observed in the experimental decay data [in the ratio of the ρ and $K^*(891)$ widths]. The φ-f' row is independent of the ρ and K^* rows, because of singlet-octet mixing.

Table I includes some predictions for Σ^* and Ξ^* partial widths that are not measured yet. The predicted sum of the $\pi^-\Sigma^+$ partial widths of the $\Lambda+\Sigma^0+\Sigma^0$ resonances in row 6 of the table is computed with phase-space factors corresponding to a resonance mass of 1690 MeV.

The trajectory ED to that of the $\Sigma(1382)$ is interesting. Because of the fact that KN states of all charges are exotic, the ED hypothesis requires that the $\Sigma(1382)$ contribution be cancelled by the Σ member of the $j^P=\frac{5}{2}^-$ octet, and that the Λ member of this octet be decoupled from KN states. It is seen from row 2 of Table I and from Ref. 7 that these requirements are in fair agreement with experiment. On the other hand, the $\Sigma^0(1382)$ contribution to the $\pi^-\Sigma^+$ mode may be cancelled by a combination of the Σ^0 and Λ of the $\frac{5}{2}^-$ octet. It is seen from row 1 of the table that the Λ plays a dominant role in this cancellation. Thus, the $\frac{5}{2}^-$ Λ and Σ^0 trade roles.

The data concerning the $j^P=\frac{3}{2}^+$ trajectories do not support the Veneziano choice of s_0, as is seen from the discrepancies in the last column of the first two rows of Table I. However, if a new s_0 were chosen so that the predicted recurrence widths were smaller by a factor of 3, the predicted ED widths would be smaller by $\sqrt{3}$ and would still agree satisfactorily with experiment.

Most of the experimental widths are not very accurate. However, the general agreement between the experimental and predicted widths in the fifth and sixth columns of the table is encouraging for the combination of the ED hypothesis and the Veneziano energy dependence.

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Reggeization in Supermultiplet Theories

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Methods for incorporating higher-symmetry ideas into the phenomenology of Regge poles are reviewed. These methods, which were developed originally for treating models of the "quark-excitation" type, are extended to cover also the "orbital excitation" models with particular reference to the oscillator model.

I. INTRODUCTION

ONE of the rather surprising features of the present scene in particle physics is the increasing evidence of the relevance of $SU(6)$-like symmetry ideas[1] in describing hadron spectra on the one hand and, on the other hand, the comparative disregard of such symmetries and the strong correlations they may be expected to provide among residue functions—even as a crude guiding principle—by those working in Regge phenomenology.[2]

One possible reason for this disregard could be that detailed experimental confirmation (from decay data) of the validity of higher symmetries for coupling parameters and residues exists for the low-lying $SU(6)$ states only. A second and more practical reason is perhaps the nonavailablity of a simple, consistent, and detailed formalism embodying the marriage of Regge ideas to higher symmetries.[2] A beginning was made in this direction in a series of recent papers. Unfortunately, although the discussion was general, the details of the formalism were given for one specific model of Reggeized higher symmetries, specifically, the model based on a quark-excitation picture for higher resonances, where along a trajectory the total *quark* content for physical states (half the number of quarks plus antiquarks) increases by integer steps in the form N, $N+1$, $N+2$, $\cdots$, and it is the quark number N which is Reggeized. It is our purpose in this paper to consider in detail the rival models based on an orbital excitation picture of two- and three-quark composites [group-theoretically, models of the type $SU(6)\otimes O_L(3)$, with a Reggeization of the orbital quantum number L].

As is well known, the quark-excitation models predict "exotic" resonances with high values of strangeness and isotopic charges, while in the orbital models only the **1**'s, **8**'s, and **10**'s of $SU(3)$ make their appearance. The physical hadron spectrum may, in the end, prove to possess features of both models; the present evidence, however, seems to favor orbital models of lesser or greater complexity with the known baryon resonances apparently grouping themselves in multiplets of $(\mathbf{56},0^+)$, $(\mathbf{56},2^+)$, $(\mathbf{70},1^-)$, $\cdots$ and meson resonances in $(\mathbf{35},0^+)$,

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[1] For a recent review, see H. Harari, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 195.

[2] The only systematic attempts in this direction that we know of are by P. G. O. Freund, Phys. Rev. **157**, 1412 (1967); R. Arnold, *ibid.* **162**, 1334 (1967); Y. Ne'eman, L. Horwitz, and N. Cabibbo, Phys. Letters **22**, 336 (1967).

(35,1^-), $\cdots$ of $SU(6)\otimes O_L(3)$. We wish to stress that the great virtue of using symmetry ideas is that we do not require the physical existence of quarks; we obtain quark-model results, with their correct *relativistic* kinematics, without actually believing that such objects exist.

The plan of the paper is as follows. In Sec. II, we review the basis of the Reggeization procedures, given any rest symmetry group for particle multiplets. The most important concept here is the notion of *generalized helicity*. This is introduced and we then define the appropriate rotation functions needed for Reggeization. These functions are a generalization of the familiar $d_{\lambda\lambda'}{}^J(\theta)$ rotation functions of the group $O_J(3)$. In Sec. III, the equivalent M-function formalism for writing amplitudes using multispinors is introduced in terms of which actual calculations are made. We wish to stress with the greatest possible emphasis that this multispinor formalism, using Bargmann-Wigner equations to describe supermultiplets, is not just a luxury. Insofar as it embodies the correct kinematics[3] and (most important) provides a natural formalism into which *symmetry-breaking effects* (due to mass splittings within a supermultiplet) can be incorporated, the multispinor M-function formalism for scattering amplitudes is an *important ingredient* of the Reggeization scheme. One wishes one could stress this enough so that the unfortunate prejudice against learning what is basically a very simple and yet extraordinarily powerful technique could be overcome. Section IV deals with the detailed description of the oscillator excitation model and its application to Reggeized meson-baryon (MB) and baryon-baryon (BB) scattering. In Sec. V, we discuss briefly the kinematic-singularity problem and the question of Tollerization versus Reggeization of physical amplitudes as a means to cope with such singularities. Section VI discusses the situation where the singularities are removed by Gribov doubling of the meson multiplets. In a separate note, the formalism of this paper is applied to the problem of charge-exchange meson-baryon scattering to see if the Reggeization of $SU(6)\otimes O(3)$-like theories with the drastic decrease in the number of residues they provide gives a reasonable fit to the data.[4] There we show that, in fact, one can correlate all known processes with a one-parameter formula.

II. REGGEIZATION SCHEME FOR HIGHER SYMMETRIES: GENERAL CONSIDERATIONS

As stated in the Introduction, there are two distinct types of models of Reggeized higher symmetries.

A. Orbital Excitation Models

Here, higher symmetries combine *intrinsic* spin and unitary spin as in the original $SU_{F,S}(6)$ proposals emanating from Wigner's $SU(4)$ (F is the unitary spin index and S is intrinsic spin), while Reggeization proceeds for orbital momentum $\mathbf{L}$. (Here $\mathbf{J}=\mathbf{L}+\mathbf{S}$.) The models we shall consider correspond to the following symmetry groups:

A. $SU(6)\otimes O_L(3)$,

A(i). $U(6)\otimes U(6)|_{F,S}\otimes O_N(4)$,

A(ii). $U(6)\otimes U(6)|_{F,S}\otimes SU_N(3)$.

(i) *O(4) orbital models.* The $U(6)\times U(6)$ intrinsic-spin–unitary-spin symmetry treats quark and antiquark spins as distinct and independent so that the intrinsic spin group contained[5] in $U(6)\otimes U(6)$ is the subgroup $SU_{Sq}(2)\otimes SU_{S\bar{q}}(2)$. As is well known, this group has the same structure as $O_S(4)$. From this point of view, a natural, though by no means essential, generalization of *orbital* angular momentum also is to consider four-dimensional orbital momenta, thereby enlarging $O_L(3)$ to $O_N(4)$, where N stands for the quantum number appearing in the eigenvalue $N(N+2)$ of the $O(4)$ Casimir operator.[6,7]

(ii) *U(3) orbital model.* A remarkable feature of the baryon spectrum known at present appears to be that all known particles belong to (56,L^{even}) and (70,L^{odd}). The fact that there appear to be two (56,0^+) multiplets and no (56,1^+) bears out the need for a radial quantum number N for classification purposes. A suggestion has been made that possibly the extra orbital degrees of freedom are associated with a harmonic-oscillator-like potential and the Reggeized quantum number is one of the Casimir operators of the three-dimensional harmonic-oscillator group $SU(3)$ rather than $O(4)$. This oscillator group $SU(3)$ is the same group familiar from

[3] The point to be reiterated is that in theories of $SU(6)$ variety the "Clebsch-Gordan" coupling coefficients contain mass-dependent kinematic factors. If the symmetry were exact, it would not matter how these things were computed. The symmetry, however, is not exact and the multispinor formalism has the advantage of explicitly stating this mass (and the kinematic) dependence. It is therefore relatively easy to take account of symmetry-breaking effects, for example, by introducing physical masses in place of mean multiplet masses in the kinematic factors.

[4] R. Delbourgo and Abdus Salam, Phys. Letters **28B**, 497 (1969).

[5] Another freedom which has not been exploited lies in that groups like $SU(6)$ and $U(6)\otimes U(6)$ admit of pseudoquark representations in addition to quark representations (pseudoquarks, like the antiquarks, carry opposite intrinsic parity to quarks). In fact, we shall see later that a Tollerization of physical amplitudes (in contrast to Reggeization) more or less forces one to take pseudoquarks seriously, and with them hadrons of unnatural intrinsic parity.

[6] The important physical example where rest states are appropriately classified in terms of a four-dimensional angular momentum group $O_N(4)$ is the case of the hydrogen atom, the four-dimensional character of the group being a reflection of the extra symmetries possessed by the Coulomb $1/r$ potential. As is well known, hydrogen atom energy levels fall on Regge trajectories in an N versus E^2 plot, where the principal quantum number N determines $N(N+2)$, the Casimir operator of the orbital $O_N(4)$. [$N=\frac{1}{2}(k_1+k_2)$, where k_1 and k_2 are the 2 three-dimensional angular momenta associated with the two independent subgroups $O_{k_1}(3)$ and $O_{k_2}(3)$ which make up $O_N(4)$: $O_N(4)\approx O_{k_1}(3)\times O_{k_2}(3)$. For the hydrogen atom, only states with $k_1=k_2$ are realized and $N=0, 1, 2, \cdots$.] The possible existence of such an orbital $O_N(4)$ in hadron spectroscopy has been speculated by Barut and Kleinert (see Ref. 7). The fact that two (56,0^+) multiplets appear to be known seems to bear out the need for a radial quantum number like N for classification purposes (see further under the "oscillator model").

[7] A. O. Barut and H. Kleinert, Phys. Rev. **161**, 1464 (1967).

nuclear physics shell-model spectroscopy. It is discussed in detail further on.

B. Quark-Excitation Model

In a different category and contrasting with the spin-orbit coupling models considered above is the quark-excitation model which was treated in detail in the earlier papers.[8] Here one starts the rest symmetry $U(6)\otimes U(6)$ and Reggeizes one or more of the Casimir operators of this group. One of the simplest cases was the Reggeization of *total quark number* N, the physical particles lying along two master trajectories in N versus $(\text{mass})^2$ plot and $2N$ taking the values 3, 5, 7, $\cdots$ for baryons and 2, 4, 6, $\cdots$ for mesons.

Reggeization procedure. Now, even though the physical ideas behind the two types of models A and B are different, the techniques for applying Regge ideas to the high-energy behavior of scattering amplitudes are very similar. So we shall state these in generality for any particle classification group G.

(1) Neglecting small deviations from a mean mass, assume that *all hadron* states (at rest) can be classified as representations of a (rest) symmetry group G.

$G=SU(6)\otimes O(3)$	for orbital models of type A
$=[U(6)\otimes U(6)]\otimes O(4)$	for orbital models of type A(i)
$=[U(6)\otimes U(6)]\otimes U(3)$	for orbital models of type A(ii)
$=U(6)\otimes U(6)$	for quark-excitation models of type B.

(2) A significant empirical feature of the spectroscopy is that only some rather simple representations of these groups appear to be realized in nature—in general, these are representations characterized by just one quantum number N (Casimir invariant of G) besides baryon number.

(3) For every rest symmetry group G, there exists a generalized helicity subgroup which we shall denote as G_W—the generalized helicity[9] being denoted by W.

(4) The importance of the generalized helicity subgroup lies in that, if the symmetry were exact for three-point vertices, W spin must be conserved. Labeling physical states with N and W [in analogy with J and λ for $G=\mathrm{SU}_J(2)$], we thus have, for the three-point function,

$$\langle W|T(E)|W_1W_2\rangle=\sum_{\zeta}\langle\zeta W|W_1W_2\rangle T_{W_1W_2}(E). \quad (1)$$

Here $\langle\zeta W|W_1W_2\rangle$ denotes the Clebsch-Gordan coefficient which in G_W couples $D^{W_1}\otimes D^{W_2}$ to D^W. (In general, there may be more than one independent coupling, so we have included a parameter ζ to distinguish among them.)

(5) W spin is also conserved for collinear scattering processes (forward scattering). Thus,

$$\langle W_3W_4|T(E)|W_1W_2\rangle = \sum_{\zeta\zeta' W}\langle W_3W_4|\zeta' W\rangle T_{\zeta'\zeta}(E)\langle\zeta W|W_1W_2\rangle. \quad (2)$$

(6) The noncollinear four-point functions exhibit conservation of coplanar symmetry which for models A, A(i), A(ii) is $SU(3)$, $[U(3)\otimes U(3)]\otimes O(2)$,

$$[U(3)\otimes U(3)]\otimes U(1),$$

and for model B is $U(3)\otimes U(3)$.

(7) If we assume only that the subgroup symmetries (1)–(6) hold as empirical facts (at least to a fair approximation), there is the mathematical theorem[10] that we may express a nonforward scattering amplitude in terms of a *complete* set of suitably defined functions $d_{WW'}{}^N(\theta)$ as follows:

$$\langle W_3W_4|T(E,\theta)|W_1W_2\rangle = \sum_{N\zeta W\zeta' W'}\langle W_3W_4|\zeta' W'\rangle d_{WW'}{}^N(-\theta) \times T_{\zeta' W',\zeta W}{}^N(E)\langle\zeta W|W_1W_2\rangle. \quad (3)$$

Here,

$$d_{W'W}{}^N(\theta)=\langle NW'|e^{-i\theta J_2}|NW\rangle \quad (4)$$

are the generalized rotation functions—the matrix elements of the space rotation operator $e^{i\theta J_2}$—for the group G. The expansion theorem used above relies on the completeness notion which requires that we sum over a *one-parameter* family D^N of representations of G, since we are dealing with a function $T(\theta)$ of just one variable θ.

[8] R. Delbourgo, Abdus Salam, M. A. Rashid, and J. Strathdee, Phys. Rev. **170**, 1477 (1968); R. Delbourgo, A. Salam, and J. Strathdee, *ibid.* **172**, 1727 (1968); R. Delbourgo and H. A. Rashid, *ibid.* **176**, 2074 (1968).

[9] All rest symmetry groups G must contain the subgroups $SU_J(2)\otimes SU_F(3)$ [$SU_J(2)=SU_L(2)\oplus SU_S(2)$]. Embedding $SU_J(2)$ into the Lorentz structure $SL(2,C)$, one identifies the conventional helicity subgroup as that subgroup of $SU_J(2)$ whose elements commute with the Lorentz boosting operator J_{03}. To define generalized helicity, one may likewise imbed the respective rest symmetry groups G into the appropriate relativistic structures:

A	$SL_S(6,C)\otimes O_L(3,1)$,
A(i)	$\tilde{U}_S(12)\otimes O_N(4,1)$ or $\tilde{U}_S(12)\otimes O_N(4,2)\approx\tilde{U}_S(12)\otimes U_N(2,2)$,
A(ii)	$\tilde{U}_S(12)\otimes U_N(3,1)$,
B	$U_N(6,6)$.

The corresponding generalized helicity subgroups are

A	$U(3)\times U(3)\,|_W\otimes O(2)$,
A(i)	$U_W(6)\otimes O(3)$,
A(ii)	$U_W(6)\otimes U(2)$,
B	$U_W(6)$.

Note that we are making the important distinction between $\tilde{U}_S(12)$ and $U(6,6)$. In its original formulation, $\tilde{U}(12)$ was the symmetry group combining *intrinsic spin* (S) and unitary spin (F), while we regard $U(6,6)$ as a noncompact rest symmetry which combines *total spin* (J) and (F). The two $U_W(6)$ groups in models A(i), A(ii), and model B are therefore different groups; they refer, respectively, to intrinsic spin S (which has to be combined with the orbital angular momentum to give J) and to J itself.

[10] For a fuller discussion, see Delbourgo, Salam, and Strathdee, Ref. 8.

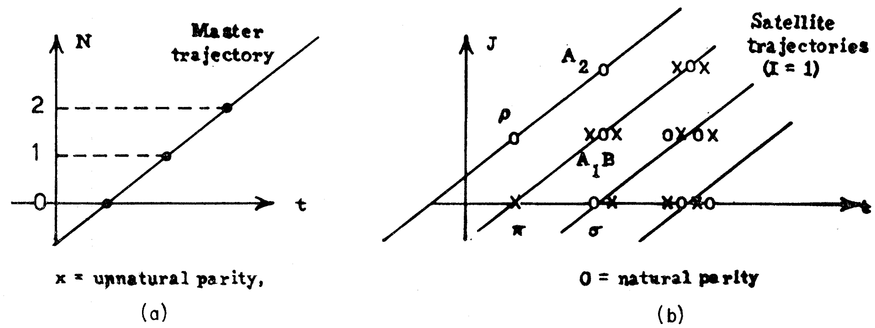

FIG. 1. Trajectories from $O(4)_N$ excitations. (○=normal parity; ×=abnormal parity.)

(8) We connect the expansion (7) with assumption (1) if we now assume that $T^N(E)$ exhibits poles in the complex N Casimir plane, corresponding to supermultiplets of group G; this reduces Eq. (3) to the form

$$\langle W_3W_4|T(E,\theta)|W_1W_2\rangle = \sum_{N\zeta W\zeta' W'} \langle W_3W_4|\zeta' W'\rangle \times g_{W_3W_4\zeta' W'}{}^N[d_{WW'}{}^N(-\theta)/(E^2-m_N{}^2)] \times g_{\zeta W W_1W_2}{}^N\langle \zeta W|W_1W_2\rangle . \quad (5)$$

(9) We can now pass to a Regge amplitude by making a Sommerfeld-Watson transformation:

$$\lim_{W\to\infty} \langle W_3W_4|T(E,\theta)|W_1W_2\rangle \sim \sum_{\zeta W\zeta' W'} g_{\zeta W' W_3W_4}{}^\alpha \times [d_{W'W}{}^\alpha(\theta)/\sin\pi\alpha(E)] g_{\zeta W W_1W_2}{}^\alpha \langle \zeta W|W_1W_2\rangle , \quad (6)$$

where $\alpha(m_N{}^2)=N$ is the supermultiplet trajectory function.

(10) The points on a "master" trajectory $\alpha(m_N{}^2)$ represent particles of differing spin values which the supermultiplet groups together. In Refs. 8 and 11, the mathematical reduction problem of expressing the general rotation functions $d^N(\theta)$ in terms of the Legendre polynomials $P^L(\theta)$ or $P^J(\theta)$ and their derivatives $[d^N(\theta)=\sum_{J,K} a_{J,K}P^J(\theta)]$ was discussed in detail. Physically this means that one master trajectory gives rise to a number of equally spaced satellite trajectories labeled with the parameter κ (in the conventional Regge ReJ-m^2 plane), all parallel to the master trajectory in the exact symmetry limit. (These satellites are not to be confused with the daughter trajectories considered by Freedman and Wang and by Toller.) In the $O(4)$ orbital scheme for mesons, for example, the following schematic picture may hold (see Fig. 1):

$N=0$	$S^P=1^-,0^-$;	$L^P=0^+$;	$J^P=0^-,1^-$				
$N=1$	$S^P=1^-,0^-$;	$L=1^-,0^+$;	$J^P=2^+,1^+,0^+$;	$1^+,1^-,0^-$			
$N=2$	$S^P=1^-,0^-$;	$L=2^+1^-0^+$;	$J^P=3^-,2^-,1^-,0^-$;	$2^+,1^+,0^+$;	$1^-,0^-$;	$2^-,1^+,0^-$.	

Note the rather obvious but extremely important circumstance that the leading satellite trajectory with the 0^- particles on it is automatically shifted downwards by one unit of J from the leading vector-tensor trajectory. The very high-energy behavior, naturally, is always dominated by the leading trajectory if the selection rules allow it to be exchanged.

(11) To take account of trajectory shifts due to symmetry breaking, we need mass formulas which, in general, may have the form (with L in place of J for the orbital models)

$$M^2=M^2(N,J,F) = M_0{}^2(N)+M_1{}^2(F)+JM_2{}^2(F) , \quad (7)$$

where $N=J+K$, and F denotes the $SU(3)$ labels (including I and Y). To incorporate trajectory shifts due to symmetry breaking in the formalism, one may go back to the formula

$$T\approx \int \frac{dN}{\sin\pi N}\frac{b^N d^N(-\theta)}{t-M^2(N)} , \quad (8)$$

write $d^N(-\theta)=\sum a_{KJ}P^J(-\theta)$, and, as an ansatz, replace $M^2(N)$ by $M^2(N,J,F)$, obtaining

$$T\sim \sum_K \int \frac{dJ}{\sin\pi J}\frac{b^{J+K}a_{KJ}P_J(-\theta)}{t-M^2(K,J,F)} . \quad (9)$$

The satellite trajectory functions $\alpha(K,J,F)$ are given as solutions of $t=M^2(K,J,F)$.

Unfortunately, no completely reliable theoretical method exists for computing these trajectory shifts. We

[11] R. Delbourgo, K. Koller, and R. Williams, J. Math. Phys. **10**, 957 (1969).

must therefore, at present, introduce the precise trajectory functions as part of empirical input. The utility of the supermultiplet Reggeization schemes is thus impaired, except for the hope that the residues are not so strongly affected by symmetry breaking as the trajectories. This appears to be the case for meson-baryon scattering (see Ref. 4).

(12) The rotation functions were computed in a previous publication[11] for a number of groups for simple classes of representations. As a general rule, a rotation function $d_{WW'}^N(\theta)$ is a sum of derivatives of the *basic* function $d_{[1][1]}^N$ (the function which appears in superscalar scattering with the exchange of a multiplet labeled with the quantum number N). This is analogous to the statement that the $d_{\lambda\lambda'}^J(\theta)$ in three dimensions can be expressed as sums of derivatives of $P_J(\theta)$. We list in the adjoining table these basic rotation functions for symmetry groups and representations of interest. G is the multiplet symmetry at rest, G_W the generalized helicity subgroup, $\mathcal{G}$ the embedding covariant group, and N labels the (one-parameter) class of representations (more precisely, we indicate the Young tableaux to which N refers).

(a) $G=U(\nu)\otimes U(\nu),\quad G_W=U(\nu),\quad \mathcal{G}=U(\nu,\nu)$.

For representations (W_N,W_N) corresponding to Young tableaux $(N,0,0,\cdots,0;\,N,N,N,\cdots,N)$,

$$d^N(\theta)\propto C_N^{\frac{1}{2}\nu}(\cos\theta)\,. \tag{10a}$$

(b) $G=U(2\nu),\quad G_W=U(\nu)\otimes U(\nu),\quad \mathcal{G}=SL(2\nu,c)$.

For representations (W_N) described by tableaux $(N+1,\,N,\,\cdots,\,N)$,

$$d^N(\theta)\propto C_N^{\nu-\frac{1}{2}}(\cos\theta)\,. \tag{10b}$$

(c) $G=U(\nu),\quad G_W=U(\nu-1),\quad \mathcal{G}=U(\nu,1)$.

For representations (N) described by tableaux $(N,0,0,\cdots,0)$,

$$d^N(\theta)=(\cos\theta)^N\,. \tag{10c}$$

(d) $G=O(\nu),\quad G_W=O(\nu-1),\quad \mathcal{G}=O(\nu,1)$.

For representations (N) described by $(N,0,0,\cdots,0)$,

$$d^N(\theta)\propto C_N^{\frac{1}{2}\nu-1}(\cos\theta)\,. \tag{10d}$$

Proofs of statements (a), (b), and (d) are already in print[11]; a proof of (c) is given further on. The Reggeized components of models A, A(i), A(ii), and B are the cases (d) with $\nu=3$, (d) with $\nu=4$, (c) with $\nu=3$, and (a) with $\nu=6$, respectively.

III. COVARIANT FORMALISM FOR SCATTERING AMPLITUDES AND M FUNCTIONS

So far we have worked with the helicity formalism. In principle, all we need now are general expressions for rotation functions $d_{WW'}^N(\theta)$ in terms of derivatives of $d_{[1][1]}^N$ of Sec. II and formulas for the general Clebsch-Gordan coefficients $\langle W|W_1W_2\rangle$, etc. One can proceed perfectly well by listing these things with the use of sophisticated group-theory methods, including the spin-orbit coupling coefficients needed in models A, A(i), and A(ii). It so happens that one of the simplest ways of making these computations is to work *ab initio* in terms of an M-function approach using a multispinor formalism. Since this has the additional merit of exhibiting manifest covariance, of allowing crossing to be performed with ease, of automatically incorporating the threshold and other mass-dependent kinematic factors,[12] from now on we shall abandon the helicity framework and work consistently with the M functions.

A. Wave Functions of Particle Multiplets

Consider first the wave functions of the particle multiplets for the various models:

Model A: *Orbital excitations.* Represent $O(3)$ multiplets of $L=0,\,1,\,2,\,\cdots$ by *symmetric traceless* tensors

$$\phi(p),\quad \phi_\mu(p),\quad \phi_{(\mu_1\mu_2)}(p),\;\cdots,$$

with the restrictions

$$\begin{aligned} p_\mu\phi_\mu&=0\,,\\ p_{\mu_1}\phi_{(\mu_1\mu_2\cdots\mu_L)}&=0\,,\\ p^2\Phi_{(\mu_1\cdots\mu_L)}&=m_L{}^2\Phi_{(\mu_1\cdots\mu_L)}\,. \end{aligned} \tag{11}$$

A(i): *Hydrogen-like excitations.* Represent $SU(2)\otimes SU(2)\approx O(4)$ multiplets belonging to the representation $(\frac{1}{2}N,\frac{1}{2}N)$, $N=0,\,1,\,2,\,\cdots$, by the multispinors

$$\Phi,\;\Phi_\alpha{}^\beta,\;\Phi_{(\alpha_1\alpha_2)}{}^{(\beta_1\beta_2)},\;\cdots,$$

symmetric in α's and β's separately, satisfying $\alpha,\,\beta=1,\,2,\,3,\,4$, Bargman-Wigner equations:

$$\begin{aligned}(p-m)_{\gamma_i}{}^{\alpha_i}&\Phi_{(\alpha_1\cdots\alpha_i\cdots\alpha_N)}{}^{(\beta_1\cdots\beta_N)}(p)\\ &=\Phi_{(\alpha_1\cdots\alpha_N)}{}^{(\beta_1\cdots\beta_i\cdots\beta_N)}(p+m)_{\beta_i}{}^{\gamma_i}=0\,. \end{aligned}\tag{12}$$

A(ii). Represent $SU(3)$ multiplets belonging to the representation[13] $[N]$, $N=0,\,1,\,2,\,\cdots$, by the fields

$$\phi(p),\quad \phi_\mu(p),\quad \phi_{\mu_1\mu_2}(p),\;\cdots,$$

which are symmetric but *not traceless* in their indices. The equations they satisfy are the same as for the $O(3)$ case:

$$\begin{aligned} p_\mu\phi_\mu&=0\,,\\ p_{\mu_1}\phi_{(\mu_1\mu_2\cdots\mu_N)}&=0\,,\\ p^2\phi_{(\mu_1\cdots\mu_N)}&=m_N{}^2\phi_{(\mu_1\cdots\mu_N)}\,. \end{aligned} \tag{13}$$

So much for the orbital part of the representations in the models A, A(i), and A(ii). For the intrinsic-spin-

[12] As stated in the Introduction, the explicit appearance of the mass factors permits one to devise mass-breaking prescriptions.

[13] See P. G. O. Freund, Nuovo Cimento **58**, 519 (1968). The $O(3)$ decompositions of these tensors is as follows:

	Dimensionality N_{C_3}
$[0]\rightarrow J=0$	1
$[1]\rightarrow J=1$	3
$[2]\rightarrow J=2,\,0$	6
$[3]\rightarrow J=1,\,3$	10, and so on.

unitary-spin part, we employ the $\tilde{U}(12)$ formalism for models A(i) and A(ii) using the multispinors

$$\Phi_B{}^A(p),\quad A,B=1,\cdots,12$$

for mesons $(6,\bar{6})$ of rest symmetry $U_S(6)\otimes U_S(6)$ and

$$\Psi_{(ABC)}(p),\quad A,B,C=1,\cdots,12$$

(symmetric A, B, C) for baryons (56,1) and $\Psi_{[AB]C}$ antisymmetric in $[A,B]$ and satisfying the cyclic condition

$$\Psi_{[AB]C}+\Psi_{[BC]A}+\Psi_{[CA]B}=0$$

for the (70,1).

The Bargmann-Wigner equations[14] are

$$\begin{aligned}(p-m)_C{}^A\Phi_A{}^B=\Phi_A{}^B(p+m)_B{}^C=0\,,\\ (p-m)_D{}^A\Psi_{(ABC)}=(p-m)_D{}^A\Psi_{[AB]C}=0,\ \text{etc.}\end{aligned}\tag{14}$$

Combining the spin–unitary-spin *and* the orbital degrees of freedom, the tensors have the final forms:

Model A(i):

$$\begin{aligned}&\Phi_B{}^A{}_{(\alpha_1\cdots\alpha_N)}{}^{(\beta_1\cdots\beta_N)} &&\text{for } (6,\bar{6},N)\,,\\ &\Psi_{(ABC)(\alpha_1\cdots\alpha_N)}{}^{(\beta_1\cdots\beta_N)} &&\text{for } (56,1,N)\,,\\ &\Psi_{[AB]C(\alpha_1\cdots\alpha_N)}{}^{(\beta_1\cdots\beta_N)} &&\text{for } (70,1,N)\,,\end{aligned}$$

with the equations of motion (12) and (14) stated earlier, and

Model A(ii):

$$\begin{aligned}&\Phi_B{}^A{}_{(\mu_1\cdots\mu_N)} &&\text{for } (6,\bar{6},N)\,,\\ &\Psi_{(ABC)(\mu_1\cdots\mu_N)} &&\text{for } (56,1,N)\,,\\ &\Psi_{[AB]C(\mu_1\cdots\mu_N)} &&\text{for } (70,1,N)\,,\end{aligned}$$

satisfying Eqs. (13) and (14). We shall be concentrating on model A(ii) in what follows.

Model B: *The quark excitation model* was dealt with in detail in Ref. 8. To complete the discussion for the case of the group $U_J(6)\times U_J(6)$, the appropriate multispinors belonging to the fully symmetrical representation (the so-called Feynman representation) are

$$\begin{aligned}&\Phi_A{}^B,\Phi_{(A_1A_2)}{}^{(B_1B_2)},\Phi_{(A_1A_2A_3)}{}^{(B_1B_2B_3)},\cdots &&\text{for mesons}\,,\\ &\Psi_{(A_1A_2A_3)}\Psi_{(A_1A_2A_3A_4)}{}^B,\cdots &&\text{for baryons}\,,\end{aligned}$$

satisfying Eqs. (14).

B. Three-Point Couplings

Given three multiplets of G for any of these models, one can easily write down the G_W invariant couplings in M-function form by noting that the three-particle momenta transform as G_W scalars. The rule then is to saturate all indices among themselves and with momentum tensors; the number of different ways of doing this giving the different types of couplings one can construct. To illustrate, consider the simple case of a quark $(6,1;0)_{-\frac{1}{2}p+q}$ and an antiquark $(1,\bar{6};0)_{\frac{1}{2}p+q}$ coupling with the $(6,\bar{6};N)_p$ meson multiplets.

A(i) $$\begin{aligned}\mathfrak{L}_{\text{eff}}=&\bar{v}^A(\tfrac{1}{2}p+q)u_B(-\tfrac{1}{2}p+q)\\ &\times[G_0\delta_A{}^Bq_D{}^C+G_1\delta_A{}^C\delta_D{}^B]q_{\beta_1}{}^{\alpha_1}\\ &\cdots q_{\beta_N}{}^{\alpha_N}\Phi_C{}^D{}_{(\alpha_1\cdots\alpha_N)}{}^{(\beta_1\cdots\beta_N)}(p)\,;\end{aligned}\tag{15a}$$

A(ii) $$\begin{aligned}=&\bar{v}^A(\tfrac{1}{2}p+q)u_B(-\tfrac{1}{2}p+q)[G_0\delta_A{}^Bq_D{}^C+G_1\delta_A{}^C\delta_D{}^B]\\ &\times q_{\mu_1}\cdots q_{\mu_N}\Phi_C{}^D{}_{(\mu_1\cdots\mu_N)}(p)\,;\end{aligned}\tag{15b}$$

B $$\begin{aligned}\mathfrak{L}_{\text{eff}}=&\bar{v}^A(\tfrac{1}{2}p+q)u_B(-\tfrac{1}{2}p+q)\\ &\times[G_0\delta_A{}^Bq_{B_1}{}^{A_1}+G_1\delta_A{}^{A_1}\delta_{B_1}{}^B]q_{B_2}{}^{A_2}\\ &\cdots q_{B_N}{}^{A_N}\Phi_{(A_1\cdots A_N)}{}^{(B_1\cdots B_N)}(p)\,.\end{aligned}\tag{15c}$$

We can cast all models in the differential form

$$\mathfrak{L}_{\text{eff}}=\bar{v}^Au_B[g_0\delta_A{}^B+mg_1(\partial/\partial q_B{}^A)]\Phi_{(N)}(p,q)\,,\tag{16}$$

where

$$\begin{aligned}\Phi_{(N)}(p,q)&\\ =&\mu^{-(N+1)}q_B{}^Aq_{\beta_1}{}^{\alpha_1}\cdots q_{\beta_N}{}^{\alpha_N}\\ &\times\Phi_A{}^B{}_{(\alpha_1\cdots\alpha_N)}{}^{(\beta_1\cdots\beta_N)}(p) &&(17a)\\ =&\mu^{-(N+1)}q_B{}^Aq_{\mu_1}\cdots q_{\mu_N}\Phi_A{}^B{}_{(\mu_1\cdots\mu_N)}(p) &&(17b)\\ =&\mu^{-N}q_{B_1}{}^{A_1}q_{B_2}{}^{A_2}\cdots q_{B_N}{}^{A_N}\Phi_{(A_1\cdots A_N)}{}^{(B_1\cdots B_N)}(p) &&(17c)\end{aligned}$$

is the coupling corresponding to scattering of supersinglets for each of these models. Note the distinction between $q_B{}^A$ and $q_\beta{}^\alpha$. For the $\tilde{U}(12)$, case $q_B{}^A=q_\beta{}^\alpha\delta_b{}^a$ where a, b refer to $SU(3)$ indices and α, β to $\tilde{U}(4)$ indices.

C. Supermultiplet Exchange Contributions to Four-Point Couplings and Computation of Rotation Functions

Just as the Legendre functions $d_{\lambda'\lambda}{}^J(\theta)$ can be "calculated" by considering the exchange of a spin-J particle in an M-function framework, so can the generalized rotation functions $d_{W'W}{}^N(\theta)$ for each of the models by exchanging a supermultiplet of quantum number N using the covariant couplings (15) written earlier. The procedures were illustrated in great detail in Ref. 8 for model B; here we quickly go over the methods again for models A(i) and A(ii). The basic functions $d_{[1][1]}{}^N(\theta)$ arise from scattering of supersinglets. Combining the three-point couplings with the $[N]$ propagators

A(i) $$\begin{aligned}&\langle\Phi_A{}^B{}_{(\alpha_1\cdots\alpha_N)}{}^{(\beta_1\cdots\beta_N)}(p)\Phi_{B'}{}^{A'}{}_{(\beta_1'\cdots\beta_N')}{}^{(\alpha_1'\cdots\alpha_N')}(-p)\rangle(p^2-M^2)\\ &\quad=(p+M)_A{}^{A'}(p-M)_{B'}{}^B\sum_{\alpha,\beta}(p+M)_{\alpha_1}{}^{\alpha_1'}\cdots(p+M)_{\alpha_N}{}^{\alpha_N'}(p-M)_{\beta_1'}{}^{\beta_1}\cdots(p-M)_{\beta_N'}{}^{\beta_N}\,,\end{aligned}\tag{18a}$$

A(ii) $$\begin{aligned}&\langle\Phi_A{}^B{}_{(\mu_1\cdots\mu_N)}(p)\Phi_{B'}{}^{A'}{}_{(\mu_1'\cdots\mu_N')}(-p)\rangle(p^2-M^2)\\ &\quad=(p+M)_A{}^{A'}(p-M)_{B'}{}^B\sum_{\mu}(-g_{\mu_1\mu_1'}+p_{\mu_1}p_{\mu_1'}/M^2)\cdots(-g_{\mu_N\mu_N'}+p_{\mu_N}p_{\mu_N'}/M^2)\,,\end{aligned}\tag{18b}$$

[14] A complete and detailed discussion of the $\tilde{U}(12)$ multispinors appears in *Proceedings of the International Seminar in High-Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965).

we get

$$(p^2-M^2)\langle\Phi_{(N)}(p,q)\Phi_{(N)}(-p,q')\rangle$$
$$=(|\mathbf{q}||\mathbf{q}'|/\mu^2)^{N+1}\times\cos\theta\, C_N{}^1(\cos\theta) \quad \text{A(i)}$$
$$=(|\mathbf{q}||\mathbf{q}'|/\mu^2)^{N+1}\times\cos\theta(\cos\theta)^N \quad \text{A(ii)}, \quad (19)$$

where

$$|\mathbf{q}||\mathbf{q}'|\cos\theta=-q\cdot q'+q\cdot pq'\cdot p/M^2, \qquad (20)$$

showing that the excitation functions are $C_N{}^1(\cos\theta)$ and $(\cos\theta)^N$ for $O(4)$ and $U(3)$, respectively; both multiplied into a $U(6)\otimes U(6)$ spin factor $(\cos\theta)=d_{[1][1]}{}^{(6,\bar{6})}\times(\cos\theta)$ coming from the $(6,\bar{6})$ piece.[15]

More complicated functions can be discovered by differentiation; for example, in quark-antiquark scattering the derived d^N can be recovered from

$$[g_0\delta_A{}^B+mg_1(\partial/\partial q_B{}^A)]$$
$$\times[g_0\delta_{B'}{}^{A'}+mg_1(\partial/\partial q'_{A'}{}^{B'})]d_{[1][1]}{}^N(\theta) \quad (21)$$

by contraction over external wave functions. We thus have a complete M-function substitute for quark-antiquark scattering of the helicity formulation of the earlier section. More complicated cases of physical interest are described in Sec. IV for model A(ii).

IV. DETAILS OF MESON-BARYON AND BARYON-BARYON SCATTERING IN THE OSCILLATOR MODEL

In this section, we wish to discuss the harmonic-oscillator model in detail. From the slender evidence available, it appears that baryons group themselves as

$$\left.\begin{matrix}(56,0)\\(56,2)\end{matrix}\right\}\quad(70,1)$$

with (56,1) apparently missing. It would thus seem that we are realizing the quantum numbers $N=0$, 2, 4, of an $SU(3)$ group, corresponding to representations

$$L=0\,,$$
$$L=0,\ 2\,,$$
$$L=0,\ 2,\ 4,\ \text{etc.}$$

Also, we know that $d_{[1][1]}{}^N\approx(\cos\theta)^N$ for all models in the asymptotic limit. So, we can in any case regard the oscillator model as *representative* of all models in the high-energy limit. Of course, to lower orders the models will differ from one another.

A. Wave Functions

Hereafter, we work entirely in the covariant framework provided by the auxiliary group $\tilde{U}(12)\otimes U(3,1)$. For M-function purposes, we adopt the fields

$$\phi_A{}^B{}_{(\mu_1\cdots\mu_N)},\quad \psi_{(ABC)(\mu_1\cdots\mu_{2N})},\quad \text{and}\ \psi_{[AB]C,(\mu_1\cdots\mu_{2N+1})}\,.$$

If we are interested in the Lorentz group components within the supermultiplets, we make, in the first step, the decompositions

$$\Phi_A{}^B{}_{(\mu_1\cdots\mu_N)}(p)$$
$$=(1/2\sqrt{2}M)[(\gamma\cdot p+M)$$
$$\times(\gamma_5P_{(\mu_1\cdots\mu_N)}-\gamma_\mu V_{\mu(\mu_1\cdots\mu_N)})]_A{}^B, \quad (22)$$

$$u_{ABC(\mu_1\cdots\mu_N)}(p)$$
$$=(1/2\sqrt{2}M)[(\gamma\cdot p+M)\gamma_\mu C]_{\alpha\beta}D_{(abc)\gamma\mu(\mu_1\cdots\mu_N)}$$
$$+(1/6\sqrt{2}M)\{[(\gamma\cdot p+M)\gamma_5C]_{\alpha\beta}$$
$$\times\epsilon_{abd}N_c{}^d{}_{\gamma(\mu_1\cdots\mu_N)}+\text{perms}\}, \quad (23)$$

$$u_{[AB]C(\mu_1\cdots\mu_N)}(p)$$
$$=(1/4M)[(\gamma\cdot p+M)\gamma_\mu C]_{\alpha\beta}\epsilon_{abd}\mathfrak{N}_c{}^d{}_{\gamma\mu(\mu_1\cdots\mu_N)}$$
$$+(1/2\sqrt{2})M[(\gamma\cdot p+M)\gamma_5C]_{\alpha\beta}\mathfrak{D}_{(abc)\gamma(\mu_1\cdots\mu_N)}$$
$$+[1/2(\sqrt{6})M]\{[(\gamma\cdot p+M)\gamma_5C]_{\beta\gamma}\epsilon_{bcd}\mathfrak{N}_a{}^d{}_{\alpha(\mu_1\cdots\mu_N)}$$
$$-[(\gamma\cdot p+M)\gamma_5C]_{\alpha\gamma}\epsilon_{acd}\mathfrak{N}_b{}^d{}_{\beta(\mu_1\cdots\mu_N)}\}$$
$$+(\epsilon_{abc}/12M)\{[(\gamma\cdot p+M)\gamma_5C]_{\alpha\gamma}\mathcal{Y}_{\beta(\mu_1\cdots\mu_N)}$$
$$+[(\gamma\cdot p+M)\gamma_5C]_{\beta\gamma}\mathcal{Y}_{\alpha(\mu_1\cdots\mu_N)}\}, \quad (24)$$

wherein the (N) excitations of the basic (quark) spin fields $P(0^-)$, $V(1^-)$, $N(\frac{1}{2}^+)$, $D(\frac{3}{2}^+)$, $\cdots$ is explicitly exhibited. In the second step, the excitations are reduced under $O(3)_L$ according to the further decomposition

$$\phi_{(\mu_1\cdots\mu_N)}(p)$$
$$=\hat{\phi}_{(\mu_1\cdots\mu_N)}{}^{(L=N)}(p)+\left[\frac{2}{N(N-1)(2N-1)}\right]^{1/2}$$
$$\times\sum_{kl}d_{\mu_k\mu_l}\hat{\phi}_{(\mu_1\cdots\bar{k}\bar{l}\cdots\mu_N)}{}^{(L=N-2)}(p)$$
$$+\text{lower orbital momenta}; \quad (25)$$

$d_{\mu\nu}\equiv-g_{\mu\nu}+p_\mu p_\nu/M^2$ and $\hat{\phi}$ are traceless in their indices. ($\bar{k}$ indicates that μ_k is absent, etc.) In the third step, the product of each of these $SU(2)_L$ irreducible components with the $SU(2)_S$ components is reduced out into total $SU(2)_J$ components. If we take a given orbital excitation L, the decomposition reads as follows: For fields of (quark) spin 1, $\frac{1}{2}$, and $\frac{3}{2}$ (the only cases of physical interest),

$$\phi_{\mu(\mu_1\cdots\mu_L)}=\phi_{(\mu\mu_1\cdots\mu_L)}{}^{(L+1)}+\frac{1}{[L(L+1)]^{1/2}}\sum_k\frac{p_\lambda}{M}\epsilon_{\lambda\mu\mu_k\nu}\phi_{(\mu_1\cdots\bar{k}\cdots\mu_L\nu)}{}^{(L)}$$
$$+\frac{1}{L}\left(\frac{2L-1}{2L+1}\right)^{1/2}\left[\sum_k d_{\mu\mu_k}\phi_{(\mu_1\cdots\bar{k}\cdots\mu_L)}{}^{(L-1)}-\frac{2}{2L-1}\sum_{kl}d_{\mu_k\mu_l}\phi_{(\mu\mu_1\cdots\bar{k}\bar{l}\cdots\mu_L)}{}^{(L-1)}\right], \quad (26)$$

[15] Actually, if Bose statistics is taken into account, only odd N values for A(i) and A(ii) and even N values for B are permitted.

$$\psi_{(\mu_1\cdots\mu_L)}=\psi_{(\mu_1\cdots\mu_L)}{}^{(L+\frac{1}{2})}+\frac{1}{[L(4L-1)]^{1/2}}\sum_k w_{\mu_k}\psi_{(\mu_1\cdots\bar{k}\cdots\mu_L)}{}^{(L-\frac{1}{2})}\,, \tag{27}$$

$$\begin{aligned}\psi_{\mu(\mu_1\cdots\mu_L)}&=\psi_{(\mu\mu_1\cdots\mu_L)}{}^{(L+\frac{3}{2})}\\&+\frac{1}{[L(L+3)]^{1/2}}\Big[\sum_k\frac{p_\lambda}{M}\epsilon_{\lambda\mu\mu_k\nu}\psi_{(\mu_1\cdots\bar{k}\cdots\mu_L\nu)}{}^{(L+\frac{1}{2})}+\sum_k w_{\mu_k}\psi_{(\mu\mu_1\cdots\bar{k}\cdots\mu_L)}{}^{(L+\frac{1}{2})}-\tfrac{1}{3}Lw_\mu\psi_{(\mu_1\cdots\mu_L)}{}^{(L+\frac{1}{2})}\Big]\\&+\frac{1}{L}\Big(\frac{2L-1}{2L}\Big)^{1/2}\Big[\tfrac{2}{3}\sum_k d_{\mu\mu_k}\psi_{(\mu_1\cdots\bar{k}\cdots\mu_L)}{}^{(L-\frac{1}{2})}-\frac{2}{2L-1}\sum_{kl}d_{\mu_k\mu_l}\psi_{(\mu\mu_1\cdots\bar{k}\bar{l}\cdots\mu_L)}{}^{(L-\frac{1}{2})}-\tfrac{1}{3}\sum_k\frac{p_\lambda}{M}\epsilon_{\lambda\mu\mu_k\nu}w_\nu\psi_{(\mu_1\cdots\bar{k}\cdots\mu_L)}{}^{(L-\frac{1}{2})}\Big]\\&+\Big(\frac{6}{(L+3)!}\Big)^{1/2}\Big[\sum_{kl}d_{\mu\mu_k}w_{\mu_l}\psi_{(\mu_1\cdots\bar{k}\bar{l}\cdots\mu_L)}-\frac{2}{2L-1}w_\mu\sum_{kl}d_{\mu_k\mu_l}\psi_{(\mu_1\cdots\bar{k}\bar{l}\cdots\mu_L)}-\frac{2}{2L-1}\sum_{klm}d_{\mu_k\mu_l}w_{\mu_m}\psi_{(\mu\mu_1\cdots\bar{k}\bar{l}\bar{m}\cdots\mu_L)}\Big],\end{aligned} \tag{28}$$

where $\psi_{(\mu_1\cdots\mu_N)}{}^{(N+\frac{1}{2})}$ are Rarita-Schwinger fields of spin $N+\frac{1}{2}$ and we have used above the abbreviation for the relativistic spin operator $w_\mu\equiv p_\lambda\sigma_{\lambda\mu}\gamma_5/M$. In the exact-symmetry limit, we can summarize these reductions by sets of J trajectories, as depicted in Figs. 2–5, where we have assumed exchange degeneracy for ease of drawing.

B. Three-Point Couplings

The great importance of reductions (22)–(28) is apparent when one wishes to compare matrix elements of particular spin components, primarily in the three-point couplings which are relevant for correlating various decay parameters or Regge residues. As we mentioned earlier, these three-point vertices are constructed in a G_W-invariant manner by forming index invariants between fields and momenta.[16] For hadrons like $(6,\bar{6};0)$, $(6,\bar{6};1)$, $(56,1;0)$, $(70,1;1)$ and a meson supermultiplet of excitation N which we later Reggeize, we list below the effective Lagrangians of these hadrons, writing our

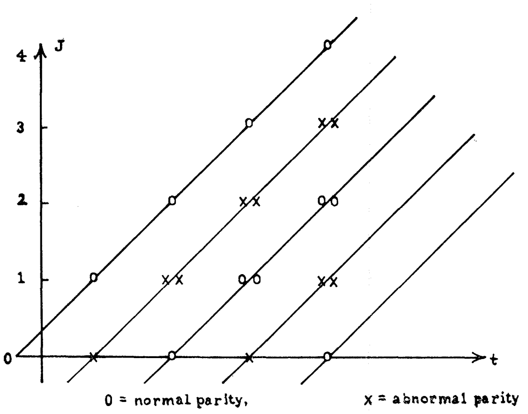

FIG. 2. Reggeized $SU(3)$ nonets of mesons from oscillator model $(6,\bar{6};N)$. (○ = normal parity, ✕ = abnormal parity.)

[16] The over-all coupling constants depend intrinsically on the excitation numbers of the three interacting particles (N_1,N_2,N_3), and we may even suspect that when one excitation number becomes large there is a corresponding falloff in the residue; but we do not attempt to investigate this aspect.

couplings in a differential notation:

$(6,\bar{6};0)_{\frac{1}{2}p+q}-(6,\bar{6};0)_{\frac{1}{2}p-q}-(6,\bar{6};N)_p$:

$$\begin{aligned}\mathcal{L}&=\mu^{1-N}\Phi_A{}^B(\tfrac{1}{2}p+q)\Phi_C{}^D(\tfrac{1}{2}p-q)\\&\times\Big[h_0{}^{(-)}\delta_B{}^C\delta_D{}^A+h_0'{}^{(-)}\mu^{-2}q_B{}^Aq_D{}^C\\&+\mu h_1{}^{(+)}\Big(\delta_B{}^C\frac{\partial}{\partial q_A{}^D}+\delta_D{}^A\frac{\partial}{\partial q_C{}^B}\Big)\\&+\mu h_1{}^{(-)}\Big(\delta_B{}^C\frac{\partial}{\partial q_A{}^D}-\delta_D{}^A\frac{\partial}{\partial q_C{}^B}\Big)\Big]\Phi_{(N)}(p,q);\end{aligned} \tag{29}$$

$(6,\bar{6};1)_{\frac{1}{2}p+q}-(6,\bar{6};0)_{\frac{1}{2}p-q}-(6,\bar{6};N)_p$:

$$\begin{aligned}\mathcal{L}&=\mu^{1-N}\Phi_A{}^B{}_\mu(\tfrac{1}{2}p+q)\Phi_C{}^D(\tfrac{1}{2}p-q)\\&\times\Big[(h_{00}\delta_B{}^C\delta_D{}^A+\mu^{-2}h_{00}'q_B{}^Aq_D{}^C)q_\mu\\&+(\mu^2h_{01}\delta_B{}^C\delta_D{}^A+h_{01}'q_B{}^Aq_D{}^C)\frac{\partial}{\partial q_\mu}\\&+\mu q_\mu\Big(h_{10}\delta_B{}^C\frac{\partial}{\partial q_A{}^D}+h_{10}'\delta_D{}^A\frac{\partial}{\partial q_C{}^B}\Big)\\&+\mu^3\frac{\partial}{\partial q_\mu}\Big(h_{11}\delta_B{}^C\frac{\partial}{\partial q_A{}^D}+h_{11}'\delta_D{}^A\frac{\partial}{\partial q_C{}^B}\Big)\Big]\Phi_{(N)}(p,q);\end{aligned} \tag{30}$$

$(56,1;0)_{\frac{1}{2}p-q}-(\overline{56},1;0)_{\frac{1}{2}p+q}-(6,\bar{6};N)_p$:

$$\begin{aligned}\mathcal{L}=m^{-N}\bar{u}^{(ACD)}(\tfrac{1}{2}p+q)u_{(BCD)}(-\tfrac{1}{2}p+q)&\\\times[g_0\delta_A{}^B+mg_1(\partial/\partial q_B{}^A)]\Phi_{(N)}(p,q);&\end{aligned} \tag{31}$$

$(56,1;0)_{\frac{1}{2}p-q}-(\overline{70},1;1)_{\frac{1}{2}p+q}-(6,\bar{6};N)$:

$$\begin{aligned}\mathcal{L}=m^{-N}\bar{u}^{[AC]D}{}_\mu(\tfrac{1}{2}p+q)u_{(BCD)}(-\tfrac{1}{2}p+q)&\\\times[g_0q_\mu+m^2g_1(\partial/\partial q_\mu)](\partial/\partial q_B{}^A)\Phi_{(N)}(p,q).&\end{aligned} \tag{32}$$

This set of formulas are some of the most crucial ones in this paper.

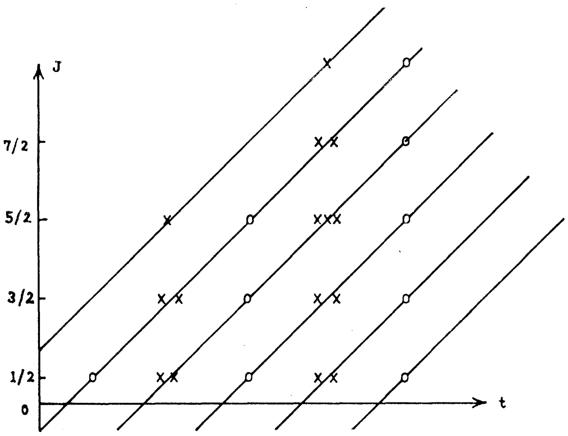

FIG. 3. Reggeized $SU(3)$ octets of positive-parity baryons (56,1; $2N$) and negative-parity baryons (70,1; $2N+1$) from oscillator model.

As before, we have used here the abbreviation

$$\Phi_{(N)}(p,q)\equiv q_{\mu_1}\cdots q_{\mu_N}q_B{}^A\Phi_A{}^B{}_{(\mu_1\cdots\mu_N)}(p) \qquad (33)$$

for the fully contracted meson field of excitation N. The superscripts $(\pm)$ on the couplings h for meson coupling (with no baryons involved) refer to the even and odd N values of the exchange mesons. Bose statistics tells us that $h^+=0$ when N is odd and $h^-=0$ when N is even.[17]

C. Meson-Baryon (MB) and Baryon-Baryon (BB) Scattering

The two most important cases concern meson-baryon (MB) and baryon-baryon (BB) scattering. We investigate the pure symmetry limit (with all masses degenerate).

If we apply the coupling rules embodied by formulas (29)–(32), it is always possible to express the covariant M functions in the form

$$T=D_qD_{q'}\Delta_{(N)}(p;\,q,q')\,, \qquad (34)$$

where D stand for various differential operators whose order is *governed by the external excitation numbers*, and where $\Delta_{(N)}$ is the fully contracted propagator which occurs in the scattering of supersinglets:

$$\Delta_{(N)}=(|\mathbf{q}|\,|\mathbf{q}'|)^{N+1}[(\cos\theta_t)^{N+1}/(p^2-M_N{}^2)]\,, \qquad (35)$$

$$|\mathbf{q}|\,|\mathbf{q}'|\,\cos\theta_t\equiv -q\cdot q'+q\cdot pq'\cdot p/M_N{}^2\,. \qquad (20)$$

In fact, $(\cos\theta)(\cos\theta)^N$ is the direct product of the basic representation functions $d_{[1][1]}{}^{(6,\bar{6})}(\theta)d_{[1][1]}{}^N(\theta)$ for $[U(6)\otimes U(6)]\otimes U(3)$ from which the general

$$d_{[W][W']}{}^{(6,\bar{6})}(\theta)d_{[W][W']}{}^N(\theta)$$

follow by the differentiations. Let us list the first few derivatives for later use:

$$\frac{\partial}{\partial q_A{}^B}\Delta_{(N)}=\frac{(\mathbf{q}\cdot\mathbf{q}')^N}{p^2-M^2}\frac{[(M+p)q'(M-p)]_B{}^A}{4M^2}\,, \qquad (36a)$$

$$\frac{\partial}{\partial q_\mu}\Delta_{(N)}=\frac{N(\mathbf{q}\cdot\mathbf{q}')^N|\mathbf{q}'|}{p^2-M^2}\left(-q_\mu'+\frac{p\cdot q'}{M^2}p_\mu\right), \qquad (36b)$$

$$\frac{\partial^2}{\partial q_\mu\partial q_A{}^B}\Delta_{(N)}=\frac{N(\mathbf{q}\cdot\mathbf{q}')^{N-1}|\mathbf{q}'|^2}{p^2-M^2}\left(-q_\mu+\frac{p\cdot q'}{M^2}p_\mu\right)\times\frac{[(M+p)q'(M-p)]_B{}^A}{4M^2}\,, \qquad (36c)$$

$$\frac{\partial^2}{\partial q_A{}^B\partial q'_{B'}{}^{A'}}\Delta_{(N)}=\frac{(\mathbf{q}\cdot\mathbf{q}')^N}{p^2-M^2}(M+p)_B{}^{B'}(M-p)_{A'}{}^A\,. \qquad (36d)$$

These Reggeize through the replacement

$$\frac{(\mathbf{q}\cdot\mathbf{q}')^N}{p^2-M^2}\rightarrow\frac{(\mathbf{q}\cdot\mathbf{q}')^{\alpha-1}}{\sin\pi(\alpha-1)}\quad(N\rightarrow\alpha-1)\,. \qquad (37)$$

It is well known that (37) gives poles at nonsense N

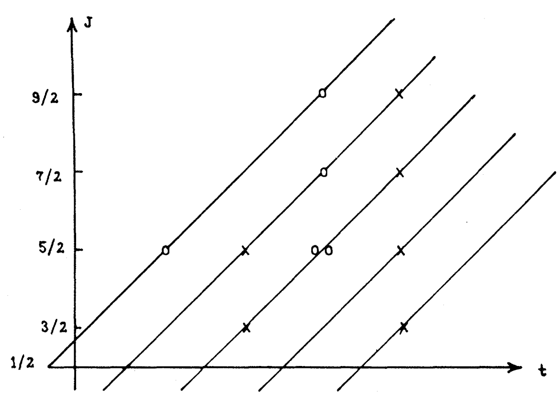

FIG. 4. Reggeized $SU(3)$ decuplets of baryons from oscillator model.

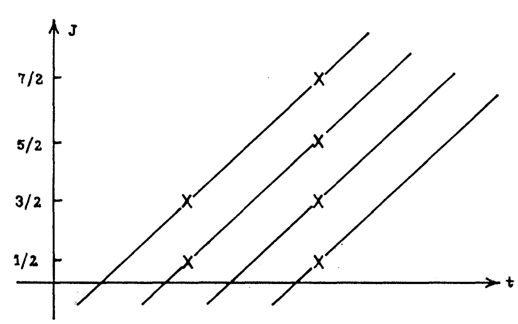

FIG. 5. Reggeized $SU(3)$ singlets of baryons from oscillator model.

[17] The subscripts on the dimensionless coupling constants refer to excitation numbers of the $U(6)\otimes U(2)_W$ subgroup representations; for instance, h_{11} in (30) corresponds to the (**35**;**3**) component of the exchanged meson, the "0" referring to singlets of the group, "1" to the first multiplet, "2" to the second, and so on. Note that, in contrast to model B, models A(i) and A(ii) can have no **405** component of $U(6)_W$ in couplings to the exchanged mesons. Also, observe the very small number of constants that appear, particularly for the baryons. This is the most powerful predictive feature of supermultiplet theory.

values ($\alpha=0, -1, -2, \cdots$). A recent suggestion to avoid these poles has been to replace $1/\sin\pi(\alpha-1)$ by $\Gamma(1-\alpha)$. This corresponds to introducing the Gell-Mann ghost-eliminating mechanism. It may be possible to devise other mechanisms for eliminating nonsense J values, but they would not fit into the orbital excitation picture so simply.

The above is all the apparatus one needs for studying the physical amplitudes of interest. We now summarize the formulas for BB and MB scattering:

$$T_{BB}=\bar{u}^{ACD}(\tfrac{1}{2}p+q')u_{BCD}(-\tfrac{1}{2}p+q)\bar{u}^{B'C'D'}(-\tfrac{1}{2}p+q')u_{A'C'D'}(\tfrac{1}{2}p+q')$$
$$\times m^{-2(N+1)}[g_0\delta_A{}^B+mg_1(\partial/\partial q_B{}^A)][g_0\delta_{B'}{}^{A'}+mg_1(\partial/\partial q_{A'}{}^{B'})]\Delta_{(N)}, \quad (38)$$

$$T_{MB}=\bar{u}^{ACD}(\tfrac{1}{2}p+q)u_{BCD}(-\tfrac{1}{2}p+q)\Phi_{A'}{}^{B'}(\tfrac{1}{2}p+q')\Phi_{C'}{}^{D'}(\tfrac{1}{2}p-q')(m\mu)^{-(N+1)}$$
$$\times\left[g_0\delta_B{}^A+mg_1\frac{\partial}{\partial q_A{}^B}\right]\left[h_0{}^{(-)}\delta_{B'}{}^{C'}\delta_{D'}{}^{A'}+\mu^{-2}h_0'{}^{(-)}q'_{B'}{}^{A'}q'_{D'}{}^{C'}\right.$$
$$\left.+\mu h_1{}^{(+)}\left(\delta_{B'}{}^{C'}\frac{\partial}{\partial q'_{A'}{}^{D'}}+\delta_{D'}{}^{A'}\frac{\partial}{\partial q'_{C'}{}^{B'}}\right)+\mu h_1{}^{(-)}\left(\delta_{B'}{}^{C'}\frac{\partial}{\partial q'_{A'}{}^{D'}}-\delta_{D'}{}^{A'}\frac{\partial}{\partial q'_{C'}{}^{B'}}\right)\right]\Delta_{(N)}. \quad (39)$$

These are the master formulas of this paper. For definitions of derivatives and wave functions $\Phi_B{}^A, \cdots$, see (22)–(28).

To illustrate the use of these master formulas, consider the charge-exchange MB processes. For those amplitudes, the couplings g_0 and h_0 are not relevant since they only describe elastic processes. Thus,

$$T_{\text{c.e.}}=(m\mu)^{-N-1}g_1h_1{}^{(+)}\bar{u}^{ACD}u_{BCD}\{\Phi,\Phi\}_{B'}{}^{A'}\frac{\partial}{\partial q_B{}^A}\frac{\partial}{\partial q'_{A'}{}^{B'}}\Delta_{(N)}$$
$$+(m\mu)^{-N-1}g_1h_1{}^{(-)}\bar{u}^{ACD}u_{BCD}[\Phi,\Phi]_{B'}{}^{A'}\frac{\partial}{\partial q_B{}^A}\frac{\partial}{\partial q'_{A'}{}^{B'}}\Delta_{(N)}. \quad (40)$$

Recall that $(\pm)$ superscripts refer to even and odd N. As is well known, after Reggeization a signature factor needs to be introduced into the formalism (the simple argument of Reggeization in Sec. II does not automatically produce this). In what follows, whenever we write $g^{\pm}$, we shall assume that N-signature projections $\frac{1}{2}(1\pm e^{i\pi N})$ are to be included. Performing the differentiation and simplifying,

$$T_{\text{c.e.}}=[g_1h_1{}^{(+)}/(t-M^2)]\bar{u}\Gamma_+\{\Phi,\Phi\}\Gamma_-u(\mathbf{q}\cdot\mathbf{q}')^N+[g_1h_1{}^{(-)}/(t-M^2)]\bar{u}\Gamma_+[\Phi,\Phi]\Gamma_-u(\mathbf{q}\cdot\mathbf{q}')^N \quad (41)$$

with

$$(\Gamma_\pm)_A{}^B=(1/2M)(M\pm p)_A{}^B \quad (42)$$

and $t\equiv p^2=M^2$ at the pole. Let us focus our attention on only those reactions where the incoming meson is pseudoscalar and the target is a nucleon, while allowing the final meson and nucleon to be any other member of the $(6,\bar{6})$ and (56,1) multiplets. Breaking up the outgoing mesons in formula (41) into vector and pseudoscalar parts and separating out $SU(3)$ components, the Reggeized amplitude reads

$$T_{\text{Regge}}=[1+M/2\mu]\bar{u}^{ACD}(\Gamma_+q'\Gamma_-/\mu)_\alpha{}^\beta u_{BCD}$$
$$\times[\tfrac{1}{2}\beta_-(1-e^{i\pi\alpha_-})\Gamma(1-\alpha_-)(P,P)_F(\mathbf{q}\cdot\mathbf{q}'/\mu m)^{\alpha_--1}+\tfrac{1}{2}\beta_+(1+e^{i\pi\alpha_+})\Gamma(1-\alpha_+)(P,P)_D(\mathbf{q}\cdot\mathbf{q}'/\mu m)^{\alpha_+-1}]_a{}^b$$
$$+[1+M/2\mu]\bar{u}^{ACD}(\Gamma_+[q',\gamma_\lambda]\gamma_5\Gamma_-/2\mu)_\alpha{}^\beta u_{BCD}[\tfrac{1}{2}\beta_-(1-e^{i\pi\alpha_-})\Gamma(1-\alpha_-)(P,V_\lambda)_D(\mathbf{q}\cdot\mathbf{q}'/\mu m)^{\alpha_--1}$$
$$+\tfrac{1}{2}\beta_+(1+e^{i\pi\alpha_+})\Gamma(1-\alpha_+)(P,V_\lambda)_F(\mathbf{q}\cdot\mathbf{q}'/\mu m)^{\alpha_+-1}]_a{}^b+[1+M/2\mu]\bar{u}^{ACD}(\Gamma_-\gamma_\lambda\gamma_5\Gamma_+)_\alpha{}^\beta u_{BCD}$$
$$\times[\tfrac{1}{2}\beta_-(1-e^{i\pi\alpha_-})\Gamma(1-\alpha_-)(P,V_\lambda)_F(\mathbf{q}\cdot\mathbf{q}'/\mu m)^{\alpha_--1}+\tfrac{1}{2}\beta_+(1+e^{i\pi\alpha_+})\Gamma(1-\alpha_+)(P,V_\lambda)_D(\mathbf{q}\cdot\mathbf{q}'/\mu m)^{\alpha_+-1}]_a{}^b. \quad (43)$$

We have distinguished between the two possible trajectories α_- and α_+ associated with the two signatures. The $(\mathbf{q}\cdot\mathbf{q}')^{\alpha_+-1}$ or $(\mathbf{q}\cdot\mathbf{q}')^{\alpha_--1}$ oscillator factors are characteristic orbital effects. Making a further decomposition into octet (N) and decuplet (D) pieces of the **56**, one gets

$$T_{PN\to PN}=\{a_F(\bar{N}N)_F+a_D(\bar{N}N)_D+\mu^{-1}[b_F(\bar{N}q'N)_F+b_D(\bar{N}q'N)_D]\}$$
$$\times[\tfrac{1}{2}\beta_-\Gamma(1-\alpha_-)(1-e^{i\pi\alpha_-})(P,P)_F(\mathbf{q}\cdot\mathbf{q}'/\mu m)^{\alpha_--1}+\tfrac{1}{2}\beta_+\Gamma(1-\alpha_+)(1+e^{i\pi\alpha_+})(P,P)_D(\mathbf{q}\cdot\mathbf{q}'/\mu m)^{\alpha_+-1}] \quad (44)$$

with $M^2=t$,

$$a_D=\left(1+\frac{M}{2\mu}\right)\left(1+\frac{2m}{M}\right)\frac{\mathbf{q}\cdot\mathbf{q}'}{m\mu}, \quad a_F=\left(\frac{2}{3}-\frac{M}{2m}\right)a_D, \quad (45a)$$

$$b_D=\left(1+\frac{M}{2\mu}\right)\left(1+\frac{2m}{M}\right)\left(1-\frac{M^2}{4m^2}\right),\quad b_F=\tfrac{2}{3}b_D. \tag{45b}$$

D, F having the usual $SU(3)$ connotations, as previously,

$$T_{PN\to PD}=\left(1+\frac{M}{2\mu}\right)\left(1+\frac{2m}{M}\right)\frac{1}{m^2\mu}\epsilon_{\mu\nu k\lambda}q_\mu'q_\nu p_k\bar{D}_\lambda N[\tfrac{1}{2}\beta_-(1-e^{i\pi\alpha_-})\Gamma(1-\alpha_-)(P,P)_F(\mathbf{q}\cdot\mathbf{q}'/\mu m)^{\alpha_--1}$$
$$+\tfrac{1}{2}\beta_+(1+e^{i\pi\alpha_+})\Gamma(1-\alpha_+)(P,P)_D(\mathbf{q}\cdot\mathbf{q}'/\mu m)^{\alpha_+-1}], \tag{46}$$

$$T_{PN\to VN}=\left(1+\frac{M}{2\mu}\right)\bar{u}\left\{\frac{\mathbf{q}\cdot\mathbf{q}'}{M^2\mu}p_\lambda-\frac{i\sigma_{k\lambda}q_k'}{\mu}+\frac{\{p,[q',\gamma_\lambda]\}}{4M\mu}\right\}\gamma_5u[\tfrac{1}{2}\beta_-(1-e^{i\pi\alpha_-})\Gamma(1-\alpha_-)(P,P_\lambda)_D(\mathbf{q}\cdot\mathbf{q}'/\mu m)^{\alpha_--1}$$
$$+\tfrac{1}{2}\beta_+(1+e^{i\pi\alpha_+})\Gamma(1-\alpha_+)(P,V_\lambda)_F(\mathbf{q}\cdot\mathbf{q}'/\mu m)^{\alpha_+-1}]+\left(1+\frac{M}{2\mu}\right)\left(1+\frac{2m}{M}\right)\bar{u}\gamma_5u\cdot\frac{p_\lambda}{M}$$
$$\times[\tfrac{1}{2}\beta_-(1-e^{i\pi\alpha_-})\Gamma(1-\alpha_-)(P,V_\lambda)_F(\mathbf{q}\cdot\mathbf{q}'/\mu m)^{\alpha_--1}+\tfrac{1}{2}\beta_+(1+e^{i\pi\alpha_+})\Gamma(1-\alpha_+)(P,V_\lambda)_D(\mathbf{q}\cdot\mathbf{q}'/\mu m)^{\alpha_+-1}]. \tag{47}$$

These are the amplitudes in the exact-symmetry limit with all masses degenerate. The following significant features may be noted:

(a) Barring differences due to signature, there is a common residue and trajectory function occurring in the characteristic combination

$$\left(1+\frac{2m}{M}\right)\left(1+\frac{M}{2\mu}\right)\beta\left(\frac{\mathbf{q}\cdot\mathbf{q}'}{m\mu}\right)^{\alpha-1}\Gamma(1-\alpha). \tag{48}$$

(b) The signature factors $(1\pm e^{i\pi\alpha_\pm})$ multiplied into $\Gamma(1-\alpha_\pm)$ mean that the even α-signature amplitudes vanish for $\alpha=-1, -3, \cdots$ and odd α-signature amplitudes vanish for $\alpha=0, -2, \cdots$. Hence, the vector $(-)$ trajectory gives amplitude zeros for $\alpha=0, -2, \cdots$ and the tensor $(+)$ trajectory for $\alpha=-1, -3, \cdots$; this applies to all the Regge amplitudes and not just the spin-flip components. We may therefore expect *dips* in the cross sections at these places. This is indeed borne out experimentally. Such dips are well known for π-N charge-exchange processes; the important remark from our point of view is that $\pi N\to\eta N$ also appears to show such dips at even-signature positions.

(c) In the forward direction, we expect to reproduce the predictions of Carter *et al.*,[18] since $SU(6)_W$ is conserved in this limit. Thus, $PN\to PD$ vanish owing to the M_1 nature of the coupling to DN of the *vector* and also the *tensor* trajectory.

V. TOLLERIZATION OF AMPLITUDES AND KINEMATIC SINGULARITIES

Before we compare our results with experiment, we must consider the $t\to 0$ limit where kinematic singularities make their appearance whenever spins are involved. This is shown quite clearly in the characteristic product

$$\left(1+\frac{2m}{\sqrt{t}}\right)\left(1+\frac{\sqrt{t}}{2\mu}\right)\beta(t),$$

which is not an analytic function of t, near $t=0$. As is well known, two types of mechanism have been proposed to remove these singularities: the first evasion—the statement that $\beta(t)$ must have a compensating zero; the second is the addition of conspiring trajectories. The most elegant formulation of conspiracies is the one proposed by Toller where one expands scattering amplitudes not in terms of a complete set of rotation functions of the rest group G, but in terms of the covariant embedding group $\mathcal{G}$ [in Toller's case, $G=O(3)$ and $\mathcal{G}=O(3,1)$]. A Tollerization procedure then replaces Reggeization; this, as is well known, leads to the parent and daughter phenomena.

For the quark-excitation model, Tollerization was studied in detail in Ref. 8. The rotation functions of the embedding groups for models studied here are the following:

		Embedding non-compact group	Rotation functions	
A	$\Rightarrow$	$O(3,1)$	$C_{\mathfrak{N}}^{1}(\cosh\zeta)$	
A(i)	$\Rightarrow$	$U(2,2)$	$C_{\mathfrak{N}}^{3/2}(\cosh\zeta)$	(49)
A(ii)	$\Rightarrow$	$U(3,1)$	$(\cosh\zeta)^N$	
B	$\Rightarrow$	$U(6,6)$	$C_{\mathfrak{N}}^{11/2}(\cosh\zeta)$.	

An alternative solution—and one not as general as Toller's—is to introduce conspiring trajectories following Gribov.[19] This is the solution most suited to the multispinor formalism for hadrons. It arises from the natural possibility of doubling afforded by quarks and pseudoquarks within a multispinor framework (this is the doubling first introduced by Gribov).

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[18] J. Carter, J. Coyne, S. Meshkov, D. Horn, M. Kugler, and H. J. Lipkin, Phys. Rev. Letters 15, 373 (1965).

[19] V. N. Gribov, L. Okun, and I. Pomeranchuk, Zh. Eksperim. i Teor. Fiz. 45, 1114 (1963) [English transl.: Soviet Phys.—JETP 18, 769 (1964).

For mesons, for example, one is led to consider two trajectories coinciding at $M=t=0$ with identical residues but otherwise distinct, which correspond to $(6,\bar{6})$ and $(\bar{6},6)'$, the primes indicating the pseudoquark composites.

In accordance with these ideas, we must therefore add to the amplitudes (43) extra terms with the sign of M reversed, corresponding to $(6,\bar{6})\to(\bar{6},6)'$. For the MB scattering amplitudes, then, we typically meet the combination

$$\frac{1}{2}\left(1+\frac{2m}{M}\right)\left(1+\frac{M}{2\mu}\right)\beta(M)\Gamma(1-\alpha)s^{\alpha-1} +\frac{1}{2}\left(1-\frac{2m}{M}\right)\left(1-\frac{M}{2\mu}\right)\beta'(M)\Gamma(1-\alpha')s^{\alpha'-1}. \quad (50)$$

Taking $\alpha(0)=\alpha'(0)$, $\beta(0)=\beta'(0)$, the $M=\sqrt{t}$ singularity disappears. In fact, if we suppose that for moderately small spacelike t, $\alpha'(t)=\alpha(t)$ and $\beta\approx\beta'\approx$const, the combination of terms[20] could sum to[21]

$$\left(1+\frac{m}{\mu}\right)\beta\Gamma(1-\alpha)s^{\alpha-1}.$$

Before applying our formalism to elastic processes, one has to make up one's mind about the Pomeranchukon. In the absence of any fundamental understanding of vacuum exchange, it is probably fair to regard the Pomeranchukon as a [fixed, $SU(3)$ scalar?] pole with $\alpha(0)=1$ which occurs in elastic processes to describe the background effects of inelastic channels via unitarity.

The remaining problem is symmetry breaking. We know it to be very important as far as trajectory shifts, which govern high-energy behaviors, are concerned. Failing a reliable theory of mass splitting between members of a supermultiplet, the only course open at present is to take the positions of the trajectories as empirical input (most significantly for the pion). With regard to the residues, one may hope that they do not change drastically, and this is what seems to be borne out by our preliminary analysis of data (Ref. 4). If this had not worked, our first prescription would have been to use physical masses in place of mean masses in kinematic factors.

VI. REGGEIZED MESON-BARYON SCATTERING AND COMPARISON WITH EXPERIMENT

We now list the final formulas and their main features for charge-exchange scattering processes that are dominated by the vector ($C=-1$) and tensor ($C=+1$) leading trajectories; namely, those in which both the initial and final meson are pseudoscalar. We shall, here, make an assumption outside the supermultiplet schemes proper—that of exchange degeneracy, i.e., $\alpha_+=\alpha_-$, $\beta_+=\beta_-$. The final expressions are as follows:

$T_{PN\to PN}$: Flip amplitude is

$$B=\left(1+\frac{m}{\mu}\right)\left(1-\frac{t}{4m^2}\right)\times(\bar{N}q'N)_{D+(2/3)F}\frac{\beta}{\mu}\Gamma(1-\alpha)\left(\frac{s}{2m\mu}\right)^{\alpha-1}\times[\tfrac{1}{2}(1-e^{i\pi\alpha})(PP)_F+\tfrac{1}{2}(1+e^{i\pi\alpha})(PP)_D] \quad (51a)$$

and the nonflip amplitude is

$$A'\approx A+\frac{s}{2m\mu}\left(1-\frac{t}{4m^2}\right)^{-1}B =\left(1+\frac{t}{4m\mu}\right)(\bar{N}N)_F\beta\Gamma(1-\alpha)\left(\frac{s}{2m\mu}\right)^{\alpha}\times[\tfrac{1}{2}(1-e^{i\pi\alpha})(PP)_F+\tfrac{1}{2}(1+e^{i\pi\alpha})(PP)_D], \quad (51b)$$

which enter in the differential cross section as

$$\frac{d\sigma}{dt}=\frac{1}{16\pi s^2}\left[\left(1-\frac{t}{4m^2}\right)|a'|^2 -\frac{t}{4m^2}\left(1-\frac{t}{4m^2}\right)^{-1}\left|\frac{sb}{2m\mu}\right|^2\right] \quad (52)$$

and give

$$\left(\frac{d\sigma}{dt}\right)_{PN\to PN}=\frac{|\beta\Gamma(1-\alpha)|^2}{16\pi s^2}\left(1-\frac{t}{4m^2}\right)\left(\frac{s}{2m\mu}\right)^{2\alpha}\times\left\{\left(1+\frac{t}{4m\mu}\right)^2-\frac{t}{4m^2}\left[\frac{g_{D+(2/3)F}}{g_F}\left(1+\frac{m}{\mu}\right)\right]^2\right\}\times|\tfrac{1}{2}(1-e^{i\pi\alpha})h_F+\tfrac{1}{2}(1+e^{i\pi\alpha})h_D|^2, \quad (53)$$

the F and D suffixes carrying the normal $SU(3)$ meanings.

$T_{PN\to PD}$: The baryon vertex is of the expected M_1 *type* all along the trajectory, as may be seen in the amplitude

$$C=\left(1+\frac{\bar{m}}{\mu}\right)\frac{1}{\bar{m}^2\mu}\epsilon_{\mu\nu\kappa\lambda}q_\mu'q_\nu p_\kappa\bar{D}_\lambda N\beta\Gamma(1-\alpha)\left(\frac{s}{2m\mu}\right)^{\alpha-1}\times[\tfrac{1}{2}(1-e^{i\pi\alpha})(PP)_F+\tfrac{1}{2}(1+e^{i\pi\alpha})(PP)_D]. \quad (54)$$

Here, $\bar{m}$ is the mean of octet (N) and decuplet (D) masses. When this is substituted into the cross section

$$\frac{d\sigma}{dt}=-\frac{1}{6\pi s^2}\left(1-\frac{t}{4\bar{m}^2}\right)\frac{s^2t}{\bar{m}^4\mu^2}|c|^2,$$

[20] There is the familiar problem of existence of particles on the Gribov-doubled trajectories. We have no new ideas on this except the conventional one that possibly the relevant residues vanish.

[21] The difference between Toller and Gribov conspiracies lies in that Toller would demand, in addition to Gribov doubling, the trajectories corresponding to $(36,1)^+$ and $(1,36)^+$.

one gets

$$\left(\frac{d\sigma}{dt}\right)_{PN\to PD} \approx -\frac{t|\beta\Gamma(1-\alpha)|^2}{6\pi s^2\bar{m}^2}\left(1-\frac{t}{4\bar{m}^2}\right)\left(\frac{s}{2\bar{m}\mu}\right)^{2\alpha}\left(1+\frac{\bar{m}}{\mu}\right)^2 \times|\tfrac{1}{2}(1-e^{i\pi\alpha})h_F+\tfrac{1}{2}(1+e^{i\pi\alpha})h_D|^2.$$

To make comparison with experiment for the $Y=0$ charge-exchange processes

$$\pi^-p\to\pi^0n,\quad K^-p\to\bar{K}^0n,\quad \pi^-p\to\eta n,$$
$$\pi^+p\to\pi^0N^{*++},\quad K^+p\to K^0N^{*++},\quad \pi^+p\to\eta N^{*++},$$

for which considerable data exist, we have used mean masses $\langle\mu\rangle\approx 0.45$ GeV/c, $\langle m\rangle\approx 1.15$ GeV/c, and $\langle\bar{m}\rangle\approx 1.3$ GeV/c in the kinematic factors and the exchange-degeneracy approximation $\alpha_+\approx\alpha_-\approx\alpha\approx 0.5+t$, in units of $(\mathrm{GeV}/c)^2$; we also took a constant residue β. Considering these very rough approximations, the agreement with the data is quite encouraging (Ref. 4).

In summary, the model has the following characteristics:

(1) It is based on the excitation scheme

$$[U(6)\otimes U(6)]_S\otimes U(3)_N,$$

though for Regge exchange of meson trajectories only the leading orbital excitation $L=N=\alpha-1$ is significant.

(2) The relativistic kinematics is provded by the embedding group $\tilde{U}(12)_S\otimes U(3,1)_N$ and produces significant kinematic factors in the Clebsch-Gordan coefficients.

(3) From the quark-antiquark nature of the mesons, $1/\sqrt{t}$ kinematic factors are encountered. A Gribov doubling of the exchanged trajectory $(6,\bar{6})$ and $(\bar{6},6')$ has been employed to eliminate these $1/\sqrt{t}$ singularities.

(4) At nonsense J values we have used the Gell-Mann mechanism for eliminating ghosts. Perhaps other mechanisms could be constructed, but they do not appear to fit so naturally into the excitation picture. Our model predicts zeros in the π charge-exchange reactions at $\alpha=0, 2, \cdots$, zeros in $\pi^\pm\to\eta$ reactions at $\alpha=-1, -3, \cdots$, and no significant dips in the K charge-exchange reactions. This last fact is due to cancellations of the signature factors in the Gell-Mann mechanism.

(5) The assumed exchange degeneracy can be relaxed by using separate α_+, β_+ and α_-, β_- for the even- and odd-signature pieces in place of the assumed common $\alpha=0.5+t$ and β.

(6) Inasmuch as we have neglected any possible t dependence of β, our simplified model admits no parameters except the one constant β. When β is fixed by the high-energy data and the Regge formula is extrapolated to the vector-meson mass, one finds $(g_{\rho\pi\pi})_{\mathrm{expt}}=2(g_{\rho\pi\pi})_{\mathrm{theoret}}$. This is a failure of the model, and we have no explanation for it.

(7) A turnover effect at small t is predicted by the model because of the largeness of the spin flip relative to the flipless amplitude: $(b/a)(s/2m\mu)=-(5/3)(1+m/\mu)\approx -6$. Moreover, the opposite relative sign of the two and the fact that a' is a polynomial in t means that an extension of the model to elastic scattering cross-section differences will give rise to a crossover effect. However, the positions of turnover and crossover points are incorrectly given by the model (which uses mean masses) at $t\approx 0.05$ and 0.5 GeV/c. It is possible that mass shifts play an important role in altering these points.

(8) Density-matrix calculations, which have not been discussed, constitute important tests of the model. Observe, too, that in its present form the simple pole picture used in this section is unable to explain polarization effects as it deals with a *single* Regge amplitude.

(9) Reactions dominated by pion exchange, e.g., $T_{PN\to VN}$, $T_{NN\to NN}$, which will form the subject of a separate note, show interesting new features owing to the singular character of the pion residue and its nearness to the scattering region. For instance, in nucleon-nucleon charge-exchange scattering, the π and ρ contributions together give terms of the form

$$d\sigma/dt\approx[s^{\alpha_\pi}\Gamma(-\alpha_\pi)]^2+[s^{\alpha_\rho}\Gamma(1-\alpha_\rho)]^2 \qquad (55)$$

so that for small t and moderately large s the pion dominates and *gives the observed sharp forward peak*, but at larger values of t the ρ takes over. The reason why the pion gives a finite nonzero cross section even in the forward limit $t\to 0$ is that the amplitude $\bar{u}\boldsymbol{p}\gamma_5 u\bar{u}'\boldsymbol{p}\gamma_5 u/p^2=(4m^2/t)\bar{u}\gamma_5 u\bar{u}'\gamma_5 u$ carries a desirable singular residue $1/t$, even though there is Gribov doubling.[22] The actual detailed analysis of pion-exchange reactions will be treated elsewhere.

[22] This $1/t$ singularity is of course cancelled by the t factor occurring in the nucleon $\gamma_5\otimes\gamma_5$ piece.