs. [For low applied voltage, this curve corresponds to $j=j(V_c+E_\text{I}L)$.] Curve 3 represents the theoretical current-voltage characteristic of a finite sample with a slightly blocking contact exhibiting an undeformed moving domain far away from both electrodes [see Eq. (10)].

where E_I is the value of the electric field outside of the undeformed moving domain [Fig. 8(a)], and where V_c represents the voltage drop near the cathode [Fig. 8(a)]. Curve 2 in Fig. 8(b) represents the current as function of the potential E_cL+V_c, while curve 3 gives the current as function of the total applied voltage [V given by (10)]. An undeformed moving domain in a finite sample can only exist if the applied voltage V is larger than the value V_m defined by the condition

$$\partial V/\partial j=0 \quad \text{at} \quad V=V_m. \qquad (11)$$

From the first condition ($j>j_\infty$) one concludes that only an upper range of σ_c—namely, in the range $\sigma_\infty<\sigma_c<\sigma_h$—allows undeformed moving domains to exist, with $\sigma_\infty=j_\infty/E_\text{II}(j_\infty)$. The lower limit σ_∞ must be larger than σ_l by a finite amount. From the second condition ($V>V_m$) one concludes that either the width of the cathode domain has to be larger than a critical value d_m (related to V_m) or, if $d_m>d_c$, type-1 domains have to occur before type-2 domains are possible. This seems to be predominantly the case for Gunn diodes. For the transition to the Gunn domain the current must decrease below the saturation value and a small high-field domain at the cathode must remain (see, e.g., Ref. 44). With increasing applied voltage, the current must decrease further [see curve 3 of Fig. 8(b)], and therefore the field at the cathode and the width of the remaining domain adjacent to the cathode must decrease. This forces E_I to decrease and necessitates a strong increase of the domain potential, forcing the solution curve of the undeformed moving domain to approach E_III. The Gunn domain becomes flat-topped and the additional applied voltage merely increases its width.

If σ_c is rather close to σ_h, the critical width d_c [Eq. (9)] is rather large, since $|\gamma|\ll 1$. If a Gunn domain is formed in this range, $|\gamma|$ can drastically increase, since the current decreases. This causes d_c to decrease, and, in principle, the remaining domain at the cathode can become unstable while the Gunn domain moves through the crystal.

From the preceding remarks the following picture of self-induced Gunn oscillations can be drawn: The conductivity "at the cathode boundary" and the applied voltage determine the current through the crystal. If the cathode is slightly blocking, high-field domains occur directly adjacent to the cathode and become unstable if the domain width is increased somewhat above the Debye length [Eq. (9)]. If the boundary conductivity lies in the range $\sigma_\infty<\sigma_c<\sigma_h$ and the applied voltage is large enough ($V>V_m$ and domain width larger than d_c), self-induced Gunn domains can occur. However, it is certain that for $\sigma_c\leq\sigma_\infty$ or $\sigma_c\geq\sigma_h$ or $V<V_m$, no Gunn domains can exist in a homogeneous crystal.

7. CONCLUSION

The field of directions, which have proved to be very useful for the case of field-dependent carrier concentration, is applied to the case of field-dependent mobility. There is no difficulty in extending this method to the case where both the carrier concentration and the mobility are field-dependent. Stationary high-field domains adjacent to the cathode and (or) to the anode are predicted in n-type Gunn diodes with their width and shape depending on the contact properties. The occurrence of self-induced instabilities in Gunn diodes is only possible in samples with a slightly blocking cathode ($\sigma_l<\sigma_c<\sigma_h$). The classical Gunn effect (undeformed moving domains) can only occur if the contact fulfills the condition $\sigma_l<\sigma_\infty<\sigma_c<\sigma_h$ (very slightly blocking contact). Finally, it is predicted that with increasing applied voltage, a stable domain first occurs adjacent to the cathode. Then the domain becomes unstable and finally becomes stable again, at even higher voltages. This prediction can only be fulfilled provided that the domain amplitude remains small (σ_c is rather close to σ_h).

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