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$\alpha = 0$ Wrong-Signature Pole in πN Charge Exchange*

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We make use of partial conservation of the axial-vector current and the finite-energy sum rules to investigate poles at the wrong-signature points in πN scattering amplitudes, and show that a wrong-signature pole at $\alpha = 0$ appears only in the helicity-flip amplitude.

USE is made of the partial conservation of the axial-vector current (PCAC) and the finite-energy sum rules (FESR)[1] to investigate poles at the wrong-signature points in πN scattering amplitudes. Our use of PCAC follows closely Adler's derivation of the consistency condition[2] on the $A^{(+)}$ amplitude.[3] We apply this method to the charge-exchange amplitudes. This allows us to show that a wrong-signature pole at $\alpha = 0$ appears only in the helicity-flip amplitude. This result is in accord with what one would expect from the Gribov-Pomeranchuk[4] argument.

1. WRONG-SIGNATURE FESR

Let us start with an explanation of the concept of the wrong-signature FESR. A general scattering amplitude $G(\nu)$ is real analytic in the complex ν plane cut along the real axis. In general it will have both a right- and a left-hand cut (RHC and LHC). In order to apply to it Regge behavior, we have to separate G into its symmetric and antisymmetric parts:

$$G(\nu) = G^{(s)}(\nu) + G^{(a)}(\nu), \quad G^{(s,a)}(\nu) = \tfrac{1}{2}[G(\nu) \pm G(-\nu)]. \quad (1)$$

Let us now define $g^{(s,a)}(\nu)$ as real analytic functions in the complex ν plane with a cut that is equal to the RHC of $G^{(s,a)}$. Moreover,

$$G^{(s,a)}(\nu) = g^{(s,a)}(\nu) \pm g^{(s,a)}(-\nu). \quad (2)$$

A Regge pole contributing to $g^{(s)}(\nu)$ ($g^{(a)}(\nu)$) is said to have positive (negative) signature. A Regge representation is equivalent to an asymptotic expansion:

$$g(\nu) \simeq \sum_i -C_i \frac{(e^{-i\pi}\nu)^{\alpha_i}}{\sin\pi\alpha_i}. \quad (3)$$

We write the individual Regge poles in (3) in a form that exhibits a RHC from 0 to ∞ with an imaginary part equal to $C_i \nu^{\alpha_i}$ above the cut. Now one can prove FESR for both positive and negative moments of Img:

$$S_n \equiv \frac{1}{N^{n+1}} \int_{\mathrm{RHC}}^{N} \nu^n \,\mathrm{Im}g(\nu) d\nu \simeq \sum_i \frac{C_i N^{\alpha_i}}{\alpha_i + n + 1},$$
$$S_{-m-1} \equiv N^m \int_{\mathrm{RHC}}^{N} \frac{\mathrm{Im}g(\nu)}{\nu^{m+1}} d\nu \simeq \sum_i \frac{C_i N^{\alpha_i}}{\alpha_i - m} + \frac{N^m \pi g^{(m)}(0)}{m!}. \quad (4)$$

Although $\mathrm{Im}g^{(s,a)}(\nu) = \mathrm{Im}G^{(s,a)}(\nu)$ and the FESR in (4) are a direct consequence of (3), still only half of them can be proved directly from $G^{\pm}(\nu)$ and its asymptotic expansion. This corresponds to the fact that a Regge pole at a wrong-signature point will appear as a pole in (3) but as a regular term in the expansion of G.[5] Moreover, there exists complete freedom in adding an odd (even) power series in ν to $g^s(\nu)$ ($g^{(a)}(\nu)$), thus changing the corresponding derivative of g at the origin that appears in the negative moment FESR. We distinguish therefore between right-signature FESR, namely, the even (odd) moments of $\mathrm{Im}G^{(a)}(\nu)$ ($\mathrm{Im}G^{(s)}(\nu)$) and wrong-signature FESR that involve the respective odd (even) moments. In Eqs. (4) one can replace Img by ImG. For the right-signature FESR one finds $2g^{(m)}(0) = G^{(m)}(0)$. In the wrong-signature case, $g^{(m)}(0)$ is arbitrary.

In the wrong-signature FESR we have to distinguish between non-negative and negative moments. In the first case the FESR can be evaluated from direct experimental data and Regge parameters can be extracted from the sum rule. We might encounter here wrong-signature fixed poles that are not observed in the scattering amplitude.[5] The case of negative moments S_{-m-1} is still more complicated since $g^{(m)}(0)$ is arbitrary.

The evaluation of the first wrong-signature FESR for the $B^{(-)}$ πN amplitude, namely, $\int_{\mathrm{RHC}}^{N} \mathrm{Im}B^{(-)}(\nu) d\nu$, has shown[1] a very strong wrong-signature pole near $\alpha = 0$ for a wide range of t. Since that happens to be a nonsense point for $B^{(-)}$, this pole cannot affect $B^{(-)}$ itself.

Note that this sum rule reads

$$\frac{\pi g_{NN\pi}^2}{2M} + \int_{\mathrm{th}}^{N} \mathrm{Im}B^{(-)}(\nu,t) d\nu \simeq \sum_i C_i \frac{N^{\alpha_i}}{\alpha_i}. \quad (5)$$

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[2] S. L. Adler, Phys. Rev. **137**, B1022 (1965).

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[4] V. N. Gribov and I. Y. Pomeracnhuk, Phys. Letters **2**, 239 (1962); S. Mandelstam and L. L. Wang, Phys. Rev. **160**, 1490 (1967).

[5] At a wrong-signature point of g^A (g^S), α is even (odd). For a discussion of the appearance of these poles in superconvergence relations, see J. H. Schwarz, Phys. Rev. **159**, 1269 (1967). The wrong-signature FESR turn in the limit $N \to \infty$ into Schwarz superconvergence relations.

The Born term contributes to the usual $\nu B^{(-)}$ amplitude an additional factor of $\nu_B=-\mu^2/2M+t/2M$ that disappears in the FESR (5). Therefore the Born term in (5) is unusually large and dominates the left-hand side. In the Regge expansion, one has therefore to add a pole near $\alpha=0$ with $C\sim\alpha$ such that it compensates the effect of the Born term.

If we want to ask ourselves whether a pole at $\alpha\simeq 0$ shows up also in $A^{(-)}$ or $A'^{(-)}$, we have to evaluate S_{-1} for these amplitudes. Constructing the corresponding $a^{(-)}$, $a'^{(-)}$ amplitudes S_{-1} reads

$$\int_{\mathrm{RHC}}^{N}\frac{\mathrm{Im}a^{(-)}(\nu)}{\nu}d\nu\simeq\sum_i C_i\frac{N^{\alpha_i}}{\alpha_i}+\pi a^{(-)}(0).$$

However, $a^{(-)}(0)$ is undetermined since $A^{(-)}(\nu)=a^{(-)}(\nu)-a^{(-)}(-\nu)$. Hence we encounter an ambiguity in the definition of the amplitude that we want to investigate.

This ambiguity can be overcome by a suitable definition of the πN amplitudes. We will define them as those amplitudes that describe the pseudoscalar component of the axial four-vector current scattering off a nucleon. This definition allows us to construct an unambiguous sum rule and to investigate the point in question. It is done within the usual PCAC framework that we review and use in the next section.

2. ADLER'S CONSISTENCY CONDITION

Let us consider the amplitude

$$T^{\alpha\beta}=\lim_{q_2^2\to\mu^2}(q_2^2-\mu^2)(q_1^2-\mu^2)(-q_1^\nu)\int e^{iq_2\cdot x}\langle N(p_2)|\times\{\theta(x_0)[\phi^\beta(x),\mathfrak{F}_\nu^{5,\alpha}(0)]-S_\nu^{\alpha\beta}(x)\}\times|N(p_1)\rangle d^4x\equiv q_1^\nu T_\nu^{\alpha\beta}. \quad (6)$$

N denotes nucleon states, ϕ the pion field, $\mathfrak{F}_\nu^{5,\alpha}$ the axial-vector current, and $S_\nu^{\alpha\beta}$ the seagull term. $T_\nu^{\alpha\beta}$ describes a four-point function involving an axial four-vector, and $T^{\alpha\beta}$ is its pseudoscalar component. Its absorptive part is given by

$$t^{\alpha\beta}=\lim_{q_2^2\to\mu^2}(q_2^2-\mu^2)(q_1^2-\mu^2)(-q_1^\nu)\times\int e^{iq_2\cdot x}\langle N(p_2)|[\phi^\beta(x),\mathfrak{F}_\nu^{5,\alpha}(0)]|N(p_1)\rangle d^4x, \quad (7)$$

where α and β are isospin indices and μ is the pion mass. Note that pion number two is on its mass shell, whereas q_1 was left free. Choosing $q_1^2=\mu^2$, we obtain an expression that we identify with the T matrix for πN scattering:

$${}_{\mathrm{out}}\langle N(p_2)\pi_\beta(q_2)|N(p_1)\pi_\alpha(q_1)\rangle_{\mathrm{in}}=\delta_{if}+i(2\pi)^4\times\delta^4(p_1+q_1-p_2-q_2)(g_{NN\pi}/M\mu^2g_A)T^{\alpha\beta}. \quad (8)$$

The assumption of PCAC states that the amplitude $T^{\alpha\beta}$ is smooth in the variable q_1. Therefore, even if we choose $q_1^2=0$, which we are going to do now, we still get the same value for $T^{\alpha\beta}$ with good accuracy (on the order of 10–15%).

Our independent variables are $\nu=(s-u)/4M$, t and q_1^2. Let us go to the limit $q_1^2=0$ and $t=\mu^2$. Then $T^{\alpha\beta}$ reduces to a simple form whose analytic structure is known to us. In order to understand this point we note that $T_\nu^{\alpha\beta}$ defined in (6) can be expanded in terms of eight invariant amplitudes:

$$T_\nu^{\alpha\beta}=\bar{u}(p_2)[\sum_{i=1}^{8}R_i^{\alpha\beta}(\nu,t,q_1^2)O_\nu^i]u(p_1), \quad (9)$$

where

$$O_\nu^1=\tfrac{1}{2}[\boldsymbol{q}_2,\gamma_\nu],\quad O_\nu^2=(p_1+p_2)_\nu,\quad O_\nu^3=(q_2)_\nu,$$
$$O_\nu^4=M\gamma_\nu,\quad O_\nu^5=\boldsymbol{q}_1(p_1+p_2)_\nu,\quad O_\nu^6=\boldsymbol{q}_1(q_2)_\nu, \quad (10)$$
$$O_\nu^7=(q_1)_\nu,\quad O_\nu^8=\boldsymbol{q}_1(q_1)_\nu.$$

On the other hand, $T^{\alpha\beta}$ can be expanded as

$$(g_{NN\pi}/M\mu^2g_A)T^{\alpha\beta}=\bar{u}(p_2)[A^{\alpha\beta}(\nu,t,q_1^2)+B^{\alpha\beta}(\nu,t,q_1^2)\tfrac{1}{2}(\boldsymbol{q}_1+\boldsymbol{q}_2)]u(p_1).$$

The connection between A,B and the various $R_i^{\alpha\beta}$ can be readily established. In the limit $q_1^2=0$ and $t=\mu^2$, we have $q_1\cdot q_2=0$ and

$$A^{\alpha\beta}(\nu,0,0)=(2g_{NN\pi}/\mu^2g_A)\nu\times[R_1^{\alpha\beta}(\nu,\mu^2,0)+R_2^{\alpha\beta}(\nu,\mu^2,0)]\equiv\nu G^{\alpha\beta}. \quad (11)$$

Both the amplitudes A and G are free of kinematical singularities. We will choose $G^{\alpha\beta}$ as the amplitude for which we write our FESR. Above threshold, we have, of course, $\mathrm{Im}G^{\alpha\beta}(\nu)=\nu^{-1}\,\mathrm{Im}A^{\alpha\beta}(\nu)$; however, G might have an additional singularity at $\nu=0$. Moreover, we know the structure of $G^{\alpha\beta}$ in terms of a RHC and LHC as defined in Eq. (7). Therefore we can define the functions g with only the RHC.

Let us now treat separately the cases of symmetric $\{\alpha\beta\}$ and antisymmetric $[\alpha\beta]$ combinations, to be denoted by $(+)$ and $(-)$, respectively. $A^{(+)}$ is symmetric in ν, and therefore $G^{(+)}$ is antisymmetric in ν. $g^{(+)}$ has a pole (the s-channel Born term) at $\nu=0$ that corresponds to a pole in $G^{(+)}$ and a low-energy limit in $A^{(+)}$. This is the famous Adler consistency condition:

$$A^{(+)}(\nu=0,\ t=\mu^2,\ q_1^2=0)=g_{NN\pi}^2/M. \quad (12)$$

For the $(-)$ combination we encounter the same Born pole in $g^{(-)}$, however, since $G^{(-)}$ is symmetric in ν, it gets canceled out in $G^{(-)}$. Therefore we find an asymmetric situation for A as well as for G, whereas both[6]

$$\lim_{\nu\to0}g^{(+)}(\nu)=\lim_{\nu\to0}g^{(-)}(\nu)=g_{NN\pi}^2/2M\nu. \quad (13)$$

[6] Note that for ($\nu\to0$, $t\to\mu^2$, $q_1^2\to0$) the values $\lim q_1^\nu O_\nu^i\times R_i^{\alpha\beta}(\nu,t,q_1^2)$ are ambiguous. Nevertheless, the amplitude $A^{\alpha\beta}(0,\mu^2,0)$ that is a well-defined combination of the $q_1^\nu O_\nu^i\times R_i^{\alpha\beta}(\nu,t,q_1^2)$ is independent of the limiting process. This is true also for the amplitude with one cut only

TABLE I. Numerical results for $S_{-1}^{(+)}$.

N (BeV)	Integral (mb BeV)	P	P'
2.0	71	45	66
3.0	103	67	75
4.0	133	89	82
5.0	162	111	87
6.0	189	133	92
16.0	444	353	125
22.0	592	486	138

The FESR can be used to restore the symmetric situation encountered in Eq. (13).

3. USE OF FESR

Let us evaluate S_{-1} of $A^{(+)}$, which is also S_0 of $g^{(+)}$. We assume that

$$\mathrm{Im}A=\nu\,\mathrm{Im}G\to\sum_i \beta_i\nu^{\alpha_i}. \tag{14}$$

Then S_0 of $g^{(+)}$ reads

$$\int_{\mathrm{RHC}}^N \mathrm{Im}g^{(+)}(\nu)d\nu\simeq\sum_i\frac{\beta_i^{(+)}N^{\alpha_i}}{\alpha_i}, \tag{15}$$

whereas S_{-1} of $A^{(+)}$ is

$$\int_{\mathrm{RHC}}^N \frac{\mathrm{Im}A^{(+)}(\nu)}{\nu}d\nu\simeq\sum_i\frac{\beta_i^{(+)}N^{\alpha_i}}{\alpha_i}+\tfrac{1}{2}\pi A^{(+)}(0). \tag{16}$$

Note that the left-hand side of (15) includes, in addition to the left-hand side of (16), the Born term $-\pi g_{NN\pi}^2/2M$, which equals $-\pi a^{(+)}(0)=-\frac{1}{2}\pi A^{(+)}(0)$. Therefore the two equations are consistent. To evaluate the sum rule, it is simpler to apply it to $A'^{(+)}$ amplitude defined as $A'=A+\nu B/(1-t/4M^2)$. A' is the amplitude corresponding to no helicity flip in the t channel. We see that at $t=\mu^2$, $A'(0)=A(0)$. We note that B contributes a Born term too. If we designate now the high-energy behavior as $\mathrm{Im}A'\to\sum\gamma_i\nu^{\alpha_i}$, we find

$$\frac{\pi g_{NN\pi}^2}{2M(1-\mu^2/4M^2)}+\int_{\mathrm{th}}^N\frac{\mathrm{Im}A'^{(+)}}{\nu}d\nu \simeq\sum_i\gamma_i^{(+)}\frac{N^{\alpha_i}}{\alpha_i}+\frac{\pi g_{NN\pi}^2}{2M}. \tag{17}$$

To evaluate it, we make the approximation that

$$\mathrm{Im}A'^{(+)}(\nu,t=\mu^2,q_1^2=0)\approx\mathrm{Im}A'^{(+)}(\nu,t=\mu^2,q_1^2=\mu^2) \approx\mathrm{Im}A'^{(+)}(\nu,t=0,q_1^2=\mu^2) \tag{18}$$

for ν above threshold. The great advantage of the point $t=0$, $q_1^2=\mu^2$ is the possibility to use total cross sections in the calculations. The first step in (18) corresponds to the usual PCAC approximation which we already mentioned before, and which we expect to hold within 10–15%. A similar off-mass-shell correction is expected for the Born terms. The second step can be judged from phase-shift solutions. The CERN phase shifts[7] imply a variation of about 10–15% in the value of $A'^{(\pm)}$ below 2 BeV. We conclude that within 25% the following equality should hold:

$$\frac{\pi g_{NN\pi}^2}{2M}+\frac{1}{2}\int_{\mathrm{th}}^N\frac{k}{\nu}[\sigma^t(\pi^-p)+\sigma^t(\pi^+p)]d\nu \simeq\sum_i\gamma_i^{(+)}\frac{N^{\alpha_i}}{\alpha_i}+\frac{\pi g_{NN\pi}^2}{2M}. \tag{19}$$

The numerical results for the finite integral and the Regge part[8] are given in Table I. The errors for the Regge terms are 5% for the Pomeranchuk trajectory and 30% for the P', whereas the off-mass-shell corrections for the Born term are of the order of 20 mb BeV. Within these errors the sum rule is verified.

In a similar fashion we can treat S_0 of $g^{(-)}$. Denoting its Regge expansion by

$$\nu\,\mathrm{Im}g^{(-)}\to\sum_i\beta_i^{(-)}\nu^{\alpha_i}, \tag{20}$$

the analog of (15) is

$$\int_{\mathrm{RHC}}^N\mathrm{Im}g^{(-)}(\nu)d\nu\simeq\sum_i\beta_i^{(-)}\frac{N^{\alpha_i}}{\alpha_i}. \tag{21}$$

The integral involves a Born term below threshold and can be rewritten as

$$-\frac{\pi g_{NN\pi}^2}{2M}+\int_{\mathrm{th}}^N\frac{\mathrm{Im}A^{(-)}(\nu,t=\mu^2,q_1^2=0)}{\nu}d\nu \simeq\sum_i\beta_i^{(-)}\frac{N^{\alpha_i}}{\alpha_i}. \tag{22}$$

Alternatively, from (13) we have $a^{(-)}(0)=\pi g_{NN\pi}^2/2M$, and (22) follows.

If one uses only the conventional ρ Regge pole in the sum on the right-hand side, then the equality fails badly. The situation is very similar to what happened in Eq. (5) to the $B^{(-)}$ amplitude. The value of $\pi g_{NN\pi}^2/2M$ is much bigger than the integral (at $N=2$ BeV) and this calls for a term at $\alpha=0$ and $\beta^{(-)}=\eta\alpha$. η will be roughly equal to $-\pi g_{NN\pi}^2/2M$. This is once again a nonsense wrong-signature pole. We note that if we consider the amplitude $A'^{(-)}$ instead of $A^{(-)}$, then we find a linear combination of (22) and (5) leading to

$$\frac{\pi g_{NN\pi}^2}{2M(1-\mu^2/4M^2)}+\int_{\mathrm{th}}^N\frac{\mathrm{Im}A'^{(-)}}{\nu}d\nu \simeq\sum_i\gamma_i^{(-)}\frac{N^{\alpha_i}}{\alpha_i}+\frac{\pi g_{NN\pi}^2}{2M}. \tag{23}$$

[7] A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968).

[8] We use the semiexperimental high-energy fit of K. J. Foley *et al.*, Phys. Rev. Letters **19**, 330 (1967).

This equality is verified at $t=0$, neglecting terms of order μ^2/M^2:

$$\frac{\pi g_{NN\pi}^2}{2M}+\frac{1}{2}\int_{\rm th}^{N}\frac{k}{\nu}[\sigma^t(\pi^-p)-\sigma^t(\pi^+p)]d\nu \simeq\sum_i \gamma_i^{(-)}\frac{N^{\alpha_i}}{\alpha_i}+\frac{\pi g_{NN\pi}^2}{2M}. \quad (24)$$

The numerical comparison given in Table II is based on the assumption that the Regge sum can be given by one effective ρ pole.[8] The error of the ρ term is of 20–30% for $2<N<20$, whereas the Born-terms off-mass-shell correction is of the order of 20 mb BeV.

TABLE II. Numerical results for $S_{-1}^{(-)}$.

N (BeV)	Integral (mb BeV)	ρ
4.0	1.9	7.2
6.0	4.5	9.5
8.0	6.8	11.5
10.0	8.8	13.4
14.0	12.4	17.0
18.0	15.7	20.0
22.0	18.8	23.2

4. DISCUSSION

In his calculations Adler used two methods:

(1) Fixed momentum transfer at $\nu_B=0$ $(t=2\mu^2)$ dispersion relations with phase shifts for A.

(2) Forward dispersion relations for A'. Both dispersion relations included subtractions on the physical cut, thus enhancing the relevent part of the spectrum which was always near threshold (below 700 MeV lab energy). In the second method almost all the contribution to $\pi a^{(+)}(0)$ came from the nucleon Born term. By using the FESR we take the other extreme, namely, we enhance the Regge part of the sum rule. Therefore we can look for the validity of the sum rule for different cutoffs and we have the following phenomena:

(i) In $S_{-1}^{(+)}$ the integral on the physical cut, the Born term, the Regge term, and $\pi a^+(0)$ are of the same order of magnitude for $N>3$ BeV. The sum rule (17) is just a requirement

$$\frac{1}{2}\int_{\rm th}^{N}\frac{k}{\nu}[\sigma(\pi^-p)+\sigma(\pi^+p)]d\nu\simeq\beta_P\frac{N^{\alpha_P}}{\alpha_P}+\beta_{P'}\frac{N^{\alpha_{P'}}}{\alpha_{P'}}, \quad (25)$$

which is experimentally valid. For small cutoffs, both the integral and the Regge part are small (relative to the Born term). We see that even for $N\to$ threshold, the sum rule (17) is fulfilled, in the approximated form

$$\pi a^+(0)\approx\frac{\pi g_{NN\pi}^2}{2M(1-t/4M^2)}. \quad (26)$$

(ii) In $S_{-1}^{(-)}$ the integral and the Regge part are small (relative to the nucleon Born term) even for high N. (For $N=600$ BeV the ρ pole would give half of the Born term.) Remembering the off-mass-shell error of the Born term as well as of the $\pi a^{(-)}$ term, the sum rule (23) is verified for all N. There is, therefore, no room for a strong fixed pole at $\alpha=0$ for the $A'^{(\pm)}$ amplitudes. [By "strong" we mean of the order of magnitude of the Born term or so, as was found in $B^{(-)}$.] This makes sense, since $\alpha=0$ is a sense point for the no-helicity-flip amplitudes. In other words, one can say that our analysis shows that the wrong-signature pole at $\alpha=0$ is necessary only when $\alpha=0$ is a nonsense point [as in $B^{(-)}$]. This is, of course, in agreement with the Gribov-Pomeranchuk mechanism for fixed poles in the partial waves.[4]