with $l \geq n+1$. This expression (B5) is a sum of two loops corresponding to an original degenerate-mass pair with the addition of $l-n$ scalar vertices, each with zero momentum transfer k_i. A detailed examination shows that a lemma analogous to the previous one holds for the nondegenerate-mass case, and thus if each $\bar{L}$ in (B5) satisfies its WI then the corresponding WI for n-pf is also naïve.

In searching for WI anomalies with respect to nondegenerate n-pf's, we note that in their δm expansions, those loops with $l \geq 6$ already satisfy their individual WI's; those loops with $l \leq 5$ can be modified according to the Eqs. (51)–(55). Our previous minimal solution in the degenerate-mass case has been examined term by term in $\bar{L}$. It therefore guarantees our minimal solution again in the present nondegenerate case. It is worthwhile to note that the linear independence of the W's and Z's (introduced in Sec. III) makes the term-by term balancing of individual $\bar{L}$ WI's easily understood.

Let us attempt to clarify what has been said so far. In the *normal*-parity case, each $\bar{L}$ either satisfies its NWI or can be modified to do so; therefore, by our preceding arguments, the corresponding nondegenerate-mass n-pf's can be defined so as to satisfy their WI's. The anomalies present in the *abnormal* nondegenerate-mass WI's are the same as those in the degenerate case. This is true because an abnormal-parity loop with any scalar vertices has no m^{-1} terms, and thus the δm expansion shows that no new anomalies are introduced by the mass breaking.

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Quark Model and the Pomeranchuk Theorem*

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It has been pointed out that the quark model may not be compatible with the Pomeranchuk theorem. We investigate one such model in more detail and discuss the implications of violation of the Pomeranchuk theorem, if any.

I. INTRODUCTION

THE equal-time commutation relation (ETCR) of the axial-vector currents has led to the Adler-Weisberger sum rule,[1] upon using the partially conserved axial-vector current (PCAC) assumption[2] and the infinite-momentum technique.[3] Applying a similar operation on the matrix element of the ETCR which involves the divergence of the axial-vector current, and using the subtraction method that has been described in Ref. 4, one can derive a superconvergence sum rule[5] for the zero-mass-pion nucleon scattering amplitude. In particular, it has been pointed out[4] that the ETCR of pion fields which are considered as bound states of the quark-antiquark system may be incompatible with the Pomeranchuk theorem[6] (P theorem). In this paper, we discuss such a possibility further and elaborate on its experimental implications.

In Sec. II, the model is specified, and in Sec. III, we show that the violation of the P theorem is related to the nonvanishing bare masses of quarks. Section IV is devoted to a discussion concerning the validity of the P theorem.

II. QUARK MODEL

We consider a quark model in which the interaction Lagrangian respects the $SU(3)\times SU(3)$ symmetry and the violation of the symmetry is due only to the mass terms. This is the model that has been discussed by several authors.[7–11]

The interaction Lagrangian may be taken as[12,13]

$$\begin{aligned}-\mathcal{L}_I = {} & g_V\bar\psi i\gamma_\mu\psi\bar\psi i\gamma_\mu\psi + g_A\bar\psi i\gamma_\mu\gamma_5\psi\bar\psi i\gamma_\mu\gamma_5\psi \\ & + G(\bar\psi i\gamma_\mu\lambda_j\psi\bar\psi i\gamma_\mu\lambda_j\psi - \bar\psi i\gamma_\mu\gamma_5\lambda_j\psi\bar\psi i\gamma_\mu\gamma_5\lambda_j\psi) \\ & + G'(\bar\psi\lambda_j\psi\bar\psi\lambda_j\psi - \bar\psi\gamma_5\lambda_j\psi\bar\psi\gamma_5\lambda_j\psi \\ & \quad + \bar\psi\lambda_0\psi\bar\psi\lambda_0\psi - \bar\psi\gamma_5\lambda_0\psi\bar\psi\gamma_5\lambda_0\psi)\,, \qquad (1)\end{aligned}$$

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or

$$-\mathcal{L}_I'=f_V\bar{\psi}i\gamma_\mu\psi v_\mu+f_A\bar{\psi}i\gamma_\mu\gamma_5\psi a_\mu,$$

where $\psi\equiv(\psi_1,\psi_2,\psi_3)$, and v_μ (a_μ) represent quark fields and a unitary singlet vector (axial-vector) field, respectively, and λ_j ($j=1, 2, \cdots, 8$) are the Gell-Mann spin matrices[2] and $\lambda_0=(\sqrt{\tfrac{2}{3}})\mathbf{1}$. For massless quark fields, both the vector and axial-vector currents,

$$J_{\mu j}(x)=\bar{\psi}i\gamma_\mu\lambda_j\psi \tag{2}$$

and

$$J_{\mu j}{}^5(x)=\bar{\psi}i\gamma_\mu\gamma_5\lambda_j\psi,\quad j=0, 1, 2, \cdots, 8 \tag{3}$$

satisfy the continuity equation, and the corresponding charges are conserved.

The breaking of the symmetry is assumed to be entirely due to mass terms[14]

$$-\mathcal{L}_m=m_0{}^1(\bar{\psi}_1\psi_1+\bar{\psi}_2\psi_2)+m_0{}^3\bar{\psi}_3\psi_3, \tag{4}$$

where $m_0{}^i$ ($i=1, 2, 3$) stands for the bare mass of the ith quark. Then, using the equation of motion, one can easily calculate the divergence of the axial-vector currents:

$$\partial_\mu J_{\mu j}{}^5(x)=W_j\bar{\psi}i\gamma_5\lambda_j\psi+D\delta_{j8}(\sqrt{\tfrac{2}{3}})\bar{\psi}i\gamma_5\psi \qquad (j \text{ not summed over}), \tag{5}$$

where

$$\begin{aligned} W_j&=2m_0{}^1 && (j=1, 2, 3)\\ &=m_0{}^1+m_0{}^3 && (j=4, 5, 6, 7)\\ &=\tfrac{2}{3}(m_0{}^1+2m_0{}^3) && (j=8), \end{aligned} \tag{6}$$

and

$$D=\tfrac{2}{3}\sqrt{2}(m_0{}^1-m_0{}^3). \tag{7}$$

If one compares Eqs. (5)–(7) with the parameters of Ref. 9, where the definitions

$$-\mathcal{L}_m=u_0+cu_8 \tag{4$'$}$$

and

$$u_j=\kappa\bar{\psi}\lambda_j\psi\quad (j=0, 1, 2, \cdots, 8) \tag{8}$$

have been used (κ being a quantity that has the dimension of mass), one obtains the relations

$$\kappa=(\sqrt{\tfrac{2}{3}})(2m_0{}^1+m_0{}^3) \tag{9}$$

and

$$c=\sqrt{2}(m_0{}^1-m_0{}^3)/(2m_0{}^1+m_0{}^3), \tag{10}$$

or, equivalently,

$$m_0{}^1/m_0{}^3=(1+c/\sqrt{2})/(1-\sqrt{2}c). \tag{10$'$}$$

The value of the parameter c has been estimated[9] to be $\cong -1.25$, which leads to the mass ratio

$$m_0{}^1/m_0{}^3\approx 0.042, \tag{11}$$

according to Eq. (10′).

[14] The $SU(2)$ symmetry is assumed to be exact, so that the proton-type quark and the neutron-type quark have the same bare mass ($m_0{}^1=m_0{}^2$).

Combining the PCAC condition for pion fields and Eq. (5), i.e.,

$$\begin{aligned} \partial_\mu J_{\mu\ 1\pm i2}{}^5(x)&=f_\pi\mu_\pi{}^2\varphi_\pi{}^{(\pm)} && (12)\\ &=2m_0{}^1\bar{\psi}i\gamma_5\tau_\pm\psi, && (5') \end{aligned}$$

where f_π stands for the matrix element of the $\pi\rightarrow\mu\nu$ decay, one can observe that the condition

$$m_1{}^0=c+\sqrt{2}=0 \tag{13}$$

corresponds to the case of vanishing pion mass. The fact that the value of c seems to be close to $-\sqrt{2}$ in reality is, therefore, considered as a manifestation of smallness of the physical pion mass.[9]

III. EQUAL-TIME COMMUTATOR AND THE POMERANCHUK THEOREM

The PCAC assumption for the pion, Eq. (12), and the requirement of the quark model, Eq. (5), enable us to calculate the ETCR for pion fields as follows:

$$\left[\int \varphi_\pi{}^{(+)}(\mathbf{x},x_0)d^3x,\ \int \varphi_\pi{}^{(-)}(\mathbf{y},x_0)d^3y\right]=C_\pi 2I_3, \tag{14}$$

where I_3 is the third component of the isospin operator and

$$C_\pi=(2m_0{}^1/f_\pi\mu_\pi{}^2)^2. \tag{15}$$

Taking the matrix element of Eq. (14) for a one-proton state, and inserting a complete set of states between the two pion fields, we obtain the relation[15]

$$C_\pi=\frac{2m}{\pi}\frac{|\mathbf{p}|}{p_0}\int_{\mu_\pi+\mu_\pi{}^2/2m}^{\infty}\frac{d\nu}{(p_0{}^2+2m\nu)^{1/2}}\times\frac{k_0\sigma_{\pi p}{}^{(-)}(\nu_L,k_0{}^2)}{(\mu^2-k_0{}^2)^2}, \tag{16}$$

where m and $(\mathbf{p},p_0)$ denote the mass and momentum four-vector, respectively, of the proton, and

$$\begin{aligned} k_0&=(\mathbf{p}^2+W^2)^{1/2}-(\mathbf{p}^2+m^2)^{1/2},\\ \nu_L&=k_0p_0/m,\\ \nu&=(W^2-m^2)/2m=\nu_L+k_0{}^2/2m, \end{aligned} \tag{17}$$

and

$$\sigma_{\pi p}{}^{(-)}(\nu,k_0{}^2)=\tfrac{1}{2}[\sigma_{\pi^-p}(\nu,k_0{}^2)-\sigma_{\pi^+p}(\nu,k_0{}^2)].$$

The variable W stands for the center-of-mass energy of the inserted intermediate states, ν_L is the laboratory energy of the π-p system, k_0 is the fictitious mass of the pion, and $\sigma_{\pi^\pm p}(\nu_L,k_0{}^2)$ is the total cross section of the $\pi^\pm$-p reaction. For $C_\pi=0$, it has been shown[4] that Eq. (16) leads to a superconvergence sum rule for the antisymmetric part of the π-p scattering amplitude, by using an appropriate subtraction method.

[15] The integral should be understood as a Cauchy principal-value integral (see Refs. 1 and 4).

For nonvanishing C_π, one can proceed in a slightly different way. Using the relation

$$dk_0=\frac{WdW}{(\mathbf{p}^2+W^2)^{1/2}}=\frac{md\nu}{(p_0^2+2m\nu)^{1/2}}, \tag{18}$$

Eq. (16) may be written as

$$C_\pi=-\frac{2}{\pi}\frac{|\mathbf{p}|}{p_0}\int_{(k_0)_{\min}}^{\infty}dk_0\frac{k_0\sigma_{\pi p}^{(-)}(k_0p_0/m,k_0^2)}{(\mu^2-k_0^2)^2}, \tag{16'}$$

where

$$(k_0)_{\min}=[p_0^2+2m\mu_\pi(1+\mu_\pi/2m)]^{1/2}-p_0. \tag{19}$$

If one takes the limit $p_0\to\infty$ in Eq. (16′), assuming that interchange of the limit and integration is permissible, one obtains[16]

$$C_\pi=-\frac{2}{\pi}\int_0^{\infty}dk_0\frac{k_0\sigma_{\pi p}^{(-)}(\infty,k_0^2)}{(\mu^2-k_0^2)^2}. \tag{20}$$

From Eq. (20), it is concluded that

$$\sigma_{\pi p}^{(-)}(\infty,k_0^2)\not\equiv 0, \tag{21}$$

provided that C_π is not zero. This implies the violation of the P theorem for the scattering of a pion with mass k_0 (on any target with nonvanishing isospin). However, the question remains whether the physical quantity

$$\sigma_{\pi p}^{(-)}(\infty,\mu_\pi^2)\equiv\sigma_{\pi p}^{(-)}(\infty) \tag{22}$$

is nonvanishing. We have no answer to this question so far. Instead, we present an argument that

$$\sigma_{\pi p}^{(-)}(\infty,0)\equiv\lim_{k_0\to 0}\sigma_{\pi p}^{(-)}(\infty,k_0^2)=0. \tag{23}$$

In order to see this constraint satisfied, we consider the matrix element of the ETCR of the axial charges $\int J_{0(\pm)}{}^5(x)d^3x$, i.e.,

$$1=g_A{}^2\left(1-\frac{m^2}{p_0^2}\right)+\frac{2mf_\pi^2\mu_\pi^4}{\pi}\frac{|\mathbf{p}|}{p_0}\times\int_{\mu_\pi+\mu_\pi^2/2m}^{\infty}\frac{d\nu}{(p_0^2+2m\nu)^{1/2}}\frac{\sigma^{(-)}(\nu_L,k_0^2)}{k_0(\mu^2-k_0^2)^2}, \tag{24}$$

where g_A is the renormalization constant of the axial-vector current for a nucleon state. Taking the limit $p_0\to\infty$ in Eq. (24) after subtracting $\sigma^{(-)}(\infty,k_0^2)$ (and adding it), we obtain the modified Adler-Weisberger relation

$$1=g_A{}^2+\frac{2f_\pi^2}{\pi}\int_{\mu_\pi+\mu_\pi^2/2m}^{\infty}d\nu\frac{\sigma_{\pi p}^{(-)}(\nu,0)-\sigma_{\pi p}^{(-)}(\infty,0)}{\nu}+\frac{2f_\pi^2\mu_\pi^4}{\pi}\int_0^{\infty}dk_0\frac{\sigma_{\pi p}^{(-)}(\infty,k_0^2)}{k_0(\mu^2-k_0^2)^2}. \tag{25}$$

The condition (23) immediately follows from the requirement that the second integral in Eq. (25) should exist at $k_0=0$. The term $\sigma_{\pi p}^{(-)}(\infty,0)$ of the first integral in Eq. (25), then, may be dropped accordingly.[17]

Coming back to the discussion of the physical quantity $\sigma_{\pi p}^{(-)}(\infty)$, we might point out that if the variation of $\sigma^{(-)}(\infty,k_0^2)$ around $k_0=0\sim\mu_\pi$ is small, then $\sigma_{\pi p}^{(-)}(\infty)$ is likely to be nonvanishing. Anyway, because of the lack of any compelling reason for $\sigma_{\pi p}^{(-)}(\infty)$ being zero, we assume that this is not the case. Then, normalizing the function $\sigma_{\pi p}^{(-)}(\infty,k_0^2)$ as

$$\sigma_{\pi p}^{(-)}(\infty,k_0^2)=\sigma_{\pi p}^{(-)}(\infty)h_{\pi p}(k_0^2/\mu_\pi^2), \tag{26}$$

where

$$h_{\pi p}(1)=1 \quad\text{and}\quad h_{\pi p}(0)=0, \tag{27}$$

we rewrite Eq. (20) as

$$C_\pi=\eta_{\pi p}\sigma_{\pi p}^{(-)}(\infty)/\pi\mu_\pi^2, \tag{28}$$

where

$$\eta_{\pi p}=P\int_0^{\infty}dx\frac{h_{\pi p}(x)}{(1-x)^2} \tag{29}$$

is a dimensionless quantity.

A similar discussion can easily be extended to other processes such as the $K^\pm$-p and $K^\pm$-n scattering. From the PCAC assumption,

$$\partial_\mu J_{\mu 4\pm i5}{}^5=f_K\mu_K{}^2\varphi_K{}^{(\pm)} \tag{30}$$

$$=(m_0{}^1+m_0{}^3)\bar\psi i\gamma_5\lambda_{4\pm i5}\psi, \tag{31}$$

one obtains the ETCR

$$\left[\int\varphi_K{}^{(+)}(\mathbf{x},x_0)d^3x,\int\varphi_K{}^{(-)}(\mathbf{y},x_0)d^3y\right]=C_K(I_3+\tfrac{3}{2}Y), \tag{32}$$

which, in turn, leads to the relations

$$C_K=\tfrac{1}{2}\eta_{Kp}\frac{\sigma_{Kp}{}^{(-)}(\infty)}{\pi\mu_K{}^2}=\eta_{Kn}\frac{\sigma_{Kn}{}^{(-)}(\infty)}{\pi\mu_K{}^2}. \tag{33}$$

Here, f_K stands for the amplitude of the $K\to\mu\nu$ decay, μ_K and $\varphi_K{}^{(\pm)}$ stand for the mass and field operator of the kaon, respectively, Y is the hypercharge

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[16] In Ref. 4, the relation

$$C_\pi=-\sigma_{\pi p}^{(-)}(\infty,0)/\pi\mu_\pi^2 \tag{20'}$$

has been derived. This equation is true if $\sigma_{\pi p}^{(-)}(\infty,k_0^2)$ is constant over the variable k_0^2. We will see in the subsequent argument that this would not be the case. See Eq. (23).

[17] The condition (23) could be different from the condition

$$\lim_{\nu\to\infty}\sigma^{(-)}(\nu,0)=0. \tag{23'}$$

If this equation is satisfied, we get the ordinary Adler-Weisberger relation as usual. Comparing it with Eq. (25), we have a constraint condition for $\sigma^{(-)}(\infty,k_0^2)$:

$$\int_0^{\infty}dk_0\frac{\sigma^{(-)}(\infty,k_0^2)}{k_0(\mu^2-k_0^2)^2}=0,$$

as a result of Eq. (23′).

operator, and

$$C_K = [(m_0^1 + m_0^3)/f_K \mu_K^2]^2. \tag{34}$$

The quantities defined for K-nucleon scattering,

$$\sigma_{KN}^{(-)}(\nu_L, k_0^2) = \tfrac{1}{2}[\sigma_{K^-N}(\nu_L, k_0^2) - \sigma_{K^+N}(\nu_L, k_0^2)], \tag{35}$$

$$\sigma_{KN}^{(-)}(\infty, k_0^2) = \sigma_{KN}^{(-)}(\infty) h_{KN}(k_0^2/\mu_K^2), \tag{36}$$

with

$$h_{KN}(1) = 1, \quad h_{KN}(0) = 0, \tag{37}$$

and

$$\eta_{KN} = P \int_0^\infty dx \frac{h_{KN}(x)}{(1-x)^2}, \tag{38}$$

are understood analogously from the case of the π-p scattering. [Corresponding equations are (17), (26), (27), and (29).]

We now assume an approximate $SU(3)$ symmetry defined as

$$h_{\pi p}(x) = h_{Kp}(x) = h_{Kn}(x), \tag{39}$$

and, consequently,

$$\eta_{\pi p} = \eta_{Kp} = \eta_{Kn}. \tag{40}$$

This assumption may be motivated by the earlier assumption adopted in Sec. II that the $SU(3) \times SU(3)$ symmetry breaking originates from the mass term only. Then, we obtain definite predictions

$$\sigma_{Kp}^{(-)}(\infty) = 2\sigma_{Kn}^{(-)}(\infty) \tag{41}$$

and

$$R = \frac{\sigma_{\pi p}^{(-)}(\infty)}{\sigma_{Kp}^{(-)}(\infty)} = \frac{C_\pi \mu_\pi^2}{2 C_K \mu_K^2} = \frac{1}{2}\left(\frac{2m_0^1}{m_0^1 + m_0^3} \frac{f_K \mu_K}{f_\pi \mu_\pi}\right)^2. \tag{42}$$

Using Eq. (11) with the experimental value[18] $f_K/f_\pi \approx 1.28$, we obtain

$$R = 0.066. \tag{43}$$

We should note, however, that R is a rapidly changing function of c or m_0^1/m_0^3. If, for example, one changes the value of c from -1.25 to -1, then one finds that $m_0^1/m_0^3 = 0.13$, which in turn gives the value for R as $\frac{1}{2}$. Equation (41) is a part of the Johnson-Treiman relation[19] and Eq. (42) becomes the rest of it when the complete $SU(3)$ symmetry is assumed.

IV. DISCUSSION

A. Experimental Total-Cross-Section Data

A small violation of the P theorem is hard to detect experimentally at the present time. Since, however, experiments at higher energy in the near future may shed light on this subject, we discuss some implications of our prediction. Equations (41) and (42) show the following:

(i) The signs of the quantities $\sigma_{\pi p}^{(-)}(\infty)$, $\sigma_{Kp}^{(-)}(\infty)$, and $\sigma_{Kn}^{(-)}(\infty)$ are the same.

(ii) $\sigma_{Kp}^{(-)}(\infty)/\sigma_{Kn}^{(-)}(\infty) = 2$.

These are qualitatively in accord with the tendency of experimental data,[20] if significant parts of the differences of the cross sections, $\sigma_{\pi p}^{(-)}(\nu)$, $\sigma_{Kp}^{(-)}(\nu)$, and $\sigma_{Kn}^{(-)}(\nu)$, at $\nu \approx 20$ GeV, are attributed to the violation of the P theorem.

(iii) The ratio $R = \sigma_{\pi p}^{(-)}(\infty)/\sigma_{Kp}^{(-)}(\infty)$ very much depends on the parameter c or m_0^1/m_0^3, as was discussed at the end of the last section.

Incidentally, we estimate the order of magnitude of the bare masses m_0^1 and m_0^3 by setting the dimensionless quantities $\eta_{\pi p} = \eta_{Kp} = 1$ and assuming that $\sigma_{Kp}^{(-)}(\infty) = 2$ mb, and $R = 0.066$ or $\frac{1}{2}$. Equations (28) and (33), then, lead to the values

$$m_0^1 \approx 3 \text{ MeV or } 9 \text{ MeV}$$

and

$$m_0^3 \approx 70 \text{ MeV}.$$

Although this estimate is a crude one, it is worthwhile to mention that such small values for bare quark masses are qualitatively compatible with those of earlier works.[7–11,21]

B. Real Part of the Forward Elastic Amplitude

The proof of the P theorem[6,22] is based on the assumption that either $\mathrm{Re}f/\mathrm{Im}f$ or f/ν be bounded at high energy, where f represents the forward elastic amplitude. The violation of the P theorem, then, implies the dominance of $\mathrm{Re}f$ at high energy. In order to see this, we write down the once-subtracted dispersion relation for the antisymmetric amplitude $f^{(-)}(\nu)$ of π-p scattering,[23]

$$\frac{\mathrm{Re}f^{(-)}(\nu)}{\nu} = \frac{\mathrm{Re}f^{(-)}(\mu_\pi)}{\mu_\pi} - \frac{2f^2(\nu^2 - \mu_\pi^2)}{(\nu^2 - \nu_B^2)(\mu_\pi^2 - \nu_B^2)} + \frac{\nu^2 - \mu_\pi^2}{2\pi^2} \int_{\mu_\pi}^\infty \frac{d\nu' \sigma^{(-)}(\nu')}{(\nu'^2 - \nu^2)(\nu'^2 - \mu_\pi^2)^{1/2}}, \tag{44}$$

[18] E.g., J. D. Bjorken and M. Nauenberg, Ann. Rev. Nucl. Sci. 18, 229 (1968).

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where

$$f^2=(g^2/4\pi)(\mu_\pi/2m)^2=0.081$$

stands for the π-N coupling constant, and

$$\nu_B=\mu_\pi^2/2m.$$

The last term of Eq. (44) may be written as

$$\frac{\nu^2-\mu_\pi^2}{2\pi^2}\int_{\mu_\pi}^{\infty}\frac{d\nu'}{\nu'^2-\nu^2}\frac{\sigma^{(-)}(\nu')-\sigma^{(-)}(\infty)}{(\nu'^2-\mu_\pi^2)^{1/2}}$$
$$-\frac{\sigma^{(-)}(\infty)}{2\pi^2}\frac{(\nu^2-\mu_\pi^2)^{1/2}}{\nu}\ln\left|\frac{\nu+(\nu^2-\mu_\pi^2)^{1/2}}{\mu_\pi}\right|.$$

Hence, the real part of $f^{(-)}(\nu)$ has the asymptotic form

$$\mathrm{Re}f^{(-)}(\nu)\underset{\nu\to\infty}{\longrightarrow}-\frac{\sigma^{(-)}(\infty)}{2\pi^2}\nu\ln\frac{2\nu}{\mu_\pi}. \tag{45}$$

The ratio $\mathrm{Re}f/\mathrm{Im}f$ for $\pi^\pm p$ scattering is, therefore, given by

$$\alpha_\pm(\nu)=\frac{\mathrm{Re}f_{\pi^\pm p}(\nu)}{\mathrm{Im}f_{\pi^\pm p}(\nu)}\to\pm\frac{2\sigma_{\pi p}^{(-)}(\infty)}{\pi\sigma_{\pi^\pm p}(\infty)}\ln\frac{2\nu}{\mu_\pi}. \tag{46}$$

In particular,

(iv) $\alpha_\pm$ grow logarithmically with the incident energy and have opposite signs asymptotically.

Since the present experimental data[24] do not show such behavior for $\alpha_\pm$, we might suspect that $\sigma_{\pi p}^{(-)}(\infty)$ is much smaller than 1 mb, which is half of the difference of the $\pi^\pm p$ cross sections at the highest energy available. In fact, for $\sigma_{\pi p}^{(-)}(\infty)=1$ mb, $\sigma_{\pi^+p}(\infty)\approx20$ mb, and $\nu=20$ GeV, we obtain $\alpha_+=0.15$. If, on the other hand, we take the value of Eq. (43) for the ratio R and assume $\sigma_{Kp}^{(-)}(\infty)=2$ mb, then at $\nu=20$ GeV, $\alpha_+=0.01$ is the contribution due to the (non-Regge) logarithmic term, Eq. (46). [$\sigma_{\pi p}^{(-)}(\infty)$ thus obtained would be 0.07 mb.] If this is the case, we have to go to a much higher energy in order to be able to detect the violation of the P theorem from the measurement of the real part of the amplitude. It might be pertinent to measure the real part of the K-nucleon forward-scattering amplitude at high energy to test the validity of the P theorem.

Finally, we note that the sum rule for the coupling constant f, to be derived by taking the limit $\nu\to\infty$ in Eq. (44) using Eq. (45), is modified as follows, for the case $\sigma_{\pi p}^{(-)}(\infty)\neq0$:

$$\frac{\mathrm{Re}f^{(-)}(\mu_\pi)}{\mu_\pi}=\frac{2f^2}{\mu_\pi^2-\nu_B^2}+\frac{1}{2\pi^2}\int_{\mu_\pi}^{\infty}\frac{d\nu'[\sigma_{\pi p}^{(-)}(\nu')-\sigma_{\pi p}^{(-)}(\infty)]}{(\nu'^2-\mu_\pi^2)^{1/2}}. \tag{47}$$

[24] K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. D. Platner, C. A. Quarles, and E. H. Wilson, Phys. Rev. Letters **19**, 193 (1967); **19**, 662(E) (1967).

Since, for small values of $\sigma_{\pi p}^{(-)}(\infty)$, the sum rule (47) is in practice not much different from the original ones,[25] we expect the value of f^2, deduced from the old sum rule, to be affected very little.

C. Non-Regge Behavior of the Amplitude

The violation of the P theorem and the logarithmic asymptotic behavior for the real part of the forward elastic amplitude, Eq. (45), imply that the Regge theory cannot be applied for the whole amplitude. It is possible, however, to derive such non-Regge behavior as a special limiting case of a Regge amplitude.

We notice, first, that the limit $\alpha\to1$ in the Regge pole amplitude with odd signature,

$$f_\alpha^{(-)}=\beta\frac{1-e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}\left(\frac{\nu}{\nu_0}\right)^{\alpha(t)}$$
$$=\beta[\tan\tfrac{1}{2}\pi\alpha(t)+i]\left(\frac{\nu}{\nu_0}\right)^{\alpha(t)}, \tag{48}$$

where t stands for momentum transfer squared, leads to a singularity for the $\mathrm{Re}f_\alpha^{(-)}$. If, however, we put $\alpha\to1$ in the formula

$$f_\alpha^{(-)}=-\frac{1}{2i}\int\frac{\beta_l/(l-\alpha)}{\sin\pi l}P_l(-\cos\theta_t)(1-e^{-i\pi l}), \tag{49}$$

where the integration path is to encircle α, then we get the asymptotic form,

$$f_\alpha^{(-)}\approx\bar{\beta}[-(2/\pi)\ln(\nu/\nu_0)+i]\nu/\nu_0, \tag{50}$$

which is identical to that of (45).

Another way of formally deriving Eq. (50) is to take a δ function as a weight function of the Regge cut formula. Using the formula

$$\tan\tfrac{1}{2}\pi\alpha\,\delta(\alpha-1)=-\frac{2}{\pi}\frac{\delta(\alpha-1)}{1-\alpha}=-\frac{2}{\pi}[\delta'(\alpha-1)+A\delta(\alpha-1)],$$

where A is an arbitrary constant, we have

$$\int^1(\tan\tfrac{1}{2}\pi\alpha+i)\left(\frac{\nu}{\nu_0}\right)^\alpha\delta(\alpha-1)d\alpha\approx\left(-\frac{2}{\pi}\ln\frac{\nu}{\nu_0}+i\right)\frac{\nu}{\nu_0}, \tag{51}$$

which is of the same form as Eq. (50). We stress that these formal manipulations still do not permit the accommodation of the asymptotic behavior (50) in the Regge scheme. Instead, we have to Reggeize the amplitude after subtracting such an asymptotic form. Whether such a form can be obtained from the contour integration over an infinitely large semicircle in the complex angular momentum plane for the Watson-Sommerfeld formula is yet to be seen.

[25] M. L. Goldberger, H. Miyazawa, and R. Oehme, Phys. Rev. **96**, 986 (1955).

D. Unitarity Restriction

Does the asymptotic behavior (46) violate unitarity? It depends on the t dependence of the elastic amplitude. For an elastic amplitude $f(\nu,t)$, the unitarity restriction reads

$$\sigma_{\rm el} \propto \frac{1}{\nu}\int_0^{-t_0} |f(\nu,t)|^2 \frac{dt}{\nu} \leq \sigma_{\rm tot}(\nu)\,, \tag{52}$$

where $\sigma_{\rm el}$ and $\sigma_{\rm tot}$ represent the elastic and total cross section, respectively, and $-t_0$ is the momentum transfer below which the amplitude has a significant magnitude. The asymptotic behavior

$$f(\nu,t) \approx \beta(t)\nu^{\alpha_1(t)}(\ln\nu)^{\alpha_2(t)}\,, \tag{53}$$

where

$$\alpha_i(t) = 1 + a_i t + \cdots \quad \text{for small } t,\ i=1,\,2\,, \tag{54}$$

clearly violates the condition (52), since the left-hand side of Eq. (52) behaves as $\ln\nu$ asymptotically, while $\sigma_{\rm tot}$ is assumed to be bounded by a constant.

If, however, $f(\nu,t)$ has the asymptotic form, e.g.,

$$f(\nu,t) \approx \beta(t)\nu^{\alpha_1(t)}(\ln\nu)^{\alpha_2(t)}g(\nu t) + \text{ordinary Regge amplitude}, \tag{55}$$

where

$$g(0)=1\,,\quad g(\infty)=0\,, \tag{56}$$

and

$$\int_0^\infty |g(z)|^2 dz < \infty\,,$$

then the unitarity condition (52) could be satisfied: For nonvanishing t and $\nu \to \infty$, the contribution of the logarithmic term is negligible. In fact, for $\beta(t)=\alpha_1(t) = \alpha_2(t)=1$, we have

$$\sigma_{\rm el}(\nu) \propto \frac{1}{\nu^2}\int_0^{-t_0} |f(\nu,t)|^2 dt \to \frac{(\ln\nu)^2}{\nu} \times \int_0^\infty |g(x)|^2 dx \to 0\,, \tag{57}$$

as far as the contribution of the first term of Eq. (55) is concerned.

Note added in manuscript. In view of the strong assumption made in this article concerning the violation of the P theorem, we add a remark for the opposite case. If the quantity $\sigma_{\pi p}^{(-)}(\infty,k_0^2)$ contains a factor $(k_0^2-\mu_\pi^2)$, so that the P theorem is preserved for the physical cross section, we may proceed in the following way. Instead of Eqs. (26) and (36), etc., we assume that

$$\sigma_{\pi p}^{(-)}\left(\frac{k_0 p_0}{m},\mu_\pi^2\right) \xrightarrow[p_0\to\infty]{} \beta_{\pi p}^{(-)}\left(\frac{k_0 p_0}{m}\right)^{\alpha-1},\quad 0<\alpha<1 \tag{A1}$$

and

$$\frac{\sigma_{\pi p}^{(-)}(k_0p_0/m,\,k_0^2)}{\sigma_{\pi p}^{(-)}(k_0p_0/m,\,\mu_\pi^2)} \xrightarrow[p_0\to\infty]{} 1+\left(\frac{k_0p_0}{m}\right)^{1-\alpha}\bar{h}_{\pi p}\left(\frac{k_0^2}{\mu_\pi^2}\right), \tag{A2}$$

where

$$\bar{h}_{\pi p}(1)=\bar{h}_{\pi p}(0)=0\,. \tag{A3}$$

Analogous equations apply to K-nucleon scattering. Then, with the assumption of approximate $SU(3)$ symmetry,

$$\bar{h}_{\pi p}(x)=\bar{h}_{Kp}(x)=\bar{h}_{Kn}(x)\,, \tag{A4}$$

we obtain the relations

$$\lim_{\nu\to\infty}\frac{\sigma_{Kp}^{(-)}(\nu)}{\sigma_{Kn}^{(-)}(\nu)}=\frac{\beta_{Kp}^{(-)}}{\beta_{Kn}^{(-)}}=2 \tag{41$'$}$$

and

$$\bar{R}=\lim_{\nu\to\infty}\frac{\sigma_{\pi p}^{(-)}(\nu)}{\sigma_{Kp}^{(-)}(\nu)}=\frac{\beta_{\pi p}^{(-)}}{\beta_{Kp}^{(-)}}=\frac{1}{2}\left(\frac{2m_0^1}{m_0^1+m_0^3}\,\frac{f_K\mu_K}{f_\pi\mu_\pi}\right)^2. \tag{42$'$}$$

These are of a weaker form than the relations (41) and (42), and are closer to those of Johnson and Treiman.[19] The discussion of Sec. IV A can be applied in a similar manner if one replaces Eqs. (41) and (42) by Eqs. (41′) and (42′).

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