for $20 \lesssim s \lesssim 60$ BeV^2. The second term gives the rate of decrease with energy of $\sigma_{\rm tot}$. Equation (17) fits the experimental values[12] of $\sigma_{\rm tot}(pp)$ well.

[12] K. J. Foley *et al.*, Phys. Rev. Letters **11**, 425 (1963).

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Regge Poles in Scalar-Vector Field Theory*

DON SHAPERO
Laboratory of Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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It is shown that to fourth order, the asympotic behavior of the photon-meson helicity amplitudes in scalar electrodynamics may be viewed as arising from two nearby Regge poles. The photon-meson scattering amplitude is examined to fourth order in the limit $(-z) \to \infty$, where z is the cosine of the scattering angle. The two highest powers of $\ln(-z)$ are retained. By comparing these forms order by order with those predicted by the presence of two Regge poles, two trajectories are obtained, one of which passes through the meson.

I. INTRODUCTION

It has long been known that bound states in potential theory appear as poles in complex J in the partial-wave amplitudes for the scattering of the bound particle. It seems natural to expect similar behavior in relativistic field theory. Early investigations of scalar field theory showed that such poles indeed exist, but that they cannot be associated with the particles in the theories studied. This is true because the poles occur near $J=-1$ in scalar theories.

It was later observed[1] that spin-1 particles could move the location of the pole up by one unit of angular momentum. In the case of spin-$\frac{1}{2}$–spin-1 theory, this led[2] to the conclusion that the spin-$\frac{1}{2}$ particle did in fact lie on a Regge trajectory. Thus, what in zero order appears to be a fixed singularity of the form δ_{L0} (where $L=J-\frac{1}{2}$) actually turns out to be a moving pole of the form $-\alpha/(L-\alpha)$. Crucial to this result is the presence of a nonsense channel and the unique factorization of the Born approximation. In the case of spin-0–spin-1 field theory, the same conclusion cannot be drawn, because the Born amplitude does not factor, so that the residues of the Regge pole fail to factor. Therefore the pole cannot be associated with a single physical particle.[3]

In the present paper we show that in the spin-0–spin-1 case, the asymptotic behavior of the scattering amplitude may be viewed as arising from two nearby Regge poles. One of the trajectories passes through the spin-0 particle and the other does not. We use the same method to determine the trajectories as that employed in Ref. 2. The presence of a Regge pole leads to an asymptotic form for scattering amplitudes of $(-z)^\alpha$ as $(-z)$ becomes large. If we suppose that α is of the order of e^2, then this asymptotic form has the expansion

$$\beta(-z)^\alpha=\beta\sum_m \alpha^m[\ln(-z)]^m/m!.$$

This expansion is compared order by order with the perturbation expansion of the scattering amplitude under study. In our case, we assume the presence of two Regge poles and determine the parameters α_1, α_2, β_1, and β_2 by calculating the asymptotic behavior of the helicity amplitudes for photon-meson scattering to fourth order in scalar electrodynamics. We give the photon a mass λ to avoid infrared difficulties.

II. SCALAR-VECTOR SCATTERING IN PERTURBATION THEORY

We will consider the scattering of a massive vector meson from a scalar meson in the field theory with an interaction of the form

$$L_i=-ie:\phi^\dagger\overleftrightarrow{\partial}_\mu\phi:A^\mu+e^2:A_\mu A^\mu::\phi^\dagger\phi:,$$

where ϕ is the field of the scalar meson and A^μ is the field of the vector meson.[4]

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[1] M. Gell-Mann and M. L. Goldberger, Phys. Rev. Letters **9**, 275 (1962).

[2] M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B145 (1964).

[3] M. Gell-Mann, M. L. Goldberger, F. E. Low, V. Singh, and F. Zachariasen, Phys. Rev. **133**, B161 (1964).

[4] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill Book Co., New York, 1965).

The leading second-order diagrams[3] as $z\rightarrow\infty$ in the gauge of Ref. 2 are shown in Fig. 1. The asymptotic second-order helicity amplitudes corresponding to these diagrams are, using the notation of Gell-Mann *et al.*[2] and the conventions of Jacob and Wick,[5]

$$8\pi W f_{00}{}^{+(2)}\sim -2e^2[a^2/(s-m^2)-2\lambda^2/k^2],$$
$$8\pi W f_{00}{}^{-(2)}\sim 0,$$
$$8\pi W f_{10}{}^{+(2)}\sim -2e^2[-ab/(s-m^2)-2\lambda\omega/\sqrt{2}k^2]/z,$$
$$8\pi W f_{10}{}^{-(2)}\sim 0,$$
$$8\pi W f_{11}{}^{+(2)}\sim -2e^2[b^2/(s-m^2)-\omega^2/k^2]/z,$$
$$8\pi W f_{11}{}^{-(2)}\sim -2e^2/z.$$

The mass of the scalar meson is m and that of the vector meson is λ. The definitions of a and b are given in the Appendix.

The leading fourth-order diagrams are shown in Fig. 2. We omit the contact term in the vector-meson propagator in our calculations. This is permissible since the theory is invariant with respect to internal gauge. The sum of the diagrams in Fig. 2 is finite under these circumstances, and we can proceed to extract the leading terms as t becomes large. The contributions of Figs. 2(a) and 2(c) dominate that of Fig. 2(b).

The leading terms for both Figs. 2(a) and 2(c) are of the form $(\ln t)^2$. However, the leading term of Fig. 2(a) exactly cancels that of Fig. 2(c). This result can easily be obtained using the methods of Federbush and Grisaru.[6] Thus, we must obtain the next to the leading term for these diagrams. This can be done by examining the Mellin transform with respect to $(-t)$ of the amplitudes for Figs. 2(a) and 2(c).[7] This yields the following fourth-order contributions to the helicity amplitudes:

$$8\pi W f_{00}{}^{+(4)}\sim -2G\ln(-t)[\tfrac{1}{2}a^2I_0+2acI_1+2c^2I_2+f\lambda^2/k^2],$$
$$8\pi W f_{00}{}^{-(4)}\sim 0,$$
$$8\pi W f_{10}{}^{+(4)}\sim -2G\ln(-t)[-\tfrac{1}{2}abI_0-(bc-ad)I_1+2cdI_2+f\lambda\omega/\sqrt{2}k^2]/z,$$
$$8\pi W f_{10}{}^{-(4)}\sim 0,$$
$$8\pi W f_{11}{}^{+(4)}\sim -2G\ln(-t)[\tfrac{1}{2}b^2I_0-2bdI_1+2d^2I_2+f\omega^2/k^2]/z,$$
$$8\pi W f_{11}{}^{-(4)}\sim Gf\ln(-t)/z,$$

where

$$G=4\pi^2e^4/(2\pi)^4,$$
$$f=I'-\ln(\lambda^2)-\tfrac{1}{2},$$

and I_n and a, b, c, and d are defined in the Appendix.

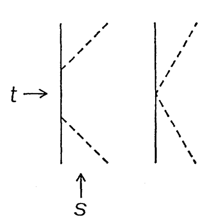

FIG. 1. Leading second-order diagrams. Vector-meson lines are dashed; scalar-meson lines are solid.

[5] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).
[6] P. Federbush and M. Grisaru, Ann. Phys. (N. Y.) **22**, 263 (1963).
[7] R. J. Eden *et al.*, *The Analytic S-Matrix* (Cambridge University Press, Cambridge, England, 1966).

III. ASYMPTOTIC BEHAVIOR PREDICTED BY PRESENCE OF TWO REGGE POLES

Assume the presence of two Regge poles with trajectory functions $\alpha_1{}^+$ and $\alpha_2{}^+$ in the partial-wave amplitudes $F_{ij}{}^{J+}$:

$$F_{00}{}^{J+}=\alpha_1{}^+(\alpha_1{}^++1)\eta_0{}^2/(J-\alpha_1{}^+)+\alpha_2{}^+(\alpha_2{}^++1)\eta_0'^2/(J-\alpha_2{}^+),$$
$$F_{01}{}^{J+}/[J(J+1)]^{1/2}=\eta_0\eta_1/(J-\alpha_1{}^+)+\eta_0'\eta_1'/(J-\alpha_2{}^+),$$
$$F_{11}{}^{J+}=\eta_1{}^2/(J-\alpha_1{}^+)+\eta_1'^2/(J-\alpha_2{}^+).$$

Assume also that F^- has one simple pole with residue $\kappa_1{}^2$. This implies the following for the scattering amplitudes:

$$f_{00}{}^+=(2\alpha_1{}^++1)\bar{P}_{\alpha_1{}^+}(-z)\alpha_1{}^+(\alpha_1{}^++1)\eta_0{}^2/\sin\pi\alpha_1{}^+ +(2\alpha_2{}^++1)\bar{P}_{\alpha_2{}^+}(-z)\alpha_2{}^+(\alpha_2{}^++1)\eta_0'^2/\sin\pi\alpha_2{}^+,$$
$$f_{10}{}^+=-(2\alpha_1{}^++1)\bar{P}_{\alpha_1{}^+}'(-z)\eta_0\eta_1/\sin\pi\alpha_1{}^+ -(2\alpha_2{}^++1)\bar{P}_{\alpha_2{}^+}'(-z)\eta_0'\eta_1'/\sin\pi\alpha_2{}^+,$$
$$\begin{aligned}f_{11}{}^+=&(2\alpha_1{}^++1)[\bar{P}_{\alpha_1{}^+}'(-z)-z\bar{P}_{\alpha_1{}^+}''(-z)]\\&\times\eta_1{}^2/\alpha_1{}^+(\alpha_1{}^++1)\sin\pi\alpha_1{}^+\\&+(2\alpha_2{}^++1)[\bar{P}_{\alpha_2{}^+}'(-z)-z\bar{P}_{\alpha_2{}^+}''(-z)]\\&\times\eta_1'^2/\alpha_2{}^+(\alpha_2{}^++1)\sin\pi\alpha_2{}^+\\&-(2\alpha^-+1)\bar{P}_{\alpha^-}''(-z)\kappa_1{}^2/\alpha^-(\alpha^-+1)\sin\pi\alpha^-.\end{aligned}$$

At integral J, $\bar{P}_J=P_J$. At other J, $\bar{P}_J$ is defined by Eq. (B5) of Ref. 2. Using the formula

$$\bar{P}_\alpha(z)\sim\Gamma(\alpha+\tfrac{1}{2})(2z)^\alpha/\pi^{1/2}\Gamma(\alpha+1)=N(\alpha)z^\alpha,$$

which is asymptotically correct as $z\rightarrow\infty$, we obtain the formulas

$$f_{00}{}^+\sim(2\alpha_1{}^++1)\alpha_1{}^+(\alpha_1{}^++1)\eta_0{}^2N(\alpha_1{}^+)(-z)^{\alpha_1{}^+}/\sin\pi\alpha_1{}^+ +(2\alpha_2{}^++1)\alpha_2{}^+(\alpha_2{}^++1)\eta_0'^2N(\alpha_2{}^+)(-z)^{\alpha_2{}^+}/\sin\pi\alpha_2{}^+,$$
$$f_{10}{}^+\sim-(2\alpha_1{}^++1)\eta_0\eta_1N(\alpha_1{}^+)\alpha_1{}^+(-z)^{\alpha_1{}^+-1}/\sin\pi\alpha_1{}^+ -(2\alpha_2{}^++1)\eta_0'\eta_1'N(\alpha_2{}^+)\alpha_2{}^+(-z)^{\alpha_2{}^+-1}/\sin\pi\alpha_2{}^+,$$
$$\begin{aligned}f_{11}{}^+\sim&(2\alpha_1{}^++1)\eta_1{}^2N(\alpha_1{}^+)(\alpha_1{}^+)^2\\&\times(-z)^{\alpha_1{}^+-1}/\alpha_1{}^+(\alpha_1{}^++1)\sin\pi\alpha_1{}^+\\&+(2\alpha_2{}^++1)\eta_1'^2N(\alpha_2{}^+)(\alpha_2{}^+)^2\\&\times(-z)^{\alpha_2{}^+-1}/\alpha_2{}^+(\alpha_2{}^++1)\sin\pi\alpha_2{}^+.\end{aligned}$$

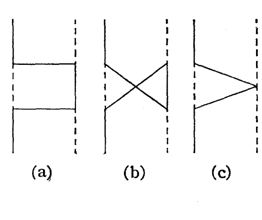

FIG. 2. Leading fourth-order diagrams.

These expressions can be rewritten in the form

$$8\pi W f_{00}{}^{+} \sim \beta_{00}(-z)^{\alpha_1{}^+} + \beta_{00}{}'(-z)^{\alpha_2{}^+}$$
$$8\pi W f_{10}{}^{+} \sim -\beta_{10}(-z)^{\alpha_1{}^+-1} - \beta_{10}{}'(-z)^{\alpha_2{}^+-1},$$
$$8\pi W f_{11}{}^{+} \sim \beta_{11}(-z)^{\alpha_1{}^+-1} + \beta_{11}{}'(-z)^{\alpha_2{}^+-1},$$

where $\beta_{ij}=\gamma_i\gamma_j$, and the γ_i are proportional to η_i.

If we now assume that α is of order e^2, we can expand these formulas in a perturbation series:

$$8\pi W f_{00}{}^{+} \sim \beta_{00} + \beta_{00}{}' + \beta_{00}\alpha_1{}^+ \ln(-t) + \beta_{00}{}'\alpha_2{}^+ \ln(-t) + \cdots,$$
$$8\pi W f_{10}{}^{+} \sim z^{-1}[\beta_{10} + \beta_{10}{}' + \beta_{10}\alpha_1{}^+ \ln(-t) + \beta_{10}{}'\alpha_2{}^+ \ln(-t) + \cdots],$$
$$8\pi W f_{11}{}^{+} \sim -z^{-1}[\beta_{11} + \beta_{11}{}' + \beta_{11}\alpha_1{}^+ \ln(-t) + \beta_{11}{}'\alpha_2{}^+ \ln(-t) + \cdots].$$

IV. TRAJECTORY AND RESIDUE FUNCTIONS FOR TWO REGGE POLES IN SCALAR-VECTOR SCATTERING

We have calculated the following six numbers:

$$8\pi W f_{00}{}^{+(2)} = A = \beta_{00} + \beta_{00}{}',$$
$$z 8\pi W f_{10}{}^{+(2)} = B = \beta_{10} + \beta_{10}{}',$$
$$-z 8\pi W f_{11}{}^{+(2)} = C = \beta_{11} + \beta_{11}{}',$$
$$8\pi W f_{00}{}^{+(4)}/\ln(-t) = D = \beta_{00}\alpha_1{}^+ + \beta_{00}{}'\alpha_2{}^+,$$
$$z 8\pi W f_{10}{}^{+(4)}/\ln(-t) = E = \beta_{10}\alpha_1{}^+ + \beta_{10}{}'\alpha_2{}^+,$$
$$-z 8\pi W f_{11}{}^{+(4)}/\ln(-t) = F = \beta_{11}\alpha_1{}^+ + \beta_{11}{}'\alpha_2{}^+.$$

If we suppose that α_1 and α_2 are given, then these equations can be solved in pairs:

$$\beta_{00} = (\alpha_2{}^+A - D)/(\alpha_2{}^+ - \alpha_1{}^+),$$
$$\beta_{00}{}' = (\alpha_1{}^+A - D)/(\alpha_1{}^+ - \alpha_2{}^+),$$
$$\beta_{10} = (\alpha_2{}^+B - E)/(\alpha_2{}^+ - \alpha_1{}^+),$$
$$\beta_{10}{}' = (\alpha_1{}^+B - E)/(\alpha_1{}^+ - \alpha_2{}^+),$$
$$\beta_{11} = (\alpha_2{}^+C - F)/(\alpha_2{}^+ - \alpha_1{}^+),$$
$$\beta_{11}{}' = (\alpha_1{}^+C - F)/(\alpha_1{}^+ - \alpha_2{}^+).$$

We now require that β and β' factor. That is,

$$\beta_{ij} = \gamma_i\gamma_j,$$
$$\beta_{ij}{}' = \gamma_i{}'\gamma_j{}'.$$

This occurs when $\beta_{01}{}^2 = \beta_{00}\beta_{11}$ and $(\beta_{01}{}')^2 = \beta_{00}{}'\beta_{11}{}'$. This requirement leads to quadratic equations for α_1 and α_2:

$$(\alpha_1{}^+B - E)^2 = (\alpha_1{}^+A - D)(\alpha_1{}^+C - F),$$
$$(\alpha_2{}^+B - E)^2 = (\alpha_2{}^+A - D)(\alpha_2{}^+C - F).$$

Since the equation is the same for $\alpha_1{}^+$ and $\alpha_2{}^+$, they must be the two roots of the quadratic form

$$(B^2 - AC)x^2 + (AF + DC - 2BE)x + (E^2 - DF).$$

These two roots are given by the formula

$$\alpha_{\pm}{}^{+} = \{-(AF+DC-2BE) \pm [(AF+DC-2BE)^2 - 4(B^2-AC)(E^2-DF)]^{1/2}\}/2(B^2-AC).$$

Since the Born approximation fails to factor, $B^2 \neq AC$, and this expression is well defined.

Near $s=m^2$, A has the form $A_0/(s-m)^2$. Referring to the formula for $\alpha_{\pm}{}^{+}$, we see that $\alpha_{+}{}^{+}(m)^2=0$, so that the positive root is the trajectory of the meson.

Relating the residues of the Regge poles to coupling constants in the usual way,[1] we find that the helicity-0–helicity-0–spin-0 coupling constant is given by

$$g_{000}{}^2 = \alpha_+{}^+\beta_{00}/8\pi s\alpha_+{}^{+\prime}.$$

At $s=m^2$, β_{00} is of the form $A_0/(s-m^2)$, so that for the $+$ trajectory,

$$\alpha_+{}^+\beta_{00} \to \alpha_+{}^{+\prime}A_0,$$

and

$$g_{000}{}^2(m^2) = A_0/8\pi m^2 = 3(e^2/4\pi).$$

It is this singularity of the underlying field theory at $s=m^2$ which leads to a coupled Regge particle. If the trajectory functions pass through zero at other places, a coupled particle will not result.

V. CONSISTENCY IN SIXTH ORDER

In sixth order, we speculate that f_{00} will approach $(\ln t)^2$ as t becomes large. If this is true, then, using the residue and trajectory functions obtained in fourth order, we can predict the leading sixth-order asymptotic form. But this form can also be calculated from the appropriate sixth-order diagrams. This calculation would be very lengthy, but it could be done. If the two forms agree, then the idea that there are two Regge poles present would be reinforced.

If the two forms do not agree, then it would be necessary to introduce new parameters. One way to do this might be the following: In order to renormalize the theory, one must introduce an interaction of the form $g(\phi^\dagger\phi)^2$, where $g=Ae^2+Ze^4$. If one chooses $A=0$, then Z is a divergent constant. Under these circumstances the extra interaction does not affect asymptotic behavior. If, on the other hand, A is chosen to be a finite constant, then a Z can still be found which renormalizes the theory (for certain values of A, in fact, Z is 0), but now the asymptotic behavior will depend on A. The fourth-order diagrams depending on A are shown in Fig. 3. These diagrams, which lack a power of t in the numerator, have the asymptotic

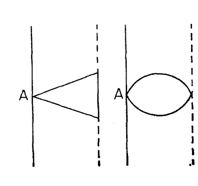

FIG. 3. Leading fourth-order diagrams depending on A.

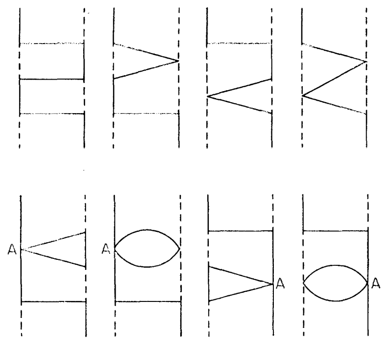

FIG. 4. Some leading sixth-order diagrams.

form $(\ln t)^2/t$. Thus Figs. 2(a) and 2(c) still dominate in fourth order.

Some relevant sixth-order diagrams are shown in Fig. 4. In an actual calculation, it would be necessary to add in all diagrams with crossed subdiagrams and also radiative corrections to fourth-order diagrams. By adjusting the parameter A, the helicity amplitudes calculated from these diagrams might be made equal to the Regge form.

VI. CONCLUSIONS

We have seen that, to fourth order, the asymptotic behavior of the scalar-vector helicity amplitudes in scalar electrodynamics can be viewed as arising from two nearby Regge poles, one of which passes through the scalar particle. This is possible because in second order, f_{00} approaches a constant as t becomes large, and in fourth order, by virtue of a cancellation of the leading $\ln^2$ terms, f_{00} approaches $\ln t$. Thus, by a choice of trajectory functions which is uniquely determined by requiring factorization of residues, we can produce this asymptotic behavior by two Regge poles.

In sixth order, a consistency problem arises, which may be dealt with by a method suggested by renormalization requirements. The final answer to this question depends upon a detailed investigation of sixth-order asymptotic behavior.

ACKNOWLEDGMENT

The author would like to express his appreciation to Professor K. Johnson for suggesting this problem and, in particular, for suggesting the method discussed of introducing an extra parameter in sixth order.

APPENDIX

$$a=k\lambda[1-(s-m^2)/k^2]/\omega\,,$$

$$b=(s-m^2)/\sqrt{2}k\,,$$

$$c=\lambda(s-m^2+\lambda^2)/2\omega k\,,$$

$$d=(s-m^2+\lambda^2)/2\sqrt{2}k\,,$$

$$I_n=\int_0^1 dx\, x^n/[m^2x+\lambda^2(1-x)-sx(1-x)]\,,$$

$$I'=\int_0^1 dx\,\ln[m^2x+\lambda^2(1-x)-sx(1-x)]$$

$$=\ln(m^2)-(m^2-\lambda^2-s)I_1-2sI_2\,,$$

$$\bar{P}_J=-\pi^{-1}Q_{-J-1}\tan J\pi\,.$$