it follows that

$$A_n=(\nu+n)\sum_{m=0}^{\leq\frac{1}{2}n}2^{\nu+n-2m}\frac{\Gamma(\nu+n-m)}{m!}b_{n-2m}. \quad \text{(B4)}$$

These equations form the basis for determining $\gamma_n(s)$, for if we define

$$B(s,y)=y^{-\alpha(s)-\frac{1}{2}}H(s,y) \quad \text{(B5)}$$

and expand

$$B(s,y)=\sum_{n=0}^{\infty}b_n(s)y^n, \quad \text{(B6)}$$

it then follows that

$$\gamma_n(s)=(-1)^n[-\alpha(s)-\tfrac{1}{2}+n]\times\sum_{m=0}^{\leq\frac{1}{2}n}2^{-\alpha(s)-\frac{1}{2}+n-2m}\frac{\Gamma(-\alpha(s)-\frac{1}{2}+n-m)}{m!}b_{n-2m}. \quad \text{(B7)}$$

The factor $(-1)^n$ is due to the presence of $I_{n+\nu}(z)$ in (B1) rather than $J_{n+\nu}(z)$.

It is possible, using a similar procedure to that of Ref. 11, to give integral representations for the signatured partial-wave amplitudes directly. We shall not do this here.

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Spin-1 Electrodynamics with an Electric Quadrupole Moment*†

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The quantum theory of the electromagnetic interactions of a charged spin-1 particle is investigated, within the framework of canonical field theory. An earlier treatment of this problem by Lee and Yang is generalized so as to include an arbitrary quadrupole moment. Some difficulties connected with the lack of relativistic covariance of the theory can be overcome, without using Lee and Yang's "ξ-limiting formalism," provided one allows for direct scattering among the charged particles.

I. INTRODUCTION

THE quantum theory of the electromagnetic interactions of a charged spin-1 particle has been treated by many authors.[1–4] Although it is well known that a spin-1 particle could have both an arbitrary magnetic moment and an electric quadrupole moment, for simplicity the additional quadrupole moment was generally excluded in the past.[5] In this paper, we will attempt a systematic study of the general case, i.e., we will consider a stable spin-1 particle which has a charge, magnetic moment, and electric quadrupole moment of arbitrary values, and its invariant couplings with the electromagnetic field. No higher moments have to be considered, since general arguments show that a particle of spin S has $2S+1$ intrinsic multipole moments.[6]

Although from past experience with the simpler cases, we expect this to be a rather complicated theory, its study is obviously desirable. For one thing, it is important to see if such a general theory can be constructed at all, within the framework of canonical field theory. A negative result would cast doubt on the possibility for a spin-1 particle to have an arbitrary quadrupole moment, at least within the framework of canonical field theory. In Sec. III, we choose a trial interaction term, which can describe a spin-1 particle with arbitrary quadrupole moment. We find that it is possible to construct a consistent theory, provided we add further interaction terms. This is similar to the treatment of Nakamura and Tzou[3] (see Sec. II) for the case of an arbitrary magnetic moment. A more physical motiva-

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[1] In this paper, we will use the canonical formalism for the spin-1 field as given in G. Wentzel, *Quantum Theory of Fields* (Wiley-Interscience, Inc., New York, 1949).

[2] A systematic study of the theory, which includes the effects of an arbitrary magnetic moment, was given by T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962). Both the canonical formalism and their ξ limiting formalism were used to derive the Feynman rules.

[3] For an alternative to the Lee-Yang theory, see M. Nakamura, Progr. Theoret. Phys. (Kyoto) 33, 279 (1965); K. H. Tzou, Nuovo Cimento 33, 286 (1964).

[4] The Feynman rules can be found in R. P. Feynman, Phys. Rev. **76**, 769 (1949). Other fundamental papers are S. Kanesawa and S. Tomonaga, Progr. Theoret. Phys. (Kyoto) 3, 101 (1948); T. Kinoshita and Y. Nambu, *ibid.* 5, 473 (1950); 5, 749 (1950); C. N. Yang and G. Feldman, Phys. Rev. **79**, 972 (1950). The latter two references use the β formalism.

[5] The general form for the $S=1$ electromagnetic vertex function on the mass shell can be found in V. Glaser and B. Jakšić, Nuovo Cimento 5, 1197 (1957). Some effects of an arbitrary quadrupole moment were treated by J. A. Young and S. A. Bludman, Phys. Rev. **131**, 2326 (1963).

[6] See, for example, L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Addison-Wesley Publishing Co., Inc., Reading, Mass., 1965), p. 262.

tion for a study of the general theory is that it might be relevant in connection with the nonet of vector mesons. Experimentally, their electric properties are not known[7] (except for the charge), so that arbitrary values for the intrinsic multipole moments may be needed in theoretical models.

It turns out that the requirement of the relativistic covariance of the theory is rather difficult to satisfy. Indeed, already for the case of an arbitrary magnetic moment, which was first introduced by Corben and Schwinger,[8] Lee and Yang[2] found that the resulting S matrix was explicitly noncovariant.[9] Lee and Yang proposed a method to obtain a covariant theory, which they called the "ξ-limiting process." The covariant theory was identified as that theory which was obtained as the formal limit $\xi \rightarrow 0^+$ of some covariant, but unphysical (for $\xi \neq 0$) theory. This continuous limit to the physical theory might be questionable, however, since it involves a discontinuous change in the number of dynamical degrees of freedom of the system. An alternative method was later proposed by Nakamura and Tzou.[3] They showed that a covariant theory could be achieved within the usual canonical formalism if an additional invariant interaction term was appropriately added. To lowest order, this interaction gives rise to the direct scattering of two charged particles. In the classical limit $\hbar \rightarrow 0$ it vanishes, in agreement with the correspondence principle. Classical electrodynamics gives no hint of this interaction, whose origin must be understood as being necessary to satisfy the requirements of relativistic quantum theory in this particular case.[10] Likewise, the generalization to an arbitrary quadrupole moment and its interaction will require additional interaction terms (counterterms) to obtain a relativistic theory.

In Sec. II, the electromagnetic interactions of a charged spin-1 particle that possesses an arbitrary magnetic moment will be reviewed. The theory will be that of Nakamura and Tzou.

In Sec. III, the theory will be extended to allow for an arbitrary quadrupole moment. The question of the relativistic covariance of the theory and the related question of counterterms will be studied.

[7] For the neutral vector mesons ρ^0, ω, and ϕ, a simple symmetry argument shows that their moments are zero. If A is any self-conjugate particle and j_μ is odd under charge conjugation, then $(A|j_\mu|A) = -(A|j_\mu|A)$, so that $(A|j_\mu|A)=0$, as well as all of the moments of the current. (I thank G. Wentzel for pointing this out to me.)

[8] H. C. Corben and J. Schwinger, Phys. Rev. **58**, 953 (1940).

[9] For the simpler case, when the Corben-Schwinger interaction is absent, there is no difficulty with the covariance of the theory. This was shown in the paper of S. Kanesawa and S. Tomonaga (Ref. 4).

[10] S. G. Brown and S. A. Bludman, Phys. Rev. **161**, 1505 (1967), have given support for the need of such a counterterm in the quantum theory. They have shown that the Dirac-Schwinger covariance condition can be unambiguously satisfied when the interaction includes the counterterms, but fails if the counterterm is absent.

II. NAKAMURA-TZOU THEORY

A. Lagrangian

We turn our attention to the electromagnetic interactions of a charged spin-1 particle, allowing first for an arbitrary magnetic moment. The theory will essentially be that of Nakamura and Tzou,[3] while the derivation of the Feynman rules will follow closely the work of Lee and Yang.[2] This will be needed to prepare the way for the general case, treated in Sec. III, where an arbitrary quadrupole moment will be introduced.

Let the charged spin-1 particles be described by the fields φ_μ, φ_μ^* and the electromagnetic field by the vector potential A_μ.[11] We will also need the four dimensional curls of these fields, denoted by $G_{\mu\nu}$, $G_{\mu\nu}^*$, and $F_{\mu\nu}$, respectively ($G_{\mu\nu} = \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu$, etc.). The Lagrangian density of the fields and their interaction is given by

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_1 + \mathcal{L}_2, \\ \mathcal{L}_1 &= -\tfrac{1}{2}\hat{G}_{\mu\nu}^*\hat{G}_{\mu\nu} - m^2 \varphi_\mu^* \varphi_\mu, \\ \mathcal{L}_2 &= -\tfrac{1}{4}\hat{F}_{\mu\nu}\hat{F}_{\mu\nu}, \end{aligned} \tag{2.1}$$

where

$$\begin{aligned} \hat{G}_{\mu\nu} &= G_{\mu\nu} - ie(A_\mu \varphi_\nu - A_\nu \varphi_\mu), \\ \hat{G}_{\mu\nu}^* &= G_{\mu\nu}^* + ie(A_\mu \varphi_\nu^* - A_\nu \varphi_\mu^*), \\ \hat{F}_{\mu\nu} &= F_{\mu\nu} + ie\kappa(\varphi_\mu^* \varphi_\nu - \varphi_\nu^* \varphi_\mu). \end{aligned} \tag{2.2}$$

Here e is the charge and $1+\kappa$ the gyromagnetic ratio of the particle.

$\mathcal{L}_1$ can be obtained from the free Lagrangian by the familiar rule $\partial_\mu \rightarrow \partial_\mu \mp ieA_\mu$; $\mathcal{L}_2$ gives the Corben-Schwinger interaction $\sim ie\kappa F_{\mu\nu}\varphi_\mu^*\varphi_\nu$ plus a direct scattering term between two charged particles. This latter term is the counterterm of the Nakamura-Tzou theory.

It is interesting to note the similar structure of the interactions written in $\mathcal{L}_1$ and $\mathcal{L}_2$. Both contain the minimal number of derivatives (namely, one) that is consistent with gauge invariance.

The field equations that follow from the Lagrangian (2.1) for the charged and electromagnetic fields are, respectively,

$$D_\mu \hat{G}_{\mu\nu} - m^2 \varphi_\nu + ie\kappa \hat{F}_{\mu\nu}\varphi_\mu = 0, \tag{2.3}$$

$$\partial_\mu \hat{F}_{\mu\nu} - ie(\hat{G}_{\mu\nu}^* \varphi_\mu - \varphi_\mu^* \hat{G}_{\mu\nu}) = 0. \tag{2.4}$$

The gauge derivative $D_\mu = \partial_\mu - ieA_\mu$ has been introduced. For other notation, see Eq. (2.2).

Taking the gauge derivative D_ν of (2.3) and summing over ν, the divergence equation follows:

$$m^2 D_\nu \varphi_\nu = \tfrac{1}{2} ie(1-\kappa)\hat{F}_{\mu\nu}\hat{G}_{\mu\nu}. \tag{2.5}$$

Use has been made of the identity $[D_\mu, D_\nu] = -ieF_{\mu\nu}$, and the equation of motion for the electromagnetic field (2.4).

[11] We use the notation $x_\mu = (x_\kappa, it)$; Greek indices run from 1 to 4; Latin indices from 1 to 3. Repeated indices are to be summed over. The asterisk is the Hermitian conjugation $\times(-)^n$, where n is the number of occurrences of the index 4. Natural units are used ($\hbar = c = 1$).

Associated with the multipole moments of the particles are the operators

$$\mu_i=\tfrac{1}{2}\epsilon_{ij\kappa}\int d^3x\ x_j j_\kappa(\mathbf{x}),$$

$$Q_{ij}=\int d^3x\ (3x_ix_j-x^2\delta_{ij})\rho(\mathbf{x}).$$

The intrinsic magnetic moment μ and quadrupole moment Q are defined as the matrix element of these operators for a positively charged particle at rest and with the spin eigenstate $S_3=+1$:

$$\mu\equiv(\mathbf{p}=0,\ S_3=1|\mu_3|\mathbf{p}=0,\ S_3=1)=(1+\kappa)e/2m,$$
$$Q\equiv(\mathbf{p}=0,\ S_3=1|Q_{33}|\mathbf{p}=0,\ S_3=1)=-2\kappa e/m^2.$$

It is seen that the charge and magnetic moment can be arbitrary, but once chosen, the quadrupole moment is fixed.

B. Hamiltonian

One can now pass to the Hamiltonian dynamics in the usual way.[1] The momenta conjugate to the fields φ_α, φ_α^*, and A_α will be called π_α, π_α^*, and P_α, respectively, and are given by

$$\begin{aligned}i\pi_\alpha&=\partial\mathfrak{L}/\partial(\partial_4\varphi_\alpha)=\hat{G}_{\alpha4}^*,\\ i\pi_\alpha^*&=\partial\mathfrak{L}/\partial(\partial_4\varphi_\alpha^*)=\hat{G}_{\alpha4},\\ iP_\alpha&=\partial\mathfrak{L}/\partial(\partial_4A_\alpha)=\hat{F}_{\alpha4}.\end{aligned}\tag{2.6}$$

$\pi_4=\pi_4^*=0$ reflects the fact that the spin-1 particle has three dynamical degrees of freedom. Their conjugate fields φ_4 and φ_4^* can be expressed as functions of the independent variables by setting $\nu=4$ in Eq. (2.3) and using (2.6). The so-called constraint equations then follow:

$$\begin{aligned}m^2\varphi_4&=i\mathbf{D}\cdot\boldsymbol{\pi}^*-e\kappa\mathbf{P}\cdot\boldsymbol{\varphi},\\ m^2\varphi_4^*&=i\mathbf{D}^*\cdot\boldsymbol{\pi}+e\kappa\mathbf{P}\cdot\boldsymbol{\varphi}^*.\end{aligned}\tag{2.7}$$

If the counterterm $\sim\kappa^2$ had not been included in the Lagrangian, the constraint equations would take a rather complicated form. This can be seen in the paper of Lee and Yang[2] [see their Eq. (A25), Appendix B]. In particular, the dependent fields φ_4, φ_4^* cannot be expressed as polynomial functions of the independent fields. This is rather important in connection with the covariance of the theory (see the remarks at the end of Sec. II D).

The Hamiltonian density $\mathfrak{H}$ can then be expressed as a function of only independent variables, substituting for φ_4, $\partial\boldsymbol{\varphi}/\partial t$, etc., Eqs. (2.6) and (2.7). The interaction part is identified as $\mathfrak{H}-\mathfrak{H}\ (e=0)$.

C. Interaction Representation

We pass to the interaction representation to obtain the Feynman rules. In this representation, the fields satisfy the free-field equation of motion [see Eqs. (2.3), (2.4), (2.6), and (2.7) with $e=0$]. In terms of the free fields, the interaction Hamiltonian $\mathfrak{H}_I$ is given by $\mathfrak{H}_I=-\mathfrak{L}_I+\mathfrak{N}$, where

$$\begin{aligned}\mathfrak{L}_I=\mathfrak{L}-\mathfrak{L}(e=0)&=ieA_\mu(G_{\mu\nu}^*\varphi_\nu-\varphi_\nu^*G_{\mu\nu})\\ &\quad-\tfrac{1}{2}e^2(A_\mu\varphi_\nu^*-A_\nu\varphi_\mu^*)(A_\mu\varphi_\nu-A_\nu\varphi_\mu)\\ &\quad-ie\kappa F_{\mu\nu}\varphi_\mu^*\varphi_\nu+\tfrac{1}{4}e^2\kappa^2(\varphi_\mu^*\varphi_\nu-\varphi_\nu^*\varphi_\mu)^2,\\ \mathfrak{N}=-e^2(A_l\varphi_4^*-A_4\varphi_l^*)&(A_l\varphi_4-A_4\varphi_l)\\ -(e^2/m^2)A_jG_{4j}^*A_kG_{4k}&-\tfrac{1}{2}e^2\kappa^2(\varphi_l^*\varphi_4-\varphi_4^*\varphi_l)^2\\ &+(e^2\kappa^2/m^2)A_kF_{4j}(G_{4k}^*\varphi_j-G_{4\kappa}\varphi_j^*).\end{aligned}\tag{2.8}$$

Besides the noncovariant terms that appear in the interaction Hamiltonian contained in the $\mathfrak{N}$ term, some of the propagators also contain noncovariant parts[12]:

$$\langle T[\varphi_\mu(x)\varphi_\nu^*(0)]\rangle_{\text{vac}}=\mathfrak{D}_{\mu\nu}(x)+(i/m^2)\delta_{\mu4}\delta_{\nu4}\delta^4(x),$$
$$\begin{aligned}&\langle T[G_{\mu\nu}(x)G_{\alpha\beta}^*(0)]\rangle_{\text{vac}}\\ &=-\partial_\mu\partial_\alpha\mathfrak{D}_{\nu\beta}-\partial_\nu\partial_\beta\mathfrak{D}_{\mu\alpha}+\partial_\nu\partial_\alpha\mathfrak{D}_{\mu\beta}\\ &\quad+\partial_\mu\partial_\beta\mathfrak{D}_{\nu\alpha}+i(\delta_{\mu4}\delta_{\alpha4}\delta_{\nu\beta}+\delta_{\nu4}\delta_{\beta4}\delta_{\mu\alpha}\\ &\qquad-\delta_{\mu4}\delta_{\beta4}\delta_{\nu\alpha}-\delta_{\nu4}\delta_{\alpha4}\delta_{\mu\beta})\delta^4(x),\end{aligned}$$
$$\begin{aligned}&\langle T[F_{\mu\nu}(x)F_{\alpha\beta}(0)]\rangle_{\text{vac}}\\ &=(-\delta_{\nu\beta}\partial_\mu\partial_\alpha-\delta_{\mu\alpha}\partial_\nu\partial_\beta+\delta_{\mu\beta}\partial_\nu\partial_\alpha\\ &\quad+\delta_{\nu\alpha}\partial_\mu\partial_\beta)\tfrac{1}{2}D_F(x)+i(\delta_{\mu4}\delta_{\alpha4}\delta_{\nu\beta}+\delta_{\nu4}\delta_{\beta4}\delta_{\mu\alpha}\\ &\qquad-\delta_{\mu4}\delta_{\beta4}\delta_{\nu\alpha}-\delta_{\nu4}\delta_{\alpha4}\delta_{\mu\beta})\delta^4(x),\end{aligned}$$

where

$$\begin{aligned}\mathfrak{D}_{\mu\nu}(x)&=[\delta_{\mu\nu}-(1/m^2)\partial_\mu\partial_\nu]\tfrac{1}{2}\Delta_F(x),\\ \Delta_F&=-i(8\pi^4)^{-1}\int d^4k\frac{1}{k^2+m^2-i\epsilon}e^{ik\cdot x},\\ D_F&=-i(8\pi^4)^{-1}\int d^4k\frac{1}{k^2}e^{ik\cdot x},\\ \delta^4(x)&=\delta^3(\mathbf{x})\delta(t).\end{aligned}\tag{2.9}$$

D. Feynman Rules and Relativistic Covariance

The Feynman rules for calculating the S matrix can now be derived using the well-known procedures of Dyson and Wick. Following Lee and Yang[2] [see their Appendices B and C], this will be done in two steps. First, the propagators and the interaction Hamiltonian are replaced by an equivalent set, equivalent with respect to the resulting S matrix (the Lee-Yang equivalence theorem). The new propagators are the covariant part of the propagators given in (2.9), and the new interaction Hamiltonian is given by

$$\mathfrak{H}_I'=-\mathfrak{L}_I+\delta H,$$

where

$$\delta H=\tfrac{1}{2}i\delta^4(0)\ln[\det(1+A)].\tag{2.10}$$

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[12] A useful identity is $T\{A(x)B(x')\}=\tfrac{1}{2}[A(x),B(x')]_++\tfrac{1}{2}\epsilon(x-x')[A(x),B(x')]_-$, for A and B boson fields. We use this formula to define the T product at the singular point $x=x'$. The propagator functions given in Eq. (2.9) can also be found in Ref. 2.

Here A is an (11×11) symmetric matrix characteristic of the system (see Appendix A). The Dyson-Wick methods can then be applied to the equivalent Hamiltonian (2.10), and the Feynman rules easily obtained.

For the Lagrangian (2.1), an explicit calculation shows that $\det(1+A)=1$, so that $\delta H=0$. The covariance of the S matrix then follows.

The condition $\det(1+A)=1$ implies that the dependent fields φ_4, $\partial\varphi/\partial t$, etc., can be written as polynomial functions of the canonical fields. Inspection of Eqs. (2.6) and (2.7) shows that this is so.

The Feynman rules for this case can be found in the papers of Nakamura and Tzou.[3] The three- and four-vertex functions are the matrix elements of $i\mathfrak{L}_I$ [see Eq. (2.8)].

The vertex functions for the general interaction are listed in Fig. 1. The above vertices can be obtained by setting $\lambda=0$ there.

III. QUADRUPOLE INTERACTION

A. Lagrangian

The foregoing theory will now be extended to include an arbitrary quadrupole moment. The Lagrangian density of the system is now

$$\mathfrak{L}=\mathfrak{L}_1+\mathfrak{L}_2+\mathfrak{L}_3. \qquad (3.1)$$

$\mathfrak{L}_1+\mathfrak{L}_2$ have already been studied in Sec. II; $\mathfrak{L}_3$ is taken to have the symmetrical form

$$\mathfrak{L}_3=(ie\lambda/m^2)\hat{F}_{\mu\nu}\hat{G}_{\mu\rho}{}^*\hat{G}_{\nu\rho}, \qquad (3.2)$$

with λ some constant [see Eq. (2.2) for notation].

The intrinsic magnetic and quadrupole moments are now

$$\mu=(1+\kappa+\lambda)e/2m, \quad Q=2(\lambda-\kappa)e/m^2.$$

Our criterion for choosing the interaction given by Eq. (3.2) is that it is the simplest one that describes an arbitrary quadrupole moment and contains no more than first derivatives of the vector fields φ_μ, $\varphi_\mu{}^*$, and A_μ. Thus we omit from our theory an interaction of the form $\hat{G}_{\mu\nu}{}^*\varphi_\lambda\partial_\lambda\hat{F}_{\mu\nu}+\text{H.c.}$, which might otherwise be chosen, since it is closer to the classical form for a quadrupole interaction. Although this means a certain loss of generality, our concern is to see if the addition of an arbitrary quadrupole moment can lead to a consistent relativistic theory.

Up to this point we have been careful to avoid using such phrases as "minimal interaction" or "anomalous moments" because of the known ambiguity of these notions for the $S=1$ case.[13] This is borne out by the fact that the interactions contained in both $\mathfrak{L}_1$ and $\mathfrak{L}_2$ seem to be equally fundamental interactions for all values of the parameter κ. However, $\mathfrak{L}_1+\mathfrak{L}_2$ can now be contrasted with $\mathfrak{L}_3$, which has a more complex structure. Also, as will be shown below, the attempts to satisfy the relativistic requirements are more difficult for $\mathfrak{L}_3$ than for $\mathfrak{L}_1+\mathfrak{L}_2$. This could lead one to conclude that in some sense $\mathfrak{L}_1+\mathfrak{L}_2$ is a minimal interaction ($\lambda=0$), while $\mathfrak{L}_3$ is nonminimal. A more precise definition of minimality, and which is in agreement with the above observations, is that by minimality we mean that the interaction Lagrangian contains the *minimal number of derivatives* consistent with the symmetries of the system. Such a definition has also been advocated by Lee.[14]

We note that the field equations for the charged and electromagnetic fields are now, respectively,

$$D_\mu\hat{G}_{\mu\nu}{}'-m^2\varphi_\nu+ie\kappa\hat{F}_{\mu\nu}{}'\varphi_\mu=0, \qquad (3.3)$$

$$\partial_\mu\hat{F}_{\mu\nu}{}'-ie(\hat{G}_{\mu\nu}{}'^*\varphi_\mu-\varphi_\mu{}^*\hat{G}_{\mu\nu}{}')=0, \qquad (3.4)$$

where the effect of $\mathfrak{L}_3$ ($\sim\lambda$) appears through the quantities

$$\begin{aligned}\hat{G}_{\mu\nu}{}'&=\hat{G}_{\mu\nu}+(ie\lambda/m^2)(\hat{F}_{\mu\rho}\hat{G}_{\nu\rho}-\hat{F}_{\nu\rho}\hat{G}_{\mu\rho}),\\ \hat{F}_{\mu\nu}{}'&=\hat{F}_{\mu\nu}-(ie\lambda/m^2)(\hat{G}_{\mu\rho}{}^*\hat{G}_{\nu\rho}-\hat{G}_{\nu\rho}{}^*\hat{G}_{\mu\rho})\end{aligned} \qquad (3.5)$$

[$\hat{G}_{\mu\nu}$ and $\hat{F}_{\mu\nu}$ are defined in (2.2)].

The conjugate momenta can be easily obtained by the replacement $G\rightarrow G'$, $G^*\rightarrow G'^*$, and $F\rightarrow F'$ in Eq. (2.6):

$$\begin{aligned}i\pi_\alpha&=\hat{G}_{\alpha4}{}'^*=\hat{G}_{\alpha4}{}^*-(ie\lambda/m^2)(\hat{F}_{\alpha\rho}\hat{G}_{4\rho}{}^*-\hat{F}_{4\rho}\hat{G}_{\alpha\rho}{}^*),\\ i\pi_\alpha{}^*&=\hat{G}_{\alpha4}{}'=\hat{G}_{\alpha4}+(ie\lambda/m^2)(\hat{F}_{\alpha\rho}\hat{G}_{4\rho}-\hat{F}_{4\rho}\hat{G}_{\alpha\rho}),\\ iP_\alpha&=\hat{F}_{\alpha4}{}'=\hat{F}_{\alpha4}-(ie\lambda/m^2)(\hat{G}_{\alpha\rho}{}^*\hat{G}_{4\rho}-\hat{G}_{4\rho}{}^*\hat{G}_{\alpha\rho}),\end{aligned} \qquad (3.6)$$

while the constraint equations (2.7) remain unchanged:

$$\begin{aligned}m^2\varphi_4&=i\mathbf{D}\cdot\boldsymbol{\pi}^*-e\kappa\mathbf{P}\cdot\boldsymbol{\varphi},\\ m^2\varphi_4{}^*&=i\mathbf{D}^*\cdot\boldsymbol{\pi}+e\kappa\mathbf{P}\cdot\boldsymbol{\varphi}.\end{aligned} \qquad (3.7)$$

The desirable feature of the constraint equation (3.7), that the dependent fields φ_4 and $\varphi_4{}^*$ can be expressed as polynomial functions of the canonical fields, is maintained here. However, as can be seen from (3.6), the same is not true for the time derivatives of the fields. This will cause difficulties connected with the relativistic covariance of the theory.

B. Interaction Representation and Relativistic Covariance of the Theory

As has already been pointed out, we expect that our theory will not be relativistically covariant. Indeed, already for the simpler interactions treated in Sec. II, we found that only electromagnetic interactions were not enough, but also direct interactions among the charged particles were needed to obtain a relativistic theory. Notwithstanding the fact that some direct interaction among the charged particles has already been included in $\mathfrak{L}_3$ through the use of $\hat{F}_{\mu\nu}$ rather than $F_{\mu\nu}$, we will see that the theory is not covariant. This lack of covariance shows up in the derivation of the Feynman

[13] See the article by G. Wentzel, in *Preludes in Theoretical Physics*, edited by A. De-Shalit, H. Feshbach, and L. VanHove (North-Holland Publishing Co., Amsterdam, 1966).

[14] T. D. Lee, Phys. Rev. 140, B967 (1965).

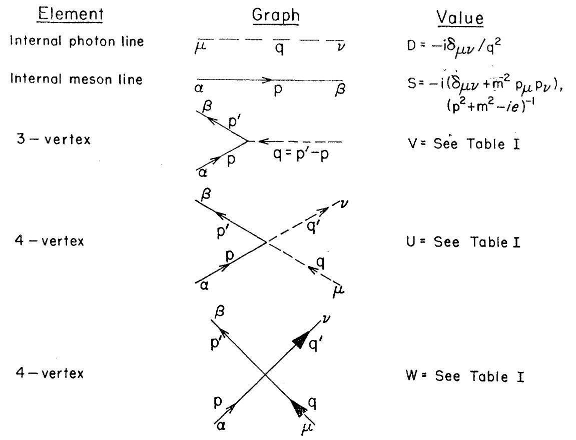

FIG. 1. Feynman rules for some of the simpler graphs associated with the interaction Lagrangian given in Eq. (3.13). Only the values of the three- and four-vertex functions are listed (see Table I). Higher-order vertex functions, as well as the vertices associated with the counterterms $\mathcal{L}_{CT}$, are not included. The rules are given in the momentum representation.

rules that correspond to $\mathcal{L}_3$. As in Secs. II B and II C, the canonical formalism will be used.

We first need the Hamiltonian of the system as a function of the canonical variables $\boldsymbol{\varphi}$, $\boldsymbol{\pi}$, etc. This can be found in the standard way.[1] We note that it will be an infinite series in the parameter $e\lambda/m^2$ because of the presence of denominators when the Eqs. (3.6) are inverted. We then pass to the interaction representation to obtain the Feynman rules. In this representation, the interaction Hamiltonian is a function of free-field variables. It, too, is an infinite series in $e\lambda/m^2$. The general form of the interaction Hamiltonian will be $-\mathcal{L}_I+\mathfrak{N}$, where all the terms that transform noncovariantly under Lorentz transformations appear in $\mathfrak{N}$. $\mathcal{L}_I$ is given simply by

$$\mathcal{L}_I=\mathcal{L}-\mathcal{L}\ (e=0)\,, \tag{3.8}$$

and is a covariant function.

As in Sec. II C, we replace the interaction Hamiltonian by an equivalent one which is to be used in conjunction with the covariant parts of the propagators for calculating the S matrix. Since our interaction, with the added term $\mathcal{L}_3$, is still contained in the class of interactions considered by Lee and Yang,[2] their equivalence theorem can be applied here. We find, then, that the equivalent Hamiltonian is given by

$$\mathcal{H}_I'=-\mathcal{L}_I+\delta H\,,$$

where $\mathcal{L}_I=\mathcal{L}-\mathcal{L}$ $(e=0)$ as before, and

$$\delta H=\tfrac{1}{2}i\delta^4(0)\ \ln[\det(1+A)].$$

A is an (11×11) matrix characteristic of the system.

In an explicit calculation, which is given in Appendix A, we find that $\det(1+A)\neq1$ and, in particular,

$$\delta H=-\tfrac{1}{2}i\delta^4(0)(2e\lambda/m^2)^2[2\hat{G}_{kl}{}^*\hat{G}_{kl}+\hat{F}_{kl}\hat{F}_{kl}]+O(e^3\lambda^3)\,. \tag{3.9}$$

A $\delta H\neq0$ is clearly an undesirable aspect of the theory. Not only does δH transform noncovariantly under Loretz transformations, but it also breaks the unitarity of the S matrix and is divergent.

An attempt to amend the theory so that δH is eliminated, at least to lowest order in $e\lambda$, and an indication of how this could be extended to higher orders will now be given.

C. Counterterms

We amend the Lagrangian $\mathcal{L}$ given in (3.1) by suitable counterterms. These counterterms will be designated collectively by $\mathcal{L}_{CT}$, and the theory will now be given by the Lagrangian $(\mathcal{L}+\mathcal{L}_{CT})$. $\mathcal{L}_{CT}$ is determined by the requirement that the resultant theory be relativistic in the sense that the δH terms of this theory are zero.

Let us first take $\mathcal{L}_{CT}$ to have the form

$$\mathcal{L}_{CT}=\mathcal{L}_a+\mathcal{L}_b+\mathcal{L}_c+\mathcal{L}_d\,, \tag{3.10}$$

$$\mathcal{L}_a=a(e\lambda/m^2)^2\hat{G}_{\mu\sigma}{}^*\hat{G}_{\nu\sigma}\hat{G}_{\mu\rho}{}^*\hat{G}_{\nu\rho}\,, \tag{3.11a}$$

$$\mathcal{L}_b=b(e\lambda/m^2)^2\hat{G}_{\mu\sigma}{}^*\hat{G}_{\nu\sigma}\hat{G}_{\nu\sigma}{}^*\hat{G}_{\mu\sigma}\,, \tag{3.11b}$$

$$\mathcal{L}_c=c(e\lambda/m^2)^2\hat{F}_{\mu\sigma}\hat{G}_{\nu\sigma}{}^*\hat{F}_{\mu\rho}\hat{G}_{\nu\rho}\,, \tag{3.11c}$$

$$\mathcal{L}_d=d(e\lambda/m^2)^2\hat{F}_{\mu\sigma}\hat{G}_{\nu\sigma}{}^*\hat{F}_{\nu\rho}\hat{G}_{\mu\rho}\,, \tag{3.11d}$$

where a, b, c, and d are constants to be determined. The calculations are contained in Appendices B and C. Briefly, the new interaction Hamiltonian is found which now includes $\mathcal{L}_{CT}$. The equivalent Hamiltonian is then constructed, with $\mathcal{L}_{CT}$ contributing a term, called δH_{CT}, to the δH already found (3.9). We then demand that to lowest order $(e^2\lambda^2)$, $\delta H_{CT}+\delta H=0$. This uniquely determines the four constants

$$a=2,\quad b=-\tfrac{3}{2},\quad c=-2,\quad d=4. \tag{3.12}$$

In the next order $O(e^3\lambda^3)$, however, $\delta H_{CT}+\delta H$ is nonvanishing, so the theory must still be judged unsatisfactory. A reasonable next step is the construction of further counterterms, i.e., we choose additional counterterms which result in $\delta H_{CT}+\delta H=0$ to $O(e^3\lambda^3)$, while not affecting the lower-order calculations. For example, for the $O(e^3\lambda^3)$ term, counterterms containing a product of five field operators would be appropriate, a typical one being $\sim(e\lambda/m^2)^3FFFG^*G$.

Concerning still higher-order terms in $\delta H_{CT}+\delta H$, we have no reason to believe that they are all zero at this stage. But by proceeding as indicated above, by adding appropriate counterterms, the theory could be made covariant to all orders in $e\lambda$. We wish to point out that if the above study of the lower-order terms is any indication, we would expect that additional counterterms are needed for each order of $\delta H_{CT}+\delta H$. Thus we expect that $\mathcal{L}_{CT}$ will have to contain products of field operators of arbitrarily high order to finally obtain a relativistic theory.

TABLE I. The values of the three-[a] and four-vertex functions associated with the interaction Lagrangian (3.13). The notation is given in Fig. 1. ($\lambda'=\lambda/m^2$.)

$$V=ie[\delta_{\alpha\beta}(p+p'-\lambda'p'\cdot qp+\lambda'p\cdot qp')_\mu-\delta_{\alpha\mu}(p-\kappa q-\lambda'p'\cdot qp+\lambda'p'\cdot pq)_\beta-\delta_{\beta\mu}(p'+\kappa q+\lambda'p\cdot qp'-\lambda'p'\cdot pq)_\alpha+\lambda'p_\mu p_\alpha' q_\beta-\lambda'p_\mu' q_\alpha p_\beta]$$

$$\begin{aligned}U=&-ie^2(2\delta_{\mu\nu}\delta_{\alpha\beta}-\delta_{\alpha\mu}\delta_{\beta\nu}-\delta_{\alpha\nu}\delta_{\beta\mu})+ie^2\lambda'\{\delta_{\mu\nu}\delta_{\alpha\beta}(q-q')\cdot(p'-p)-\delta_{\alpha\mu}\delta_{\beta\nu}(q\cdot p'+q'\cdot p)\\&+\delta_{\alpha\nu}\delta_{\beta\mu}(q\cdot p+q'\cdot p')-\delta_{\alpha\beta}[q_\nu(p'-p)_\mu-q_\mu'(p'-p)_\nu]+\delta_{\mu\nu}[p_\beta(q-q')_\alpha-p_\alpha'(q-q')_\beta]+\delta_{\beta\nu}(q_\alpha p_\mu'+q_\alpha' p_\mu+q_\alpha' p_\mu'-q_\mu' p_\alpha')\\&-\delta_{\alpha\nu}(q_\beta' p_\mu-q_\mu' p_\beta+q_\beta p_\mu+q_\beta' p_\mu')-\delta_{\mu\beta}(p_\nu' q_\alpha-p_\alpha' q_\nu+p_\nu q_\alpha+p_\nu' q_\alpha')+\delta_{\mu\alpha}(p_\nu q_\beta-p_\beta q_\nu+q_\beta p_\nu'+q_\beta' p_\nu)\}\end{aligned}$$

$$\begin{aligned}W=&ie^2\kappa^2(2\delta_{\beta\nu}\delta_{\alpha\mu}-\delta_{\beta\alpha}\delta_{\nu\mu}-\delta_{\beta\mu}\delta_{\nu\alpha})\\&-ie^2\kappa\lambda'\{\delta_{\nu\beta}\delta_{\mu\alpha}(q'+p')\cdot(q+p)-\delta_{\nu\alpha}\delta_{\mu\beta}(q'\cdot q+p'\cdot p)-\delta_{\alpha\beta}\delta_{\mu\nu}(p'\cdot q+q'\cdot p)+\delta_{\mu\nu}(q_\beta' q_\alpha-q_\alpha' q_\beta+q_\beta p_\alpha'+q_\alpha' p_\beta)\\&+\delta_{\alpha\beta}(p_\nu q_\mu'+p_\mu' q_\nu+p_\nu' p_\mu-p_\mu' p_\nu)-\delta_{\mu\alpha}(q_\beta p_\nu'+p_\nu q_\beta'+q_\nu q_\beta'+p_\beta p_\nu')-\delta_{\nu\beta}(p_\mu' q_\alpha+q_\alpha' p_\mu+q_\mu' q_\alpha+p_\alpha' p_\mu)\\&+\delta_{\mu\beta}(p_\nu' q_\alpha-p_\alpha' q_\nu+q_\nu q_\alpha'+p_\alpha' p_\nu)+\delta_{\alpha\nu}(q_\beta' p_\mu-q_\mu' p_\beta+p_\beta p_\mu'+q_\mu' q_\beta)\}\end{aligned}$$

[a] When the charged particles are *on the mass shell*, the three-vertex function V reduces to $V\rightarrow ie[\delta_{\alpha\beta}(p+p')_\mu+(1+\kappa+\lambda)(\delta_{\alpha\mu}q_\beta-\delta_{\beta\mu}q_\alpha)+(\lambda/m^2)(q_\alpha q_\beta-\frac{1}{2}q^2\delta_{\alpha\beta})(p+1p')_\mu]$. This is in agreement with the expressions written down on general invariance grounds by A. Zichichi, S. M. Berman, N. Cabibbo, and R. Gatto, Nuovo Cimento **24**, 170 (1962). See also Glaser and Jakšić (Ref. 5).

D. Feynman Rules

The vertex functions for the general interaction are listed in Fig. 1 and Table I. They are the matrix elements of

$$\begin{aligned}&i\mathcal{L}_I+i\mathcal{L}_{CT}\\&=-eA_\mu(G_{\mu\nu}{}^*\varphi_\nu-\varphi_\nu{}^*G_{\mu\nu})\\&\quad-\tfrac{1}{2}ie^2(A_\mu\varphi_\nu{}^*-A_\nu\varphi_\nu{}^*)(A_\mu\varphi_\nu-A_\nu\varphi_\mu)\\&\quad+e\kappa F_{\mu\nu}\varphi_\mu{}^*\varphi_\nu+ie^2\kappa^2(\varphi_\mu{}^*\varphi_\nu-\varphi_\nu{}^*\varphi_\mu)^2\\&\qquad-(e\lambda/m^2)\hat{F}_{\mu\nu}\hat{G}_{\mu\rho}{}^*\hat{G}_{\nu\rho}+i\mathcal{L}_{CT}.\qquad(3.13)\end{aligned}$$

IV. SUMMARY

We have demonstrated that it is possible, in principle, to construct a relativistic theory of the general electromagnetic interactions of a charged spin-1 particle. Our particular interest was the inclusion of a quadrupole moment of arbitrary value. In order to construct such a relativistic theory, however, it was necessary to include terms beyond those suggested by the classical interaction of multipole moments with the electromagnetic field. In particular, we needed both nonelectromagnetic interactions, involving products of the charged field variables only, and higher-order electromagnetic interactions. The latter interactions contained the electromagnetic field in a nonlinear way. These additional terms (counterterms) were explicitly found to lowest order $O(e^2\lambda^2)$, with their strength uniquely determined by the requirement that to this order the resulting theory be relativistic. It was then indicated how the higher-order terms could be constructed in a similar manner, and how the theory could then be made covariant to all orders in $e\lambda$. These counterterms have physical significance, and, for example, additional scattering terms are predicted which are due to them.

The problem of the renormalizability of the theory still remains. Our point of view throughout the paper has been that it is meaningful to consider the two problems of the relativistic covariance of the theory and its renormalizability separately. While the structure of our theory is rather complicated, we are at least reassured that a positive answer could be given to the problem of the relativistic covariance.

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APPENDIX A

In this appendix, we will calculate the value of $\ln[\det(1+A)]$. For the purpose of this paper, it will be sufficient to evaluate it to $O(e^3\lambda^3)$. This work will use the general results found in the Lee-Yang paper (see their Appendix C).

The symmetric matrix A_{ab} is defined by

$$A_{ab}=\partial^2\mathcal{L}_I/\partial\psi_a\partial\psi_b,\qquad(A1)$$

where $\mathcal{L}_I=\mathcal{L}-\mathcal{L}(e=0)$. The 11 Hermitian variables ψ_a ($a=1,\cdots,11$) are defined by

$$\begin{aligned}\psi_j&=iF_{4j},\\\psi_{3+j}&=(1/\sqrt{2})[G_{4j}-G_{4j}{}^*],\\\psi_{6+j}&=-(i/\sqrt{2})[G_{4j}+G_{4j}{}^*],\qquad(A2)\\\psi_{10}&=(m/\sqrt{2})[\varphi_4-\varphi_4{}^*],\\\psi_{11}&=-(im/\sqrt{2})[\varphi_4+\varphi_4{}^*],\end{aligned}$$

where $j=1, 2, 3$.

In (A1), $\mathcal{L}_I$ is to be understood as a function of the ψ_a, while all other variables like φ, φ^*, G_{ij}, etc., are to be treated as constants.

Using Eqs. (3.1), (A1), and (A2), the symmetric matrix A can be found for our theory. Written in block form, it is

$$A=\begin{vmatrix}0&a_{ij}&b_{ij}&R_i&I_i\\&0&c_{ij}&S_i&J_i\\&&0&T_i&K_i\\&&&U&V\\&&&&W\end{vmatrix}.\qquad(A3)$$

We have used a mixed notation: a, b, and c are (3×3) matrices, R, S, T, I, J, and K are (3×1) columns, and

U, V, and W are (1×1) elements. The elements of the (3×3) matrices are

$$a_{ij}=-\sqrt{2}e\lambda'(\hat{G}_{ij}+\hat{G}_{ij}^*),$$
$$b_{ij}=i\sqrt{2}e\lambda'(\hat{G}_{ij}-\hat{G}_{ij}^*),$$
$$c_{ij}=2e\lambda'\hat{F}_{ij}.$$

The elements of the (3×1) columns are

$$R_i=(e\kappa/\sqrt{2}m)(\varphi_i+\varphi_i^*)+(ie^2\lambda'/\sqrt{2}m)A_j(\hat{G}_{ij}-\hat{G}_{ij}^*),$$
$$I_i=-(ie\kappa/\sqrt{2}m)(\varphi_i-\varphi_i^*)+(e^2\lambda'/\sqrt{2}m)A_j(\hat{G}_{ij}+\hat{G}_{ij}^*),$$
$$S_i=(e^2\lambda'/m)\hat{F}_{ij}A_j+(e^2\kappa\lambda'/2m)(\varphi_j^*+\varphi_j)(\hat{G}_{ij}^*+\hat{G}_{ij}),$$
$$J_i=-(e/m)A_i+(ie^2\kappa\lambda'/2m)(\varphi_j^*-\varphi_j)(\hat{G}_{ij}^*+\hat{G}_{ij}),$$
$$T_i=(e/m)A_i+(ie^2\kappa\lambda'/2m)(\varphi_j^*+\varphi_j)(\hat{G}_{ij}^*-\hat{G}_{ij}),$$
$$K_i=(e^2\lambda'/m)A_j\hat{F}_{ij}-(e^2\kappa\lambda'/2m)(\varphi_j^*-\varphi_j)(\hat{G}_{ij}^*-\hat{G}_{ij}).$$

The (1×1) elements are

$$U=(e^2\kappa^2/2m^2)(\varphi_i+\varphi_i^*)^2-(ie^3\kappa\lambda'/m^2)A_j(\varphi_i+\varphi_i^*)(\hat{G}_{ij}^*-\hat{G}_{ij})+e^2A^2/m^2,$$
$$V=(ie^2\kappa^2/2m^2)(\varphi_i^*\varphi_i^*-\varphi_i\varphi_i)+(e^3\kappa\lambda'/m^2)A_j(\varphi_i^*G_{ij}^*+\varphi_i\hat{G}_{ij}),$$
$$W=-(e^2\kappa^2/2m^2)(\varphi_i-\varphi_i^*)^2+(ie^3\kappa\lambda'/m^2)A_j(\varphi_i^*-\varphi_i)(\hat{G}_{ij}^*+\hat{G}_{ij})+e^2A^2/m^2.$$

(i and j run from 1 to 3, and summation is understood over repeated indices). $\lambda'=\lambda/m^2$.

The value of $\ln\det(1+A)$ is needed. We first perform column and row manipulations and are able to transform $1+A$ into diagonal block form:

$$(1+A)\rightarrow\begin{vmatrix}9\times9 & 0\\ 0 & 2\times2\end{vmatrix}.$$

The (9×9) part of $(1+A)$ is not affected by this transformation, but the (2×2) part is changed. The determinant of the new (2×2) piece $=1$. We are then left with a symmetric (9×9) determinant to evaluate, i.e.,

$$\det(1+A)\rightarrow\det\begin{vmatrix}1 & a & b\\ a^T & 1 & c\\ b^T & c^T & 1\end{vmatrix}.$$

For calculational purposes, the following identity is useful:

$$\ln\det(1+X)=\operatorname{Tr}\ln(1+X)=\operatorname{Tr}(X-\tfrac{1}{2}X^2+\tfrac{1}{3}X^3-+\cdots).$$

With

$$X=\begin{vmatrix}0 & a & b\\ a^T & 0 & c\\ b^T & c^T & 0\end{vmatrix},$$

we find, to the required order, that

$$\ln\det(1+A)=-4e^2\lambda'^2(2\hat{G}_{ij}^*\hat{G}_{ij}+\hat{F}_{ij}\hat{F}_{ij})-i16e^3\lambda'^3\hat{F}_{ij}\hat{G}_{jk}^*\hat{G}_{ki}+O(e^4\lambda^4).$$

This gives the result in Eq. (3.9).

APPENDIX B

In this appendix, we will generalize the Lee-Yang equivalance theorem to include a larger class of interactions. This is necessary to treat the counterterms that were added in Sec. III C.

We take for a model Lagrangian

$$L=\tfrac{1}{2}\dot{Q}_a^2+\tfrac{1}{2}\lambda A_{ab}\dot{Q}_a\dot{Q}_b+\tfrac{1}{2}\lambda^2R_{ab}\dot{Q}_a\dot{Q}_b+\tfrac{1}{4}\lambda^2M_{abcd}\dot{Q}_a\dot{Q}_b\dot{Q}_c\dot{Q}_d+C,\quad\text{(B1)}$$

where all indices run from 1 to N. This Lagrangian represents a system of N coupled oscillators, with A, R, M, and C functions of the coordinates Q_a only. λ is the coupling constant, which we take to be small, and assume that expansions can be performed in terms of it.

If we set $A_{ab}'=\lambda A_{ab}+\lambda^2R_{ab}$, we see that (B1) differs from the Lee-Yang Lagrangian [their Eq. (A41)] in an essential way by the inclusion of a term which is of fourth order in the velocities $\dot{Q}_a$.

The canonical momenta are

$$P_a=\partial L/\partial\dot{Q}_a=(\delta_{ab}+A_{ab}')\dot{Q}_b+\lambda^2M_{abcd}\dot{Q}_b\dot{Q}_c\dot{Q}_d.\quad\text{(B2)}$$

This equation can be solved for $\dot{Q}_a$ as a power series in λ.

For the purpose of this paper, we will only need the equivalent Hamiltonian to $O(\lambda^2)$. This requires the expression for the interaction part of the Hamiltonian to the same order. The interaction Hamiltonian of Lee and Yang [their (A47)] is modified by the additional term in (B1), and to $O(\lambda^2)$ the interaction Hamiltonian in the interaction representation is given by

$$H_{\text{int}}=-\tfrac{1}{2}\left[\frac{A'}{1+A'}\right]_{ab}\dot{Q}_a\dot{Q}_b-\tfrac{1}{4}\lambda^2M_{abcd}\dot{Q}_a\dot{Q}_b\dot{Q}_c\dot{Q}_d+O(\lambda^3).\quad\text{(B3)}$$

The vacuum expectation values of the T products in the interaction representation, given by Lee and Yang,[2] are the following:

$$\langle T[Q_n(t)Q_m(0)]\rangle_{\text{vac}}=\tfrac{1}{2}\delta_{nm}s(t),$$
$$\langle T[\dot{Q}_n(t)Q_m(0)]\rangle_{\text{vac}}=\tfrac{1}{2}\delta_{nm}\dot{s}(t),\quad\text{(B4)}$$
$$\langle T[\dot{Q}_n(t)\dot{Q}_m(0)]\rangle_{\text{vac}}=-\tfrac{1}{2}\delta_{nm}\ddot{s}(t)-i\delta_{nm}\delta(t),$$

where

$$s(t)=e^{-it}\quad\text{for}\quad t\geqslant0$$
$$=e^{it}\quad\text{for}\quad t<0,$$
$$\dot{s}=ds/dt,\quad\ddot{s}=d^2s/dt^2.$$

To obtain the equivalent Hamiltonian, all Feynman diagrams are calculated, but *only* the explicit $\delta(t)$ part of the propagators [see Eq. (B4)] is used for an internal line. The equivalent Hamiltonian calculated in this way will then have the form

$$H_{\text{equiv}}=-L_{\text{int}}+\delta H.$$

δH is the term due to those Feynman diagrams in which

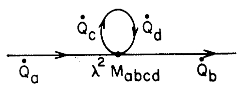

FIG. 2. Feynman diagram associated with Eq. (B6) of Appendix B.

an internal line begins and ends at the same point. More technically, it is due to the T product of operators which are at the same instant of time.

In Eq. (B3), let us set

$$H_{\text{int}}=\text{I}+\text{II},$$

where

$$\text{I}=-\tfrac{1}{2}[(A'/1+A')]_{ab}\dot{Q}_a\dot{Q}_b,$$
$$\text{II}=-\tfrac{1}{4}\lambda^2 M_{abcd}\dot{Q}_a\dot{Q}_b\dot{Q}_c\dot{Q}_d.$$

The δH due to I in the above equation has already been calculated by Lee and Yang. We note their result (with A_{ab} replaced by $A_{ab}'=\lambda A_{ab}+\lambda^2 R_{ab}$):

$$\delta H_{\text{I}}=\tfrac{1}{2}i\delta(0)\ \text{Tr}[\ln(1+A')_{ab}]. \tag{B5}$$

To order λ^2, the contribution of II to δH can be found by calculating the diagram of Fig. 2:

$$\begin{aligned}\delta H_{\text{II}}&=-(6/4)\lambda^2 M_{abcd}\dot{Q}_a\dot{Q}_b\langle T[\dot{Q}_c(0)\dot{Q}_d(0)]\rangle_{\text{vac}}\\&=\lambda^2(i/2)\delta(0)\times\tfrac{3}{2}\sum_c M_{abcc}\dot{Q}_a\dot{Q}_b,\end{aligned} \tag{B6}$$

where only the $\delta(t)$ part of the propagator was used in the last line. In order to obtain a sensible theory, we must impose the condition that

$$\sum_c M_{abcc}\dot{Q}_a\dot{Q}_b=0. \tag{B7}$$

Otherwise, contracting the remaining two operators in (B6) gives an additional

$$\delta H_{\text{II}}'\sim\lambda^2(i/2)^2\delta(0)\delta(0)\times\tfrac{3}{2}\sum_{a,c} M_{aacc}.$$

It is to forbid such terms that (B7) has been imposed.

APPENDIX C

We are now able to calculate the coefficients of the counterterms of Sec. III C, which were denoted as a, b, c, and d there. We determine them by requiring that they result in $\delta H_{\text{CT}}+\delta H=0$ to order $e^2\lambda^2$. The results of Appendix B for particle dynamics are generalized to the present case of field dynamics by identifying the $\dot{Q}_a$ as the Hermitian part of the 9 fields F_{4j}, G_{4j}, and G_{4j}^*. More specifically,

$$\begin{aligned}&\dot{Q}_j\to iF_{4j},\\&\dot{Q}_{3+j}\to(1/\sqrt{2})(G_{4j}-G_{4j}^*),\\&\dot{Q}_{6+j}\to-(i/\sqrt{2})(G_{4j}+G_{4j}^*).\end{aligned} \tag{C1}$$

From Appendix B, Eqs. (B5) and (B7), we find that the equations that must be satisfied for $\delta H_{\text{CT}}+\delta H=0$ to order λ^2 are

$$\text{Tr}A=0, \tag{C2}$$

$$\text{Tr}(A-\tfrac{1}{2}R^2)=0, \tag{C3}$$

$$\sum_c M_{abcc}\dot{Q}_a\dot{Q}_b=0. \tag{C4}$$

The explicit form for A_{ab} is given in Appendix A; only the (9×9) submatrix is needed here. Since it is traceless, Eq. (C2) is satisfied. R_{ab} and M_{abcd} can be found by comparing the counterterms Eq. (3.11) with the model Lagrangian of Appendix B, Eq. (B1), and making use of Eqs. (C1) above.

After a straightforward calculation, using the A, R, and M so found, Eq. (C3) results in the pair of equations

$$\begin{aligned}(2a+4b+c)\hat{G}_{ij}^*\hat{G}_{ij}&=-4\hat{G}_{ij}^*\hat{G}_{ij},\\c\hat{F}_{ij}\hat{F}_{ij}&=-2\hat{F}_{ij}\hat{F}_{ij},\end{aligned} \tag{C5}$$

while Eq. (C4) results in the pair of equations

$$\begin{aligned}(12a+16b+4c+2d)\hat{G}_{4j}^*\hat{G}_{4j}&=0,\\(4c+2d)\hat{F}_{4j}\hat{F}_{4j}&=0.\end{aligned} \tag{C6}$$

The solution of (C6) and (C5) is

$$a=2,\quad b=-\tfrac{3}{2},\quad c=-2,\quad d=4.$$

This gives the results quoted in Eq. (3.12).