

corresponds to $\Gamma_{\rho\pi\pi}=110$ MeV. $g_S^2/4\pi$ is determined from the observed width $\Gamma_{\eta'K_0\bar{K}_0}=80$ MeV and the formula

$$\Gamma_{\eta'K_0\bar{K}_0}=\frac{1}{6}(g_S^2/4\pi)(\chi_{K\bar{K}\eta'})^2P, \quad (23)$$

with

$$P=\frac{1}{2}(S_{\eta'}-4S_K)^{1/2}.$$

The resulting cubic equation for $g_S/\sqrt{4\pi}$ has a single real root giving $g_S^2/4\pi=9$.

From Table I, we obtain $R_{\eta'}=2.25$, in agreement with the experimental upper limit $R_{\eta'}\leq 2.3$. A choice of $g^2/4\pi>2.15$ results in an even stronger suppression of the $\pi\pi$ mode of η' . As to the remaining $S_8P_8P_8$ couplings, no precise experimental data are yet available. Qualitatively, however, in the case of π' , the predicted ratio $(\chi_{KK\pi'}/\chi_{\pi\eta\pi'})^2=2.2\times 10^2$ concurs with the fact that $K^\pm K^0$ is the only mode seen.

A priori, our model does give a rather simplified dynamical picture of the 0^+ mesons. Thus, it could be expected to offer little more than a suppression mechanism for the η' decay, and it is successful in this respect. But, in addition, we find remarkably close agreement with experiments.

However, the validity of our results is sensitive to at least two kinds of dynamical effects not included in our simple model. First, the large deviations in the coupling shifts indicate that we should consider not just the mass shifts but also the coupling-shift feedback terms, certainly in the scalar-exchange part of the model. Second, the higher-order effects of nonlinear mass differences could either significantly alter our results or leave them intact through the same compensating mechanism which saves the decuplet equal-spacing rule.¹³ The second alternative may possibly explain why the GMO rule in particular works so well in accounting for experimental facts. The study of such phenomena should be part of any future effort to improve on the present model.

ACKNOWLEDGMENT

The author is grateful to Professor K. C. Wali for suggesting this problem and for his unfailing interest and advice throughout the course of the work.

¹³ F. J. Ernst, R. L. Warnock, and K. C. Wali, Phys. Rev. **141**, B1354 (1966).

Calculation of $\text{Re}\epsilon$ for the $K^0\text{-}\bar{K}^0$ System

A. N. KAMAL* AND B. G. KENNY†

Rutherford High-Energy Laboratory, Chilton, Didcot, Berkshire, England

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An attempt is made to estimate the contribution to $\text{Re}\epsilon$ for the $K^0\text{-}\bar{K}^0$ system from the existence of CP nonconservation in the $|\Delta I|>\frac{1}{2}$ $K^0\rightarrow 2\pi$ amplitude. In doing so, we have used as input three parametrizations of the S -wave $I=2$ $\pi\text{-}\pi$ scattering phase shift δ_2 . These, in turn, were used to calculate the self-energy contribution to the complex-mass matrix for the $K^0\text{-}\bar{K}^0$ system. It is found that the contribution of the above amplitude to $\text{Re}\epsilon$, although an order of magnitude larger than previous estimates by Truong and Barshay, is smaller than experimental measurements of $\text{Re}\epsilon$ by at least a factor of 2.

I. INTRODUCTION

THE discovery¹ of $K_L^0\rightarrow\pi^+\pi^-$ established CP nonconservation. When the mode $K_L^0\rightarrow\pi^0\pi^0$ was first detected,² the ratio $\Gamma(K_L^0\rightarrow\pi^0\pi^0)/\Gamma(K_L^0\rightarrow\pi^+\pi^-)$ strongly suggested that the source of CP nonconservation was in the $|\Delta I|>\frac{1}{2}$, $K^0\rightarrow 2\pi$ mode.³ Such

a model of CP nonconservation was first put forward by Truong.⁴ Some of the consequences of this model were worked out by Truong,⁴ and subsequently by Barshay.⁵ One of the predictions of this model was that the ratio $\Gamma(K_L^0\rightarrow\pi^0\pi^0)/\Gamma(K_L^0\rightarrow\pi^+\pi^-)$ be close to 2. This was borne out by the original experiments² on $K_L^0\rightarrow\pi^0\pi^0$, but more recent experiments⁶ suggest that this ratio is somewhat less than 2, but still large enough to indicate the presence of CP nonconservation

* On leave of absence from the University of Alberta, Edmonton, Canada. Present address: University of Alberta.

† Present address: Australian National University, Canberra, Australia.

¹ J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).

² J. M. Gaillard, F. Krienan, W. Galbraith, A. Hussri, M. R. Jane, N. H. Lipman, G. Manning, T. Ratcliffe, P. Day, A. G. Parham, B. T. Payne, A. C. Sherwood, H. Faissner, and H. Reithler, Phys. Rev. Letters **18**, 20 (1967); J. W. Cronin, P. F. Kunz, W. S. Risk, and P. C. Wheeler, *ibid.* **18**, 25 (1967).

³ By this we mean that if we follow the notation of T. T. Wu and C. N. Yang [Phys. Rev. Letters **13**, 380 (1964)] and choose

the phase of K^0 so that the amplitude for $K^0\rightarrow 2\pi$ ($I=0$) is real, apart from the phase δ_0 due to the final-state interaction, then the amplitude for $K^0\rightarrow 2\pi$ ($I=2$) is complex, apart from the phase δ_2 due to the final-state interaction.

⁴ T. N. Truong, Phys. Rev. Letters **13**, 358a (1964).

⁵ S. Barshay, Phys. Rev. **149**, 1229 (1966). Note that this paper contains corrections to Ref. 4.

⁶ For example, see J. W. Cronin, Rapporteur's talk in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 281.

in the $|\Delta I| > \frac{1}{2}$, $K^0 \rightarrow 2\pi$ mode. Furthermore, Barshay's prediction for the charge asymmetry in the semileptonic decay of K_L^0 , namely,

$$\frac{[\Gamma(K_L^0 \rightarrow \pi^- l^+ \nu) - \Gamma(K_L^0 \rightarrow \pi^+ l^- \bar{\nu})]}{[\Gamma(K_L^0 \rightarrow \pi^- l^+ \nu) + \Gamma(K_L^0 \rightarrow \pi^+ l^- \bar{\nu})]},$$

was between 30 and 60 times smaller than the experimentally measured values^{7,8} assuming $\Delta S = \Delta Q$.⁹

In this paper, we suppose that a source of the CP nonconservation is in the $|\Delta I| > \frac{1}{2}$, $K^0 \rightarrow 2\pi$ mode.³ We calculate $\text{Re}\epsilon$, which is proportional to the charge asymmetry mentioned above, and find that our estimate is larger by an order of magnitude than that of Barshay. Our calculation differs in an important respect from that of Truong⁴ and Barshay.⁵ These authors make what amounts to a dynamical assumption on the π - π scattering. We do not make this rather restrictive assumption. This point is discussed later in this paper. Finally, we wish to emphasize that we are only evaluating the contribution of CP nonconservation in the $|\Delta I| > \frac{1}{2}$, $K^0 \rightarrow 2\pi$ mode.

II. SOME RELEVANT FORMALISM

We introduce the complex 2×2 mass matrix $\mathbf{M} - \frac{1}{2}i\mathbf{\Gamma}$,¹⁰ and define K_S^0 and K_L^0 as the eigenstates of this matrix with short and long lifetimes, respectively. They satisfy

$$\begin{aligned} (\mathbf{M} - \frac{1}{2}i\mathbf{\Gamma})|K_S^0\rangle &= (M_S - \frac{1}{2}i\Gamma_S)|K_S^0\rangle, \\ (\mathbf{M} - \frac{1}{2}i\mathbf{\Gamma})|K_L^0\rangle &= (M_L - \frac{1}{2}i\Gamma_L)|K_L^0\rangle, \end{aligned} \quad (1)$$

where M_S , M_L , Γ_S , and Γ_L are the masses and decay rates of the short- and long-lived neutral kaons. Using as our basis states $|K^0\rangle$ and $|\bar{K}^0\rangle$, we write

$$\begin{aligned} |K_S^0\rangle &= [2(1 + |\epsilon|^2)]^{-1/2}[(1 + \epsilon)|K^0\rangle + (1 - \epsilon)|\bar{K}^0\rangle], \\ |K_L^0\rangle &= [2(1 + |\epsilon|^2)]^{-1/2}[(1 + \epsilon)|K^0\rangle - (1 - \epsilon)|\bar{K}^0\rangle]. \end{aligned} \quad (2)$$

To a good approximation, the parameter ϵ is related to the off-diagonal matrix elements of $\mathbf{M} - \frac{1}{2}i\mathbf{\Gamma}$ in the $|K^0\rangle - |\bar{K}^0\rangle$ basis by¹⁰

$$\epsilon = (-2 \text{Im}M_{12} + i \text{Im}\Gamma_{12})/\Gamma_S(1 - i \tan\delta), \quad (3)$$

where

$$\tan\delta = 2[(M_L - M_S)/(\Gamma_S - \Gamma_L)]. \quad (4)$$

⁷ D. Dorfan, J. Enstrom, D. Raymond, M. Schwartz, S. Wojcicki, D. H. Miller, and M. Paciotti, Phys. Rev. Letters **19**, 987 (1967).

⁸ S. Bennett, D. Nygren, H. Saal, J. Steinberger, and J. Sunderland, Phys. Rev. Letters **19**, 993 (1967).

⁹ If we drop the assumption that $\Delta S = \Delta Q$, then the enhancement factor due to the presence of $\Delta S = -\Delta Q$ amplitudes for the charge asymmetry in the decays of $K_L^0 \rightarrow \pi^\pm e^\pm \nu$ is 1.06 ± 0.06 . This was reported by S. Bennett, D. Nygren, H. Saal, J. Sunderland, J. Steinberger, and K. Kleinknecht, Phys. Letters **27B**, 244 (1968). This means that we shall not be much in error if we assume for simplicity that $\Delta S = +\Delta Q$.

¹⁰ N. Byers, S. McDowell, and C. N. Yang, in *Proceedings of the Seminar on High-Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965).

Using^{11,12}

$$(M_L - M_S)/(\Gamma_S - \Gamma_L) = 0.46 \pm 0.02, \quad (5)$$

we have

$$\delta = (42.7 \pm 1.3)^\circ. \quad (6)$$

In general, M_{12} and Γ_{12} , the off-diagonal matrix elements of \mathbf{M} and $\mathbf{\Gamma}$, receive contributions from all allowed states that connect K^0 to \bar{K}^0 ; and M_{12} may receive a contribution from a direct $\Delta S = 2$ "superweak" $K^0 - \bar{K}^0$ transition.¹³ In the present paper, we attempt to calculate the contribution of 2π , $I = 2$ intermediate states to $\text{Im}M_{12}$ and $\text{Im}\Gamma_{12}$, assuming CP is not conserved in this channel, and thereby evaluate $\text{Re}\epsilon$. For simplicity, we set $\delta = 45^\circ$ so that

$$\text{Re}\epsilon = -(\text{Im}M_{12} + \frac{1}{2} \text{Im}\Gamma_{12})/\Gamma_S. \quad (7)$$

We define the usual amplitudes for K^0 decay into $I = 0$ and 2 states³

$$\begin{aligned} \langle 2\pi, I = 0 | H_W | K^0 \rangle &= A_0 e^{i\delta_0}, \\ \langle 2\pi, I = 2 | H_W | K^0 \rangle &= A_2 e^{i\phi} e^{i\delta_2}, \end{aligned} \quad (8)$$

where A_0 and A_2 are real, ϕ is the CP -nonconserving phase, and δ_0 and δ_2 are the S -wave $I = 0$ and 2 π - π scattering phase shifts. We then have

$$\Gamma_S = 2A_0^2 = \Gamma_0 \quad (9)$$

to a good approximation, and

$$\Gamma_{12} = A_2^2 e^{2i\phi}, \quad (10a)$$

so that

$$\text{Im}\Gamma_{12} = 2A_2^2 \sin\phi = \Gamma_2 \sin\phi, \quad (10b)$$

where we have assumed that ϕ is a small angle, and Γ_0 and Γ_2 are the decay rates for $K_S^0 \rightarrow 2\pi$ in the $I = 0$ and 2 states. Assuming that the CP -nonconserving phase ϕ is constant for kaons off the mass shell, we define, analogously,

$$\text{Im}M_{12} = M_2 \sin\phi. \quad (11)$$

Then

$$\text{Re}\epsilon = -[(M_2 + \frac{1}{2}\Gamma_2)/\Gamma_0] \sin\phi. \quad (12)$$

Following the approach of Barger and Kazes,¹⁴ we relate M_2 to Γ_2 by

$$2M_2/\Gamma_2 = -\text{Re}\Sigma(M^2)/\text{Im}\Sigma(M^2), \quad (13)$$

where

$$\Sigma(M^2) = \text{const} \times \int_0^\infty \frac{|F(k)|^2 k^2 dk}{(k^2 + \mu^2)^{1/2} (k^2 + \mu^2 - \frac{1}{4}M^2 - i\epsilon)}, \quad (14)$$

μ and M being the pion and kaon masses. $\Sigma(M^2)$ is the self-energy contribution to the kaon propagator (evaluated at the kaon mass) which, after angular

¹¹ C. Alff-Steinberger, W. Heuer, K. Kleinknecht, C. Rubbia, A. Scribano, J. Steinberger, M. J. Tannenbaum, and K. Tittel, Phys. Letters **21**, 595 (1966).

¹² M. Bott-Bodenhausen, X. de Bouard, D. G. Cassel, D. Dekkers, R. Felst, R. Mermod, I. Savin, P. Scharff, M. Vivargent, T. R. Willits, and K. Winter, Phys. Letters **20**, 212 (1966).

¹³ L. Wolfenstein, Phys. Rev. Letters **13**, 562 (1964).

¹⁴ V. Barger and E. Kazes, Phys. Rev. **124**, 279 (1961).

integration and integration over k_0 yields Eq. (14), assuming that the kaon and pion fields are coupled locally as in Ref. 14. $F(k)$ is the form factor describing the $\bar{K}-\pi-\pi$ vertex where the pions are in an $I=2$ state. The form-factor structure is generated by the strong interactions of the pions in an $I=2$, S -wave state. If we assume that the form factor has no zeros, and that no $\pi-\pi$ bound state exists with quantum numbers $I=2$, $J=0$, then, assuming elastic unitarity, we have¹⁴

$$F(k) = \exp\left(\frac{2k^2}{\pi} P \int_0^\infty \frac{\delta_2(k') dk'}{k'(k'^2 - k^2)}\right). \quad (15)$$

In the usual notation,¹⁰

$$\eta_{+-} = \epsilon + \epsilon', \quad \eta_{00} = \epsilon - 2\epsilon', \quad (16)$$

where

$$\epsilon' = (\sqrt{\frac{1}{2}}) i e^{i(\delta_2 - \delta_0)} \text{Im} A_2 / A_0, \quad (17a)$$

so that

$$|\epsilon'| = (\sqrt{\frac{1}{2}}) |A_2 / A_0| \sin\phi. \quad (17b)$$

Taking

$$M_2 = n\Gamma_2, \quad (18)$$

we have from Eq. (12)

$$\text{Re}\epsilon = -\left(n + \frac{1}{2}\right) (\Gamma_2 / \Gamma_0) \sin\phi. \quad (19)$$

Then

$$|\text{Re}\epsilon| = \left|n + \frac{1}{2}\right| |A_2 / A_0| |A_2 / A_0| \sin\phi \quad (20a)$$

$$= \sqrt{2} \left|n + \frac{1}{2}\right| |A_2 / A_0| |\epsilon'|. \quad (20b)$$

Now from Eq. (16), we have

$$3\epsilon' = \eta_{+-} - \eta_{00}, \quad (21)$$

so that we get

$$3|\epsilon'| \leq |\eta_{+-}| + |\eta_{00}|, \quad (22a)$$

$$\leq 3|\eta_{+-}|. \quad (22b)$$

Hence,¹

$$|\epsilon'| \leq |\eta_{+-}| \approx 2 \times 10^{-3}. \quad (23)$$

Taking¹⁵ $|A_2 / A_0| = 0.044$, and setting $|\epsilon'|$ at the upper limit of 2×10^{-3} , we have

$$|\text{Re}\epsilon| \leq \left|n + \frac{1}{2}\right| \times 1.24 \times 10^{-4}. \quad (24)$$

We may estimate an upper limit for $|\text{Re}\epsilon|$ from Eqs. (13) and (24) using

$$n = -\text{Re}\Sigma(M^2) / 2 \text{Im}\Sigma(M^2). \quad (25)$$

III. DETAILS OF CALCULATION

In the calculations of Truong⁴ and Barshay,⁵ the model of $\pi-\pi$ scattering was assumed to be such that

$$\text{Re}\Sigma(k^2) / \text{Im}\Sigma(k^2) = \cot\delta_2(k^2). \quad (26)$$

Such a condition is satisfied for only a restricted class of phase shifts.¹⁶ In particular, it is satisfied both by a resonance parametrization for $\delta_2(k^2)$ and a Chew-

Mandelstam¹⁷ type of effective-range parametrization with positive scattering length. We do not make a dynamical assumption of this nature. From what is known of δ_2 ,¹⁸ it appears to be negative and very nearly constant for a wide range of energy up to 900 MeV in the dipion center-of-mass system. It is clear then, that Eq. (26), which holds in the models of Truong⁴ and Barshay,⁵ does not apply in this case, since the right-hand side will be almost constant over a considerable energy range, while the left-hand side is, in general, a function of k^2 .

If δ_2 goes to a negative limit asymptotically, the form factor increases with energy and the integral for $\text{Re}\Sigma(M^2)$ [see Eq. (14)] diverges, and a cutoff is needed to secure a finite value of $\text{Re}\Sigma(M^2)$. We have attacked the problem of evaluating n by using different parametrizations of the phase shift δ_2 , all of which have the same negative asymptotic limit for δ_2 . Thus, a cutoff was used to obtain a finite value of $\text{Re}\Sigma(M^2)$. One might hope that a natural cutoff would be provided by the inelastic channels. The threshold for a nucleon-antinucleon state would be at 2 BeV in the total center-of-mass energy. We have chosen a cutoff of 15 pion masses corresponding to a center-of-mass momentum of approximately 2 BeV for each pion. If one assumes, as we have done in the following calculations, that the phase shift reaches an asymptotic limit of $-m\pi$, then the asymptotic behavior of $F(k)$ would be k^{2m} . Thus, the sensitivity of the result to the cutoff would increase as the asymptotic value of $|\delta_2|$ increases. For the values of m that we choose ($\frac{1}{8}$ and $\frac{1}{4}$), the results are not very sensitive to the value of the cutoff.

We have chosen three different parametrizations of δ_2 , all of which are adjusted to have the same scattering length and asymptotic behavior. In particular, we have chosen a range of scattering lengths a_2 ,

$$0.1 \leq -a_2 \mu \leq 0.5 \quad (27)$$

and

$$\delta_2 \rightarrow -\frac{1}{8}\pi \quad \text{or} \quad -\frac{1}{4}\pi, \quad (28)$$

as $k \rightarrow \infty$, for the asymptotic behavior. The three parametrizations are

$$\delta_2 = ck / (k+b), \quad (29a)$$

$$\begin{aligned} \delta_2 &= a_2 k & k < k_0 \\ &= \delta_\infty & k > k_0, \end{aligned} \quad (29b)$$

$$\delta_2 = ck(k+a) / (k^2 + b^2). \quad (29c)$$

The first two parametrizations have two parameters which are fixed uniquely by the specification of the scattering length and the asymptotic behavior. The third parametrization has an additional parameter which was fixed by demanding that, at its minimum value, δ_2 overshoot the asymptotic value of $-\frac{1}{8}\pi$ or

¹⁷ G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).

¹⁸ W. D. Walker, J. Carroll, A. Garfinkel, and B. Y. Oh, Phys. Rev. Letters **18**, 630 (1967).

¹⁵ Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).

¹⁶ T. N. Truong, Phys. Rev. Letters **17**, 1102 (1966).

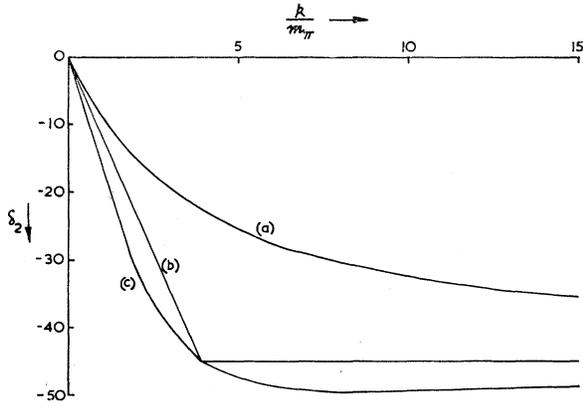


FIG. 1. The behavior with respect to pion center-of-mass momentum of the phase shifts δ_2 corresponding to the parametrizations introduced in Eqs. (29a)–(29b). All are normalized to have the same scattering length $a = -0.2\mu^{-1}$ and asymptotic value -45° .

$-\frac{1}{4}\pi$ by only 10%. This was done to ensure reasonably smooth behavior of δ_2 with respect to k . In Fig. 1, we show the behavior of the different parametrizations with respect to k for a scattering length $a_2 = -0.2\mu^{-1}$ and an asymptotic phase shift of $-\frac{1}{4}\pi$. We may note that the current-algebra prediction for a_2 is $-0.06\mu^{-1}$.¹⁹

IV. CONCLUSIONS

In Table I, we show the variation of $-n$ with respect to the scattering lengths for different values of the asymptotic phase shift and for the three different parametrizations. It can be seen that n is quite sensitive to both the low-energy behavior and the asymptotic behavior of the phase shifts. In general, $|n|$ is larger for larger values of the phase shift. We may compare the phase shifts we have used with the results of Walker *et al.*,¹⁸ where the value of δ_2 is about -20° at a center-of-mass energy of 900 MeV, corresponding to $k \approx 3m_\pi$. Since the results of Walker *et al.* suggest that δ_2 is fairly constant between 600 and 900 MeV, one might be inclined to favor the case where we have chosen an asymptotic value of $-\frac{1}{8}\pi$ for δ_2 . However, because of our ignorance of what happens at significantly higher energies, we have chosen an additional asymptotic value of $-\frac{1}{4}\pi$ for δ_2 . It is clear from Table I that at

TABLE I. Values of $-n$ for different scattering lengths and asymptotic phase-shift values, with the parametrizations [Eqs. (29a)–(29b)] of the phase shift δ_2 .

δ_2 $a_2\mu$	(a)		(b)		(c)	
	$-\frac{1}{8}\pi$	$-\frac{1}{4}\pi$	$-\frac{1}{8}\pi$	$-\frac{1}{4}\pi$	$-\frac{1}{8}\pi$	$-\frac{1}{4}\pi$
-0.1	1.26	1.32	1.65	1.67	1.88	2.13
-0.2	1.45	1.72	2.19	3.24	2.24	4.27
-0.3	1.57	2.06	2.09	4.62	2.22	5.48
-0.4	1.64	2.33	1.98	5.55	2.15	5.83
-0.5	1.69	2.55	1.94	5.76	2.09	5.76

¹⁹ S. Weinberg, Phys. Rev. Letters 17, 616 (1966).

most we may have

$$|n + \frac{1}{2}| \approx 5, \quad (30)$$

so that, from Eq. (24), we obtain

$$|\text{Re}\epsilon| \leq 0.6 \times 10^{-3}. \quad (31)$$

We may compare this with the experimental values for $\text{Re}\epsilon$ which assume that there is no $\Delta S = \Delta Q$ amplitude in the semileptonic decays of K_L^0 .⁹ It is found that

$$\begin{aligned} \text{Re}\epsilon &= (1.12 \pm 0.18) \times 10^{-3} \quad (\text{Ref. 8}) \\ \text{Re}\epsilon &= (2.0 \pm 0.7) \times 10^{-3} \quad (\text{Ref. 7}). \end{aligned} \quad (32)$$

Thus, even by choosing the most optimistic values of $|\epsilon'|$ and $|n|$, we find that our prediction for $|\text{Re}\epsilon|$ is too low by at least a factor of 2. In addition, our calculation is subject to uncertainties by having to use a cutoff, but the sensitivity to the cutoff is not too great for the asymptotic limits of δ_2 chosen.

As can be deduced from Table I, the value of n (and hence of $|\text{Re}\epsilon|$) is quite sensitive to the detailed structure of δ_2 at low energies. Although in this paper we have constrained δ_2 in parametrization (29c) not to overshoot the asymptotic value of δ_2 by more than 10% at its minimum value, this was simply a device to fix the third parameter in such a way that δ_2 would have reasonably smooth behavior with respect to k . If we drop this constraint and allow δ_2 to drop off much faster and considerably overshoot the asymptotic value before returning to it, then much larger values of $|n|$ could be obtained. However, such behavior seems unlikely,¹⁸ and we mention it only to further illustrate the sensitivity of the value of $|n|$ to the low-energy behavior of δ_2 .

If we believe the $I=2$, S -wave $\pi\text{-}\pi$ scattering length predicted by current algebra,¹⁹ our most optimistic estimate of $|\text{Re}\epsilon|$ will drop, by a factor of 3, to 0.2×10^{-3} . It may be noted that even this estimate is an order of magnitude larger than the value of $\text{Re}\epsilon = 0.3 \times 10^{-4}$ estimated by Barshay.⁵

In conclusion, although we have estimated $\text{Re}\epsilon$ to be an order of magnitude larger than did Truong and Barshay,⁵ it would seem that a CP -nonconserving $K^0 \rightarrow 2\pi$, $I=2$ amplitude by itself is not enough to give a value of $\text{Re}\epsilon$ compatible with experiment, although this amplitude may contribute substantially to $\text{Re}\epsilon$. This would suggest that other contributions to the imaginary part of the off-diagonal term in the mass matrix are important. Such contributions could arise from CP nonconservation in the $K^0 \rightarrow 3\pi$ mode, or the $\Delta S = -\Delta Q$ semileptonic modes or a possible CP -nonconserving $|\Delta S|=2$ superweak transition¹³ between K^0 and \bar{K}^0 .

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