Model for Higher-Order Weak Interactions

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The most divergent terms to all orders in G_F for $\Delta S=0$ nonleptonic transitions are considered in a model of weak interactions. The analysis confirms and extends the conclusions of an earlier study of the model which showed that although terms of higher order in G_F diverged, they were compatible with a value of a cutoff Λ such that $G_F\Lambda^2 \sim 1$.

INTRODUCTION

One of us[1] has recently proposed a model of weak interactions, which obeys, in lowest order, all the usual selection rules and predicts rates in agreement with experiment. The mediators of the weak interaction are three intermediate vector bosons $W_a^{\pm}$, $W_b^{\pm}$, $W_c^{0,\bar{0}}$ with their respective antiparticles. The model also contains, in addition to the usual leptons, two neutral massive (mass $\gtrsim 500$ MeV) leptons λ_μ and λ_e.

Denoting by J_σ^i the hadronic $V-A$ current with $SU(3)$ transformation properties indicated by the superscript, by j_σ^+ the $V-A$ leptonic current, and by $\tilde{j}_\sigma^+$ the $V-A$ lepton current obtained substituting λ_e, λ_μ for ν_e, ν_μ in j_σ^+, we consider the following weak-interaction Hamiltonian:

$$H_{\text{weak}}=g[J_\sigma^{\pi^+}W_{a,\sigma}^- + J_\sigma^{K^+}W_{b,\sigma}^- + (J_\sigma^{K^0}+\gamma V_\sigma^0)W_{c,\sigma}^{\bar{0}} + j_\sigma^+(W_{a,\sigma}^-\cos\theta + W_{b,\sigma}^-\sin\theta) + \tilde{j}_\sigma^+(-W_{a,\sigma}^-\sin\theta + W_{b,\sigma}^-\cos\theta)]+\text{H.c.}, \quad (1)$$

where V_σ^0 is the baryonic number current and, assuming all the W mesons have the same mass, $g^2/M_W^2 = G_F/\sqrt{2}$.

As was shown in I, the model predicts the correct rates and selection rules for leptonic and semileptonic processes; in addition, the matrix elements for semileptonic strangeness-changing decays, resulting from higher-order weak interactions (such as $K_L^0 \to \mu^+\mu^-$) are more convergent than in the usual theory, being proportional to $G_F^2 m_{\lambda_\mu}^2$ rather than the usual $G_F^2\Lambda^2$, where Λ is a cutoff.[2,3]

Now leptonic strangeness-changing decays clearly have octet transformation properties since V^0 is an $SU(3)$ singlet, and their magnitude is determined by adjusting the free parameter γ ($\gamma=1$ corresponds to an enhancement of nonleptonic over semileptonic rates because of the absence of the Cabibbo angle in the hadron currents). In addition, the fact that V^0 commutes at equal times with the $SU(3)$ currents and their divergences, makes higher-order strangeness-changing nonleptonic processes more convergent than in the usual theory.[3]

Nonleptonic $\Delta S=0$ processes are, however, quadratically divergent, the coefficient of the quadratic divergence being determined by the equal-time commutator[4]

$$[J_0^{\pi^+}(x),\partial_\sigma J_\sigma^{\pi^-}(0)]_{x_0=0}+[J_0^{K^+}(x),\partial_\sigma J_\sigma^{K^-}(0)]_{x_0=0} +[J_0^{K^0}(x),\partial_\sigma J_\sigma^{\bar{K}^0}(0)]_{x_0=0}=R(\mathbf{x})\delta(\mathbf{x}). \quad (2)$$

Assuming the Gell-Mann model for $SU(3)\times SU(3)$ breaking,[5] namely, that

$$H=\bar{H}+H_B=\bar{H}+\epsilon_0 u_0+\epsilon_8 u_8, \quad (3)$$

where $\bar{H}$ is invariant under $SU(3)\times SU(3)$ and u_0, u_8 are scalar densities belonging to the $(3,\bar{3})+(\bar{3},3)$ representation of $SU(3)\times SU(3)$, we calculated in I, the divergences of the current and evaluated the commutator, finding

$$R(0)=4[\epsilon_0 u_0(0)+\epsilon_8 u_8(0)], \quad (4)$$

so that, to lowest order, we had no parity or isospin violation and in fact found that the weak divergence just corresponded to a redefinition of the strong breaking. The matrix element for a transition $A\to B$ was given by

$$T_{A\to B}=(G_F/\sqrt{2})(\Lambda^2/4\pi^2) \times\langle B|4[\epsilon_0 u_0(0)+\epsilon_8 u_8(0)]|A\rangle, \quad (5)$$

where Λ is a cutoff of the momenta integral.

In this paper we wish to explore the possibility of extending this result to higher orders. The question is whether in nth order the most divergent part of the $A\to B$ transition matrix element preserves its attractive lowest-order features. We shall explore a free-quark model, along the lines of the recent work of Gatto, Sartori, and Tonin.[6] In Sec. II we will give a general treatment of the problem, leaving to the Appendices some of the technical details. In Sec. III, the question of coupling to leptons will be considered.

I. QUARK-MODEL CALCULATIONS

In studying the most divergent diagrams, one can neglect altogether the current V^0; furthermore, the breaking of $SU(3)\times SU(3)$ is simply due to the quark

* Supported in part by the U. S. Atomic Energy Commission.
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[2] B. L. Ioffe and E. P. Shabalin, Yadern. Fiz. 6, 603 (1966) [English transl.: Soviet J. Nucl. Phys. 6, 828 (1967)].
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[6] R. Gatto, G. Sartori, and M. Tonin, Nuovo Cimento Letters 1, 1 (1969); Phys. Letters 28B, 128 (1968).

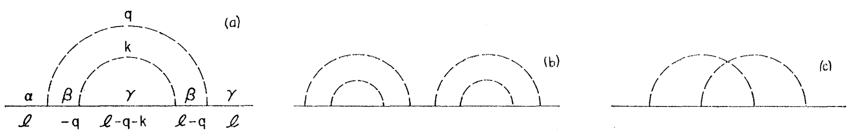

FIG. 1. Self-mass graphs in the quark model interacting weakly with W mesons.

masses, so we have

$$H=\bar{H}+m_N(\bar{\mathcal{P}}\mathcal{P}+\bar{\mathfrak{N}}\mathfrak{N})+m_\Lambda(\bar{\lambda}\lambda)\,, \qquad (6)$$

where $\mathcal{P}$, $\mathfrak{N}$, and λ are now taken to be the three quark fields. Neglecting the lepton coupling to W's for the moment, we see that the most divergent diagrams are of the types shown in Figs. 1(a) and 1(b). In Fig. 1(a), α,β, and γ are quark labels and l, q, and k are momenta. Diagrams like 1(c) are forbidden by the conservation principles in the theory when we neglect V^0. Now the graphs of 1(a) and 1(b) are diagonal, namely, they only connect $\mathcal{P}$ to $\mathcal{P}$, $\mathfrak{N}$ to $\mathfrak{N}$, and λ to λ. Furthermore, the γ_5 invariance of the W couplings ensures that the graphs of Figs. 1(a) and 1(b) depend only on the mass of the external particle. For instance, the most divergent part of the graph in 1(a) goes as

$$\int\int\frac{d^4q d^4k}{q^2k^2}\bar{u}\left(\boldsymbol{q}(1+\gamma_5)\frac{\boldsymbol{l}-\boldsymbol{q}+im_\beta}{(l-q)^2}\boldsymbol{k}(1+\gamma_5)\frac{\boldsymbol{l}-\boldsymbol{q}-\boldsymbol{k}+im_\gamma}{(l-q-k)^2}\boldsymbol{k}(1+\gamma_5)\frac{\boldsymbol{l}-\boldsymbol{q}+im_\beta}{(l-q)^2}\boldsymbol{q}(1+\gamma_5)\right)u\,, \qquad (7)$$

but of course $(1+\gamma_5)M(1-\gamma_5)$ equals zero. So what we find, to nth order, is that the most divergent diagrams contribute mass shifts proportional to the masses of the external particles, i.e., are of the form

$$a_n(G_F\Lambda^2)^n[m_N(\bar{\mathcal{P}}\mathcal{P}+\bar{\mathfrak{N}}\mathfrak{N})+m_\lambda(\bar{\lambda}\lambda)]\,, \qquad (8)$$

which is proportional to the breaking of $SU(3)\times SU(3)$ in (5). Diagrams like those of Fig. 1(b) are just iterations of the diagrams of Fig. 1(a).

II. GENERAL HIGHER-ORDER WEAK DIVERGENCES

In general, the leading divergence to nth order in G_F is obtained by keeping only the $q_\mu q_\nu$ part of the n W propagators involved in the process. The matrix element for the nth-order contribution to a nonleptonic transition is proportional to[7]

$$(iG_F)^n\int\cdots\int\prod_{j=1}^{n}e^{iq_j(x_j-y_j)}\frac{q_{j,\mu_j}q_{j,\nu_j}}{q_j^2+M_W^2}d^4x_jd^4y_j\frac{d^4q_j}{(2\pi)^4}\sum_{\{i\}}\langle B|T\{J_{\mu_1}{}^{i_1}(x_1)J_{\nu_1}{}^{i_1}(y_1)\cdots J_{\mu_n}{}^{i_n}(x_n)J_{\nu_n}{}^{i_n}(y_n)\}|A\rangle$$

$$=(iG_F)^n\int\cdots\int\prod_{j=1}^{n}\frac{e^{iq_j(x_j-y_j)}}{q_j^2+M_W^2}d^4x_jd^4y_j\frac{d^4q_j}{(2\pi)^4}\frac{\partial}{\partial x_{j,\mu_j}}\frac{\partial}{\partial y_{j,\nu_j}}\sum_{\{i\}}\langle B|T\{J_{\mu_1}{}^{i_1}(x_1)J_{\nu_1}{}^{i_1}(y_1)\cdots J_{\mu_n}{}^{i_n}(x_n)J_{\nu_n}{}^{i_n}(y_n)\}|A\rangle\,, \qquad (9)$$

where we have for the moment neglected diagrams involving lepton loops. (We shall, however, return to these later.) In the above summation over the $SU(3)$ indices of the currents, we have gone over to a Cartesian labeling of the currents, i.e.,

$$W_a{}^-=W^1-iW^2\,,\quad W_b{}^-=W^4-iW^5\,,\quad W_c{}^0=W^6-iW^7. \qquad (10)$$

This allows us to write the hadron current part of H_{weak} as

$$H_{\mathrm{weak}}{}^{\mathrm{hadron}}=g\sum_{i=1,2,4,5,6,7}(J_\sigma{}^iW_\sigma{}^i)+\gamma V_\sigma{}^0W_{c,\sigma}{}^0+\mathrm{H.c.}, \qquad (11)$$

where of course we neglect the term involving V^0 in calculating leading divergences. The summation over an arbitrary i_k in (9) then runs over the $SU(3)$ indices 1,2,4,5,6,7. Alternatively, one may introduce fictitious fields W^3 and W^8, so that

$$\sum_{i=1,2,4,5,6,7}J_\sigma{}^iW_\sigma{}^i=\sum_{i=1}^{8}J_\sigma{}^iW_\sigma{}^i-J_\sigma{}^3W_\sigma{}^3-J_\sigma{}^8W_\sigma{}^8. \qquad (12)$$

Let us begin our analysis of (9) by considering the term obtained when all but one of the differential operators act on the time ordering. The result of this is a chain of equal-time commutators

$$\sum_i\sum_{\text{all permutations}}[[\cdots[\partial_\sigma J_\sigma{}^{i_m},J_0{}^{i_k}]\cdots]J_0{}^{i_l}], \qquad (13)$$

where all the commutators are taken at equal time. The divergence of the current must appear in the innermost

[7] See J. Iliopoulos [CERN Report No. Th. 981 (unpublished)] for a discussion, among other points, of what factors are necessary to write such an amplitude.

commutator, since terms of the form

$$[[\cdots[J_0^{i_m},J_0^{i_k}]\cdots] \tag{14}$$

are canceled by terms obtained from the permutation that interchanges i_m and i_k, leaving all other indices unchanged.

If we assume that the Schwinger terms are c numbers, we may calculate the commutators of the integrated current densities,

$$\langle A|S_n|B\rangle=\sum_{\{i\}}\sum_{\text{perm}}\langle B|[\cdots[\dot{F}_-^{i_m},F_-^{i_k}]\cdots]|A\rangle, \tag{15}$$

where $F_\pm^i=F^i\pm F_5^i$ are the generators of the $SU(3)\times SU(3)$ algebra. Each one of these multiple commutators, involving n indices, $i_1\cdots i_n$ leads to a divergent term proportional to $(G_F\Lambda^2)^n$, since the expression in (9) reduces to

$$(iG_F)^n\int\prod_{j=1}^{n}\frac{d^4q_j}{(2\pi)^4(q_j^2+M_W^2)}\langle B|S_n|A\rangle =\left(\frac{iG_F\Lambda^2}{16\pi^2}\right)^n\langle B|S_n|A\rangle. \tag{16}$$

What we want to show now is that the lowest-order result, in which we found $R(x)$ proportional to $H_B(x)$ as given in (2), (3), in fact carries through to higher order; namely, S_n is proportional to H_B.

To do this, it will be convenient to use (12) to rewrite the weak-interaction Hamiltonian. We see, then, that there will be three types of terms in S_n with $S_n=S_{n,\text{I}}+S_{n,\text{II}}+S_{n,\text{III}}$: $S_{n,\text{I}}$, terms where indices run over $1\cdots8$; $S_{n,\text{II}}$, terms with only F^3 and F^8; and $S_{n,\text{III}}$, terms with even number of F^3 and/or F^8, others being F^i, where $i=1\cdots8$. Consider first $S_{n,\text{I}}$. H_B transforms like a member of an irreducible representation of the $SU(3)\times SU(3)$ generated by $F_\pm^i$; for generality let us say it is the α member of the irreducible representation labeled by a so that $H_B=O_\alpha^{(a)}$. Then we have

$$\dot{F}_-^j=[F_-^j,O_\alpha^{(a)}]=\sum_{\alpha'}\langle a\alpha'|F_-^j|a\alpha\rangle O_{\alpha'}^{(a)} \tag{17}$$

and

$$[F_-^k,[F_-^j,O_\alpha^{(a)}]]=\sum_{\alpha'\alpha''}\langle a\alpha'|F_-^j|a\alpha\rangle \times\langle a\alpha''|F_-^k|a\alpha'\rangle O_{\alpha''}^{(a)} =\sum_{\alpha''}\langle a\alpha''|F_-^kF_-^j|a\alpha\rangle O_{\alpha''}^{(a)}, \tag{18}$$

so that it becomes clear that for $S_{n,\text{I}}$ we must consider the operator

$$C=\sum_{\{i\}}\sum_{\text{perm}}F_-^{i_n}F_-^{i_n}\cdots F_-^{i_1}F_-^{i_1}. \tag{19}$$

We shall now show that this is either equal to zero or is a Casimir operator of the $SU(3)\times SU(3)$ group, thereby implying that $S_{n,\text{I}}$ is in fact proportional to matrix elements of H_B, since the whole sequence of commutators then merely rotates H_B back into itself. To show that the operator in (19) is either zero or a Casimir operator, we need only consider its commutator with an arbitrary F_-^k, since of course F_+^k commutes with all F_-^i:

$$[F_-^k,C]=\sum_{\{i\}}\sum_{\text{perm}}([F_-^k,F_-^{i_n}]F^{i_n}\cdots F^{i_1} +F_-^{i_n}[F_-^k,F_-^{i_n}]\cdots F_-^{i_1}\cdots +F_-^{i_n}\cdots[F_-^k,F_-^{i_1}]). \tag{20}$$

Now each index i_j occurs twice, and the corresponding commutators cancel each other:

$$\begin{aligned}\cdots[F_-^k,F_-^j]\cdots F_-^j\cdots+\cdots F_-^j\cdots[F_-^k,F_-^j]\cdots &=2if^{kjm}(\cdots F_-^m\cdots F_-^j\cdots+\cdots F_-^j\cdots F_-^m\cdots)\\ &=2if^{kjm}(\cdots F_-^m\cdots F_-^j\cdots-\cdots F_-^m\cdots F_-^j\cdots)\\ &=0,\end{aligned} \tag{21}$$

where the interchange of j and m necessary to obtain the cancellation was made possible by the fact that both indices run from one to eight. The cancellation does not depend on the ordering of the generators and must therefore be true for all terms obtained by permutations. This, then, shows that C is either zero or a Casimir operator and, hence, that if $S_{n,\text{I}}$ is nonvanishing, it is proportional to H_B.

For $S_{n,\text{II}}$, one must consider commutators of the form

$$[F_-^3[F_-^8\cdots[F_-^3[F_-^8,H_B]]\cdots], \tag{22}$$

which can be reordered using the fact that F_-^3 and F_-^8 commute as

$$[F_-^8[F_-^8\cdots[F_-^3[F_-^3,H_B]]\cdots]. \tag{23}$$

An explicit calculation shows that

$$[F_-^3,[F_-^3,H_B]]+[F_-^8,[F_-^8,H_B]]=\tfrac{4}{3}H_B. \tag{24}$$

Using this relation, we can then easily show that $S_{n,\text{II}}$ is also proportional to H_B. The same can be said for $S_{n,\text{III}}$, though the arguments involved are considerably more complicated and will be left to Appendix A.

Let us now turn our attention to the other terms arising from (9), obtained by letting more than one differentiation be applied to the currents rather than to the time ordering. There is a whole set of other terms which diverge like $(G_F\Lambda^2)^n$ in nth order. They are of the form

$$(iG_F)^n\int\prod_{j=1}^{n}\frac{d^4q_j}{(2\pi)^4(q_j^2+M_W^2)}\int\cdots\int d^4x_1\cdots d^4x_k \times\langle B|T\{S_{m_1}(x_1)\cdots S_{m_k}(x_k)\}|A\rangle,\quad \sum_{j=1}^{k}m_j=n, \tag{25}$$

where S_{m_j} are the operators we determined previously. Since each of the S_{m_j}'s transforms like H_B, this corre-

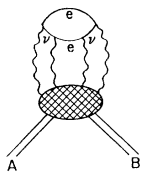

FIG. 2. Contribution of lepton loops to weak $A\to B$ transitions.

sponds to terms of higher order in the breaking of $SU(3)\times SU(3)$ as would be obtained, for instance, taking H_B to act as a spurion, by having k spurions act on the strong amplitude. Graphically, these terms are described in a free-quark model by diagrams such as those of Fig. 1(b), while S_n corresponds to a diagram like that of Fig. 1(a). In coordinate space one may envision the first type of terms as those for which

$$x_1=y_1=x_2=y_2=\cdots=x_n=y_n\,, \tag{26}$$

and the second as those for which

$$\begin{aligned} x_1=y_1=x_2=\cdots x_{m_1}&=y_{m_1}\,,\\ x_{m_1+1}=y_{m_1+1}=\cdots&=y_{m_1+m_2}\,,\\ x_{n-m_k+1}=y_{n-m_k+1}=\cdots&=x_n=y_n\,. \end{aligned} \tag{27}$$

Both diverge like $(G_F\Lambda^2)^n$; finally, we have terms for which the argument of the W propagators in coordinate space do not vanish, and we assume these to be less singular. As an example, consider a matrix element to first order in G_F, keeping only the $q_\mu q_\nu$ part of the W propagator. It is given by

$$\begin{aligned} T_{A\to B}=\tfrac{1}{2}i^2G_F\int\frac{d^4q}{(2\pi)^4}\frac{e^{iq(x-y)}}{q^2+M_W^2}d^4x\,d^4y \\ \times\sum_{i=1,2,4,5,6,7}\{\langle B|[J_0^i(x),\partial_\mu J_\mu^i(y)]|A\rangle\delta(x_0-y_0) \\ +\langle B|T\{\partial_\mu J_\mu^i(x)\partial_\nu J_\nu^i(y)\}|A\rangle\}\,. \end{aligned} \tag{28}$$

The first term in the curly brackets leads to what we would call S_1, and the second we assume to be only logarithmically divergent by making use of a Bjorken[8] limit technique. The validity of this last assumption[9] is admittedly highly questionable, particularly when we go to higher orders and assume their analogs for time-ordered products of $2n$ currents.[10,11]

III. LEPTON CONTRIBUTION

In this last section we would like to discuss briefly the heretofore neglected contribution of leptons. Diagrams such as the one in Fig. 2 also contribute divergences of the form $(G_F\Lambda^2)^n$ in nth order and hence may not be neglected. Since W_c couples to hadrons and not to leptons, a nontrivial extension of the theory is involved. We shall proceed in evaluating a graph, such as that of Fig. 2, by first doing the divergent integral over the lepton loop momentum, keeping fixed the momenta of the W mesons coupled to the hadrons. This essentially reduces the problem to the one treated in Sec. II, except that there is no longer the symmetry between coupling to W_a, W_b, and W_c; it also, of course, introduces a factor $G_F\Lambda^2$ coming from the lepton loop or $G_F\Lambda^2(G_F\Lambda^2)^m$ if the lepton loop has m W mesons inside it. The symmetry between W_a and W_b still persists, however, and in fact, as was shown in I, a transition $W_a\rightleftarrows W_b$ via an intermediate lepton loop only occurs to order $G_Fm_{\lambda\mu}^2$ and not to order $G_F\Lambda^2$, so we neglect it.

One might worry then about the possibility of a parity-violating term being present in the maximally divergent graphs, but as we shall show in Appendix B, there is none. The proportionality of these maximally divergent terms to H_B does not however hold any longer; in fact a term of the form u_3 appears. Consider the simplest set of graphs for $A\to B$ involving a lepton loop, namely, those of order G_F^2 obtained by inserting a lepton loop in the W propagator of the lowest-order diagram for $A\to B$. The matrix element for $A\to B$ is given by

$$\begin{aligned} &\frac{G_F\Lambda^2}{4\pi^2}\frac{G_F\Lambda^2}{16\pi^2}\int d\mathbf{x}\langle B|\sum_{i=1,2,3,4}[J_0^i(\mathbf{x},0),\partial_\mu J_\mu^i(0)]|A\rangle \\ &=\frac{1}{12}\left(\frac{G_F\Lambda^2}{4\pi^2}\right)^2\langle B|\{(8\epsilon_0+\sqrt{2}\epsilon_8)u_0 \\ &\quad+[(\sqrt{6})\epsilon_0+\sqrt{3}\epsilon_8]u_3+[(\sqrt{6})\epsilon_0+7\epsilon_8]u_8\}|A\rangle\,, \end{aligned} \tag{29}$$

so, although there is a u_3 term, the violation of isospin conservation, due to the presence of u_3, is quite small. As stressed in I, just as the natural expansion parameter in electrodynamics is

$$\frac{\alpha}{\pi}=\frac{1}{\pi}\frac{e^2}{4\pi}\,,$$

here it is

$$\frac{1}{\pi}\frac{g^2}{4\pi}\frac{\Lambda^2}{M_W^2}=\frac{G_F\Lambda^2}{\sqrt{2}(4\pi^2)}\,,$$

so that even if $G_F\Lambda^2\sim 1$, we still only get at most 1% effects; in this case the violation of isospin conservation appears to be considerably smaller than 1%.

IV. CONCLUSIONS

We have extended the analysis of the model of weak interactions presented in I to higher order in G_F contributions to $\Delta S=0$ hadronic transitions. We have

[8] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

[9] R. Jackiw and G. Preparata, Phys. Rev. Letters **22**, 975 (1969) and S. Adler and Wu-Ki Tung, *ibid.* **22**, 978 (1969) have shown that the results of a Bjorken-type analysis do not coincide with those of perturbation theory in certain cases.

[10] P. Olesen, Phys. Rev. **175**, 2165 (1968).

[11] In Ref. 6 it is pointed out that certain other types of divergences may appear, which could be canceled by extended version of current-algebra commutation relations, such as those proposed by R. Brandt and J. D. Bjorken, Phys. Rev. **177**, 2331 (1968). This neglect is a serious shortcoming and corresponds to not keeping derivatives of δ functions, or, as the authors of Ref. 6 say, only $\partial_0 J_0$ terms.

shown that the maximally divergent terms, behaving like $(G_F\Lambda^2)^n$, do not lead to any parity violation in this model, and, furthermore, that the rather surprising feature of the $G_F\Lambda^2$ term being proportional to the symmetry-breaking part of the strong Hamiltonian carries through to higher-order terms for diagrams not involving lepton loops.

The assumptions made are, of course, rather drastic, chiefly with regard to the validity of the analysis of degree of divergence of the various integrals. The spirit of the model, as stated in I, was really only to see whether, within the framework of using cutoffs, it was possible to construct a model for which $\Lambda\sim 1/\sqrt{G_F}$ did not lead to any obvious contradiction with experiment. The results of this paper confirm and extend the conclusions of I, in providing an affirmative answer to that question.

We should add in conclusion that there are several other models in existence at present which address themselves to this question of divergences in weak interactions, either with cancellations or counterterms,[12] or extra fields,[13,14] or an indefinite metric,[15] or new analyses of the underlying field theory[16,17] abandoning altogether the underlying $V-A$ fundamental coupling,[18–21] or giving the W mesons strong interactions.[22]

APPENDIX A

In this appendix we want to show that the term $S_{n,\mathrm{III}}$ is still proportional to H_B. We assume the following transformation properties for the scalar and pseudoscalar densities:

$$[F^i(t),u^j(0)]_{t=0}=if^{ijk}u^k(0)\,, \tag{A1}$$

$$[F^i(t),v^j(0)]_{t=0}=if^{ijk}v^k(0)\,, \tag{A2}$$

$$[F_5{}^i(t),v^j(0)]_{t=0}=id^{ijk}u^k(0)\,, \tag{A3}$$

$$[F_5{}^i(t),u^j(0)]_{t=0}=-id^{ijk}v^k(0)\,. \tag{A4}$$

Hence, from Eq. (17), we know immediately the coefficients $\langle a|F^i|b\rangle$ and $\langle a|F_5{}^i|b\rangle$, where $a,b=u^k$ or v^k; e.g., from Eq. (A1), $\langle u^k|F^i|u^j\rangle=if^{ijk}$, $\langle v^k|F^i|u^j\rangle=0$. First of all, we wish to show that there is no mixing between $SU(3)$ singlet and octet. Consider the commutator of F^i with terms with only one pair of currents summed over 3 and 8:

$$[F_-{}^i,(\cdots F_-{}^3\cdots F_-{}^3)] =\cdots[F_-{}^i,F_-{}^3]\cdots F_-{}^3\cdots+\cdots F_-{}^3\cdots[F_-{}^i,F_-{}^3]\cdots.$$

All other commutators cancelled for the same reason as Eq. (21). Similarly, we have

$$\begin{aligned}\sum_{i=1}^{8}[F_-{}^i,[F_-{}^i,(\cdots F_-{}^3\cdots F_-{}^3\cdots)]]
&=\sum_{i=1}^{8}(\cdots[F_-{}^i,[F_-{}^i,F_-{}^3]]\cdots F_-{}^3\cdots\\
&\quad+\cdots F_-{}^3\cdots[F_-{}^i[F_-{}^i,F_-{}^3]]\cdots\\
&\quad+2\cdots[F_-{}^i,F_-{}^3]\cdots[F_-{}^i,F_-{}^3]\cdots)\\
&=12(\cdots F_-{}^3\cdots F_-{}^3\cdots)\\
&\quad-4\sum_{l,i=1}^{8}(f_{i3l})^2\cdots F_-{}^l\cdots F_-{}^l.\end{aligned} \tag{A5}$$

The second term follows from the fact that

$$f_{i3l}f_{i3m}\propto\delta_{lm}.$$

A similar formula follows for terms involving $F_-{}^8$ instead of $F_-{}^3$. Then we have

$$\begin{aligned}\sum_{i=1}^{8}[F_-{}^i,[F_-{}^i,\sum_{k=3,8}(\cdots F_-{}^k\cdots F_-{}^k)]]
&=12\sum_{k=3,8}(\cdots F_-{}^k\cdots F_-{}^k)\\
&\quad-4\sum_{l,i=1}^{8}[(f_{i8l})^2+(f_{i3l})^2](\cdots F_-{}^l\cdots F_-{}^l).\end{aligned} \tag{A6}$$

From explicit calculation we obtain

$$\sum_{i=1}^{8}[(f_{i8l})^2+(f_{i3l})^2]=1\quad(l=1,2,4,5,6,7).$$

We can rewrite Eq. (A6) as

$$\begin{aligned}\sum_{i=1}^{8}[F_-{}^i[F_-{}^i,\sum_{k=3,8}(\cdots F_-{}^k\cdots F_-{}^k)]]
&=16\sum_{k=3,8}(\cdots F_-{}^k\cdots F_-{}^k\cdots)\\
&\quad-4\sum_{l=1}^{8}(\cdots F_-{}^l\cdots F_-{}^l\cdots).\end{aligned} \tag{A7}$$

Let $B_{n,k}$ be the term with k pairs of indices summing over 3, 8. Equation (A7) can then be written as

$$\sum_{i=1}^{8}[F_-{}^i[F_-{}^i,B_{n,1}]]=16B_{n,1}-4B_{n,0}\,,$$

where $B_{n,0}$ is the term with all indices summing over

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$1,\cdots,8$, which is then either a Casimir operator or zero, as we have shown.

It is then clear that, for general $B_{n,k}$, we have

$$\sum_i [F_-^i[F_-^i,B_{n,k}]]=C_kB_{n,k}-C_k'B_{n,k-1}, \quad \text{(A8)}$$

where C_k and C_k' are some constants depending on k only.

The vector charge F^i is given by

$$F^i=\tfrac{1}{2}(F_+^i+F_-^i).$$

Then by using the fact that F_+^i commutes with $B_{n,k}$, we have

$$\sum_{i=1}^{8} [F^i,[F^i,B_{n,k}]]=\tfrac{1}{4}C_kB_{n,k}-\tfrac{1}{4}C_k'B_{n,k-1}. \quad \text{(A9)}$$

Since $\langle n|F^i|u_0\rangle=0=\langle u_0|F^i|n\rangle$, we get from Eq. (A9)

$$\langle u_m|F^iF^iB_{n,k}|u_0\rangle=\tfrac{1}{4}C_k\langle u_m|B_{n,k}|u_0\rangle -\tfrac{1}{4}C_k'\langle u_m|B_{n,k-1}|u_0\rangle, \quad \text{(A10a)}$$

$$\langle v_m|F^iF^iB_{n,k}|u_0\rangle=\tfrac{1}{4}C_k\langle v_m|B_{n,k}|u_0\rangle -\tfrac{1}{4}C_k'\langle v_m|B_{n,k-1}|u_0\rangle, \quad \text{(A10b)}$$

$$\langle u_0|F^iF^iB_{n,k}|u_m\rangle=\tfrac{1}{4}C_k\langle u_0|B_{n,k}|u_m\rangle -\tfrac{1}{4}C_k'\langle u_0|B_{n,k-1}|u_m\rangle, \quad \text{(A10c)}$$

$$0=\tfrac{1}{4}C_k\langle v_0|B_{n,k}|u_0\rangle -\tfrac{1}{4}C_k'\langle v_0|B_{n,k-1}|u_0\rangle, \quad \text{(A10d)}$$

where $m=1,\cdots,8$.

From the fact that

$$\langle u_m|F^iF^i|u_n\rangle=\langle v_m|F^iF^i|v_n\rangle=3\delta_{nm},$$
$$\langle u_m|F^iF^i|v_n\rangle=\langle v_m|F^iF^i|u_n\rangle=0,$$

we can deduce from Eq. (A10a)

$$\tfrac{1}{4}(C_k-3)\langle u_m|B_{n,k}|u_0\rangle=\tfrac{1}{4}C_k'\langle u_m|B_{n,k-1}|u_0\rangle. \quad \text{(A11)}$$

Equation (A11) shows that the matrix element of $B_{n,k}$ is proportional to $B_{n,k-1}$. Therefore, by induction, we have from Eq. (A10a)

$$\langle u_m|B_{n,k}|u_0\rangle \propto \langle u_m|B_{n,k-1}|u_0\rangle \propto \cdots \propto \langle u_m|B_{n,0}|u_0\rangle. \quad \text{(A12)}$$

Because $B_{n,0}$ is either a Casimir operator or zero, we have

$$\langle u_m|B_{n,0}|u_0\rangle=0, \quad m\neq 0.$$

So, from Eq. (A12), we conclude that

$$\langle u_m|B_{n,k}|u_0\rangle=0, \quad m\neq 0. \quad \text{(A13)}$$

We can apply a similar argument for (A10b)–(A10d) to get

$$\langle v_m|B_{n,k}|u_0\rangle=\langle u_0|B_{n,k}|v_m\rangle=\langle v_0|B_{n,k}|u_0\rangle=0. \quad \text{(A14)}$$

This proves that there is no mixing between singlet and octet under B_{nk}, and there is no parity violation for the u_0 term in H_B. So we have left to show that for the u_8 term the only nonvanishing coefficient will be $\langle u_8|B_{n,k}|u_8\rangle$.

By the same reasoning as Eq. (21), it is easy to see that

$$[F^3,B_{n,k}]=[F^8,B_{n,k}] =[F_5^3,B_{n,k}]=[F_5^8,B_{n,k}]=0. \quad \text{(A15)}$$

Since $\langle n|F^3|u_8\rangle=0$ for all n, from the first equation we get

$$\sum_m \langle u_l|F^3|m\rangle\langle m|B_{n,k}|u_8\rangle=0, \quad \text{(A16a)}$$

$$\sum_m \langle v_l|F^3|m\rangle\langle m|B_{n,k}|u_8\rangle=0. \quad \text{(A16b)}$$

From the value of $\langle n|F^3|m\rangle$, Eq. (A16) becomes

$$f_{3lm}\langle u_m|B_{n,k}|u_8\rangle=0, \quad \text{(A17a)}$$

$$f_{3lm}\langle v_m|B_{n,k}|u_8\rangle=0. \quad \text{(A17b)}$$

For $m\neq 0,3,8$ there exists an l such that $f_{3lm}\neq 0$; thus we have

$$\langle u_m|B_{n,k}|u_8\rangle=0 \quad \text{and} \quad \langle v_m|B_{n,k}|u_8\rangle=0 \quad \text{for } m\neq 0,3,8. \quad \text{(A18)}$$

The equations for the axial charges are somewhat more complicated. However, by using the fact that $[F_5^3, u_0-2u_8]=0$, then $\langle n|F_5^3|u_0-2u_8\rangle=0$ for all n, and we get

$$\sum_m \langle v_0|F_5^3|m\rangle\langle m|B_{n,k}|u_0-\sqrt{2}u_8\rangle=0, \quad \text{(A19a)}$$

$$\sum_m \langle u_0|F_5^3|m\rangle\langle m|B_{n,k}|u_0-\sqrt{2}u_8\rangle=0, \quad \text{(A19b)}$$

$$\sum_m \langle u_3|F_5^3|m\rangle\langle m|B_{n,k}|u_0-\sqrt{2}u_8\rangle=0, \quad \text{(A19c)}$$

$$\sum_m \langle v_3|F_5^3|m\rangle\langle m|B_{n,k}|u_0-\sqrt{2}u_8\rangle=0. \quad \text{(A19d)}$$

By substituting the values of $\langle n|F_5^3|m\rangle$, we get the following equations:

$$\sqrt{2}\langle u_3|B_{n,k}|u_8\rangle=\langle u_3|B_{n,k}|u_0\rangle,$$
$$\sqrt{2}\langle v_3|B_{n,k}|u_8\rangle=\langle v_3|B_{n,k}|u_0\rangle,$$
$$-\sqrt{2}\langle v_8|B_{n,k}|u_8\rangle+\sqrt{2}\langle u_0|B_{n,k}|u_0\rangle =2\langle u_0|B_{n,k}|u_8\rangle-\langle u_8|B_{n,k}|u_0\rangle,$$
$$-\sqrt{2}\langle v_8|B_{n,k}|u_8\rangle=\sqrt{2}\langle v_0|B_{n,k}|u_0\rangle +2\langle v_0|B_{n,k}|u_8\rangle-\langle v_8|B_{n,k}|u_0\rangle.$$

However, the terms on the right-hand side are zero by Eq. (A14), so we have

$$\langle u_3|B_{n,k}|u_8\rangle=\langle v_3|B_{n,k}|u_8\rangle=\langle v_8|B_{n,k}|u_8\rangle=0 \quad \text{(A20)}$$

and

$$\langle u_8|B_{n,k}|u_8\rangle=\langle u_0|B_{n,k}|u_0\rangle. \quad \text{(A21)}$$

Thus Eqs. (A20), (A21), and (A14) all together imply that

$$S_{n,\text{III}}\propto \epsilon_0u_0+\epsilon_8u_8.$$

APPENDIX B

Here we want to prove that there is no parity violation in the diagrams involving lepton loops. In this case, the operator considered is of the form

$$B=\sum_{i=1,2,4,5}\ \sum_{\text{all permutations}} \cdots F_-{}^i\cdots F_-{}^i\cdots.$$

We now want to show that B commutes with $F_-{}^3$, i.e.,

$$[F_-{}^3,B]=0. \tag{B1}$$

This can be illustrated as follows:

$$\begin{aligned}\cdots[F_-{}^3,F_-{}^i]\cdots F_-{}^i\cdots+\cdots F_-{}^i\cdots[F_-{}^3,F_-{}^i]\cdots\\ =2if^{312}(\cdots F_-{}^2\cdots F_-{}^1\cdots+\cdots F_-{}^1\cdots F_-{}^2\cdots)\\ \text{for } i=1\\ =2if^{321}(\cdots F_-{}^1\cdots F_-{}^2\cdots+\cdots F_-{}^2\cdots F_-{}^1\cdots)\\ \text{for } i=2.\end{aligned}$$

These two terms cancel each other when we sum over $i=1,2$. The same thing happens for $i=4,5$. Thus, Eq. (B1) is true. Similarly, we can prove that

$$[F_-{}^8,B]=0. \tag{B2}$$

It is trivial to see that B commutes with $F_-{}^0$ and all $F_+{}^i$. Then, from Eqs. (B1) and (B2), we have

$$[F_5{}^0,B]=[F_5{}^3,B]=[F_5{}^8,B]=0. \tag{B3}$$

From the first equation we get

$$\sum_m \langle u_j|F_5{}^0|m\rangle\langle m|B|u_j\rangle =\sum_m \langle u_j|B|m\rangle\langle m|F_5{}^0|u_j\rangle. \tag{B4}$$

Substituting the value of $\langle a|F_5{}^0|b\rangle$ into Eq. (B4), we have

$$\langle v_j|B|u_j\rangle=-\langle u_j|B|v_j\rangle. \tag{B5}$$

The Hermitian conjugate of each term in B is of the same form but in reverse order, because each $F_-{}^i$ is Hermitian. Since B contains all the different orderings of generators, B is Hermitian. Hence we have

$$\langle u_j|B|v_j\rangle=\langle v_j|B|u_j\rangle^*. \tag{B6}$$

But every term in B contains an even number of generators; therefore, $\langle\alpha|B|\beta\rangle$ must be real, i.e.,

$$\langle v_j|B|u_j\rangle^*=\langle v_j|B|u_j\rangle. \tag{B7}$$

Combining (B3)–(B5), we get

$$\langle v_j|B|u_j\rangle=-\langle v_j|B|u_j\rangle$$

or

$$\langle v_j|B|u_j\rangle=0 \quad\text{for } j=0,\cdots,8. \tag{B8}$$

By taking an appropriate basis, we get from the second equation of (B5),

$$\sum_m \langle u_3|F_5{}^3|m\rangle\langle m|B|u_0-\sqrt{2}u_8\rangle=0, \tag{B9a}$$

$$\sum_m \langle v_3|F_5{}^3|m\rangle\langle m|B|v_0-\sqrt{2}v_8\rangle=0, \tag{B9b}$$

$$\sum_m \langle u_0|F_5{}^3|m\rangle\langle m|B|u_0-\sqrt{2}u_8\rangle=0, \tag{B9c}$$

$$\sum_m \langle u_0|F_5{}^3|m\rangle\langle m|B|u_0\rangle=-i(\sqrt{\tfrac{2}{3}})\langle u_0|B|v_3\rangle. \tag{B9d}$$

By substituting the value of $\langle a|F_5{}^3|b\rangle$, we get from Eqs. (B7a) and (B7b),

$$\langle v_8|B|u_0\rangle-2\langle v_0|B|u_8\rangle =\sqrt{2}\langle v_8|B|u_8\rangle-\sqrt{2}\langle v_0|B|u_0\rangle, \tag{B10}$$

$$\langle u_8|B|v_0\rangle-2\langle u_0|B|v_8\rangle =\sqrt{2}\langle u_8|B|v_8\rangle-\sqrt{2}\langle u_0|B|v_0\rangle. \tag{B11}$$

The right-hand side of these equations vanishes because of (B8). So from (B10) and (B11) we obtain

$$\begin{aligned}\langle v_8|B|u_0\rangle&=2\langle v_0|B|u_8\rangle=2\langle u_8|B|v_0\rangle=4\langle u_0|B|v_8\rangle\\ &=4\langle v_8|B|u_0\rangle\Rightarrow\langle v_8|B|u_0\rangle=0\\ &=\langle v_0|B|u_8\rangle,\end{aligned} \tag{B12}$$

where we have used the facts that B is Hermitian and $\langle a|B|b\rangle$ is real. From Eq. (B7d) we get

$$\langle v_3|B|u_0\rangle=-\langle u_0|B|v_3\rangle.$$

By the same argument as Eq. (B8), we have

$$\langle v_3|B|u_0\rangle=\langle u_0|B|v_3\rangle=0. \tag{B13}$$

From Eq. (B7c), we get

$$\langle v_3|B|u_0-\sqrt{2}u_8\rangle=0\Rightarrow\langle v_3|B|u_0\rangle=\sqrt{2}\langle v_3|B|u_8\rangle,$$

but the left-hand side vanishes because of Eq. (B13); hence we obtain

$$\langle v_3|B|u_8\rangle=0. \tag{B14}$$

Equations (B8) and (B12)–(B14), together with Eq. (A18), imply that there are no v terms, because $\langle v_k|B|u_0\rangle$ and $\langle v_k|B|u_8\rangle$ all vanish.