PHYSICAL REVIEW VOLUME 186, NUMBER 1 5 OCTOBER 1969

# Time-of-Flight Studies of Propagating Electron Plasma Waves in Tonks-Dattner Resonances*

S. Aksornkitti,[†] H. C. S. Hsuan, and K. E. Lonngren
*University of Iowa, Iowa City, Iowa* 52240
(Received 28 March 1969)

We describe here a series of time-of-flight experiments designed to investigate propagating electron plasma waves in a dc mercury-vapor discharge based upon the development of a phase-coherent nanosecond sine-wave burst. Signals of appreciable amplitude are observed to propagate with a finite velocity on the order of the electron thermal velocity and can be damped by increasing the electron neutral collision frequency. Multiple reflections of the signal are also detected.

If an electromagnetic wave is incident on a laboratory glass enclosed dc discharge plasma column ($\vec{k}$ and $\vec{E}$ both perpendicular to the axis of the column, $\lambda_{\text{free space}} > 2r_0$ where $r_0$ is the radius of the column), both the forward- and back-scattered fields exhibit a number of resonances as a function of average electron plasma frequency $\langle\omega_{pe}(r)\rangle$. It has been generally accepted that all of these Tonks-Dattner resonances, except possibly the largest one, could be explained as being caused by standing longitudinal electron plasma waves which are trapped in a small region between the glass wall and a radius, where $\omega_{pe} \times (r_c) = \omega_{\text{inc}}$. The above explanation was given mainly from a fluid model and appeared to give excellent quantitative agreement with experiment.[1] Recently, however, Baldwin suggested that a kinetic-theory model must be used to correctly explain the nature of the standing longitudinal waves in the inhomogeneous plasma which exists in a discharge tube.[2] In particular, Landau damping of these waves and a valid boundary condition at the wall must be included in the model. Qualitative agreement was also found between this theory and experiment.[3]

In view of the macroscopic nature of the experiments, it appears difficult to ascertain the details necessary for a complete picture of the Tonks-Dattner resonances. It is the purpose of this work to describe an experiment designed to probe into the microscopic nature of Tonks-Dattner resonances and to report on initial findings. We describe here a series of time-of-flight experiments designed to investigate propagating electron plasma waves in a dc mercury-vapor discharge based upon the development of phase-coherent nanosecond sine-wave bursts. Signals of appreciable amplitude are observed to propagate with a finite velocity on the order of the electron thermal velocity for densities corresponding to each of the resonances. Multiple reflections of the signal are also detected.

The experiments were performed using the positive column of a mercury-vapor discharge tube (i.d. ~1.1 cm, $p \sim 10^{-3}$ Torr, $T_e \sim 4$ ev) inserted in a dipolar strip line. The sine-wave burst was generated by passing a subnanosecond 10-V pulse through a bandpass filter (2–4 GHz) and was amplified with a broadband low-noise TWT amplifier (2–4.5 GHz). The signals were observed with a high-frequency sampling oscilloscope (triggered by the nanosecond pulse generator) using a $T$ junction in the coaxial line connecting the sine-wave burst generator with the plasma column strip line (see Fig. 1).

We first calibrated the discharge tube by noting the value of $I_D$ for each of the resonances of the resonance spectrum for a cw incident signal of $f \sim 2.15$ GHz, this also being the carrier frequency of the sine-wave burst. Time-of-flight experiments were then performed in a known Tonks-Dattner resonance, and typical results are shown in Fig. 2. Figure 2(a) shows both the incident burst $A$ and reflected burst $B$ from the tube in the absence of a plasma. Using the reflected burst $B$ from the glass as a reference point, we examined the received signal in the largest or main Tonks-Dattner resonance. Surprisingly, a second signal $C$ appeared, whose time delay depended on the value of $I_D$, which is proportional to $\langle\omega_p{}^2(r)\rangle$ [see Figs. 2(b) and 2(c)]. This second signal was also noticed in the next two resonances but was not observed for values of $I_D$ for which a Tonks-Dattner resonance did not exist. Due to the noise, we could not follow it in higher resonances. We interpret this to be a propagating electron plasma wave and estimate its distance of propagation $d$ in the following manner[4–6]: the radial density profile is assumed to be

$$n(r) = n(0)[1 - \alpha(r/r_0)^2]$$

$$= n(0)\{1 - \alpha[(r_0 - d)/r_0]^2\}. \qquad (1)$$

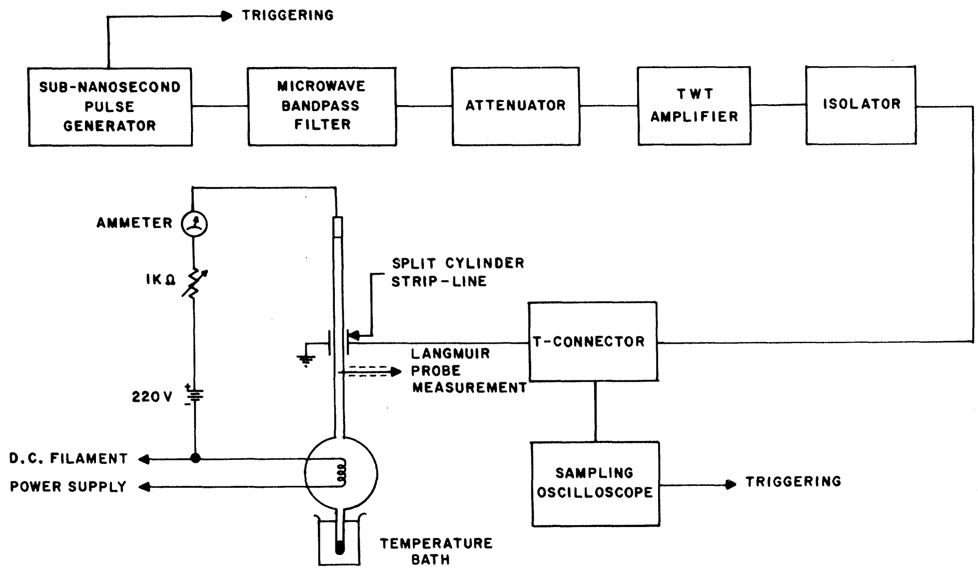


FIG. 1. Experimental setup for the generation and detection of nano-second sine-wave bursts.

The peak density can be computed from an average value $\langle n(r)\rangle$ previously measured with a microwave cavity, and is, therefore, related to $I_D$, i.e., $n(0)=KI_D$.

From Eq. (1), we write

$$n(r_c)=KI_D[(1-\alpha)+2\alpha d/r_0-\alpha d^2/r_0{}^2], \quad (2)$$

where $\omega_{pe}(r_c)=\omega_{\rm inc}$. Since $d$ is proportional to a velocity times one-half the time-of-flight, we examine the curve of $I_D{}^{-1}$ versus the time-of-flight for the peak of the envelope of the sine-wave burst (Fig. 3). Since it is linear for a large region, we can neglect the quadratic term in Eq. (2) and relabel the ordinate in terms of distance:

$$d=(r_0/2\alpha)\,\omega_{\rm inc}{}^2/\omega_p{}^2(0)-[(1-\alpha)/2\alpha]\,r_0. \quad (3)$$

From Fig. 3, the $t=0$ intercept allows us to compute $\alpha$, which was found to be $\alpha\sim 0.96$. From the slopes of these curves, we compute the group velocities to be $1.2\times10^7$ and $6.0\times10^7$ cm/sec for the main and the next resonances, respectively, which are on the order of the computed electron thermal velocity $\sim 8\times10^7$ cm/sec, where the electron temperature was measured with a Langmuir probe at the center of the tube. We also examined the amplitude of the electron plasma wave as a function of our computed $d$. The $e$-folding length in the main resonance was approximately $6\times10^{-2}$ cm.

At this point, we inquired whether it would be possible to do a more direct time-of-flight experiment by launching the electron plasma waves with one probe, and detecting with a second probe

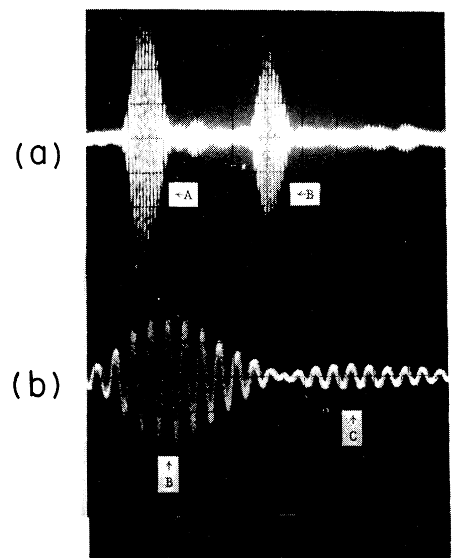


FIG. 2. (a) Detected signals with the plasma tube turned off. The signal $A$ is incident on the glass column and $B$ is the signal reflected from the column-microwave mismatch. This signal $B$ will be used as a reference in later figures. $T=5$ nsec/cm. (b) The plasma tube is tuned to the main Tonks-Dattner resonance, and a second signal $C$ is detected. $T=1$ nsec/cm. (c) $I_D$ is increased, but the column is still the main resonance as determined by an independent microwave scattering measurement with its cw frequency equal to the carrier frequency of the pulse.

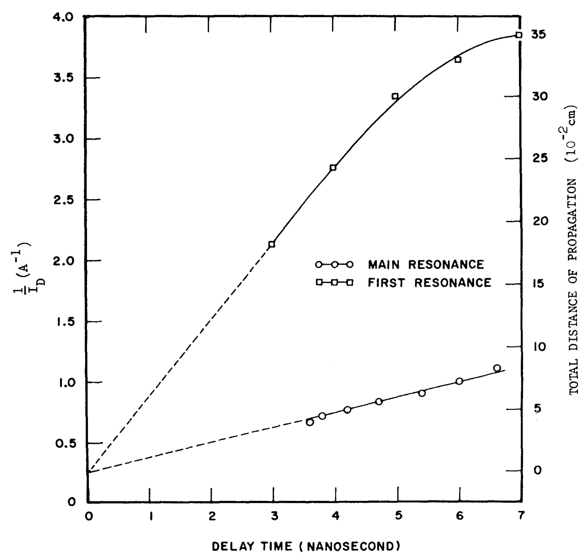


FIG. 3. Plot of $1/I_D$ versus the measured delay time of the peak of the envelope of the electron plasma wave for the main and the next resonances. Since the curves are linear over a large region, it is possible to relabel the ordinate in terms of total distance traveled. The group velocity of the wave can be found from the slopes of the curves and is found to be on the order of the electron thermal velocity.

separated by a distance $d$. This would offer the advantage of knowing the point of excitation and the distance $d$ more accurately. However, since our computed $d$ was on the order of 1 mm, an experiment was not attempted in this tube since the plasma would be grossly perturbed by the addition of the probes.

Since the distance $2d$ was computed based on our interpretation of the experiment, we must also demonstrate that the signal was due to a propagating wave and not due to just exciting a Tonks-Dattner resonance – since our sine-wave burst had approximately 8 cycles. This can be done by comparing the ratio of the total time delay of the pulse $\tau$ to the period of a Tonks-Dattner resonance $T$ and observing if the ratio $\tau/T$ is larger than or on the order of the number of sine waves in our burst.

We must first assume a dispersion relation for the electron plasma waves in an inhomogeneous plasma. We used[7]

$$\omega^2/\omega_p{}^2(r) = 1 + 3k^2 V_{\mathrm{th}}{}^2/\omega^2 , \tag{4}$$

where $V_{\mathrm{th}}$ is the thermal velocity of the electrons. The time delay $\tau$ that it takes for the pulse to traverse the distance $r_0 - r_c$ can be computed from

$$\tfrac{1}{2}\tau = \int_{r_c}^{r_0} \frac{dr}{V_g(r)} , \tag{5}$$

where $V_g(r)$ is the group velocity found from Eq. (4). The density profile near the edge of the tube [Eq. (3)] was used:

$$n(r) = 2\alpha n(0)\,[1 + (1-\alpha)/2\alpha - r/r_0] . \tag{6}$$

Performing the integration in Eq. (5), we compute

$$\tfrac{1}{2}\tau = \frac{r_0}{\sqrt{3}\,V_{\mathrm{th}}}\left[\tfrac{3}{4}\pi(\beta-\xi) + \tfrac{3}{2}(\beta-\xi)\sin^{-1}\frac{2-(\xi+\beta)}{\beta-\xi} - (1-\xi)^{1/2}(\beta-1)^{1/2}\right] , \tag{7}$$

where $\xi = r_c/r_0$,

$\beta = 1 + (1-\alpha)/2\alpha$.

Similarly, the period of the wave $T$ in a Tonks-Dattner resonance can be computed from

$$\tfrac{1}{2}T = \int_{r_c}^{r_0} \frac{dr}{V_p(r)} , \tag{8}$$

where $V_p(r)$ is the phase velocity found from Eq. (4). Again performing the integration, we obtain

$$\tfrac{1}{2}T = \frac{r_0}{\sqrt{3}\,V_{\mathrm{th}}}\left\{\left[\tfrac{1}{2}\pi - \sin^{-1}\left(\frac{\beta-1}{\beta-\xi}\right)^{1/2}\right] \times(\beta-\xi) - (1-\xi)^{1/2}(\beta-1)^{1/2}\right\} . \tag{9}$$

The ratio $\tau/T = 3$, if $\alpha = 1$; $\tau/T = 5.14$ for the experimentally estimated value of $\alpha = 0.96$; and $\tau/T = 7.6$ if $\alpha = 0.90$. Therefore, we feel confident that the burst can be separated from an excited Tonks-Dattner resonance.[8]

Since Tonks-Dattner resonances are considered to be standing waves, we should also be able to observe a multiply reflected propagating wave. This is shown in Fig. 4 where a third signal $D$ can also be found in the main resonance. Changing $I_D$, hence the distance of propagation, allows us to demonstrate that multiple reflection can occur. Since the velocity is constant, the twice-reflected signal $D$ should move twice as fast in the time scale as the other signal $C$. This criterion which was also used to demonstrate reflection of ion acoustic waves[9] is satisfied. In addition, the amplitude of $D$ was proportional to $C$ which indicates that we have not observed a third-order sheath echo.[10]

Finally, we examined the effect of increased collisions on both the propagating wave and on a normal Tonks-Dattner resonance spectrum. Since our tube was sealed, we could not add any gas as in previous afterglow experiments.[3,11] However, by placing the entire mercury-vapor plasma tube in an oven and slowly increasing the ambient temperature from room temperature to 60°C, we

(a)

(b)

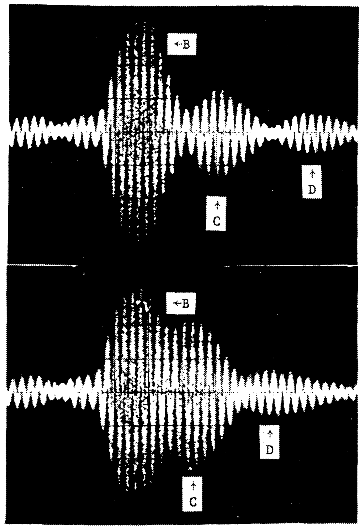


FIG. 4. Demonstration of multiple reflection of a propagating electron plasma wave. Increasing $I_D$ slightly (a) to (b) while still in the main resonance decreases the distance of propagation $2(d-\delta)$ for $C$ and $4(d-\delta)$ for $D$. Since the velocity is constant, the time changes for $C$ should be $2\tau$. This is shown in (a) and (b), where $D$ undergoes twice the time change as $C$ does for two values of $I_D$. $T=2$ nsec/cm.

observed the effects of increased collisions.[12,13] First, the propagating wave suffered increased damping. Second, the normal Tonks-Dattner resonance spectrum for an incident cw signal [i.e., $E_{SC}$ versus $\langle\omega_p{}^2(r)\rangle/\omega^2$] was first distorted and finally disappeared, leaving just an envelope, which is probably just the cold-plasma resonance at $\omega \sim \langle\omega_p(r)\rangle/\sqrt{2}$. Butterworth has also noted this distortion in the ambient temperature range $30°C \leq T \leq 36°C$.[14]

In conclusion, we have observed that radial-propagating electron plasma waves can be excited in plasma columns if, and only if, the plasma densities correspond to those of the Tonks-Dattner resonances. It is also shown that a multiple reflection does exist for these electron plasma waves. Since the main resonance corresponds to the density for the largest signal detected here, it appears that the above data confirm that besides its electromagnetic (i.e., cold plasma) character the main resonance has also temperature features. The fact that the multiply reflected signal is substantially nondispersive seems to contain some information concerning the reflection process. However, if one wants to determine experimentally whether such a reflection, one should try to probe the wave form at surfaces other than the surface where $\omega_{pe}(r_c) = \omega_{inc}$. Such an experiment could be done with an electron beam probe or a laser, but because of equipment limitations, has not been carried out in our laboratory.

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*Supported in part by the National Science Foundation.
Present address: Department of Aviation, Bangkok, Thailand.

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[2]D. E. Baldwin and J. Hirshfield, Appl. Phys. Letters 11, 175 (1967). D. E. Baldwin, Phys. Fluids, 12, 279 (1969).

[3]D. W. Ignat and D. E. Baldwin, Bull. Am. Phys. Soc. 13, 1497 (1968), paper ID9; D. W. Ignat, Ph.D. thesis, Yale University, 1969 (unpublished).

[4]To our knowledge, only one other experiment similar to this has been performed. In that experiment, Schmidt (Ref. 5) also interpreted this second signal as being caused by a propagating electron plasma wave. However, the experiment was performed using an afterglow plasma and he therefore could not isolate the main resonance as in this work. It appears that the thermal resonances in an afterglow plasma reside on a cold-plasma resonance (Ref. 3). Finally, both Schmidt's and our experiment differ from an experiment designed to examine the "ringing" of a plasma caused by the application of a nanosecond voltage pulse to an afterglow plasma, where it has been observed that the plasma rings at $\omega = \omega_p(r=0)$ (Ref. 6). We have also repeated that ringing-type experiment. This was done by removing the bandpass filter and the broadband TWT amplifier from the circuit (Fig. 1). We find the ringing frequency to be proportional to the discharge current, which attests to its value for plasma diagnostics. This method of diagnostics was also suggested independently to us by Dr. I. Alexeff.

[5]H. J. Schmidt, Appl. Phys. Letters 5, 187 (1965).

[6]D. E. Baldwin, D. M. Henderson, and J. L. Hirshfield, Phys. Rev. Letters 20, 314 (1968).

[7]C. L. Longmire, Elementary Plasma Physics (Wiley-Interscience, Inc., New York, 1963), p. 114.

[8]We also observed that the signal was not due to just the beating of two signals at $\omega_1$ and $\omega_2$: $\cos\omega_1 t + \cos\omega_2 t = 2\cos\frac{1}{2}(\omega_1+\omega_2)t\cos\frac{1}{2}(\omega_1-\omega_2)t$. This beating would require the existence of two signals at frequencies separated by (delay time)$^{-1}$. A spectrum analyzer analysis failed to reveal the existence of these two signals. We acknowledge Dr. J. L. Hirshfield for suggesting this check.

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PHYSICAL REVIEW VOLUME 186, NUMBER 1 5 OCTOBER 1969

# Flow of Helium II through Fine Pores

B. K. Jones
*Physics Department, University of California, Los Angeles, California* 90024
(Received 9 June 1969)

Measurements are reported of the flow of He II through packed carbon powders with pore sizes 10–1000 Å. Considerable lowering of the superfluid transition temperature was observed and found to correspond with measurements made with fourth sound. An analysis is presented of the temperature dependence of the flow rate near the transition temperature.

## INTRODUCTION

Experiments have shown that the superfluid properties of He II are changed considerably when it is confined in a restricted space. The adsorbed helium film provides confinement in one dimension, and confinement in two dimensions can be obtained with the helium in the pores of either Vycor glass or compressed fine powders. In the latter systems, the analysis is not easy as the geometry is not so well defined.

The average superfluid density in these systems can be measured using third sound for the adsorbed film,[1,2] and fourth sound for the fine pores.[3–5] The results show that the superfluid density is decreased from the bulk value and its temperature dependence is changed. Also, the waves can only be propagated below some temperature $T_0$ (the onset temperature), which is taken to be that at which the system undergoes the superfluid transition. Thus $T_0 = T_\lambda$ for the bulk liquid. In these systems, there appears to be a correlation between $\rho_s(T)$, the pore size or film thickness, and $T_0$; thus $T_0$ can be used as a parameter to characterize the system.

The critical velocity is a fundamental parameter in the study of superfluidity. We present here measurements of critical flow rates through fine porous powders to supplement the data on the film[2,6] and Vycor.[7,8] Of particular interest is the value of the onset temperature and the temperature dependence of the critical flow rate near the onset temperature.

## EXPERIMENT

To study the critical and supercritical flow of superfluids, two basic methods may be used. A velocity can be imposed on the fluid, and the chemical potential difference $\Delta\mu$ across a constriction observed. Alternatively, a potential can be imposed, and a velocity of flow observed; the intercept of the characteristic curve ($v_s$ versus $\Delta\mu$) is the critical velocity $v_c$. If the approach of the curve to $v_c$ is slow, then the former method is the more suitable. If, however, the flow velocity is near $v_c$ for all $\Delta\mu$, then the latter is suitable. Because the characteristic curve is found to have little curvature for helium in the absorbed film and in fine channels, and also because the method is experimentally more convenient, the second approach was taken.

The apparatus is shown schematically in Fig. 1. The specimen S is a cylindrical plug of packed carbon powder 1 cm diam. and 1 in. long inside a cylindrical copper resonator tube R. The specimens are those used for fourth-sound measurements; more details can be found in Refs. 3 and 4. The specimen forms the bottom of a copper vessel B, and is sealed to it with an indium O-ring. The vessel is suspended in a helium Dewar. The relative levels inside and outside the vessel can be altered with a Plexiglas displacer D, so that a chemical potential difference is imposed. The level of the helium inside the vessel is monitored by means of a capacitive level transducer C. This forms part of the frequency-determining section of a metal-oxide-semiconductor field-effect-transistor (MOSFET) oscillator O immersed in the main helium bath.[9] Because of the difference in the dielectric constants of liquid and gaseous helium, the oscillator frequency gives a measure of the helium-liquid level. Although this sensitivity was never required, level changes of better than 1 $\mu$ were

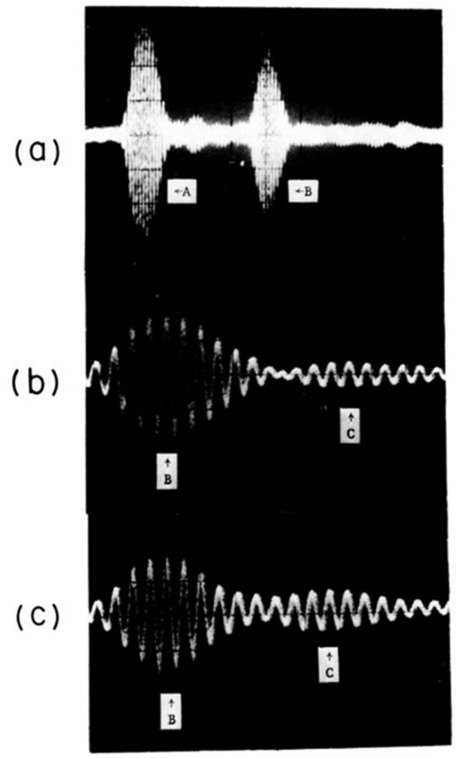


FIG. 2. (a) Detected signals with the plasma tube turned off. The signal $A$ is incident on the glass column and $B$ is the signal reflected from the column-microwave mismatch. This signal $B$ will be used as a reference in later figures. $T=5$ nsec/cm. (b) The plasma tube is tuned to the main Tonks-Dattner resonance, and a second signal $C$ is detected. $T=1$ nsec/cm. (c) $I_D$ is increased, but the column is still the main resonance as determined by an independent microwave scattering measurement with its cw frequency equal to the carrier frequency of the pulse.

(a)

(b)

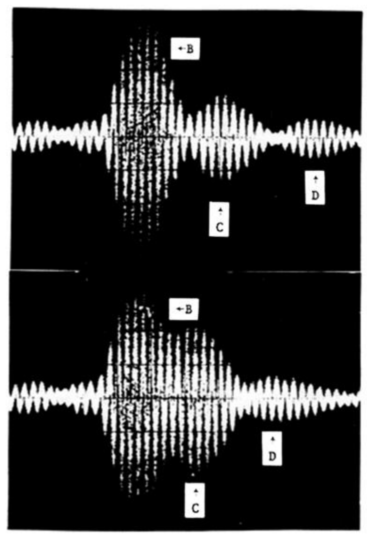


FIG. 4. Demonstration of multiple reflection of a propagating electron plasma wave. Increasing $I_D$ slightly (a) to (b) while still in the main resonance decreases the distance of propagation $2(d-\delta)$ for $C$ and $4(d-\delta)$ for $D$. Since the velocity is constant, the time changes for $C$ should be $2\tau$. This is shown in (a) and (b), where $D$ undergoes twice the time change as $C$ does for two values of $I_D$. $T=2$ nsec/cm.