Schwinger Terms, Field Algebra, the Parity-Violating Internucleon Potential, and Models of the Weak Hamiltonian

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Using the current-field identity, it is shown that the entire vector-meson-exchange contribution to the parity-violating internucleon potential V_{12} derives from Schwinger terms arising from the commutator of the $\Delta S=0$ F-spin currents and the weak Hamiltonian H_w. The vector-meson-exchange contributions are evaluated using field-algebra commutation relations and are shown to reproduce the generalized Michel potential. These results are used to express V_{12} as a function of an arbitrary model of H_w.

RECENTLY the one-pion-exchange (OPE) contribution to the parity-violating (p-v) internucleon potential V_{12} was evaluated,[1–3] and its dependence on models of the weak-interaction Hamiltonian density $H_w(x)$ explored.[3,4] It was shown[4] that the predictions for representative models of $H_w(x)$ were sufficiently different from one another that a choice among the models could be made given an experimental knowledge of OPE contribution alone. This contribution can in principle be distinguished from that of vector-meson (ρ,ω,ϕ) exchange (which, along with OPE, dominates V_{12}) by its space-spin-isospin transformation properties.[3,5] However, present experimental data exist only for nuclear transitions where both OPE and OVE (one-vector-meson exchange) may be expected to contribute, and hence it is necessary to know the model dependence of OVE in addition to that of OPE. The evaluation of the OVE contribution reduces to a calculation of the p-v amplitudes $N(p)\to N(p')+V(q)$ for $q^2\simeq 0$, if we assume that the *strong* NNV vertices are described by phenomenological Lagrangian densities such as

$$L_{\rho NN}=f_\rho \boldsymbol{\varrho}_\mu\cdot i\bar{N}\left(\gamma_\mu+\frac{i\mu_N}{2m_N}\sigma_{\mu\nu}\partial_\nu\right)\frac{\boldsymbol{\tau}}{2}N\,, \qquad (1)$$

where μ_N is the nucleon anomalous magnetic moment and $f_\rho^2/4\pi\simeq 2.4$. In the present paper these amplitudes are evaluated using the current-field identity (CFI) and are shown to derive entirely from Schwinger terms[6] arising from the commutator of the $\Delta S=0$ F-spin currents and H_w. When these Schwinger terms are evaluated using field-algebra commutation relations, the p-v NNV amplitudes are identical to those previously obtained from the factorization approximation [see Eq. (4) below]. These results are used to derive the vector-meson (1^-) exchange contribution to V_{12} as a function of the parameters which characterize an arbitrary model of H_w. Since the model dependence of pseudoscalar-meson (0^-) exchange has been derived previously, the present results permit the (presumably dominant) 0^- and 1^- contributions to V_{12} to be expressed as functions of an arbitrary model of H_w. The resulting V_{12} is given explicitly in Eqs. (13)–(17).

Consider, for the sake of definiteness, the p-v vertex $n\to p\rho^-$. The amplitude for the weak p-v emission of a ρ^- with momentum q is given by

$$\begin{aligned}A(N\to N\rho^-)&=\langle N\rho^-|H_w^{\text{p-v}}(0)|N\rangle=\epsilon_\lambda^*M_\lambda^{(+)}\\&=i\epsilon_\lambda^*(q^2+m_\rho^2)(2q_0V)^{-1/2}\int d^4x e^{-iq\cdot x}\\&\quad\times\langle N(p')|T(\rho_\lambda^{(+)}(x)H_w^{\text{p-v}}(0))|N(p)\rangle\,, \qquad (2)\end{aligned}$$

where $\rho_\lambda^{(+)}(x)$ is the operator which annihilates a ρ^+, ϵ_λ^* is the polarization vector for the emitted ρ^-, and m_ρ is its mass. On general covariance grounds, we can write

$$\begin{aligned}M_\lambda^{(+)}=i(m_N^2/2q_0p_0p_0'V^2)^{1/2}\bar{u}(p')[\gamma_\lambda h_A(q^2)\\+\sigma_{\lambda\nu}q_\nu h_E(q^2)+iq_\lambda h_P(q^2)]\gamma_5\tau^{(+)}u(p)\,. \qquad (3)\end{aligned}$$

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By isospin conservation, the term proportional to h_P makes no contribution to V_{12}. The ρ^- contribution has previously been evaluated for the Cabibbo model in the factorization approximation[7]

$$\langle N\rho^-|H_w{}^{\text{p-v}}(0)|N\rangle \simeq (G/\sqrt{2})\cos^2\theta\,\langle N|A_\lambda{}^{(+)}(0)|N\rangle \times\langle\rho^-|V_\lambda{}^{(-)}(0)|0\rangle, \quad (4)$$

which gives[8]

$$h_A(0)=(G/\sqrt{2})\cos^2\theta\,(\sqrt{2}m_\rho{}^2G_A/f_\rho), \quad (5)$$

and $h_E=0$ (by G invariance). Equations (1) and (5) yield the so-called Michel potential[9]

$$V_{12}{}^{\rho\pm}=\frac{-h_A(0)f_\rho}{8\pi\sqrt{2}m_N}\{i(1+\mu_p-\mu_n)\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\cdot[\mathbf{p}_{12},e^{-m_\rho r_{12}}/r_{12}]_- +(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot[\mathbf{p}_{12},e^{-m_\rho r_{12}}/r_{12}]_+\}T_{12}{}^{(+)}, \quad (6)$$

where $T_{12}{}^{(\pm)}=\tau_1{}^{(+)}\tau_2{}^{(-)}\pm\tau_1{}^{(-)}\tau_2{}^{(+)}$, and $\mathbf{p}_{12}=\mathbf{p}_1-\mathbf{p}_2$ is the relative momentum operator. We wish to show that Eq. (5) is an exact consequence of field algebra.[10]

Differentiating $M_\lambda{}^{(+)}$ in Eq. (2) and using the CFI[11]

$$\rho_\lambda{}^{(\alpha)}(x)=(f_\rho/m_\rho{}^2)V_\lambda{}^{(\alpha)}(x), \quad \alpha=1,2,3 \quad (7)$$

[where $V_\lambda{}^{(\alpha)}(x)$ is the isospin current], we find that

$$q_\lambda M_\lambda{}^{(+)}=(2q_0V)^{-1/2}(f_\rho/m_\rho{}^2)(q^2+m_\rho{}^2)2^{-1/2} \times\int d^4x\,e^{-iq\cdot x}\delta(x_0)\langle p|[V_0{}^{(+)}(x),H_w{}^{\text{p-v}}(0)]_-|n\rangle. \quad (8)$$

The commutation relation in Eq. (8) assumes the form

$$[V_0{}^{(\alpha)}(\mathbf{x},0),H_w{}^{\beta\gamma}(0)]_-=i\delta^3(\mathbf{x})[f^{\alpha\beta\delta}J_\lambda{}^{(\delta)}(\mathbf{x})V_\lambda{}^{(\gamma)}(0) +f^{\alpha\gamma\delta}V_\lambda{}^{(\beta)}(0)J_\lambda{}^{(\delta)}(\mathbf{x})]+\text{S.t.}, \quad \alpha,\beta,\gamma,\delta=1,\ldots,8 \quad (9)$$

for any current-current model or any schizon model to order G, where $J_\lambda{}^{(\delta)}=V_\lambda{}^{(\delta)}+A_\lambda{}^{(\delta)}$, and S.t. denotes as yet unspecified Schwinger terms arising from the commutator of V_0 with J_k ($k=1,2,3$). Defining the isospin-rotated $H_w{}^{\text{p-v}}(0)$ by $H_w{}^{\text{p-v}\prime}(0)$, we have from Eqs. (8) and (9)

$$q_\lambda M_\lambda{}^{(+)}=(2q_0V)^{-1/2}(f_\rho/m_\rho{}^2)(q^2+m_\rho{}^2) \times\left(\langle p|H_w{}^{\text{p-v}\prime}(0)|n\rangle+\int d^3x\,e^{-i\mathbf{q}\cdot\mathbf{x}}\langle p|\text{S.t.}|n\rangle\right). \quad (10)$$

We note that if $H_w{}^{\text{p-v}}$ is CP-conserving, the first term in Eq. (10) vanishes in the $SU(2)$ limit,[12] and hence the entire contribution to $q_\lambda M_\lambda{}^{(+)}$ at $q=0$ (and hence to $V_{12}{}^{\rho\pm}$) comes from Schwinger terms.

The structure of the Schwinger terms in Eq. (9) depends strongly on field-theoretic models for the vector and axial-vector currents. In the particular model of field algebra,[10] the Schwinger terms can be evaluated explicitly and the relevant commutation relations are

$$[V_0{}^{(\alpha)}(\mathbf{x},0),V_k{}^{(\beta)}(\mathbf{x}',0)]_-=if^{\alpha\beta\gamma}V_k{}^{(\gamma)}(\mathbf{x},0)\delta^3(\mathbf{x}-\mathbf{x}') -i\delta^{\alpha\beta}(m_\rho/f_\rho)^2\partial_k\delta^3(\mathbf{x}-\mathbf{x}'), \quad (11)$$
$$[V_0{}^{(\alpha)}(\mathbf{x},0),A_k{}^{(\beta)}(\mathbf{x}',0)]_-=if^{\alpha\beta\gamma}A_k{}^{(\gamma)}(\mathbf{x},0)\delta^3(\mathbf{x}-\mathbf{x}'),$$

where $V_\lambda(\mathbf{x},0)$ and $A_\lambda(\mathbf{x},0)$ are the polar and axial-vector currents. From Eqs. (10) and (11) it follows that $q_\lambda M_\lambda{}^{(+)}$ depends solely on the piece of H_w transforming as $J^{\pi+}J^{\pi-}$ ($J^{\pi\pm}=J^{1\pm i2}$). Since H_w must be a unitary symmetric function of $J^{\pi+}J^{\pi-}$, we recover the $|\Delta\mathbf{I}|=0$, 2 selection rule[5] for the $\rho^\pm$ contribution to V_{12}. It remains to evaluate $h_A(0)$ explicitly. From Eqs. (10) and (11), we have

$$q_\lambda M_\lambda{}^{(+)}=-i(2q_0V)^{-1/2}(f_\rho/m_\rho{}^2)(q^2+m_\rho{}^2)(2/\sqrt{2})(m_\rho/f_\rho)^2 \times(G\cos^2\theta/\sqrt{2})\int d^3x\,e^{-i\mathbf{q}\cdot\mathbf{x}}\partial_k\delta^3(\mathbf{x})\langle p|A_k{}^{\pi+}(0)|n\rangle$$
$$=i(m_N{}^2/2q_0p_0p_0'V^2)^{1/2}(\sqrt{2}/f_\rho)(q^2+m_\rho{}^2)(G\cos^2\theta/\sqrt{2}) \times\bar{u}(p')[\boldsymbol{\gamma}\cdot\mathbf{q}F_A(\mathbf{q}^2)+i\mathbf{q}^2F_P(\mathbf{q}^2)]\gamma_5\tau^{(+)}u(p), \quad (12)$$

where F_A and F_P are the usual axial-vector and pseudoscalar form factors of the nucleon and $F_A(0)\equiv G_A$.[13] Assuming that $|\mathbf{q}|/m_N\ll1$ [and hence retaining only terms linear in $|\mathbf{q}|$ in Eqs. (3) and (12)], we find that

$$h_A(0)=(G/\sqrt{2})\cos^2\theta\,(\sqrt{2}m_\rho{}^2G_A/f_\rho),$$

which is just Eq. (5). This establishes the claimed result and indicates that the (seemingly naive) factorization approximation of Eq. (4) is actually an exact consequence of field algebra. We note, in passing, that the term in Eq. (10) which vanishes by virtue of CP invariance contains the formally divergent contribution to the weak $NN\rho^\pm$ amplitude.[14]

[7] It is assumed, in addition, that for small q^2 the polar-vector form factors are dominated by ρ, ω, and ϕ. In terms of a W-boson model this approximation corresponds to the one-W-exchange contribution with form factors.

[8] G_A is the usual nucleon axial-vector form factor and θ is the Cabibbo angle.

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[13] The noncovariance of Eq. (12) is a direct consequence of the fact that the T product in Eq. (2) is not covariant, and may be remedied by adding to the T product an appropriate Schwinger term. See, e.g., J. D. Bjorken, Phys. Rev. **148**, 1467 (1966). Note that the contributions introduced by this "covariance" term are at least quadratic in $|\mathbf{q}|/m_N$ and hence are negligible from a practical point of view.

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TABLE I. Summary of weak Hamiltonian parameters.[a]

Model	Parameters[b] A	B	C	C'	D
Conventional	$(-1/\sqrt{2})\tan\theta$	0	0	0	0
Extra current	$(1/\sqrt{2})\cot\theta$	2	$-2/\sqrt{3}$	$-2/\sqrt{3}$	$\frac{2}{3}$
Segrè γ_5-invariant	0	0	0	0	$\frac{4}{3}$
Segrè γ_5-noninvariant	-4	0	0	$-2/\sqrt{3}$	$\frac{4}{3}$
Lee	$(1/\sqrt{8})\cot\theta$	1	0	$-\sqrt{3}$	0

[a] Models are defined in Ref. 4.
[b] We assume that $1\pm\sin^2\theta\cong 1$.

The $NN\rho^0$, $NN\phi$, and $NN\omega$ vertices may be treated in a similar fashion, the only additional complication being that the neutral vector mesons can give rise to both isoscalar and isovector amplitudes, in contrast to $\rho^\pm$ which yield only isovector amplitudes. We have calculated the p-v NNV amplitudes in the exact $SU(3)$ limit and hence have neglected any effects of ω-ϕ mixing. In addition we have assumed that the $SU(3)$-symmetric vector mass m_V is approximately equal to m_ρ in order to give the vector-meson contribution to V_{12} a realistic spatial dependence characterized by $e^{-m_\rho r_{12}}/r_{12}$. A complete discussion of the effects of $SU(3)$-symmetry breaking will be published elsewhere. The vector-meson-exchange contribution to V_{12} is then given by

$$
\begin{aligned}
V_{12}{}^V=&(-h_A f_\rho/8\pi\sqrt{2}m_N)(i\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\cdot[\mathbf{p}_{12},e^{-m_\rho r_{12}}/r_{12}]_-\\
&\times\{(1+\mu_p-\mu_n)[T_{12}{}^{(+)}+\tfrac{1}{4}B\tau_1{}^{(z)}\tau_2{}^{(z)}\\
&+\tfrac{1}{4}C\xi(\tau_1{}^{(z)}\tau_2{}^{(0)}+\tau_2{}^{(z)}\tau_1{}^{(0)})]+(1+\mu_p+\mu_n)\\
&\times(\tfrac{1}{8}\sqrt{3})C'(\tau_1{}^{(z)}\tau_2{}^{(0)}+\tau_2{}^{(z)}\tau_1{}^{(0)})+\tfrac{1}{2}\sqrt{3}D\xi\tau_1{}^{(0)}\tau_2{}^{(0)}\}\\
&+(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot[\mathbf{p}_{12},e^{-m_\rho r_{12}}/r_{12}]_+\{T_{12}{}^{(+)}+\tfrac{1}{4}B\tau_1{}^{(z)}\tau_2{}^{(z)}\\
&+\tfrac{1}{2}\sqrt{3}D\xi\tau_1{}^{(0)}\tau_2{}^{(0)}\}+[\mathbf{p}_{12},e^{-m_\rho r_{12}}/r_{12}]_+\\
&\cdot[\tfrac{1}{4}\sqrt{3}C'(\boldsymbol{\sigma}_1\tau_1{}^{(z)}\tau_2{}^{(0)}-\boldsymbol{\sigma}_2\tau_2{}^{(z)}\tau_1{}^{(0)})\\
&\qquad+\tfrac{1}{2}C\xi(\boldsymbol{\sigma}_1\tau_2{}^{(z)}\tau_1{}^{(0)}-\boldsymbol{\sigma}_2\tau_1{}^{(z)}\tau_2{}^{(0)})]),
\end{aligned}
\tag{13}
$$

where f_ρ and h_A are the constants defined in Eqs. (1) and (5), $\boldsymbol{\sigma}_{1,2}$ are the spin operators for the two nucleons, and $\tau^{(0)}$ is the 2×2 isospin-identity matrix. The constants B, C, C', and D define the contribution to $H_w{}^{\text{p-v}}(\Delta S=0)$ from the nonstrange F-spin currents (which are the only currents that contribute to $V_{12}{}^V$):

$$
\begin{aligned}
H_w{}^{\text{p-v}}(\Delta S=0)\equiv&(G/\sqrt{2})\cos^2\theta[(V_\lambda{}^{\pi+}A_\lambda{}^{\pi-}+V_\lambda{}^{\pi-}A_\lambda{}^{\pi+})\\
&+BV_\lambda{}^{\pi0}A_\lambda{}^{\pi0}+CV_\lambda{}^{\pi0}A_\lambda{}^{\eta}+C'V_\lambda{}^{\eta}A_\lambda{}^{\pi0}\\
&+DV_\lambda{}^{\eta}A_\lambda{}^{\eta}+\cdots],\\
V_\lambda{}^{\pi0}=&\mathfrak{F}_\lambda{}^{(3)},\quad V_\lambda{}^{\eta}=\mathfrak{F}_\lambda{}^{(8)},\ \text{etc.}
\end{aligned}
\tag{14}
$$

Finally, the parameter ξ in Eq. (13) is defined by

$$
\begin{aligned}
&\langle N(\nu_3)|\mathfrak{F}_{5\lambda}(\nu_2)|N(\nu_1)\rangle\\
&=\left\{\begin{pmatrix}8 & 8 & 8_s\\ \nu_1 & \nu_2 & \nu_3\end{pmatrix}D+\begin{pmatrix}8 & 8 & 8_a\\ \nu_1 & \nu_2 & \nu_3\end{pmatrix}F\right\}\bar{u}(p')\gamma_\lambda\gamma_5 u(p)\\
&\xi\equiv\frac{1}{\sqrt{(12)}}\left(1-\frac{2(1-F/D)}{1+F/D}\right).
\end{aligned}
\tag{15}
$$

A recent analysis by Brene *et al.*[15] suggests the value $F/D\cong\frac{1}{2}$ and hence $\xi\cong 1/6\sqrt{3}$. The complete contribution to V_{12} from 0^- and 1^- exchanges is then given by $V_{12}=V_{12}{}^V+V_{12}{}^\pi$, where $V_{12}{}^\pi$ is the potential[4,16]

$$
\begin{aligned}
V_{12}{}^\pi&=A\Gamma(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot[\mathbf{p}_{12},e^{-m_\pi r_{12}}/r_{12}]_- T_{12}{}^{(-)},\\
\Gamma&=\frac{1.25\times10^5\ \text{sec}^{-1/2}g_{\pi NN}m_\pi{}^{-1/2}}{8\pi\sqrt{2}m_N}.
\end{aligned}
\tag{16}
$$

In Eq. (16) the parameter A is defined by

$$
H_w{}^{\text{p-v}}=AT_{0,1,0}{}^{(8)}+T_{1,1/2,-1,2}{}^{(8)},\quad T_\nu=T_{Y,I,I_3}. \tag{17}
$$

We see that the dominant contributions to V_{12}, which derive from the exchange of 0^- and 1^- mesons, can be expressed as a function of the five parameters A, B, C, C', and D. Table I gives the values of these parameters for the models of H_w discussed in Ref. 4.

In conclusion, we wish to emphasize again that since the vector-meson contributions to V_{12} can be distinguished from those of $\pi^\pm$, the structure of the Schwinger terms in Eq. (11) can be tested experimentally through a study of the matrix elements of V_{12}. Details of calculations along these lines are presently under way and will be published elsewhere. We also wish to stress that since the Schwinger-term contribution exactly reproduces the (nonzero) contribution from W exchange,[7] our (nonvanishing) result for 1^- exchange cannot be ascribed to a neglect of possible "seagull" terms as has been suggested recently by several authors.[17]

Note added in manuscript. After completing this work, we learned that a treatment of the weak p-v $NN\rho$ vertex similar to ours has been given by Feuer.[17] We wish to thank Dr. G. Scharff-Goldhaber for bringing Dr. Feuer's work to our attention.

Note added in proof. In response to our work, Olesen and Rao (Ref. 17) have suggested that the vector-meson-exchange diagrams do not contribute to V_{12} because the Schwinger terms are canceled by seagull terms. We observe that if this were true, it would mean that the right-hand side of Eq. (4) derived from the factorization approximation should be canceled by some other terms. It is hard to see how this cancellation could occur, however, since the factorization approximation for the matrix element $\langle N_1'N_2'|V_\mu(x)A_\mu(x)|N_1N_2\rangle$ is finite and reproduces the results of a W meson pole diagram. We also note that the cancellation of seagull and Schwinger terms is generally thought to be a consequence, and not a proof, of the condition $q_\lambda M_\lambda=0$.

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[16] Recall that neutral scalar (or pseudoscalar) mesons make no contribution to V_{12} as was first noted by G. Barton, Nuovo Cimento **19**, 512 (1961).
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At present it is not clear that such a condition should be imposed on the parity-violating weak interaction NN_p vertex. These points will be discussed more fully elsewhere.

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Direct-Channel Resonances in Antiproton-Proton Elastic Scattering*

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The effect of a direct-channel resonance on antiproton-proton elastic scattering is investigated. The differential cross section has a nonresonant term, a pure resonant term, and an interference term. It is not possible to say much about the interference term. The angular shape of the pure resonant term depends upon the resonance quantum numbers C and P, as well as the spin J, but in general resembles the square of the Legendre polynomial $P_J(\theta)$. The possibility of a background amplitude in the resonant partial waves affects the enhancements in the total elastic and total cross sections.

I. INTRODUCTION

THERE has been increasing interest recently in the search for high-mass bosons appearing as direct-channel resonances in the nucleon-antinucleon system, and in particular in backward elastic scattering.[1–3]

In the present paper, we investigate the resulting antiproton-proton elastic scattering in the presence of a direct-channel resonance. One would like to be able to deduce the elasticity and quantum numbers of such a resonance from the observed angular distributions. However, the large nonresonant background amplitude, and consequent interference term, plus the variety of different quantum numbers attainable in this channel, make such deduction difficult.

The differential cross section can be written as the sum of a nonresonant (background) term, a pure resonant term, and an interference term. We can say little about the interference term. But it is still useful to study the pure resonant term, especially at backward scattering angles, where experimentally the cross section is small. We can write down explicit expressions for the pure resonant term, depending upon the quantum numbers of the resonance. We find that this term, in general, resembles $|P_J(\theta)|^2$, where J is the spin of the resonance [$P_J(\theta)$ is the Legendre polynomial, θ is the c.m. scattering angle].

The method we follow makes use of fairly standard results which can be found in papers by Blatt and Biedenharn[4] and Dalitz.[5] The results are immediately applicable to the charge exchange reaction $\bar{p}p \rightarrow \bar{n}n$ and, after correcting for a simple isospin factor, to antiproton-neutron elastic scattering. Other two-body final states, for example, $\bar{p}p \rightarrow \pi^+\pi^-$, can be investigated in a similar way.

II. GENERAL EXPRESSION FOR ELASTIC SCATTERING

We use a standard partial-wave expansion of the elastic scattering amplitude. Since proton and antiproton are both spin-$\frac{1}{2}$ particles, there are 16 spin amplitudes (we assume unpolarized beam and target). Actually, 6 of these 16 amplitudes are zero, and only 5 of the remainder are independent.

The elastic scattering amplitude is written[4,6]

$$A_\nu = i(2k)^{-1} \sum_{J,l_i,l_f} \{[4\pi(2l_i+1)]^{1/2} i^{l_i-l_f} \langle l_i s_i 0 m_s | J m_s\rangle \times\langle l_f s_f m_f m_s' | J m_s\rangle Y_{l_f}{}^{m_f}(\theta,\varphi) \times 0.5[(\delta_{fi}-S_{fi}{}^{T=1})+(\delta_{fi}-S_{fi}{}^{T=0})]\}, \quad (1)$$

where ν is a spin index, standing for (s_i,m_s,s_f,m_s'); clearly, $\nu=1$–16. Six of the A_ν are zero—say for $\nu=11$–

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