Noise in a Driven Josephson Oscillator

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The effect of noise on the microwave-induced steps in the current-voltage characteristic of a Josephson junction is investigated theoretically. The dynamic resistance of a step is obtained in the limit that the capacitance of the junction is small. The phase fluctuations of the junction are greatly reduced in the presence of the microwave signal, and the dynamic resistance can be extremely small.

I. INTRODUCTION

When microwave radiation of an appropriate frequency is applied to a Josephson junction it is found that steps of nearly constant voltage are induced in the dc current-voltage characteristic of the junction.[1] This effect was predicted by Josephson.[2] The steps occur at voltages V which are related to the applied circular frequency ν by $n\hbar\nu=2eV$, where n is an integer. This effect has been made use of recently by Parker *et al.*[3] to obtain an extremely accurate value of $2e/\hbar$. They found that for superconductor-oxide-superconductor junctions and point contacts of low resistance ($<0.1\ \Omega$) the steps had very narrow voltage widths. However, for junctions and point contacts of larger resistance, between 0.1 and 1 Ω, the width of a step varied between 10 and 200 nV. Recently, Clarke,[4] using junctions made of superconductor-copper-superconductor, put an upper limit on the width of a microwave-induced step in these junctions of 10^{-17} V.

In the present paper we investigate the effects of noise on the microwave-induced steps and obtain an expression for the current-voltage relation of a step. It was shown previously[5] that a Josephson oscillator, in the absence of any applied signal, is neutrally stable towards changes in the phase of the oscillator. This is because the oscillator is autonomous, and it costs no energy to change its phase. Fluctuations in the system cause the phase to diffuse, and it is this phase diffusion which leads to the linewidth of the emitted radiation. The phase fluctuations are analogous to the Brownian motion of a free particle.

In the present case we suppose that the external oscillator supplying microwave radiation to the junction produces monochromatic radiation with a definite phase. If the applied signal is sufficiently strong the Josephson oscillator "locks" on to the external oscillator; i.e., the phase of the Josephson oscillator is determined by the phase of the applied radiation. In this case the phase fluctuations in the Josephson oscillator are greatly reduced. The phase fluctuations are now analogous to the Brownian motion of a particle on a periodic potential. The height of the potential maxima is determined by the strength of the applied signal. Under the influence of noise the phase of the Josephson oscillator generally fluctuates around one of the minima in the potential, but occasionally may pass over the potential barrier into an adjacent minimum. This phase diffusion leads to a small broadening of the step, i.e., gives it a finite resistance.

A similar problem to the above arises in connection with the effect of noise on the dc Josephson effect and has been investigated recently by Ambegaokar and Halperin.[6] This problem can also be interpreted as the Brownian motion of a particle on a one-dimensional periodic potential. The Brownian motion of a particle in one dimension in a field of force has been considered by Kramers.[7] In the case of large viscosity, he showed that the problem can be reduced to the simpler single variable Smoluchowski equation. In the present context, large viscosity is equivalent to a small junction capacitance. This is the case considered here and in Ref. 6. The theory of the phase locking of two oscillators in the presence of noise has been investigated extensively by Stratonovitch.[8]

II. LANGEVIN EQUATIONS

We suppose that a microwave signal of circular frequency ν is applied to the Josephson oscillator. In accordance with the notation of II, we take the vector potential of the applied radiation $A_z{}^e$ in the form

$$A_z{}^e=-4(\pi\hbar c^2/\epsilon\nu l L_x L_y)^{1/2}\alpha\sin\nu t. \tag{2.1}$$

The corresponding voltage across the junction produced by the radiation is

$$V_z{}^e=4(\pi\hbar\nu l/\epsilon L_x L_y)^{1/2}\alpha\cos\nu t, \tag{2.2}$$

when l is the thickness of the oxide layer, ϵ is the dielectric constant of the oxide, and L_xL_y is the area of the junction. The parameter α^2 is then the number of photons in the cavity formed by two superconductors due to the external signal. The total vector potential describing the radiation in the junction is $A_z{}^e+A_z$

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[2] B. D. Josephson, Phys. Rev. Letters **1**, 251 (1962).
[3] W. H. Parker, D. N. Langenberg, A. Denenstein, and B. N. Taylor, Phys. Rev. **177**, 639 (1969).
[4] J. Clarke, Phys. Rev. Letters **21**, 1566 (1968).
[5] M. J. Stephen, Phys. Rev. Letters **21**, 1639 (1968); Phys. Rev. **182**, 531 (1969), hereafter referred to as II.
[6] V. Ambegaokar and B. I. Halperin, Phys. Rev. Letters **22**, 1364 (1969); **23**, 274 (E) (1969).
[7] H. Kramers, Physica **7**, 284 (1940).
[8] R. L. Stratonovitch, *Topics in the Theory of Random Noise* (Gordon and Breach, Science Publishers, Inc., New York, 1963).

where A_z is the contribution from the cavity mode closest to resonance with the Josephson frequency. As in II we suppose that the cavity modes are well separated so that we only need consider them one at a time.

The current through the junction in the weak-coupling limit is

$$J_T = j_1 \sin\theta + (2e/\hbar cl)(A_z^e + A_z) j_1 \cos\theta + J_n(V), \quad (2.3)$$

where θ is the phase difference between the two superconductors and $J_n(V)$ is the normal (quasiparticle) current. We will neglect the rapidly varying first term and make the rotating approximation on the second term. This leads to

$$J_T = 4eT\alpha \sin(\theta - \nu t) + 2eT(be^{i\theta} + b^\dagger e^{-i\theta}) + J_n(V), \quad (2.4)$$

where b and $b^\dagger$ are annihilation and creation operators for photons in the cavity mode. T is the coupling constant introduced in II,

$$T = j_1(\pi l/\hbar\epsilon\Omega L_x L_y)^{1/2}. \quad (2.5)$$

In the amplitude of the first term of (2.4) we have neglected the difference between the frequency ν and the cavity mode frequency Ω, so that it is proportional to T.

We can now write Langevin equations describing the oscillator. These are essentially identical to those obtained in II apart from the extra term in the current involving the external signal. The Langevin equations are

$$d\theta/dt = 2eV/\hbar, \quad (2.6)$$

$$CdV/dt = I - 4eT\alpha \sin(\theta - \nu t) - 2eT(be^{i\theta} + b^\dagger e^{-i\theta}) - J_n(V) + F(t), \quad (2.7)$$

$$db/dt = (-i\Omega - \tfrac{1}{2}\gamma)b + Te^{-i\theta} + f(t), \quad (2.8)$$

when V is the voltage across the junction, C is the capacitance of the junction, and γ is the cavity bandwidth. We suppose that the junction is connected to a constant-current source which supplies a current I to the junction. The noise sources are exactly the same as in II and have the correlation functions

$$\langle F(t_1)F(t_2)\rangle = 2D_q\delta(t_1 - t_2), \quad (2.9)$$

$$2D_q = eJ_n(V) \coth(eV/2kT), \quad (2.10)$$

$$\langle f(t_1)f^\dagger(t_2)\rangle = \gamma(\bar{n}+1)\delta(t_1 - t_2), \quad \langle f^\dagger(t_1)f(t_2)\rangle = \gamma\bar{n}\delta(t_1 - t_2). \quad (2.11)$$

Thus the noise source $F(t)$ arises from the shot noise associated with normal currents in the junction and the noise source f from the dissipation of radiation in the cavity. The term $\bar{n}$ is the Planck function $(e^{\hbar\Omega/kT} - 1)^{-1}$.

The Langevin equations (2.6)–(2.8) differ from those of II only by the term involving the externally applied radiation. This term is important because it explicitly involves the time and has the effect of determining the phase of the oscillator.

In order to solve (2.6)–(2.8), we use a method of slowly varying phase and set

$$\theta(t) = \nu t + \theta_0(t), \quad (2.12)$$

where $\theta_0(t)$ will be regarded as slowly varying so that the voltage $(\hbar/2e)\dot{\theta}_0(t)$ is small. We define a voltage corresponding to the frequency ν by

$$V_0 = \hbar\nu/2e, \quad (2.13)$$

so that the total voltage across the junction is

$$V = V_0 + (\hbar/2e)\dot{\theta}_0(t). \quad (2.14)$$

In (2.8) we substitute $b = b'e^{-i\theta(t)}$ and then

$$db'/dt = [i(\nu - \Omega) + i\dot{\theta}_0 - \tfrac{1}{2}\gamma]b' + T + fe^{i\theta} \quad (2.15)$$

and approximately for large γ

$$b' = (T + fe^{i\theta})/[i(\Omega - \nu) - i\dot{\theta}_0 + \tfrac{1}{2}\gamma]. \quad (2.16)$$

For most junctions, γ is the most rapid decay constant so that this is an adequate approximation. Substituting (2.16) in (2.7), we obtain the single equation

$$CdV/dt = I - 4eT\alpha \sin\theta_0 - J_s(V) - J_n(V) + G(t), \quad (2.17)$$

where

$$J_s(V) = (2eT^2\gamma)/[(\nu - \Omega)^2 + \dot{\theta}_0]^2 + \tfrac{1}{4}\gamma^2. \quad (2.18)$$

The pair tunneling current is the sum of the second and third terms on the right-hand side of (2.17). $G(t)$ is a noise source with correlation function

$$\langle G(t_1)G(t_2)\rangle = 2(D_q + D_c)\delta(t_1 - t_2), \quad (2.19)$$

where D_q is given by (2.10) and

$$2D_c = 4e^2T^2\gamma(2\bar{n}+1)/[(\nu - \Omega)^2 + \tfrac{1}{4}\gamma^2] = 2eJ_s(V_0)(2\bar{n}+1). \quad (2.20)$$

This noise source arises from dissipation of radiation in the cavity. As discussed in II, it leads to shot noise in the pair current which explains the form of (2.20). We have neglected the small voltage $(\hbar/2e)\dot{\theta}_0$ in (2.20). At temperatures such that $kT > eV_0$ we can combine the two terms in (2.19) to give

$$D = D_q + D_c = (kT/V_0)(J_s + J_n) = (kT/V_0)J, \quad (2.21)$$

where J is the total current in the junction in the absence of the microwave field.

In Eq. (2.17) we expand the currents in terms of the small voltage $(\hbar/2e)\dot{\theta}_0$. Thus

$$J_s(V) = J_s[V_0 + (\hbar/2e)\dot{\theta}_0] \simeq J_s(V_0) + (\hbar/2eR_s)\dot{\theta}_0, \quad (2.22)$$

where $R_s = (\partial J_s/\partial V)_{V_0}^{-1}$ is the dynamic resistance for supercurrent. Similarly,

$$J_n(V) = J_n(V_0) + (\hbar/2eR_n)\dot{\theta}_0. \quad (2.23)$$

Substituting (2.22) and (2.23) in (2.17), we get

$$(\hbar C/2e)\ddot{\theta}_0=\Delta I-(\hbar/2eR)\dot{\theta}_0-4eT\alpha\sin\theta_0+G(t)\,, \quad (2.24)$$

where R is the total dynamic resistance of the junction:

$$R^{-1}=R_s^{-1}+R_n^{-1}.$$

Also,

$$\Delta I=I-J_s(V_0)-J_n(V_0)\,,$$

and is the difference between the actual current I and the current that would flow in the junction in the absence of the applied signal at the voltage V_0.

Equation (2.24) is of exactly the same form as the circuit equation that would be obtained in the dc Josephson effect. In the present case, however, the height of the dc step is determined by the strength of the applied microwave radiation. In the absence of noise (2.24) has the steady-state solution

$$\sin\theta_0^0=\Delta I/j_2 \quad (\Delta I<j_2)\,, \quad (2.25)$$

where $j_2=4eT\alpha$ and is one-half the height of the dc step.

It is also of interest to investigate the frequencies of small oscillation around the steady solution (2.25). By linearizing the equation around θ_0^0 it is easily found that the frequencies of small oscillation are

$$\omega_\pm=-\frac{i}{2RC}\pm\left(\frac{2ej_2}{\hbar C}\cos\theta_0^0-\frac{1}{4R^2C^2}\right)^{1/2}. \quad (2.26)$$

These oscillations are analogous to the Josephson plasma frequency.[9] Near the top of the step where θ_0^0 approaches $\frac{1}{2}\pi$, Eq. (2.26) reduces to

$$\omega_+=-i(2eRj_2/\hbar)\cos\theta_0^0\,,$$
$$\omega_-=-i/RC. \quad (2.27)$$

The ω_- mode is damped very rapidly and would not be expected to influence the system in an important way. The ω_+ mode, on the other hand, approaches zero and will be very important in determining how the system behaves near the top of the step. We note in passing that it is this zero-frequency mode which gives rise to the linewidth of the emitted radiation discussed in II. The mode ω_+ in (2.27) is independent of the capacitance C, and this suggests that we can simplify (2.24) by setting $C=0$. The validity of this approximation is discussed further below. Then (2.24) becomes

$$(\hbar/2eR)\dot{\theta}_0=\Delta I-j_2\sin\theta_0+G(t)\,. \quad (2.28)$$

In the absence of noise the solution of this equation is simply

$$\sin\theta_0=\Delta I/j_2 \quad (\Delta I<j_2)\,,$$
$$\dot{\theta}_0=(2eR/\hbar)\left((\Delta I)^2-j_2^2\right)^{1/2} +\text{periodic terms} \quad (j_2<\Delta I). \quad (2.29)$$

[9] B. D. Josephson, in *Quantum Fluids*, edited by D. F. Brewer (North-Holland Publishing Co., Amsterdam, 1966), p. 174.

For $\Delta I<j_2$ the oscillator is phase locked to the external signal. When ΔI exceeds the height of the step j_2, an extra voltage appears across the junction proportional to the square root of the excess current.

A useful mechanical analogy to (2.28) is that of a particle executing Brownian motion in a potential proportional to $\Delta I\theta_0+j_2\cos\theta_0$. Generally the particle oscillates in one of the minima of the potential, but every now and again it may pass over the potential maximum to an adjacent minimum.

III. FOKKER-PLANCK EQUATION

To investigate the phase-locking equation (2.28) in the presence of noise, it is more convenient to consider the equivalent Fokker-Planck equation for the distribution function $P(\theta_0,t)$. From (2.28), using the properties of the noise source $G(t)$ in (2.19) and (2.21), the Fokker-Planck equation is

$$\frac{\partial P}{\partial t}=-\frac{2eR}{\hbar}\frac{\partial}{\partial\theta_0}[(\Delta I-j_2\sin\theta_0)P] +\frac{4e^2R^2}{\hbar^2}D\frac{\partial^2P}{\partial\theta_0^2}=-\frac{\partial w}{\partial\theta_0}. \quad (3.1)$$

This equation is known as the Smoluchowski equation. The solution of (3.1) required here corresponds to the situation where the phase θ_0 is diffusing at a steady rate. This corresponds to the case where w is a constant and P is periodic in θ_0 with period 2π. Such a solution has been given by Stratonovitch[8] and for completeness we will briefly outline the derivation. We introduce the dimensionless variables

$$a_1=(\hbar/2eRD)\Delta I=(\hbar\Delta I/2ekT)(V/RJ)\,, \quad (3.2)$$

$$a_2=(\hbar/2eRD)j_2=(\hbar j_2/2ekT)(V/RJ)\,, \quad (3.3)$$

where we have used (2.21). Then (3.1) takes the form

$$\partial P/\partial\theta_0-(a_1-a_2\sin\theta_0)P=-\hbar^2w/4e^2R^2D. \quad (3.4)$$

When P is normalized according to

$$\frac{1}{2\pi}\int_0^{2\pi}P(\theta_0)d\theta_0=1 \quad (3.5)$$

the extra dc voltage ΔV due to noise appearing across the junction is related to w by

$$\Delta V=\hbar w/2e. \quad (3.6)$$

The periodic solution of (3.4) given in Ref. 8 is

$$P(\theta_0)=\frac{\hbar^2w}{4e^2R^2D}\int_0^{2\pi}d\theta_0'G(\theta_0,\theta_0')\,, \quad (3.7)$$

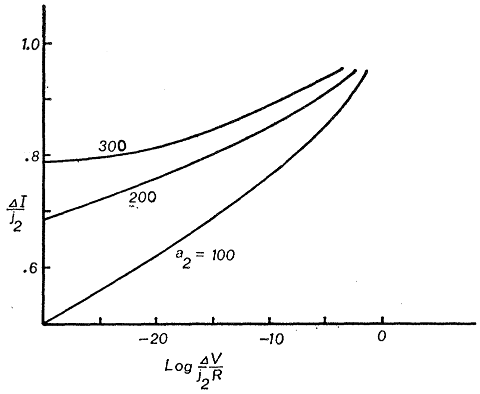

FIG. 1. Plot of $\log_{10}(\Delta V/j_2R)$ against $\Delta I/j_2$ for different values of a_2 [see Eq. (3.12)].

where the Green's function is

$$G(\theta,\theta')=\left(\frac{e^{2\pi a_1}}{e^{2\pi a_1}-1}-\eta(\theta-\theta')\right)$$
$$\times\exp[a_1(\theta-\theta')-a_2(\sin\theta-\sin\theta')],\quad(3.8)$$

where $\eta(\theta-\theta')$ is the step function

$$\eta(\theta-\theta')=1\quad \theta>\theta'$$
$$=0\quad \theta<\theta'.$$

The quantity w is determined by substituting (3.7) in the normalization condition (3.5). Hence from (3.6) we obtain the current-voltage relation for the step:

$$\Delta V=\frac{4\pi eR^2D}{\hbar}\left(\int_0^{2\pi}d\theta\int_0^{2\pi}d\theta'\,G(\theta,\theta')\right)^{-1}.\quad(3.9)$$

The validity of the approximation $C=0$ which was used in obtaining this result has been investigated by Kramers.[7] In the present notation it is necessary that

(a) $\Delta V\ll(kT/C)^{1/2}$,

(b) $(2eRC/\hbar)(kT/C)^{1/2}\ll1$.

The first condition requires that the noise voltage ΔV be much less than the thermal voltage $(kT/C)^{1/2}$. Secondly, the change in the phase produced by the thermal voltage in a relaxation time RC must be much less than one. Both these conditions can be satisfied if C is small enough.

The integral in (3.9) has been evaluated by Stratonovitch. The physically most interesting cases are the following.

(a) For very small ΔI where a_1 is small (3.9) reduces to

$$\Delta V=[R/I_0^2(a_2)]\Delta I,\quad(3.10)$$

where I_0 is a Bessel function of the second kind. Thus the dynamic resistance R_c at the center of the step is

$$R_c=R/I_0^2(a_2).\quad(3.11)$$

For $a_2=0$, and hence no microwave signal, this reduces to the dynamic resistance R of the junction. As $I_0(a_2)$ increases exponentially with a_2 this resistance can be extremely small.

(b) For large a_1 and a_2 ($a_2>a_1$) the integrals in (3.9) can be carried out by steepest descents with the result

$$\Delta V=2j_2R\cos\theta_0^0\sinh\pi a_1$$
$$\times\exp(-2a_1\theta_0^0-2a_2\cos\theta_0^0),\quad(3.12)$$

where $\sin\theta_0^0$ is given by (2.25). It is interesting to note that the prefactor in (3.12) is exactly $\hbar/e|\omega_+|$, where ω_+ is given by (2.27). Thus the phase-diffusion process corresponds to thermal activation over an energy barrier. The approximation (3.12) breaks down as a_1 approaches a_2 and $\cos\theta_0^0\to0$. It is adequate, however, over most of the interesting part of the step where $a_1<a_2$.

It should be noted that the only junction parameters appearing in (3.9) are the dynamic resistance R and j_2, one-half of the height of the microwave-induced step in the absence of noise. While j_2 is not directly measureable, it can be estimated accurately from experimental data. Thus while we have obtained (3.9) for the special case of a resonant cavity we can expect that a similar result will hold for other kinds of devices, e.g., point contacts and superconductor-metal-superconductor junctions.

For the junctions used by Clarke[4] we find from (3.3) that $a_2\simeq3.10^3$, corresponding to the values $R=10^{-7}\ \Omega$, $J=10$ mA, $V_0=10^{-9}$ V, $j_2=\frac{1}{2}$ mA, and $T=4$°K. Thus from (3.11), since $I_0(a_2)\sim e^{a_2}$ the dynamic resistance at the center of the step will be exceedingly small. From (3.12) we find that the voltage deviation from the center of the step will be about 10^{-17} V, the upper limit found by Clarke, only when $a_1/a_2\simeq0.97$ or 97% of the height of the step.

Parker *et al.*[3] found for their tunnel junctions and point contacts of low resistance ($R<0.1\ \Omega$) that, in agreement with Clarke, the microwave-induced steps had negligible voltage width. They also found that as the resistance of the junction was increased from about 0.1 to 1 Ω, the voltage width of the step increased from about 10 to 200 nV. If we assume that this measured width is at 95% of the height of the step and take $j_2=\frac{1}{2}$ mA and $R=0.1\ \Omega$, then from (3.12) we find $a_2\simeq300$. Using the above values of j_2 and R together with $J=50$ mA, $V_0=6.10^{-4}$ V, and $T=4$°K, which are typical for the junctions used in Ref. 3, we find $a_2\simeq400$. It should be noted that the dynamic resistance at the center of the step is still exceedingly small. In Fig. 1 a plot of the noise voltage ΔV, calculated from (3.12), is given for various values of the parameter a_2.

The effects of finite capacitance on the noise voltage have not been investigated in detail, but it would be expected that ΔV would be larger in this case than $C=0$.

Note added in proof. Recently, T. F. Finnegan, A. Denenstein, D. N. Langenberg, J. C. McMenamin, D. E. Novoseller, and L. Cheng [Phys. Rev. Letters **23**, 229 (1969)] have shown that the "nonvertical" steps observed by Parker *et al.* are not an intrinsic property of the high-resistance junctions. If care is taken to screen out external noise the steps are vertical in agreement with the results found here.

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Tunneling Investigation of Energy-Gap Anisotropy in Superconducting Bulk Pb†*

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A technique has been developed for tunneling from thin Pb films into Pb single crystals formed by a high-vacuum melting process. A preliminary investigation of the energy-gap anisotropy has been carried out. Data are presented from ten separate junctions, including ones on the (001) and (111) crystal facets. In all cases, two energy gaps, Δ_1 and Δ_2 ($\Delta_1<\Delta_2$), were apparent in the characteristics and differed from each other by 10–15%. The maximum variation of Δ_1 was 5% over the tunneling directions studied, while Δ_2 varied by considerably less than this. The maximum observed value of $2\Delta_2(0)$ was 2.78 ± 0.01 meV and the minimum value of $2\Delta_1(0)$ was 2.36 ± 0.01 meV. A comparison was made with a published theory of the energy-gap anisotropy. In some respects the agreement is fairly good, while in other respects it is poor. No evidence was found for the predicted structure due to critical points of the energy-gap surface in **k** space. Second-derivative measurements (d^2V/dI^2 versus V) showed an extra (compared to results from thin-film Pb junctions) peak in the group of structures associated with the transverse phonon modes.

PREVIOUS tunneling investigations[1,2] of energy-gap anisotropy in superconducting Pb have been carried out using thick-film (thickness≫coherence length) specimens. In such cases the tunneling direction is not known and this limits the information about anisotropy which can be obtained. Specimens involving a single crystal do not suffer from this limitation but the technical difficulties are considerably increased. A technique has now been developed for tunneling from thin Pb films into Pb single crystals formed by a high vacuum (10^{-8}–10^{-7} Torr) melting process.[3] Using the technique, a preliminary investigation of the energy-gap anisotropy has been carried out.

The crystals form in the shape of oblate spheroids (~1 cm maximum diameter) and the surfaces compare favorably with evaporated films as far as cleanliness and smoothness are concerned. Laue back-reflection photographs gave a mosaic spread of 1° or less and resistivity ratio measurements by the eddy-current decay method gave ratios of greater than 5000. Flat facets (1–2 mm in diam) develop on the (001) and (111) faces and are very convenient for forming tunnel junctions in these directions.

Masking of the crystal surface was achieved by painting on G.E.[4] varnish except for a narrow (~$\frac{1}{2}$ mm wide) strip. The exposed strip was then oxidized in O_2 at atmospheric pressure and 60°C for about 16 h. After cooling to room temperature two narrow Pb films[5] (1500 Å thick×$\frac{1}{2}$ mm wide) were evaporated at right angles to the oxidized strip to complete the tunnel junctions. Copper electrical leads were attached to the films and the crystal using a conducting Epoxy[6] which was cured in air at 50°C for 1–2 h. The ratio of good to shorted junctions has been about 1 to 3 thus far and in only one case were the two junctions at a given crystal both of good quality.

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[3] E. Menzel, Rept. Progr. Phys. **26**, 47 (1963).

[4] General Electric Corp. No. 7031. Thinned to about 50% with a 50:50 mixture of toluene and methanol.

[5] The energy gap Δ_F of the thin films was determined independently from thin-film specimens deposited on glass slides. $\Delta_F=1.39\pm0.005$ MeV, where the error represents the precision of the number but not absolute accuracy. The films were thin enough to make Δ_F isotropic.

[6] Eccobond 56C. Emerson and Cuming Corp., Canton, Mass.