Production of Strongly Interacting W's in Inelastic Electron-Nucleon Collisions*

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We consider the possibility of extending the search for the intermediate vector boson by observing the products of inelastic electron-nucleon collisions. If the W has a mass less than 5.1 BeV/c^2 and can interact strongly, then experiments at SLAC, which look for muons with large transverse momentum, should provide a sensitive test for its existence. If the W is not observed in the proposed experiments, then we can deduce stringent upper bounds on the W-nucleon cross section, or conclude that if a strongly interacting W exists, it must have a mass in excess of 5.1 BeV/c^2.

RECENTLY, increased interest has been directed toward the elusive W meson. There are several reasons for this interest. First, evidence has been accumulated that seems to indicate that muons detected far underground do not satisfy the secθ law.[1] Some authors feel that the existence of the W might account for this effect.[2] Second, with the advent of high-flux neutrino beams, it has become possible to search for the W in a relatively simple and systematic fashion.[3] However, the cosmic-ray experiments suffer from very poor statistics and questionable interpretations. The neutrino experiments, on the other hand, are severely limited by available beam energies and have only been able to imply that the mass of the W is greater than about 2 BeV/c^2. In this paper, we propose a search for the W using inelastic electron-nucleon scattering and consider, in particular, the experimental possibilities at the Stanford Linear Accelerator Center (SLAC). Using a 20-BeV electron beam, we can potentially create W's having a mass as great as 5.1 BeV/c^2. The W will probably decay weakly into a muon and neutrino, and impart a large transverse momentum to the μ. Hence, even though W's may be produced with a small cross section, μ's with large transverse momentum might be detectable over backgrounds. This is indeed found to be the case if, and only if W's can interact strongly with nucleons.

Consider briefly the electromagnetic processes drawn in Figs. 1(a) and 1(b). Using calculations and asymptotic formulas,[4] one can estimate that these diagrams lead to W production cross sections of, at best, 10^{-37} cm^2. Unfortunately, between the low muon counting rates that this cross section implies and the large muon background from π and K decays that occur at SLAC, one cannot hope to detect the muons produced in this way. However, there has recently been considerable speculation that the W might interact strongly,[5] as shown in Fig. 2. It is this process that could lead to W production cross sections and μ counting rates which should be easily observable. The scattering matrix will be

$$T_{fi}=\frac{g}{V}\frac{1}{\sqrt{(2E_e)}}\frac{1}{\sqrt{(2E_\nu)}}\bar{u}_\nu\gamma^\mu(1-\gamma_5)u_e \times\left(\frac{-g_{\lambda\mu}+(k_\lambda k_\mu)/m_W^2}{k^2-m_W^2}\right)\langle P_n|g'J^\lambda(0)|P\rangle, \quad (1)$$

where P=momentum of incoming nucleon, P_n=momentum of outgoing hadron state, $J^\lambda=W$ field source, and k=momentum of the internal W line. We then calculate the cross section in the usual way:

$$\frac{d\sigma}{d\Omega_\nu dE_\nu}=\frac{g^2}{(2\pi)^3}\frac{E_\nu}{E_e}\frac{m_p}{E_p}\left(\frac{1}{k^2-m_W^2}\right)^2 \times(p_\mu{}^e p_\lambda{}^\nu+p_\lambda{}^e p_\mu{}^\nu-p^e\cdot p^\nu g_{\mu\lambda}+i\epsilon_{\mu\lambda\alpha\beta}p_e{}^\alpha p_\nu{}^\beta)M^{\mu\lambda}, \quad (2)$$

where

$$M^{\mu\nu}=\frac{E_p}{m_p}\sum_n\overline{\sum_{\text{spins}}}\langle P|g'J^{\dagger\mu}(0)|P_n\rangle \times\langle P_n|g'J^\lambda(0)|P\rangle(2\pi)^4\delta(P_n-P-k), \quad (3)$$

and $\overline{\sum}_{\text{spins}}$ indicates an average over initial spins and

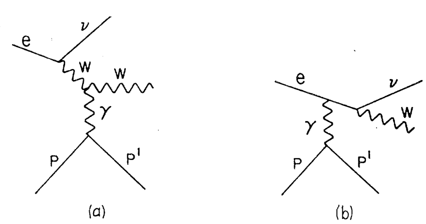

FIG. 1. Electroproduction of W on protons.

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sum over final spins. The general form of $M^{\mu\lambda}$ reads

$$M^{\mu\lambda}=P^\mu P^\lambda c_1+P^\mu k^\lambda c_2+P^\lambda k^\mu c_3+k^\lambda k^\mu c_4 +g^{\mu\lambda}c_5+i\epsilon^{\mu\lambda\alpha\beta}P_\alpha k_\beta c_6. \quad (4)$$

However, since $k_\mu \bar{u}_\nu \gamma^\mu(1-\gamma_5)u_e=0$, upon taking the lepton masses equal to zero throughout the calculation, only the first, fifth, and sixth terms contribute here. Now writing everything in lab coordinates and letting θ_ν denote the angle between the ν and the beam, we have

$$\frac{d\sigma}{dE_\nu d\Omega_\nu}=\frac{4g^2}{(2\pi)^3}E_\nu^2 m_p^2\left(\frac{1}{k^2-m_W^2}\right)^2\left[\tfrac{1}{2}\cos^2(\tfrac{1}{2}\theta_\nu)c_1 -\frac{1}{m_p^2}\sin^2(\tfrac{1}{2}\theta_\nu)c_5-\frac{(E_\nu+E_e)}{m_p}\sin^2(\tfrac{1}{2}\theta_\nu)c_6\right]. \quad (5)$$

We can complete the calculation by relating c_1, c_5, and c_6 to the off-mass-shell scattering cross sections for polarized W's on nucleons (Fig. 3). These cross sections are given in invariant form by

$$\sigma(k^2,\nu)=\frac{m_p}{2[(k\cdot P)^2-m_W^2 m_p^2]^{1/2}}\epsilon_\rho^* M^{\rho\lambda}\epsilon_\lambda, \quad (6)$$

where ϵ=polarization vector of W, k=momentum of the W, and ν=energy of the W. Extrapolating the kinematical factors such as $(k\cdot P)^2-m_W^2m_p^2$ into the region of spacelike k, we finally obtain the cross section for the desired process in terms of the variables k^2 and ν:

$$\frac{d\sigma}{dk^2 d\nu}=\frac{10^{-5}m_W^2}{\sqrt{2}(2\pi^2)E_e}\left(\frac{1}{k^2-m_W^2}\right)^2\left\{\frac{-k^2(E_e-\nu)}{2m_p^2(\nu^2-k^2)^{1/2}}\times[2\sigma_L(k^2,\nu)+\sigma_t^+(k^2,\nu)]-\frac{(k^2)^2}{8E_e m_p^2(\nu^2-k^2)^{1/2}}\times[2\sigma_L(k^2,\nu)+\sigma_t^+(k^2,\nu)]-\frac{k^2}{4E_e m_p^2}(\nu^2-k^2)^{1/2}\sigma_t^+(k^2,\nu)-\frac{k^2(2E_e-\nu)}{4m_p^2E_e}\sigma_t^-(k^2,\nu)\right\}, \quad (7)$$

where σ_L=W-nucleon cross section for longitudinal W's, and $\sigma_t^+(\sigma_t^-)$=sum (difference) of the W-nucleon cross sections for left- and right-hand polarized W's.

In order to proceed, we must make assumptions concerning σ_t^+, σ_t^-, and σ_L. In the spirit of this calculation, we make the simple assumption $\sigma_L=0$, $\sigma_t^-=0$,

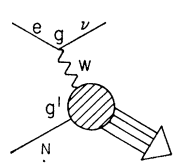

FIG. 2. Electroproduction of strong W on nucleon.

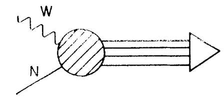

FIG. 3. Inelastic W-nucleon interaction.

and σ_t^+=constant independent of k^2 and ν. The integration over the allowed region of k^2-ν space can now easily be done. The results are given in Table I. Although we might hope to say that σ_t^+ is a sizeable strong-interaction cross section, recent cosmic-ray experiments[6] have already yielded rather small upper bounds. However, we shall see that the proposed electron experiments could lower these upper bounds considerably and provide a more sentitive test for the W.

According to recent cosmic-ray calculations, there exists a sea-level neutrino flux in the horizontal direction which is approximately[7]

$$\eta(E_\nu)=(0.029/E_\nu^3)\ \text{BeV}^{-1}\ \text{cm}^{-2}\ \text{sec}^{-1}\ \text{sr}^{-1}. \quad (8)$$

These neutrinos can then interact with nuclei in the earth and create W's as in Fig. 4. The muon which accompanies this reaction will then quickly lose its energy while traveling through the nearby rock and perhaps pass through a detector. The resultant muon flux is then approximately

$$\frac{dN}{dA\,d\Omega dt}\cong\int_0^\infty dx\int_0^\infty dE_f\int_{E_{\nu\min}}^\infty dE_\nu \times\left(\frac{0.029}{E_\nu^3}\right)(N_T)\left(\frac{1}{2}\frac{d\sigma}{dE_\mu}\right), \quad (9)$$

where $dN/dAd\Omega dt$=number of muons per cm² sr sec at the detector, N_T=Avogadro's number, E_μ=energy of the muon at creation (BeV), E_f=energy of muon at detector (BeV), x=distance (g/cm²) muon travels in rock, k=energy loss (BeV/g cm^{-2}) of muon ($E_f=E_\mu-kx$), and $E_{\nu\min}$=threshold neutrino energy for this process. Taking $d\sigma/dE_\mu$ from our earlier calculations, we can evaluate these integrals and

$$\frac{dN}{dA\,d\Omega dt}\approx\frac{N_T(0.029)}{6k(m_W+m_W^2/2m_p)}\times\sigma(E_\nu=20,\ \text{BeV}\ m_W). \quad (10)$$

Demanding that this flux be less than the experimental

TABLE I. Total W-production cross sections in electron-nucleon collisions.

m_W (BeV/c^2)	σ (cm²)
3	$(4.0\times10^{-7})\sigma_t^+$
4	$(5.4\times10^{-8})\sigma_t^+$
5	$(1.3\times10^{-9})\sigma_t^+$

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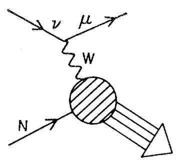

FIG. 4. Kinematics of a neutrino-induced muon reaction.

value

$$\frac{dN}{dA\,d\Omega dt} < 6\times10^{-13} \text{ per cm}^2 \text{ sr sec}, \tag{11}$$

we obtain the bounds on $\sigma(E_\nu=20 \text{ BeV}, m_W)$ listed in Table II.

We now proceed to find the desired muon spectrum for

$$e+p \rightarrow \nu+W+\text{“anything”} \rightarrow \nu+\mu+\nu+\text{“anything,”}$$

assuming that the W produced at the W-nucleon vertex emerges in the forward direction with an energy distribution essentially flat and extending from $\frac{1}{2}E_e$ to the kinematic extreme. These approximations, especially the first, are motivated by the detailed calculations.[4]

TABLE II. Bounds on W-production cross sections implied by cosmic-ray experiments.

m_W (BeV/c^2)	E_e (BeV)	σ (b)	$\sigma_t{}^+$ (b)
3	20	$<3.75\times10^{-12}$	$<9.5\times10^{-6}$
4	20	$<6.0\ \times10^{-12}$	$<1.0\times10^{-4}$
5	20	$<8.75\times10^{-12}$	$<1.8\times01^{-3}$

Finally, the physical W decays, and we compute the angular distribution of the muons produced in the process $W\rightarrow\mu\nu$. This then gives us the differential cross section $d\sigma/dE_\mu d\Omega_\mu$ for the process in Fig. 5 (cf. Fig. 6). In order to relate this to an actual experimental situation, we consider a Be target 0.3 radiation lengths (r.l.) thick, and compute a yield (number of muons/electron sr GeV/c), and compare with the SLAC background yields[8] (cf. Fig. 7). For a given m_W and E_μ, simple kinematics cuts off the theoretical curves at various maximum muon angles as shown. Actually these curves will be smeared out, since the W will be produced with some transverse momentum. However, we see that if we look for energetic muons at large angles, the experiment will be most sensitive to the existence of the W. In fact, the $m_W=5$ BeV/c^2, $E_\mu=16$ BeV curve exceeds the expected background by several orders of magnitude. Also, since a yield of about 10^{-10}/

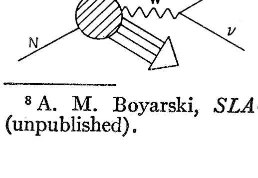

FIG. 5. Decay kinematics for electroproduced strong W.

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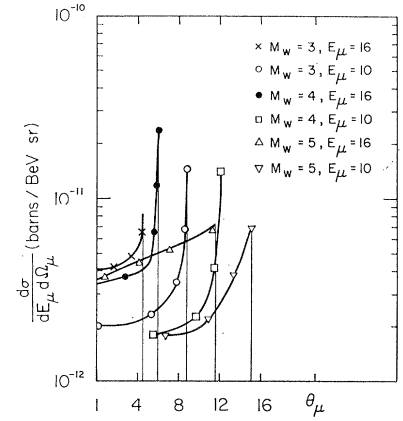

FIG. 6. Differential cross section of muons via strong W production. m_W are in BeV/c^2, E_μ in BeV.

electron sr BeV/c corresponds to a counting rate of about 1 per sec, the experimentalist should have no difficulty with absolute rates.

Up to this point, we have made several simplifications which should be pointed out. First, we have not multiplied our cross sections by the branching ratio for $W\rightarrow\mu\nu$. The rate for $W\rightarrow e\nu$ is essentially identical to the $W\rightarrow\mu\nu$ rate, so we should at least divide our results by a factor of 2. Finally, we have not taken into account the fact that the electron beam loses energy as it passes through the 0.3-radiation-length (r.l.) Be target. This effect reduces the number of very energetic W's produced, which then reduces the number of muons produced at large angles for given m_W and E_μ. However, since the target is thin, this effect is not severe and should not amount to a reduction in muon intensity in excess of a few percent.

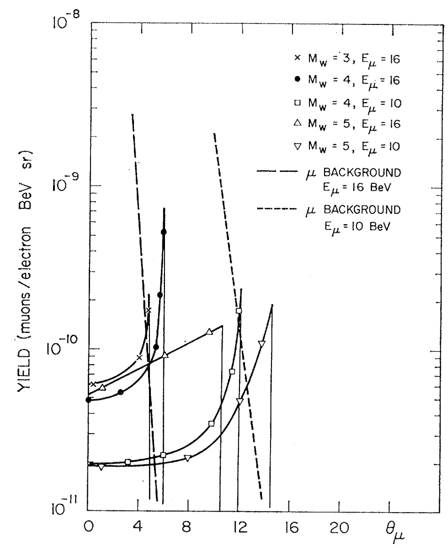

FIG. 7. Comparison of differential yields of muons with experimental backgrounds. Units are the same as in Fig. 6.

In conclusion, inelastic electron-nucleon scattering could easily and profitably be employed in the search of the W. In fact, we expect that the background μ flux should increase slowly with beam energy and maintain its fast exponential decline in scattering angle. However, the muon flux resulting from W production at larger m_W and beam energy could certainly increase in intensity and maintain its unique angular dependence without conflicting with present cosmic-ray experiments. So, a much more decisive search for W's could be made once higher-energy electron beams become available. Background muon intensities from π and K decays might also be greatly reduced by placing a lead absorber behind the target. The experiments' sensitivity to the existence of the W could also be improved by several orders of magnitude in such a way. This idea has recently been used in an experiment at Brookhaven National Laboratory[9] which looked for wide-angle muons emerging from proton-nucleon collisions. The experiment we propose, however, is preferable to its Brookhaven counterpart because electron-nucleon collisions are simpler and better understood theoretically than nucleon-nucleon collisions. The major drawback of all these experiments is, however, that even if wide-angle muons were found, it would not unambiguously imply the existence of W. The very discovery of wide-angle muons would in itself, however, be very important.

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Vector-Dominance Model and the Reaction $\gamma p \rightarrow \pi^- \Delta^{++}$†

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The vector-dominance-model (VDM) relation between the reactions (a) $\pi p \rightarrow \rho\Delta$ and (b) $\pi p \rightarrow \omega\Delta$ and the reaction (c) $\gamma p \rightarrow \pi\Delta$ is investigated within the context of the Regge-pole model. Parameters for the strong-interaction processes are determined by making a least-squares fit to the differential cross section and spin-density-matrix data. We find that good agreement with the experimental measurements can be obtained by including the π, A_1, and A_2 trajectories for reaction (a), while for reaction (b) we need the exchange of the ρ and B trajectories. Using values of the photon–vector-meson coupling constants, as determined from the leptonic decays of these mesons, and the VDM relations between the amplitudes for processes (a)–(c), we calculate the differential cross section for reaction (c). This is found to be in good agreement with the experimental data and yields a $\chi^2=3$ for nine data.

I. INTRODUCTION

RECENTLY several papers have appeared reporting on applications of the vector-dominance model (VDM) which are in disagreement with the experimental data.[1–4] The reasons why the model is successful[5–7] in certain situations and fails in others are not yet understood.

Two questionable procedures which are used in conjunction with the VDM are (i) the extrapolation of the vector-meson mass m_v to zero and (ii) the method used to eliminate the longitudinal spin component of the massive vector meson. The purpose of this paper is to investigate the basic assumptions of the VDM within the context of the Regge-pole model. This has the advantage of allowing one to work directly with the scattering amplitudes. We can thus test the fundamental suppositions inherent in the VDM, without having to approximate effects such as interference and mass extrapolations which may be the cause of why the VDM appears to fail in some instances. To this end we studied the reactions

$$\text{(a)}\ \pi^+ p \rightarrow \rho^0 \Delta^{++},$$
$$\text{(b)}\ \pi^+ p \rightarrow \omega \Delta^{++},$$
$$\text{(c)}\ \gamma p \rightarrow \pi^- \Delta^{++}.$$

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