The effects of finite capacitance on the noise voltage have not been investigated in detail, but it would be expected that ΔV would be larger in this case than $C=0$.

Note added in proof. Recently, T. F. Finnegan, A. Denenstein, D. N. Langenberg, J. C. McMenamin, D. E. Novoseller, and L. Cheng [Phys. Rev. Letters **23**, 229 (1969)] have shown that the "nonvertical" steps observed by Parker *et al.* are not an intrinsic property of the high-resistance junctions. If care is taken to screen out external noise the steps are vertical in agreement with the results found here.

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Tunneling Investigation of Energy-Gap Anisotropy in Superconducting Bulk Pb†*

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A technique has been developed for tunneling from thin Pb films into Pb single crystals formed by a high-vacuum melting process. A preliminary investigation of the energy-gap anisotropy has been carried out. Data are presented from ten separate junctions, including ones on the (001) and (111) crystal facets. In all cases, two energy gaps, Δ_1 and Δ_2 ($\Delta_1<\Delta_2$), were apparent in the characteristics and differed from each other by 10–15%. The maximum variation of Δ_1 was 5% over the tunneling directions studied, while Δ_2 varied by considerably less than this. The maximum observed value of $2\Delta_2(0)$ was 2.78 ± 0.01 meV and the minimum value of $2\Delta_1(0)$ was 2.36 ± 0.01 meV. A comparison was made with a published theory of the energy-gap anisotropy. In some respects the agreement is fairly good, while in other respects it is poor. No evidence was found for the predicted structure due to critical points of the energy-gap surface in $\mathbf{k}$ space. Second-derivative measurements (d^2V/dI^2 versus V) showed an extra (compared to results from thin-film Pb junctions) peak in the group of structures associated with the transverse phonon modes.

PREVIOUS tunneling investigations[1,2] of energy-gap anisotropy in superconducting Pb have been carried out using thick-film (thickness≫coherence length) specimens. In such cases the tunneling direction is not known and this limits the information about anisotropy which can be obtained. Specimens involving a single crystal do not suffer from this limitation but the technical difficulties are considerably increased. A technique has now been developed for tunneling from thin Pb films into Pb single crystals formed by a high vacuum (10^{-8}–10^{-7} Torr) melting process.[3] Using the technique, a preliminary investigation of the energy-gap anisotropy has been carried out.

The crystals form in the shape of oblate spheroids (~1 cm maximum diameter) and the surfaces compare favorably with evaporated films as far as cleanliness and smoothness are concerned. Laue back-reflection photographs gave a mosaic spread of 1° or less and resistivity ratio measurements by the eddy-current decay method gave ratios of greater than 5000. Flat facets (1–2 mm in diam) develop on the (001) and (111) faces and are very convenient for forming tunnel junctions in these directions.

Masking of the crystal surface was achieved by painting on G.E.[4] varnish except for a narrow (~½ mm wide) strip. The exposed strip was then oxidized in O_2 at atmospheric pressure and 60°C for about 16 h. After cooling to room temperature two narrow Pb films[5] (1500 Å thick×½ mm wide) were evaporated at right angles to the oxidized strip to complete the tunnel junctions. Copper electrical leads were attached to the films and the crystal using a conducting Epoxy[6] which was cured in air at 50°C for 1–2 h. The ratio of good to shorted junctions has been about 1 to 3 thus far and in only one case were the two junctions at a given crystal both of good quality.

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* Part of a Ph.D. thesis by B. L. Blackford, Dalhousie University, 1969 (unpublished).

‡ Supported in part by a Studentship from the National Research Council of Canada.

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[5] The energy gap Δ_F of the thin films was determined independently from thin-film specimens deposited on glass slides. $\Delta_F=1.39\pm0.005$ MeV, where the error represents the precision of the number but not absolute accuracy. The films were thin enough to make Δ_F isotropic.

[6] Eccobond 56C. Emerson and Cuming Corp., Canton, Mass.

Measurements of the characteristics were performed with the specimens immersed in liquid helium at $\simeq$1.4°K. Four-terminal connections were used throughout and first and second derivatives were plotted using the harmonic-detection method. The actual circuits used were similar to those described in detail elsewhere.[7,8]

I-V curves and first derivatives for tunneling in the [001] and [111] directions, respectively, are shown in Fig. 1. The appearance of two energy gaps (as witnessed by two steps in the *I-V* curves and two dips in the *dV/dI* curves) is a very distinctive feature and is typical of all the junctions studied. The values of $(\Delta_1+\Delta_F)$ and $(\Delta_2+\Delta_F)$ were inferred from the dips in *dV/dI*, and Δ_1 and Δ_2 were calculated using an independent knowledge of Δ_F.[5]

The two gaps are interpreted as being due to two (or more) groups of tunneling electrons coming from different parts of the Fermi surface of the Pb crystal. Bennett[9] has estimated the areas of Fermi surface of Pb which contribute to tunneling in the three main symmetry directions. In the [111] direction the second-zone hole surface has a large contributing area compared to that from the third-zone electron surface. Accordingly, in the [111] direction, the current contribution from the hole surface is expected to be considerably larger than that from the electron surface. In the [001] direction, on the other hand, the hole surface has a smaller contributing area than the electron surface. From the tunneling data the ratio I_2/I_1 (defined in Fig. 1) was found to have a maximum of 1.7 at [111] and a minimum of 0.88 at [001]. This agrees qualitatively with the above arguments provided

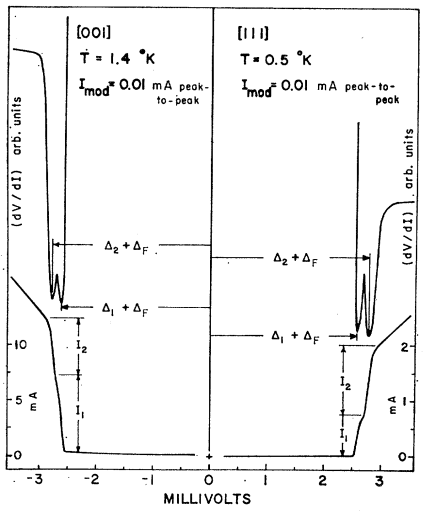

FIG. 1. *I-V* and *dV/dI-V* curves for tunneling in the [001] and [111] directions, respectively, of a Pb single crystal.

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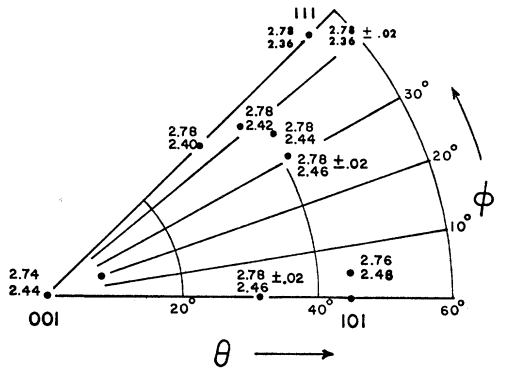

FIG. 2. Dependence of energy gaps $2\Delta_1(0)$ and $2\Delta_2(0)$ on the tunneling direction. The angles are accurate to about 2° except for [111] and [001], in which cases the junctions were on flat facets. Energy gaps are precise to ±0.01 meV unless indicated otherwise.

that Δ_2 is identified with the hole surface and Δ_1 with the electron surface. Similar arguments lead to the same identification of gaps for the [101] direction; thus we tentatively make this gap identification for all the tunneling directions.

Bennett[9] also predicted that critical points of the energy-gap surface in **k** space should produce structure in the tunneling characteristics but this has not been observed here.

Figure 2 shows the gaps $2\Delta_1(0)$ and $2\Delta_2(0)$ as a function of the tunneling direction for the junctions

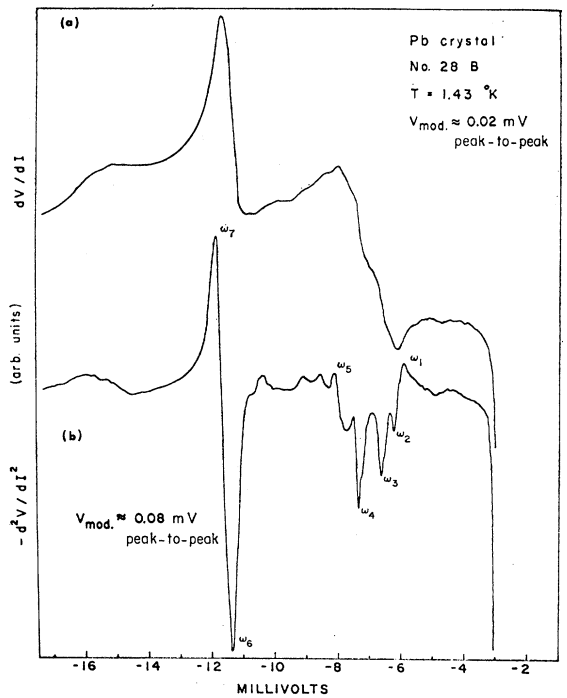

FIG. 3. (a) First- and (b) second-derivative curves for tunneling into a Pb single crystal.

studied. Generally speaking the variations are very systematic, which gives confidence in the data. $2\Delta_1$ changes by a maximum of about 5% while $2\Delta_2$ changes by considerably less than this. $2\Delta_1$ has a minimum of 2.36 meV at [111], a maximum of 2.48 meV at [101], and an intermediate value of 2.44 meV at [001]. Making a comparison with Bennett's[9] theory of the anisotropy, one finds generally that the agreement is not particularly good. A notable discrepancy is in the [001] direction, where the theory predicts an absolute maximum for $2\Delta_2$, whereas the data give a minimum.

The structures associated with phonon critical points[10] are shown in the d^2V/dI^2 curve of Fig. 3. The main new feature attributable to the single crystal is the dip labeled ω_2. The corresponding curves from Pb film specimens (obtained by other workers[10,11] and also in this work) show evidence of some structure at the same energy but certainly not a dip. Second-derivative curves were taken on six of the single-crystal specimens and all of them showed this new feature. The energies of the various peaks and dips showed no significant variation with the tunneling direction, but the relative amplitudes of the transverse and longitudinal structures did change noticeably. In the [111] direction the amplitude of the longitudinal structure compared to the amplitude of the strongest transverse structure was at a maximum.

A second feature of the single-crystal results is the suggestion of splitting of some of the structures, particularly the dips labeled ω_3 and ω_4 in Fig. 3. Splitting of this nature is expected because of the two energy gaps in the crystal, and it is disturbing that it is not more pronounced. A possible explanation comes from the fact that Δ_2 is nearly the same as Δ_F and thus the structure in d^2V/dI^2 associated with Δ_2 is enhanced by the mechanism pointed out by Rowell and Kopf.[10] In this regard it would be helpful to use a probing film (such as Al) having Δ_F different from both Δ_1 and Δ_2.

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Temperature Dependence of the Infrared Divergence in the Degenerate Fermi Gas

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A sudden change in a short-range single-particle potential excites an infinite number of low-energy particle-hole pairs in a degenerate Fermion gas. Although each order of perturbation theory diverges, the perturbation series can be summed formally. The sum agrees in the limit of zero temperature with a result due to Nozières and De Dominicis. At finite temperature, it confirms a conjecture of Anderson's. The excitation spectrum at energies that are small compared to the temperature can be interpreted in terms of a relaxation frequency, which is proportional to the temperature times the square of the phase shift.

THE ease with which low-energy pairs can be excited in a degenerate Fermi gas is responsible for many phenomena, such as superconductivity and the Kondo effect. Recently, Nozières and De Dominicis[1] have shown how the x-ray absorption and emission edges for a metal can be drastically modified by such "soft-pair" processes. In fact, the probability of producing pairs in such cases is so great that a kind of "infrared catastrophe" occurs,[2] similar to the soft-photon divergence which is well known in electrodynamic phenomena such as bremstrahlung. By treating this divergence in a consistent way, Nozières and De Dominicis found an essential change in the x-ray spectrum, with a nonanalytic dependence of the intensity on the photon energy in the vicinity of the absorption edge. In a similar way, Anderson[3] and, more recently, Anderson and Yuval[4] have discussed the role of the infrared divergence in the Kondo effect. These papers have employed an integral equation formulation of the problem, which seems to be difficult to generalize to finite temperatures. The purpose of the present note is to present an alternative treatment, by means of more elementary methods, which does not have this limitation.

In order to demonstrate the method, it will suffice to calculate the closed-loops portion of the x-ray problem treated by Nozières and De Dominicis. For the sake of definiteness, we imagine a short-range single-fermion potential which imposes a phase shift δ onto the s waves at the Fermi surface. (For simplicity, we assume all other phase shifts to be zero.) Until time $t=0$, when the

* National Science Foundation Senior Postdoctoral Fellow.
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