Regge Trajectory of the Pion in the $N\bar{N}$ Bethe-Salpeter Equation*

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In the region near $t=0$, we solve the $O(4)$ projected $N\bar{N}$ Bethe-Salpeter equation for the pion's Regge trajectory. We find that there are two Regge trajectories with the same quantum numbers as the pion which lie above $J=-1$. At $t=0$, the highest-lying trajectory has Toller quantum number $M=1$. The other Regge trajectory lies somewhat further below and has the Toller quantum number $M=0$. In addition to the intercepts, we also calculate the slopes of these trajectories.

I. INTRODUCTION

In the region near $t=0$, there are currently two approaches followed in the fitting of experimental data in the reactions $\gamma p\to\pi^+ n$, $np\to pn$, $\pi N\to\rho\Delta$, and their corresponding finite-energy sum rules. One approach proposes pion conspiracy while the other employs Regge cuts as well as poles. In this work we adopt the former approach and hence implicitly assume that the effects of the cuts are small.

With this assumption that the cuts can be neglected, experimental and theoretical evidence seems to indicate that there are two nearby Regge trajectories with the same quantum numbers as the pion. The forward peaks in pion photoproduction[1] and np charge exchange[2] have been interpreted as the exchange of a "pion" Regge trajectory with Toller quantum number $M=1$. The forward peaks in the reactions $\pi N\to\rho\Delta$ and $\pi N\to f^\circ\Delta$ are explained by the exchange of a pion Regge trajectory with $M=0$, as the polarizations of the ρ and f° have been measured to be almost totally longitudinal.[3,4] The result of finite-energy sum rule calculations have the pion Regge trajectory being $M=1$ near $t=0$.[5] However, as has been noted many times previously, the physical pion must lie on an $M=0$ trajectory.[6,7] Thus, the experimental and theoretical evidence points to the fact that the exact nature of the pion's Regge trajectory near $t=0$ is quite complicated.

In a previous publication, we have developed the formalism necessary to study the nature of the pion's Regge trajectory near $t=0$ in the context of the $N\bar{N}$ Bethe-Salpeter (BS) equation.[8] In this paper we wish to report the results of the numerical solution of the $N\bar{N}$ BS equation in the ladder approximation. We follow the work of Scotti and Wong,[9] using the exchanges of the σ, π, η, ρ, ω, and φ mesons as the forces which bind the nucleon-antinucleon system.[10] We find that there are two Regge trajectories with the same quantum numbers as the pion which lie above $J=-1$. At $t=0$ the highest-lying trajectory has the Toller quantum number $M=1$. The other trajectory which lies quite far below the $M=1$ trajectory has $M=0$. The on-shell contribution of the $M=1$ trajectory to the $\pi N\bar{N}$ vertex corresponds to that of the parent of Freedman and Wang[11] (FW) type III, while the contribution of the $M=0$ trajectory corresponds to that of the first daughter of FW type II. The $M=0$ trajectory which would contribute to the parent of FW type I is not present.

With an $M=1$ pion trajectory lying highest at $t=0$, trajectory-mixing models have been postulated in order to obtain a physical pion.[7,12] The qualitative nature of the trajectory mixing for the two trajectories which we find to be present has previously been examined.[7,8] Since the $M=0$ trajectory lies far below $J=0$, it must have a very large slope if it is to produce the physical pion. We find that the coupling of the $M=0$ trajectory to the $N\bar{N}$ system alone is insufficient to give such a large slope.

In Sec. II, we discuss the $N\bar{N}$ BS equation and briefly review its $O(4)$ projection. In Sec. III, we perform the $O(4)$ projection of the meson exchanges kernels which we use to bind the $N\bar{N}$ system. The numerical results for the two Regge trajectories are given in Sec. IV.

II. THE $N\bar{N}$ BS EQUATION

At zero-momentum transfer the scattering amplitude is invariant under transformations of the symmetry group $O(3,1)$, and hence is diagonal in the representa-

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[7] W. R. Frazer, H. M. Lipinski, and D. R. Snider, Phys. Rev. **174**, 1932 (1968).

[8] H. M. Lipinski and D. R. Snider, Phys. Rev. **176**, 2054 (1968).

[9] A. Scotti and D. Y. Wong, Phys. Rev. **138**, B145 (1965); J. S. Ball, A. Scotti, and D. Y. Wong, *ibid.* **142**, 1000 (1966).

[10] The exchange of the σ meson, even though it is a two-pion resonance, is included directly in the kernel of the equation. Since we use a simple propagator for pion exchange, we cannot expect that its iteration in the BS equation will produce the strong $I=0$ s-wave binding in the two-pion system that is associated with the σ meson.

[11] D. Z. Freedman and J. M. Wang, Phys. Rev. **160**, 1560 (1967).

[12] R. Sugar and R. Blankenbecler, Phys. Rev. Letters **20**, 1014 (1968).

tion functions of the group. As a result, Regge trajectories at $t=0$ are classified by the additional quantum numbers n and M, the Casimir operators of the symmetry group. Away from $t=0$, Regge trajectories are, in general, mixtures of representations with different n's and M's. However, if one adopts an off-shell approach and expands the off-shell scattering amplitude in a power series in t, then for a Regge trajectory with a given n and M at $t=0$, the admixing of other representations away from $t=0$ follows a regular pattern.[13] To a given power of t, only a finite number of representations are mixed in. It is possible, in this way, to study the nature of Regge trajectory away from $t=0$. Although the mixing of representations is of a general nature, it is necessary to adopt a model in order to obtain quantitative results. One such model for studying the nature of the pion's Regge trajectory is the $N\bar{N}$ BS equation.

We make use of the fact that the nucleon-nucleon BS equation can be used to obtain the results for the nucleon-antinucleon system if the signs of the odd-G-parity meson exchanges are changed.[8,9] We therefore consider nucleon-nucleon scattering in the t channel with the T matrix defined by

$$\langle\lambda_1'\lambda_2'|S|\lambda_1\lambda_2\rangle=1-i(2\pi)^4\delta^4(p_1'+p_2'-p_1-p_2)\times[M^2/(E_1'E_2'E_1E_2)^{1/2}]\langle\lambda_1'\lambda_2'|T|\lambda_1\lambda_2\rangle. \quad (2.1)$$

The off-shell amplitude M is obtained from the T matrix by removing the Dirac spinors

$$\langle\lambda_1'\lambda_2'|T(p',p,k)|\lambda_1\lambda_2\rangle=\bar{u}_{\alpha'}(\tfrac{1}{2}k+p',\lambda_1')\bar{u}_{\beta'}(\tfrac{1}{2}k-p',\lambda_2')\times M_{\alpha'\beta'\alpha\beta}(p',p,k)u_\alpha(\tfrac{1}{2}k+p,\lambda_1)u_\beta(\tfrac{1}{2}k-p,\lambda_2), \quad (2.2)$$

where $k=(\sqrt{t},0,0,0)$ is the c.m. momentum and p' and p are the final and initial relative four-momenta. The amplitude M then satisfies the BS equation

$$M_{\alpha'\beta'\alpha\beta}(p',p,k)=B_{\alpha'\beta'\alpha\beta}(p',p,k)+\frac{i}{(2\pi)^4}\int d^4q\, M_{\alpha'\beta'\gamma\delta}(p',q,k)\times S_{\gamma\gamma'}(\tfrac{1}{2}k+q)S_{\delta\delta'}(\tfrac{1}{2}k-q)B_{\gamma'\delta'\alpha\beta}(q,p,k), \quad (2.3)$$

where iS is the nucleon propagator and $-iB$ is the sum of the off-shell contributions of the one-particle-exchange diagrams. It is more convenient for the projection of the BS equation into $O(4)$ partial waves to work with the amplitude R defined by

$$R_{\alpha'\beta'\alpha\beta}(p',p,k)\equiv M_{\alpha'\beta'\gamma\delta}(p',p,k)S_{\gamma\alpha}(\tfrac{1}{2}k+p)S_{\delta\beta}(\tfrac{1}{2}k-p). \quad (2.4)$$

In order to obtain a Euclidean metric, we perform a Wick rotation of the contour of integration in Eq. (2.3) and make the definitions $k^0=ik^4$, $p^0=ip^4$, etc. The four-momenta are now Euclidean four-vectors and the integration is over four-dimensional Euclidean space. The BS equation is

$$R_{\alpha'\beta'\gamma\delta}(p',p,k)S^{-1}{}_{\gamma\alpha}(\tfrac{1}{2}k+p)S^{-1}{}_{\delta\beta}(\tfrac{1}{2}k-p)=-\tilde{B}_{\alpha'\beta'\alpha\beta}(p',p,k)+\frac{1}{(2\pi)^4}\int d^4q\, R_{\alpha'\beta'\gamma'\delta'}(p',q,k)\tilde{B}_{\gamma'\delta'\alpha\beta}(q,p,k), \quad (2.5)$$

where $\tilde{B}\equiv -B$.

We may now project the BS equation into $O(4)$ partial waves using the representation functions developed in Ref. 8 (henceforth denoted LS). We let Υ represent the set of $O(4)$ quantum numbers, $\Upsilon=\{nMJm\Sigma\omega\kappa\}$. The quantum numbers n and M are the Casimir quantum numbers of the group $O(4)$, J and m are the usual $O(3)$ angular momenta quantum numbers, Σ is the additional spin index necessary to specify the state, and ω and κ are indices which are related to the transformation of the $O(4)$ NN state under parity and particle interchange. The projected BS equation is

$$R^{\Upsilon'\Upsilon''}(P',P,t)\langle\Upsilon''|S^{-1}(\tfrac{1}{2}k+p)S^{-1}(\tfrac{1}{2}k-p)|\Upsilon\rangle=-\tilde{B}^{\Upsilon'\Upsilon}(P',P,t)+\frac{1}{(2\pi)^4}\int_0^\infty Q^3dQ\, R^{\Upsilon'\Upsilon''}(P',Q,t)\tilde{B}^{\Upsilon''\Upsilon}(Q,P,t), \quad (2.6)$$

where P', P, and Q are the magnitudes of the four-momenta p', p, and q. The matrix elements of the inverse propagator $\langle\Upsilon''|S^{-1}S^{-1}|\Upsilon\rangle$ are calculated by LS. We continue the equation in the angular momentum plane holding $n'-J=\mathcal{K}'$ and $n-J=\mathcal{K}$ integer.[8] The Regge trajectories correspond to the solutions of the homogeneous equation. The BS equation giving the projected equations for the $M=0$ and $M=1$ pion trajectories has already been given in LS. In order to solve these equations, we must still project the one-particle-exchange diagrams into $O(4)$ partial waves.

III. $O(4)$ PROJECTION OF $N\bar{N}$ BS KERNEL

A. σ Exchange

The kernel of the BS equation consists of the sum of the diagrams corresponding to the exchanges of the σ, π, η, ρ, ω, and φ mesons. The $O(4)$ kernels corresponding to the exchanges of the π, η, ρ, ω, and φ mesons can be related to the $O(4)$ kernels of scalar (σ) exchange by noting that the kernel for a meson exchange can be written as

$$f_{\alpha'\beta'\alpha\beta}(p',p,\mu_m)\times 1/[(p'-p)^2+\mu_m^2],$$

where f is the product of the numerator of the propagator and the appropriate vertex functions. In projecting a kernel of this type, we insert a complete set of states between f and the scalar propagator, calculate

[13] W. R. Frazer, F. R. Halpern, H. M. Lipinski, and D. R. Snider, Phys. Rev. 176, 2047 (1968).

the projection of f, and then sum over the intermediate states. The kernels for σ exchange have been calculated by LS, but for the sake of clarity and completeness we include them here also.

The Lagrangian for a σ meson of mass μ_σ is

$$\mathcal{L}_\sigma=(4\pi)^{1/2}g_\sigma\bar{\psi}\varphi_\sigma\psi. \tag{3.1}$$

The off-shell contribution to the kernel $\tilde{B}$ is

$$\tilde{B}_\sigma=4\pi g_\sigma^2 I\otimes I/[(p'-p)^2+\mu_\sigma^2]. \tag{3.2}$$

The kernel is independent of t and hence diagonal in the $O(4)$ representation functions. The $O(4)$ projection of the kernel is therefore of the form

$$\langle\Upsilon'|\tilde{B}_\sigma|\Upsilon\rangle=4\pi g_\sigma^2\delta_{n'n}\delta_{M'M}\delta_{J'J}\delta_{m'm}\times\tilde{B}_{\Sigma'\omega'\kappa',\Sigma\omega\kappa}{}^{(n,M)}(P',P,\mu_\sigma). \tag{3.3}$$

In the dependence upon the indices of the $O(4)$ state, it was found in LS that the kernels $\tilde{B}$ are diagonal in κ and depend only on the product of ω' and ω. As a result, we make the definition

$$\tilde{B}_{\Sigma'\omega'\kappa',\Sigma\omega\kappa}{}^{(n,M)}(P',P,\mu_\sigma)=\delta_{\kappa'\kappa}\tilde{B}_{\Sigma'\Sigma,\zeta\kappa}{}^{(n,M)}(P',P,\mu_\sigma),\quad \zeta=\omega'\omega. \tag{3.4}$$

The kernels for scalar exchange are then

$$\begin{aligned}
\tilde{B}_{00,++}{}^{(n,0)}&=\tilde{B}_{11,++}{}^{(n,0)}=\tilde{B}_{11,+-}{}^{(n,1)}\\
&=\tilde{B}_{11,+-}{}^{(n,-1)}=[1/(n+1)]F_n(z),\\
\tilde{B}_{00,+-}{}^{(n,0)}&=[1/(n+1)]zF_n(z),\\
\tilde{B}_{11,+-}{}^{(n,0)}&=[1/n(n+1)(n+2)]\{n^2z+2(n+1)\\
&\quad\times[z+(z^2-1)^{1/2}]\}F_n(z),\\
\tilde{B}_{01,--}{}^{(n,0)}&=\tilde{B}_{10,--}{}^{(n,0)}\\
&=\{-1/(n+1)[n(n+2)]^{1/2}\}\\
&\quad\times[z+(n+1)(z^2-1)^{1/2}]F_n(z),\\
\tilde{B}_{11,++}{}^{(n,1)}&=\tilde{B}_{11,++}{}^{(n,-1)}=[1/n(n+2)]\\
&\quad\times[(n+1)z+(z^2-1)^{1/2}]F_n(z),\\
\tilde{B}_{11,-+}{}^{(n,1)}&=-\tilde{B}_{11,-+}{}^{(n,-1)}=[1/n(n+2)]\\
&\quad\times[z+(n+1)(z^2-1)^{1/2}]F_n(z),
\end{aligned} \tag{3.5}$$

where

$$F_n(z)=(2\pi^2/P'P)[z+(z^2-1)^{1/2}]^{-n-1} \tag{3.6}$$

and

$$z=(P'^2+P^2+\mu_\sigma^2)/2P'P.$$

A number of these kernels possess a fixed pole at $n=0$ which is independent of the mass of the exchanged particle. These poles correspond to δ function and derivative of δ-function terms in the potential. To remove these fixed poles, we replace the scalar propagator by[14]

$$1/[(p'-p)^2+\mu_\sigma^2]-1/[(p'-p)^2+\Lambda_\sigma^2]. \tag{3.7}$$

[14] By removing these fixed poles, the connection between our treatment and the usual perturbation theory becomes unclear. We regard the BS equation obtained by subtracting these fixed poles as a model in which we are able to calculate Regge trajectories. We would expect that the trajectories calculated in this way to be independent of our model.

This cutoff not only removes the fixed poles, but makes the kernel better behaved for large P, P'. Since the kernels for the meson exchanges π, η, ρ, ω, and φ are expressed in terms of the kernels for scalar exchange, the fixed poles associated with σ meson exchange will arise in all the meson-exchange kernels. Proceeding as in the case of σ exchange, we replace the scalar propagator term in these exchanges by

$$1/[(p'-p)^2+\mu_m^2]-1/[(p'-p)^2+\Lambda_m^2], \tag{3.8}$$

where μ_m, and Λ_m are the mass and cutoff mass of the exchanged meson. In order to introduce as few parameters as possible into the problem, we set $\Lambda_m^2=C_p\mu_m^2$, where C_p is a same constant for all the exchanges. The σ-meson-exchange kernels are now the difference of kernels Eq. (3.5) corresponding to Eq. (3.8).

B. π Exchange

The π-meson Lagrangian is

$$\mathcal{L}_\pi=(4\pi)^{1/2}g_\pi\bar{\psi}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\varphi}_\pi\psi. \tag{3.9}$$

The off-shell kernel corresponding to the Lagrangian is

$$\tilde{B}_\pi=4\pi g_\pi^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\gamma_5\otimes\gamma_5\times I\otimes I/[(p'-p)^2+\mu_\pi^2]. \tag{3.10}$$

With the insertion of a complete set of states, the projection of the $\gamma_5\otimes\gamma_5$ numerator is

$$\langle\Upsilon'|\gamma_5\otimes\gamma_5|\Upsilon''\rangle=\kappa'\delta_{\Upsilon'\Upsilon''}. \tag{3.11}$$

Summing over the intermediate states, we obtain

$$\langle\Upsilon'|\tilde{B}_\pi|\Upsilon\rangle=4\pi g_\pi^2(\mathcal{P}_1-3\mathcal{P}_0)\delta_{n'n}\delta_{M'M}\delta_{J'J}\delta_{m'm}\delta_{\kappa'\kappa}\kappa\times\tilde{B}_{\Sigma'\Sigma,\zeta\kappa}{}^{(n,M)}(P',P,\mu_\pi), \tag{3.12}$$

where we have expressed $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$ in terms of the isotopic spin projection operators $\mathcal{P}$ of the two-nucleon state. In changing from the NN to the $N\bar{N}$ system, we must change the sign of the pion-exchange contribution and, since we wish to solve for the pion's Regge trajectory, we must take the $I=1$ part of the exchange Eq. (3.12). The pion contribution to the kernel of the $N\bar{N}$ BS equation for the pion is

$$\langle\Upsilon'|\tilde{B}_\pi|\Upsilon\rangle=-4\pi g_\pi^2\delta_{n'n}\delta_{M'M}\delta_{J'J}\delta_{m'm}\delta_{\kappa'\kappa}\kappa\times\tilde{B}_{\Sigma'\Sigma,\zeta\kappa}{}^{(n,M)}(P',P,\mu_\pi). \tag{3.13}$$

C. η Exchange

The Lagrangian corresponding to η-meson exchange is

$$\mathcal{L}_\eta=(4\pi)^{1/2}g_\eta\bar{\psi}\gamma_5\varphi_\eta\psi. \tag{3.14}$$

The contribution to $\tilde{B}$ is

$$\tilde{B}_\eta=4\pi g_\eta^2\gamma_5\otimes\gamma_5\times I\otimes I/[(p'-p)^2+\mu_\eta^2]. \tag{3.15}$$

The kernels are identical to those of pion exchange except that their sign does not change in going from NN to $N\bar{N}$ scattering and no isotopic spin factors are pres-

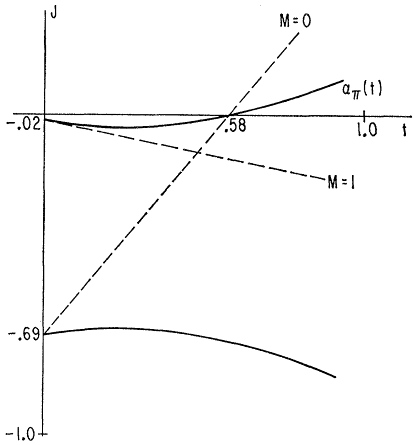

FIG. 1. Chew-Frautschi plot showing the Regge trajectories (solid lines) calculated from the simple two-trajectory-mixing model. The dashed lines are the trajectories in the absence of mixing.

ent. The projected kernel is

$$\langle\Upsilon'|\tilde{B}_\eta|\Upsilon\rangle=4\pi g_\eta^2\delta_{n'n}\delta_{M'M}\delta_{J'J}\delta_{m'm}\delta_{\kappa'\kappa}\kappa \times\tilde{B}_{\Sigma'\Sigma,\zeta\kappa}{}^{(n,M)}(P',P,\mu_\eta). \quad (3.16)$$

D. ρ Exchange

The Lagrangian is given by

$$\mathcal{L}_\rho=i\pi^{1/2}(g_{\rho1}+g_{\rho2})\bar{\psi}\gamma_\nu\boldsymbol{\tau}\cdot\boldsymbol{\varphi}_\rho{}^\nu\psi -\pi^{1/2}(g_{\rho2}/2M)(q_\nu+q_\nu')\bar{\psi}\boldsymbol{\tau}\cdot\boldsymbol{\varphi}_\rho{}^\nu\psi, \quad (3.17)$$

where $g_{\rho1}$ and $g_{\rho2}$ are the vector and tensor coupling constants, q and q' the nucleon momenta, and M the nucleon mass. The vector charge coupling contribution to the off-shell kernel is

$$\tilde{B}_{\rho C}=-\pi(g_{\rho1}+g_{\rho2})^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \times[\gamma_\nu\otimes\gamma^\nu+(1/\mu_\rho^2)(\mathbf{p}'-\mathbf{p})\otimes(\mathbf{p}'-\mathbf{p})] \times I\otimes I/[(p'-p)^2+\mu_\rho^2]. \quad (3.18)$$

To obtain the projection of the numerator, we use

$$\langle\Upsilon'|\gamma_\nu\otimes\gamma^\nu|\Upsilon''\rangle=\delta_{n'n''}\delta_{M'M''}\delta_{J'J''}\delta_{m'm''}\delta_{\omega'\omega''}\omega \times[4\delta_{\kappa'+}\delta_{\kappa''+}\delta_{\Sigma'0}\delta_{\Sigma''0}+2(-1)^{1-\Sigma}\delta_{\kappa'-}\delta_{\kappa''-}], \quad (3.19)$$

and

$$\langle\Upsilon'|\mathbf{p}\otimes I|\Upsilon''\rangle=-P\delta_{n'n''}\delta_{M'M''}\delta_{m'm''}\times\delta_{\Sigma'\Sigma''}\delta_{\omega'\omega''}\delta_{\kappa'-\kappa''}$$
$$\langle\Upsilon'|I\otimes\mathbf{p}|\Upsilon''\rangle=-\omega'P\delta_{n'n''}\delta_{M'M''}\delta_{J'J''}\times\delta_{m'm''}\delta_{\Sigma'\Sigma''}\delta_{\omega'\omega''}\delta_{\kappa'-\kappa''} \quad (3.20)$$
$$\langle\Upsilon'|\mathbf{p}\otimes\mathbf{p}|\Upsilon''\rangle=\omega'P^2\delta_{\Upsilon'\Upsilon''}.$$

Summing over the intermediate states, the contribution of ρ vector charge exchange to the $I=1$ $N\bar{N}$ equation is

$$\langle\Upsilon'|\tilde{B}_{\rho C}|\Upsilon\rangle=-\pi(g_{\rho1}+g_{\rho2})^2\delta_{n'n}\delta_{M'M}\delta_{J'J}\delta_{m'm} \times\{4\omega\delta_{\Sigma'0}\delta_{\Sigma0}\delta_{\zeta+}\delta_{\kappa+}\delta_{M0}\tilde{B}_{00,++}{}^{(n,0)}(P',P,\mu_\rho) +2\omega(-1)^{1-\Sigma}\delta_{\kappa-}\tilde{B}_{\Sigma'\Sigma,\zeta-}{}^{(n,M)}(P',P,\mu_\rho) +(1/\mu_\rho^2)[(\omega P^2+\omega'P'^2)\tilde{B}_{\Sigma'\Sigma,\zeta\kappa}{}^{(n,M)}(P',P,\mu_\rho) -2P'P\omega\delta_{\zeta+}\tilde{B}_{\Sigma'\Sigma,+\kappa}{}^{(n,M)}(P',P,\mu_\rho)]\}. \quad (3.21)$$

Because of the presence of extra momentum factors in the numerator of the kernel, an additional cutoff will need to be introduced. These momentum factors arise from the off-shell part of the spin-1 propagator which vanishes on the mass shell. We note that these terms vanish when $P'=P$.[15] We cut off these kernels by a multiplicative factor $\exp[-C_V(P'-P)^2]$.

The tensor magnetic coupling contribution to the off-shell kernel is

$$\tilde{B}_{\rho M}=\pi(g_{\rho2}/2M)^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\{(k+p'+p)\cdot(k-p'-p) +(1/\mu_\rho^2)[(k+p'+p)\cdot(p'-p)(k-p'-p)\cdot(p'-p)]\} \times I\otimes I/[(p'-p)^2+\mu_\rho^2]. \quad (3.22)$$

The projection of $\tilde{B}_{\rho M}$ into the $I=1$ $O(4)$ $N\bar{N}$ state is

$$\langle\Upsilon'|B_{\rho M}|\Upsilon\rangle=-\pi(g_{\rho2}/2M)^2\delta_{J'J}\delta_{m'm}\{[t+2P^2+2P'^2+\mu_\zeta^2+(1/\mu_\rho^2)(P^2-P'^2)^2]\delta_{n'n}\delta_{M'M}B_{\Sigma'\Sigma,\zeta\kappa}{}^{(n,M)}(P',P,\mu_\rho)$$
$$+(t/\mu_\rho^2)[P^2\tilde{B}_{\Sigma'\Sigma,\rho\kappa}{}^{(n',M')}(P',P,\mu_\delta)\langle n'M'\Sigma|\cos^2\psi|nM\Sigma\rangle+P'^2\langle n'M'\Sigma'|\cos^2\psi|nM\Sigma'\rangle\tilde{B}_{\Sigma'\Sigma,\zeta\kappa}{}^{(n,M)}(P',P,\mu_\rho)$$
$$-2P'P\sum_{n''M''}\langle n'M'\Sigma'|\cos\psi|n''M''\Sigma'\rangle\tilde{B}_{\Sigma,\Sigma'\zeta\kappa}{}^{(n'',M'')}(P',P,\mu_\rho)\langle n''M''\Sigma|\cos\psi|nM\Sigma\rangle]\}. \quad (3.23)$$

The matrix elements of $\cos\psi$ have been calculated in Ref. 16. The explicit evaluation of the kernels in which these $\cos\psi$ matrix elements are present is carried out in Ref. 17. As in the projection of the ρ vector charge coupling kernel, there are terms in Eq. (3.23) which arise from the off-shell part of the spin-1 propagator. We cut off these terms with the factor $\exp[-C_V(P'-P)^2]$. Because of the momentum factors introduced by the magnetic coupling, we associate with each magnetic coupling vertex the factor $\exp[-C_M(P'+P)^2]$.

The mixed charge-magnetic coupling of ρ exchange gives rise to the kernel

$$\tilde{B}_{\rho C,\rho M}=-i\pi(g_{\rho1}+g_{\rho2})(g_{\rho2}/2M)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\{(\mathbf{k}-\mathbf{p}-\mathbf{p}')\otimes I+I\otimes(\mathbf{k}+\mathbf{p}+\mathbf{p}'') +(1/\mu_\rho^2)[k\cdot(p-p')((\mathbf{p}-\mathbf{p}')\otimes I+I\otimes(\mathbf{p}-\mathbf{p}'')) +(P^2-P'^2)(I\otimes(\mathbf{p}-\mathbf{p}')-(\mathbf{p}-\mathbf{p}')\otimes I)]\}\times I\otimes I/[(p'-p)^2+\mu_\rho^2]. \quad (3.24)$$

[15] The mass shell corresponds to $P'=P-iM(1-t/4M^2)^{1/2}$.

[16] H. M. Lipinski and D. R. Snider, Phys. Rev. **179**, 1315 (1969).

[17] H. M. Lipinski, thesis, University of California, San Diego (unpublished).

The projection of this kernel for $I=1$ is

$$\begin{aligned}\langle\Upsilon'|\tilde{B}_{\rho C,\rho M}|\Upsilon\rangle &= -i\pi(g_{\rho 1}+g_{\rho 2})(g_{\rho 2}/2M)\delta_{J'J}\delta_{m'm}\\ &\times\{2\delta_{n'n}\delta_{M'M}\delta_{\kappa'-\kappa}[\delta_{\omega-}P(1+(1/\mu_\rho^2)(P^2-P'^2))\tilde{B}_{\Sigma'\Sigma,\zeta\kappa'}{}^{(n,M)}(P',P,\mu_\zeta)\\ &+\delta_{\omega'-}P'(1-(1/\mu_\rho^2)(P^2-P'^2))\tilde{B}_{\Sigma'\Sigma,\zeta\kappa}{}^{(n,M)}(P',P,\mu_\rho)]-i(\sqrt{t})\sum_{\Upsilon''}\langle\Upsilon'|I\otimes\gamma^4+\gamma^4\otimes I|\Upsilon''\rangle B^{\Upsilon''\Upsilon}(P',P,\mu_\rho)\\ &+[2i(\sqrt{t})/\mu_\rho^2][\delta_{\omega+}\delta_{\kappa'-\kappa}(P^2B_{\Sigma'\Sigma,\zeta\kappa'}{}^{(n',M')}(P',P,\mu_\rho)\langle n'M'\Sigma|\cos\psi|nM\Sigma\rangle\\ &-P'P\langle n'M'\Sigma'|\cos\psi|nM\Sigma'\rangle B_{\Sigma'\Sigma,\zeta\kappa'}{}^{(n,M)}(P',P,\mu_\rho))+\delta_{\omega'+}\delta_{\kappa-\kappa'}(P'^2\langle n'M'\Sigma'|\cos\psi|nM\Sigma'\rangle B_{\Sigma'\Sigma,\zeta\kappa}{}^{(n,M)}(P',P,\mu_\rho)\\ &-P'P\tilde{B}_{\Sigma'\Sigma,\zeta\kappa}{}^{(n',M')}(P',P,\mu_\rho)\langle n'M'\Sigma|\cos\psi|nM\Sigma\rangle)]\}\,. \end{aligned}\quad(3.25)$$

We use the momentum cutoffs introduced previously to cut off the appropriate kernels.

E. ω and φ Exchange

The Lagrangian for ω and φ exchange is identical to that for ρ exchange except that $\boldsymbol{\tau}\cdot\boldsymbol{\varphi}$ is replaced by φ. We neglect the magnetic coupling because of the small isoscalar anomalous magnetic moment. The kernels are then identical to those of ρ vector charge exchange except that their sign must be changed as ω and φ have odd G parity.

IV. NUMERICAL RESULTS

For the one-meson-exchange kernels we use, in addition to the known meson masses and the pion-nucleon coupling constant $g_\pi^2=14$, the meson masses and coupling constants determined from the fitting of nucleon-nucleon and nucleon-antinucleon scattering data.[9] These values are

$$\begin{aligned}&\mu_\sigma=437\text{ MeV}, && \mu_\rho=591\text{ MeV}, && g_{\varphi 1}{}^2=2.26,\\ &g_\sigma{}^2=3.05, && g_{\rho 1}{}^2=1.27, && g_{\omega 1}{}^2=2.77,\\ &g_\eta{}^2=7.4, && g_{\rho 2}{}^2=11.4. &&\end{aligned}\quad(4.1)$$

The exact solution of the BS equation is still dependent on the three cutoffs introduced in Sec. III. With these cutoffs, the BS equation is a Fredholm integral equation with the Regge poles corresponding to the zeros of the Fredholm determinant. The particular values of the cutoff parameters for which we present numerical results are

$$C_p=9,\quad C_V=1,\quad C_M=0.01;\quad(4.2)$$

although for reasonable variation of these parameters, the nature of the solution is qualitatively unchanged.

At $t=0$, the BS equation decouples as a result of the $O(4)$ symmetry and the various Regge poles are labeled by the $O(4)$ Casimir quantum numbers n and M. There are three sets of amplitudes which have the quantum numbers of the pion, one set with $M=1$ and the other two with $M=0$.[7,8] Only two of these sets of amplitudes possess a Regge pole. The $M=1$ set of amplitudes which contributes to FW type-III parent has a Regge pole which lies highest and with our choice of cutoffs is in the vicinity of $J=0$. The other Regge pole occurs in the $M=0$ set of amplitudes which contribute to FW type-II first daughter. This Regge pole lies far below the $M=1$ Regge pole at about $J=-0.7$.[18] The remaining $M=0$ set of amplitudes which would contribute to FW type-I parent does not possess a Regge pole.

Away from $t=0$, there is no longer any $O(4)$ symmetry and as a result the various sets of amplitudes are coupled together. The Regge trajectories are then obtained by the solution of the entire set of coupled equations, and the Regge trajectories cannot be classified according to a particular $O(4)$ representation as was the case at $t=0$. Because of the complexity inherent in solving the entire set of coupled equations for the Regge trajectories, it is instructive as a first approximation to neglect the coupling between the $M=0$ and $M=1$ sets of amplitudes. The trajectories then calculated can be labeled by their M quantum number. The intercepts and slopes of the two trajectories are

$$\begin{aligned}&\alpha_{M=1}(0)=-0.02, && \alpha_{M=0}(0)=-0.69,\\ &\alpha_{M=1}'(0)=-0.2\text{ BeV}^{-2}, && \alpha_{M=0}'(0)=1.2\text{ BeV}^{-2},\end{aligned}\quad(4.3)$$

where we have adjusted our cutoff parameters so that the highest trajectory lies at $J=-0.02$. In the small-t region where our perturbation solution is valid, the inclusion of the coupling between the $M=0$ and $M=1$ trajectories has little effect on the trajectories given by Eq. (4.3) since the trajectories are so far apart. However, if we extrapolate these trajectories, assuming they remain linear even for large t, then they will cross, with the $M=0$ trajectory intersecting $J=0$ at $t=0.58$ BeV^2. In order to investigate the nature of the trajectory mixing, we adopt a simple two-trajectory algebraic model. In the simplified model, the Regge trajectories are given by the solutions of the equation

$$\det\begin{pmatrix}J-\alpha_0-\alpha_1 t & -(Jt)^{1/2}\\ (Jt)^{1/2} & J-\beta_1-\beta_1 t\end{pmatrix}=0,\quad(4.4)$$

where $\alpha_0=-0.02$, $\beta_0=-0.69$, and $\alpha_1=-0.2$, $\beta_1=1.2$ are the intercepts and slopes of the $M=1$ and $M=0$

[18] The parent of this trajectory has the quantum numbers of the A_1.

trajectories. The nature of the coupling between the two trajectories is inferred from the BS equation. The solution of Eq. (4.4) is given in Fig. 1. The pion trajectory which was $M=1$ at $t=0$ would then be $M=0$ at $J=0$,[19] but with a mass which is far too large. We note that the pion trajectory cannot have a ghost at $J=0$ for negative t since it would be pure $M=1$ at that point. Further extrapolation of the trajectory for larger values[20] of negative t would not be justified as we have determined the slope of the trajectories only at $t=0$.

[19] The coupling of a trajectory which is the first daughter of type II to the $N\bar{N}$ system is $\gamma_\mu\gamma_5$.

[20] The pion trajectory calculated from the two-trajectory-mixing model crosses $J=1$ at $t=-2.4$ BeV^2.

We are led to conclude that the coupling of the pion trajectory to the $N\bar{N}$ channel alone is not sufficient to produce a physical pion with the correct mass. The inclusion of unequal-mass channels such as $\pi\rho$ and $\pi\sigma$ would tend to reduce the high mass for the pion because of the resulting increase in binding.

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Jost-Function Description for the Bethe-Salpeter Equation*

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A new formulation of the Bethe-Salpeter eigenvalue problem is presented. An integral representation of the eigenvalue condition is given which is a generalization of the familiar Jost function in nonrelativistic theory. The formalism is worked out in detail for the scalar-exchange potential and is used to examine Regge trajectories in the weak coupling limit. The detailed multiplicity of the lower trajectories is shown to depend on the behavior of the potential at the origin in coordinate space, and hence to be very potential-dependent.

I. INTRODUCTION

The Bethe-Salpeter (BS) equation has become a standard tool to use in developing an understanding of the structure of the relativistic scattering problem. Some specific problems that have been usefully discussed are the occurrence of bound states,[1] the behavior of phase shifts,[2] the properties of Regge poles,[3] and, in particular, the relevance of daughter trajectories and unequal-mass scattering.[4] A new interest has arisen in multiperipheral models[5] and the general connection between Feynman-diagram models and Regge models.[6]

In this paper a new and general approach will be developed for discussing the solution of the BS equation. It is a generalization of the Jost-function[7] approach which has proven so useful in discussions of the Schrödinger equation.[8] Since the BS equation in coordinate space involves a fourth-order differential operation, one must deal with a two-by-two matrix and its determinant rather than with the simple function used for second-order differential equations. The present paper will be restricted to a discussion at zero total energy, since the ability to expand in four-dimensional spherical harmonics will considerably simplify the discussion.

The standard BS equation looks as though it becomes singular if the potential behaves as badly as R^{-2} at the origin. However, it is well known that this is only an apparent singular behavior, not a real one. This will be discussed carefully below.

Our formalism will be applied to a perturbation calculation of the Regge trajectories at zero total energy. It will be found that, in general, the multiplicity of the lower trajectories increases quite rapidly as one goes down in the angular momentum plane. This was demonstrated for the first few trajectories by Swift,[9] Landshoff and Halliday,[10] and Caneschi.[11] A simple ex-

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