and bounding each term separately, using the condition (5.4) in the process. We merely state the result here,

$$|P_2|\le[g/\sqrt{(-p^2)}](\tfrac{1}{2}\pi+\alpha^{-1})\,. \tag{F8}$$

Thus we have shown that

$$|\mathrm{Re}H_\lambda(p^2,k^2)|\le\left(\frac{1+2gm}{m}\frac{\pi}{2}+\frac{g}{\alpha}\right)\frac{1}{\sqrt{(-p^2)}}\,. \tag{F9}$$

One can now employ bounds (F3) and (F9) to establish that B_λ is subject to the bound

$$B_\lambda\le\left(\frac{1}{\pi m}+\frac{2g}{\pi}+\frac{2g}{\pi^2\alpha}\right)\int_{-\infty}^{-\mu^2}\frac{dp^2}{\sqrt{(-p^2)}}|\Delta T(p^2)| +\frac{g}{m}\int_{-\infty}^{-\mu^2}dp^2|\Delta T(p^2)|\,. \tag{F10}$$

From the inequalities (F2) and (F10), it follows that condition (5.7) is sufficient to guarantee that $B_\lambda+C_\lambda<1$.

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Experimental Aspects of Dual Theories for Baryons*

V. BARGER AND C. MICHAEL†
Department of Physics, University of Wisconsin, Madison, Wisconsin 53706

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Formulations of baryon-exchange-degenerate schemes incorporating $SU(3)$ symmetry are presented. We discuss the relevance of the $SU(3)$ solutions to the baryon spectrum and backward-scattering data and present a more empirical broken-$SU(3)$ approach to reconcile the duality constraints with experiment.

I. INTRODUCTION

THE duality of direct-channel resonances and Regge exchange poles[1] has led in some instances to a considerable simplification in dynamical models. The combination of duality and $SU(3)$ theories gives further severe restrictions on trajectories and residue functions, but unfortunately we find that some of these constraints appear to be at variance with empirical observation. Since $SU(3)$ is a broken symmetry, it is an interesting question whether a broken exchange-degenerate $SU(3)$ theory can exist. In this paper, we investigate duality constraints for s, t, and u channels in meson-baryon scattering, seeking a solution consistent with the baryon spectra and meson-baryon scattering data.

In those pseudoscalar-meson-baryon scattering reactions for which the t channel involves exotic meson-exchange quantum numbers, the absence of forward peaks is one of the more striking empirical observations in strong-interaction scattering; similarly, backward peaks are absent to a very low level for reactions requiring the exchange of exotic baryons.[2,3] The absence of exotic exchanges, as inferred from scattering data, provides valuable insight into the dynamics of these two-body reactions through the duality principle. The duality constraints connected with t-channel exotic may be considered separately from those connected with u-channel exotic. There is already some evidence on the validity of the u-channel exotic constraints from K^+p backward-scattering data.[4] On the other hand, the empirical absence of t-channel peaks should place even more stringent requirements on the dynamics, since the allowed forward cross-section peaks are generally an order of magnitude larger than the allowed backward cross-section peaks. We consider it to be most reasonable, in view of these facts, to impose both t- and u-channel exotic duality conditions simultaneously. This we do in Secs. II and III.

A further complication in applying duality for baryon exchanges concerns the MacDowell symmetry.[3] Analytic requirements on fermion exchange amplitudes force a conspiracy at $u=0$ between exchanges of opposite parity. If we first restrict our attention to positive u (the mass region), then duality conditions can be applied to a u-channel parity-conserving helicity amplitude of definite τP quantum number[5] and we ob-

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† Present address: CERN, Geneva 23, Switzerland.

[1] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1786 (1968); C. Schmid, Phys. Rev. Letters **20**, 689 (1968); G. Veneziano, Nuovo Cimento **57A**, 190 (1968).

[2] V. Barger, Rev. Mod. Phys. **40**, 129 (1968).

[3] V. Barger and D. Cline, *Phenomenological Theories of High Energy Scattering* (W. A. Benjamin, Inc., New York, 1969).

[4] V. Barger, Phys. Rev. **179**, 1371 (1969); P. B. James, *ibid.* **179**, 1559 (1969); K. Igi and J. Storrow, Nuovo Cimento (to be published).

[5] At asymptotic energies ($s\to\infty$), fermion Regge-pole exchange contributions can be isolated according to τP quantum number through the u-channel parity-conserving helicity-amplitude formalism (cf. Ref. 3). By this separation, we can deal independently with the dominant and reflected (in $\sqrt{u}$) branches of the fermion exchange amplitudes. If instead, we were to consider directly the invariant amplitudes, A and B, then both τP branches would be involved simultaneously.

TABLE I. $SU(3)$ coefficients for 10, 8, and 1 u-channel baryon exchange amplitudes.

Reaction	u-channel couplings	Decuplet exchange (u-channel exchanges)	Octet exchange (u-channel exchanges)	Singlet exchange
		s-channel exotic		
$K^+p \to K^+p$	$(\bar{K}N)^2$	$\frac{1}{6}\Sigma^{10}$	$\frac{1}{3}(2F-1)^2\Sigma^8+\frac{1}{9}(2F+1)^2\Lambda^8$	Λ^1
$K^+n \to K^+n$	$(\bar{K}N)^2$	$\frac{1}{3}\Sigma^{10}$	$\frac{2}{3}(2F-1)^2\Sigma^8$	
$\pi^+\Sigma^+ \to \pi^+\Sigma^+$	$(\pi\Sigma)^2$	$\frac{1}{6}\Sigma^{10}$	$\frac{4}{3}F^2\Sigma^8+(4/9)(1-F)^2\Lambda^8$	Λ^1
$K^-\Sigma^- \to \pi^-\Xi^-$	$(\pi\Sigma)(K\Xi)$	$-\frac{1}{6}\Sigma^{10}$	$\frac{2}{3}F\Sigma^8+(2/9)(1-F)(4F-1)\Lambda^8$	Λ^1
$K^-\Xi^- \to K^-\Xi^-$	$(K\Xi)^2$	$\frac{1}{6}\Sigma^{10}$	$\frac{1}{3}\Sigma^8+\frac{1}{9}(4F-1)^2\Lambda^8$	Λ^1
$\bar{K}^0\Sigma^+ \to \bar{K}^0\Sigma^+$	$(K\Sigma)^2$	$\frac{1}{3}\Delta^{10}$	$\frac{2}{3}(2F-1)^2N^8$	
$\pi^-\Xi^- \to \pi^-\Xi^-$	$(\pi\Xi)^2$	$\frac{1}{3}\Xi^{10}$	$\frac{2}{3}(2F-1)^2\Xi^8$	
		t-channel exotic		
$\pi^-p \to K^+\Sigma^-$	$(\bar{K}N)(\pi\Sigma)$	$\frac{1}{6}\Sigma^{10}$	$-\frac{2}{3}F(2F-1)\Sigma^8-(2/9)(1-F)(2F+1)\Lambda^8$	Λ^1
$\pi^+\Sigma^- \to \pi^-\Sigma^+$	$(\pi\Sigma)^2$	$-\frac{1}{6}\Sigma^{10}$	$-\frac{4}{3}F^2\Sigma^8+(4/9)(1-F)^2\Lambda^8$	Λ^1
$K^-p \to K^+\Xi^-$	$(K\Xi)(\bar{K}N)$	$-\frac{1}{6}\Sigma^{10}$	$-\frac{1}{3}(2F-1)\Sigma^8-\frac{1}{9}(4F-1)(2F+1)\Lambda^8$	Λ^1
$K^-p \to K^0\Xi^0$	$(K\Xi)(\bar{K}N)$	$-\frac{1}{3}\Sigma^{10}$	$-\frac{2}{3}(2F-1)\Sigma^8$	
$K^-\Sigma^+ \to \pi^+\Xi^-$	$(K\Xi)(\pi\Sigma)$	$\frac{1}{6}\Sigma^{10}$	$-\frac{2}{3}F\Sigma^8+(2/9)(1-F)(4F-1)\Lambda^8$	Λ^1
$K^-p \to \pi^+\Sigma^-$	$(K\Sigma)(\pi N)$	$-\frac{1}{3}\Delta^{10}$	$-\frac{2}{3}(2F-1)N^8$	
$\pi^-\Sigma^+ \to K^+\Xi^-$	$(\pi\Xi)(K\Sigma)$	$-\frac{1}{3}\Xi^{10}$	$-\frac{2}{3}(2F-1)\Xi^8$	
		Other reactions		
$\pi^-p \to \pi^-p$	$(\pi N)^2$	Δ^{10}		
$\pi^+p \to \pi^+p$	$(\pi N)^2$	$\frac{1}{3}\Delta^{10}$	$\frac{2}{3}N^8$	
$K^-p \to \pi^-\Sigma^+$	$(K\Sigma)(\pi N)$	$-\Delta^{10}$		
$K^-p \to \pi^0\Lambda$	$(K\Lambda)(\pi N)$		$-(2/3\sqrt{12})(2F-1)N^8$	
$\pi^-p \to K^0\Sigma^0$	$(\bar{K}N)(\pi\Sigma)$	$\frac{1}{6}\sqrt{2}\Sigma^{10}$	$-\frac{2}{3}\sqrt{2}F(2F-1)\Sigma^8$	
$\pi^+p \to K^+\Sigma^+$	$(\bar{K}N)(\pi\Sigma)$	$-\frac{1}{6}\Sigma^{10}$	$\frac{2}{3}F(2F-1)\Sigma^8-(2/9)(1-F)(2F+1)\Lambda^8$	Λ^1
$\pi^-p \to K^0\Lambda$	$(\bar{K}N)(\pi\Lambda)$	$\frac{1}{6}\Sigma^{10}$	$(4/3\sqrt{6})(1-F)(2F-1)\Sigma^8$	

tain the usual exchange degeneracy between particles of alternating parity. For $u<0$ we label our amplitudes by continuations from the $u>0$ classification. Throughout, we assume that trajectories have no important particle manifestations when reflected to the $-\sqrt{u}$ region. With our present understanding of baryon trajectories,[6] this seems to be the most sensible alternative. A more prominent role for the reflected trajectories would require a more intricate theory than exists at present.

II. $SU(3)$ STRUCTURE OF DUALITY SOLUTIONS

To begin with, we present a brief examination of the implications of duality for baryon amplitudes when exact $SU(3)$ symmetry is imposed. For the purpose of simplification, we shall consider initially only octets on the external lines. In Table I, we show, for pseudoscalar-meson-baryon scattering, those reactions with exotic quantum numbers in the s ($PB \to PB$) or t ($\bar{B}B \to \bar{P}P$) channel. In each case the u-channel contributions from decuplet, octet, and singlet Regge exchanges are tabulated (F is the usual octet coupling parameter normalized to $F+D=1$). Assuming nonexistence of particles with exotic quantum numbers, the imaginary part of the u-channel exchange contribution must vanish when crossed to the exotic channel. This requirement is equivalent to exchange degeneracy for u-channel trajectories and residues as a function of u. Furthermore, since Regge exchange contributions are asymptotically ($s \to \infty$) separable according to the (τP) quantum number, the exchange degeneracy can be considered independently for the two (τP) possibilities:

$$\tau P=+1: \tfrac{1}{2}^+,\ \tfrac{3}{2}^-,\ \tfrac{5}{2}^+,\ \ldots (\alpha,\gamma) \text{ trajectory};$$
$$\tau P=-1: \tfrac{1}{2}^-,\ \tfrac{3}{2}^+,\ \tfrac{5}{2}^-,\ \ldots (\beta,\delta) \text{ trajectory}.$$

The dual constraints for the imaginary parts of the amplitudes in Table I are

$$\sum_{\{R=1,8,10\}} A^R=0 \tag{1}$$

for s-exotic reactions[7] and

$$\sum_{\{R=1,8,10\}} A^R\tau^R=0 \tag{2}$$

[6] V. Barger, in *Proceedings of the Coral Gables Conference on Fundamental Interactions at High Energy, 1969*, edited by T. Gudehus *et al.* (Gordon and Breach, Science Publishers, Inc., New York, 1969).

[7] The duality constraints from s-exotic reactions have been considered by R. H. Capps, Phys. Rev. Letters **22**, 215 (1969). However, his criteria are overly restrictive and select a particular solution from a broad class of possible solutions. In addition, Capps does not impose constraints from t-exotic reactions.

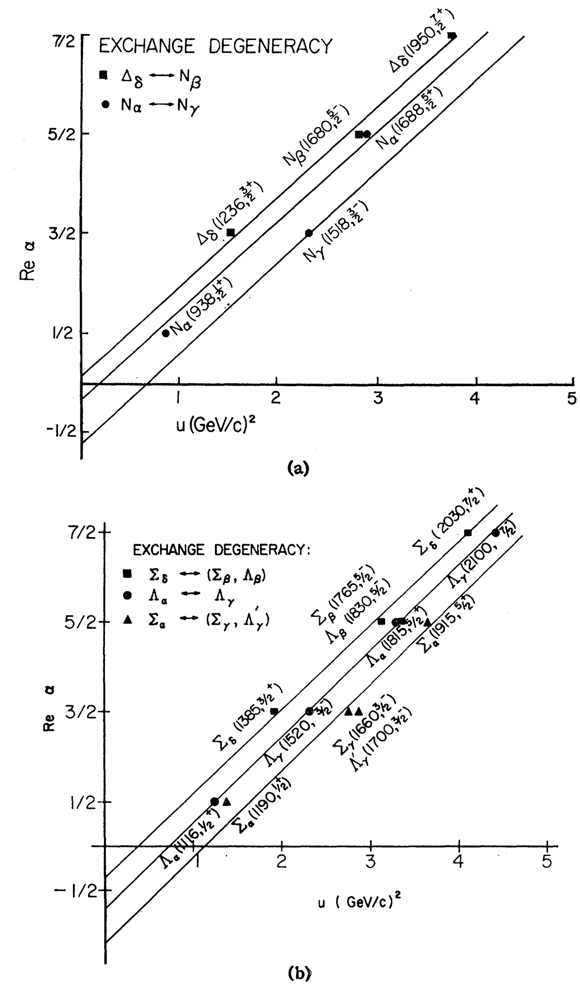

FIG. 1. Regge spin-mass plots displaying exchange-degenerate schemes for baryon trajectories. (a) $Y=+1$ baryons; (b) $Y=0$ baryons.

for t-exotic reactions. Here A^R denotes the u-channel entries in Table I. In the limit of exact $SU(3)$, two independent dual requirements are found for each of the above sets of constraints.

No s exotic.

$$[10]+2(2F-1)^2[8]=0, \qquad (3)$$

$$9[1]+(2F+1)^2[8]=0. \qquad (4)$$

No t exotic.

$$\tau_{10}[10]+2(2F-1)\tau_8[8]=0, \qquad (5)$$

$$9\tau_1[1]-(4F-1)(2F+1)\tau_8[8]=0. \qquad (6)$$

The quantities in brackets are the exchange amplitudes, with implied summation for any multiplicity of $SU(3)$ representations. The class of possible solutions to Eqs. (3)–(6) is further restricted by u-channel unitarity requirements in the particle mass region, namely,

$$\tau_{10}[10]>0, \quad \tau_8[8]>0, \quad \tau_1[1]>0. \qquad (7)$$

The most economical solutions, in terms of numbers of particles, involve three $SU(3)$ multiplets in exchange-degenerate patterns as follows:

(i) $$[8]\leftrightarrow[1], [8'],$$

with

$$(1/27)[8]_{F=1/2}=-\tfrac{1}{8}[1]=-\tfrac{1}{9}[8']_{F=1/2}, \qquad (8)$$

or with

$$[8]_{F=1}=-(9/8)[1]=-[8']_{F=0}. \qquad (9)$$

(ii) $$[8]\leftrightarrow[10], [8'],$$

with

$$\tfrac{1}{3}[8]_{F=-1/2}=-\tfrac{1}{16}[10]=-[8']_{F=-1/2}, \qquad (10)$$

or with

$$\tfrac{1}{9}[8]_{F=0}=-\tfrac{1}{16}[10]=-[8']_{F=1}. \qquad (11)$$

(iii) $$[8]\leftrightarrow[8'], [8''],$$

with one F parameter undetermined. The two-way arrows in the above equations connect states with the same τP and opposite P. Other three-multiplet cases, involving only one octet, e.g.,

$$[10]\leftrightarrow[8], [1]; \quad [1]\leftrightarrow[8], [1];$$
$$[10]\leftrightarrow[8], [10],$$

all violate unitarity. An alternative approach, based on duality diagrams,[8] or the crossing matrix,[9] indicates the existence of a solution with

$$[1], [8]\leftrightarrow[8'], [10].$$

From Eqs. (1)–(7) above, we can determine the amplitudes of this solution to be

$$\frac{9}{2(2F'+1)^2}[1]=\frac{1}{(4F'-1)^2}[8]_F =-\tfrac{1}{3}[8']_{F'}=-\frac{1}{12(2F'-1)^2}[10], \qquad (12)$$

where $F=(1-F')/(4F'-1)$, and F' is arbitrary. This general solution is also valid for decuplets on external lines, and the corresponding constraints obtained from duality are

$$\tfrac{1}{3}[8]=-[8']=-\tfrac{2}{5}[10], \qquad (13)$$

for $PD\rightarrow PD$ and

$$\frac{1}{(4F'-1)}[8]_F=[8']_{F'}=\frac{1}{(\sqrt{10})(2F'-1)}[10], \qquad (14)$$

for $PB\rightarrow PD$, where F and F' are related as before.

The general solution factorizes correctly; however, the special cases of Eqs. (9) and (11) require a negligible coupling to external decuplets in order to avoid the

[8] J. L. Rosner, Phys. Rev. Letters **22**, 689 (1969); H. Harari, *ibid* **22**, 562 (1969).

[9] J. Mandula *et al.*, Phys. Rev. Letters **22**, 1147 (1969).

consequences of factorization. We note that Eq. (9) is the solution relevant for meson-meson scattering.

III. BROKEN $SU(3)$

A. Λ-Σ Splitting

An attractive theoretical approach is to regard duality requirements as nearly exact, even with a broken $SU(3)$ regime. The extent to which this is realizable with the $SU(3)$ solutions in Sec. II will now be discussed.

The baryon trajectories from the particle mass spectra, as illustrated in Fig. 1, are grossly split according to strangeness quantum number. As a basic requirement, therefore, we want solutions in which the $Y=1$, 0, and -1 trajectories are mutually independent, while preferably retaining the $SU(3)$ structure of the residues. Since the duality constraints from Table I do not relate baryon exchanges of different strangeness, this is a consistent point of view. At a further level of $SU(3)$ symmetry breaking, we need to consider the splitting of Λ and Σ members of the same octet as well as mixing between Λ's of a singlet and octet with the same spin-parity. A careful examination of the conditions in Table I indicates that this second degree of symmetry breaking cannot be maintained without relaxing either duality or the exact $SU(3)$ structure of the residues. This leads to the conclusion that, although the physical solution may contain remnants of $SU(3)$ couplings, some departures from symmetric couplings must be anticipated.

B. Trajectory Classifications

The known baryon trajectories with which we will be particularly concerned are illustrated in Fig. 1. The apparent "rank" in terms of the exactness of the exchange degeneracy observed from the particle mass spectra is[10]

(1) $\Lambda_\alpha{}^8(1115,\tfrac{1}{2}+) \to \Lambda_\gamma{}^1(1520,\tfrac{3}{2}-) \to \Lambda_\alpha{}^8(1815,\tfrac{5}{2}+)$;

(2) $\Delta_\delta{}^{10}(1236,\tfrac{3}{2}+) \to N_\beta{}^8(1680,\tfrac{5}{2}-) \to \Delta_\delta{}^{10}(1920,\tfrac{7}{2}+)$,
$\Sigma_\delta{}^{10}(1385,\tfrac{3}{2}+) \to \Sigma_\beta{}^8(1770,\tfrac{5}{2}-) \to \Sigma_\delta{}^{10}(2030,\tfrac{7}{2}+)$;

(3) $\Sigma_\alpha{}^8(1189,\tfrac{1}{2}+) \to \Sigma_\gamma{}^8(1660,\tfrac{3}{2}-) \to \Sigma_\alpha{}^8(1910,\tfrac{5}{2}+)$;

(4) $N_\alpha{}^8(938,\tfrac{1}{2}+) \to N_\gamma{}^8(1515,\tfrac{3}{2}-) \to N_\alpha{}^8(1688,\tfrac{5}{2}+)$,
$\Sigma_\alpha{}^8(1189,\tfrac{1}{2}+) \to \Lambda_\gamma{}^8(1700,\tfrac{3}{2}-) \to \Sigma_\alpha{}^8(1910,\tfrac{5}{2}+)$,
$\Sigma_\delta{}^{10}(1385,\tfrac{3}{2}+) \to \Lambda_\beta{}^8(1830,\tfrac{5}{2}-) \to \Sigma_\delta(2030,\tfrac{7}{2}+)$.

The relative splitting from exact exchange degeneracy in these categories is estimated from Fig. 1 to be (1) $\Delta J \simeq 0$, (2) $\Delta J \simeq 0.1$, (3) $\Delta J \sim 0.25$, (4) $\Delta J \sim 0.4$. One of the puzzling questions to be answered is the remarkable degeneracy of the $\Lambda_\alpha{}^8$-$\Lambda_\gamma{}^1$ trajectories,[4] particularly in view of the fact that $\Lambda^1(1520,\tfrac{3}{2}-)$ is apparently not a pure singlet.[11] The answer to this must come from outside the framework of exact $SU(3)$.

In correspondence with the three multiplet solutions of Sec. II, the above degeneracies might be classified as

$$[8(\tfrac{1}{2}+)]_{F=1/2} \leftrightarrow [1(\tfrac{3}{2}-)],\ [8'(\tfrac{3}{2}-)]_{F=1/2},$$
$$[10(\tfrac{3}{2}+)],\ [8'(\tfrac{3}{2}+)]_{F'=1/2} \leftrightarrow [8(\tfrac{5}{2}-)]_{F=-1/2}.$$

The 8, 1, and 8′ $SU(3)$ multiplets in the $\tfrac{1}{2}+ \leftrightarrow \tfrac{3}{2}-$ sequence are already filled out by known baryons. With the value $F=\tfrac{1}{2}$, as required for both $\tfrac{1}{2}+$ and $\tfrac{3}{2}-$ octets in this solution, the Σ members are decoupled from $\bar{K}N$, in accordance with observed decay rates.[11] Furthermore, for $F=\tfrac{1}{2}$, the N and Ξ exchanges are entirely decoupled from all exotic channels, and duality thus places constraints on the degeneracy of these trajectories *only* through the assumption of exact $SU(3)$. In the physical limit of broken $SU(3)$, it is then natural to expect that the $N_\alpha \leftrightarrow N_\gamma$ (or $\Xi_\alpha \leftrightarrow \Xi_\gamma$) degeneracy will not be well maintained, in agreement with our previous empirical observations. In order to accommodate the known PD couplings of the $8(\tfrac{1}{2}+)$ in this scheme, one must use the $F' \to \tfrac{1}{2}$ limit of Eqs. (12)–(14) which leads to a mass-degenerate $10(\tfrac{1}{2}+)$ that is decoupled from PB but coupled to PD.

The 10, $8' \leftrightarrow 8$ solution for the $\tfrac{3}{2}+ \leftrightarrow \tfrac{5}{2}-$ sequence requires an octet of $\tfrac{3}{2}+$ particles (degenerate with the $\tfrac{3}{2}+$ decuplet) that has not been observed. However, this $\tfrac{3}{2}+$ octet amplitude is reduced in strength by a factor of 16 from the $\tfrac{3}{2}+$ decuplet amplitude, so its nonobservance or even nonexistence is perhaps not of crucial importance. In particular, such a state, if sufficiently coupled to many-body channels to build a reasonable width, would not be detected readily through experiments involving two-body channels. Equation (10) implies that $F=-\tfrac{1}{2}$ for the $\tfrac{5}{2}-$ octet, which is comparable to the decay rate determination[11] of $F\approx -0.16$. The exchange degeneracy of $10(\tfrac{3}{2}+)$ with $8(\tfrac{5}{2}-)$ implies a zero of the 10 residue at $\alpha=\tfrac{1}{2}$, in correspondence with the absence of an octet of $\tfrac{1}{2}-$ particles at low mass. Such a wrong-signature-sense zero at $\alpha=\tfrac{1}{2}$ in the Δ_δ Regge exchange amplitude has been suggested to account for the rapid variation in the magnitude of the Δ_δ Regge residue between $u=0$ ($\pi^- p$ elastic scattering region) and $u=M_\Delta{}^2$ (particle decay width).[12]

C. Solutions without Symmetry

Although $SU(3)$ considerations provide guidelines for formulating dual models, a more general approach seems to be necessary. Basic requirements for any dual solution are that (i) separate Λ-Λ and Σ-Σ exchange degeneracies must occur in the $\bar{K}N \to \bar{K}N$ u channel,

[10] The empirical status of exchange-degenerate baryon trajectories was previously surveyed by V. Barger and D. Cline, University of Wisconsin Report No. C00-203, 1968 (unpublished).

[11] R. D. Tripp, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 189.

[12] K. Igi *et al.*, Phys. Rev. Letters **21**, 580 (1968).

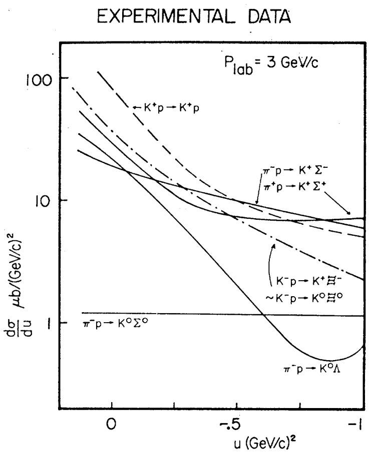

FIG. 2. Artist's conception of backward-scattering differential cross-section data at 3 GeV/c on $Y=0$ baryon exchange reactions. Data from Ref. 13. The variation in minimum momentum transfer for the different reactions is exhibited.

and (ii) Λ-Σ type degeneracies must occur in the $\pi\Sigma \rightarrow \pi\Sigma$ u channel. These requirements follow from Table I and the unitarity conditions of Eq. (7). The simplest way to satisfy (i) and (ii), consistent with the observed broken Λ, Σ mass spectra of Fig. 1, is to have different trajectories exchanged in the $\bar{K}N \rightarrow \bar{K}N$ and $\pi\Sigma \rightarrow \pi\Sigma$ u channels. However, such a solution is not compatible with empirical $\bar{K}N/\pi\Sigma$ decay branching ratios. Thus, we need to look for approximately dual solutions, where some qualitative memory (i) and (ii) is retained. Furthermore, if duality is to be well maintained by the physical solution, then couplings to exotic channels must be suppressed for those trajectories displaying an appreciable breaking of exchange degeneracy. This aspect of duality solutions was already discussed in Sec. III B for the case of the N_α-N_γ trajectory. In that instance, the N_α-N_γ amplitude was decoupled from exotic channels and thereby the trajectories were unconstrained by dualism. Examination of Fig. 1 suggests that a similar mechanism might obtain for Σ-Λ type degeneracies, since there is a significant measure of degeneracy breaking. Since, as noted above, Σ-Λ exchange degeneracies are required in the $\pi\Sigma \rightarrow \pi\Sigma$ u channel, we expect all $\pi\Sigma$ couplings to be somewhat suppressed. This observation is in moderate agreement with coupling constants from branching ratios,[11] and will enable us to understand features of the scattering data as described in Sec. IV. Such a decoupling of Σ-Λ trajectories to $\pi\Sigma$ is not allowed by $SU(3)$ solutions, as can be seen from Table I.

IV. BACKWARD SCATTERING

The particle mass spectra place constraints on the types of duality solutions that can be physically realized, and data on baryon exchange reactions allow a further selection of the physical dual solution. In order to investigate the relevance of our proposed broken-$SU(3)$ solutions, we have compiled in Figs. 2 and 3 a representative sample of experimental backward-scattering data at 3.0 GeV/c.[13]

The heirarchy of the magnitudes of the differential cross sections for strange-baryon exchange in Fig. 2 can qualitatively be explained by the following ranking of couplings:

Strong couplings.

$$\text{“}\Lambda\text{”} \rightarrow \bar{K}N, \quad \text{“}\Sigma\text{”} \rightarrow K\Xi, \quad \text{“}\Sigma\text{”} \rightarrow \pi\Lambda.$$

Medium strong couplings.

$$\text{“}\Lambda\text{”} \rightarrow \pi\Sigma, \quad \text{“}\Sigma\text{”} \rightarrow \pi\Sigma, \quad \text{“}\Sigma\text{”} \rightarrow \bar{K}N, \quad \text{“}\Lambda\text{”} \rightarrow K\Xi.$$

Here "Λ" or "Σ" are simply isospin labels for the exchanged trajectories. The empirical ordering of couplings above has some nice features that tie in with our previous arguments in Sec. III C. For example, the "Λ" $\rightarrow \pi\Sigma$ and "Σ" $\rightarrow \pi\Sigma$ couplings are suppressed as anticipated, and the "Σ" $\rightarrow \bar{K}N$ coupling is weak in agreement with decay rates. Furthermore, this ranking of the couplings explains the smallness of $\pi^-p \rightarrow K^0\Sigma^0$ near 180^0 as a doubly suppressed u-channel transition. This suppression has long been a puzzling feature of backward-scattering data, since this is the only reaction with allowed baryon exchange that does not exhibit a backward peak. It can easily be shown that the above coupling categories cannot be naturally reproduced in an $SU(3)$-symmetric scheme. The data on the $\pi^-p \rightarrow K^0\Lambda$ reaction also cause difficulties for the $SU(3)$ solutions of Sec. II, but are accommodated in this nonsymmetric scheme. The energy dependence and polarization data on $\pi^-p \rightarrow K^0\Lambda$ favor a $(\Sigma_\alpha,\Sigma_\gamma)$ exchange interpretation instead of $(\Sigma_\delta,\Sigma_\beta)$.[14] However, the $F=\frac{1}{2}$ $SU(3)$ solution of Eq. (8) for the (α,γ) octets decouples the $(\Sigma_\alpha,\Sigma_\gamma)$ from this reaction. When $SU(3)$ constraints are relaxed, as in the coupling rankings above, the $(\Sigma_\alpha,\Sigma_\gamma)$ exchange explanation can be maintained.

In Fig. 3, the backward-scattering data for nonstrange-baryon exchange reactions are summarized, again at 3 GeV/c.[15] The theoretical aspects of the πN data have already been fairly well publicized.[16] However, we wish to draw attention to some interesting implications of the existing data on $\bar{K}N \rightarrow \pi\Sigma$ reactions.

[13] [$K^+p \rightarrow K^+p$] G. S. Abrams *et al.*, University of Illinois Report No. C00-1195-156 (unpublished); W. F. Baker *et al.*, Nucl. Phys. **B9**, 249 (1969). [$\pi N \rightarrow K\Sigma$] O. I. Dahl *et al.*, Phys. Rev. **163**, 1430 (1967); R. Kofler *et al.*, *ibid.* **163**, 1479 (1967). [$\bar{K}N \rightarrow K\Xi$] J. Badier *et al.*, Saclay CEA Report No. R-3037, 1966 (unpublished); P. Dauber *et al.*, University of California Radiation Laboratory Report No. UCRL-18388, 1968 (unpublished). [$\pi p \rightarrow K\Lambda$] Extrapolated from data and fits in Ref. 14.

[14] V. Barger, D. Cline, and J. Matos, Phys. Letters **29B**, 121 (1969).

[15] [$\pi N \rightarrow \pi N$] W. F. Baker *et al.*, Nucl. Phys. **B9**, 249 (1969). [$\bar{K}N \rightarrow \pi Y$] J. Badier *et al.*, Saclay CEA Report No. R-3037, 1966 (unpublished); R. Barloutaud *et al.*, Nucl. Phys. **B9**, 493 (1969).

[16] V. Barger and D. Cline, Phys. Rev. Letters **21**, 392 (1968); **19**, 1504 (1967).

The isospin decompositions of the $\bar{K}N \to \pi\Sigma$ reactions are

$$\frac{d\sigma}{du}(K^-p \to \pi^-\Sigma^+) = |\Delta|^2,$$

$$\frac{d\sigma}{du}(K^-p \to \pi^+\Sigma^-) = (1/9)|\Delta + 2N|^2,$$

$$\frac{d\sigma}{du}(K^-n \to \pi^0\Sigma^-) = (2/9)|\Delta - N|^2.$$

For $K^-p \to \pi^-\Sigma^+$ with Δ_δ exchange, the $SU(3)$ coupling constants lead to the equality[2]

$$\frac{d\sigma}{du}(K^-p \to \pi^-\Sigma^+) = \frac{d\sigma}{du}(\pi^-p \to \pi^-p),$$

which agrees reasonably well with experiment, as illustrated in Fig. 3. The $K^-p \to \pi^+\Sigma^-$ reaction is exotic in the t channel, forcing exchange degeneracy of the u-channel amplitudes for duality. The natural baryon exchange candidates for this degeneracy are (Δ_δ, N_β), since these particles fall reasonably well on a single trajectory. However, a single exchange-degenerate pair in the u channel implies in turn that the s-channel backward-scattering cross section will have no dips[4] (the wrong-signature nonsense dips of each component are filled in by the other exchange). The present data on $K^-p \to \pi^+\Sigma^-$ suggest a dip or break near $u = -0.2$ or -0.3. but the evidence is inconclusive. A possible orgin of this behavior is the $\alpha = -\frac{1}{2}$ dip of the N_α amplitude,[16] with a badly broken (N_α, N_γ) amplitude exchange degeneracy.[17] Further evidence for a strong N_α component comes from the $K^-n \to \pi^0\Sigma^-$ reaction, which shows a pronounced dip at $u \simeq -0.15$. The ratio of the cross sections for $K^-p \to \pi^+\Sigma^-$ and $K^-n \to \pi^0\Sigma^-$ are incompatible with N_α dominance, so the actual physics of these reactions may be rather complicated. Nevertheless, we would like to stress that further experimental measurements of the $\bar{K}N \to \pi\Sigma$ processes should prove to be illuminating on the role of duality for baryons. The duality constraints on the $K^-p \to \pi^+\Sigma^-$ reaction arise entirely from the t-channel exotic assumption, and therefore a breakdown of exchange degeneracy of the u-channel baryons involved could be ascribed to t-channel exotic states.[18]

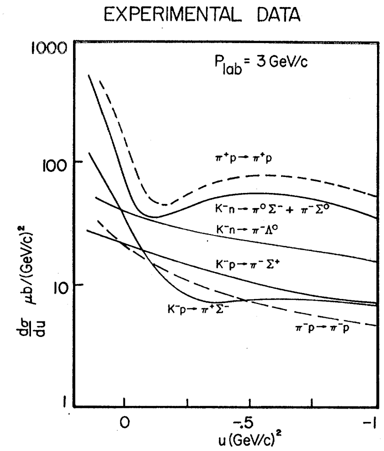

FIG. 3. Schematic representation of backward-scattering differential cross-section data at 3 GeV/c on $Y = +1$ baryon exchange reactions. Data from Ref. 15.

In summary, it seems that exact exchange degeneracy for baryons as derived from duality is not realized in nature. Alternative ways of patching up the defects of the theory include (i) exotic states in t (or possibly s) channel, (ii) undiscovered particles in the mass spectra, which lie on new trajectories that are more exchange-degenerate, and (iii) Regge cuts in addition to the exchange-degenerate poles which tend to distort the primordial exchange-degenerate pattern.[19] Whatever turns out to be the perturbing mechanism, there is, nevertheless, a considerable aesthetic appeal in the classification schemes that arise in dual theories.

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[17] Another example of a badly broken (N_α, N_γ) exchange degeneracy has been discussed by V. Barger and C. Michael, Phys. Rev. Letters **22**, 1330 (1969).

[18] Duality considerations for BB and $\bar{B}B$ scattering have led to the expectation of exotic mesons coupled to $\bar{B}B$ [J. Rosner, Phys. Rev. Letters **21**, 950 (1968).] However, the $\bar{M}M$ system has a consistent dual solution, so there exists no *a priori* reason for exotic t-channel states in $\bar{B}B \to \bar{M}M$. Eliminating t-channel exotic constraints while retaining the s-channel exotic conditions in our treatment of MB scattering allows much greater freedom in solutions.

[19] The third alternative has been advocated by C. Lovelace in connection with the Veneziano formula.