Experimental Implications of the Bardakci-Ruegg Representations for Resonance Production*

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The experimental implications of the Bardakci-Ruegg representation for the five-particle processes $K\bar{K}\pi\pi\pi$ and $K\bar{K}K\bar{K}\pi$ are investigated in the region of various resonance productions. As in the Veneziano representations, there are many spins present in high-mass resonances. The angular momentum content of the resonances and the angular distributions of the decay products are studied as a function of momentum transfer. In a number of cases, we find that there is only a very small admixture of odd daughters. In these instances, the condition under which the odd daughters vanish is nearly equivalent to certain previously obtained mass formulas.

RECENTLY, there has been much interest in the construction of generalized Veneziano representations for the N-point function.[1] The experimental implications of the Veneziano-type four-point function have been investigated by a number of authors.[2] One general feature of these crossing-symmetric amplitudes with Regge behavior is the presence of several partial waves resonating at precisely the same energy. Although one should view this representation as a first approximation, it indicates that perhaps the ρ-ϵ interference phenomenon is the rule rather than the exception. It is our purpose in this note to focus attention on this and other questions concerning resonance production in what we believe to be a meaningful model.

In the present paper, we analyze the Bardakci-Ruegg representations for the processes $\pi K \to \pi\pi K$ and $K\bar{K} \to K\bar{K}\pi$,[3] with emphasis on the features of resonance production. Though, *a priori*, there seems little reason to expect the odd daughters[4] of a resonance to be absent in the asymptotic limit, they are not present for many cases in the above processes. Even when they are present, the odd daughters appear to be dynamically different from the even ones. The angular distributions and angular momentum composition of the resonances change with varying momentum transfer in a rather complex fashion. There are, however, certain regular features of the variation which appear to be quite general.

The five-particle processes are a function of five variables which BR take to be the five subenergies s_{ij} (where $j=i\pm1$). (See Fig. 1.) In terms of these variables, the representation for the process $\pi^0{}_1K^-{}_5 \to \pi^-{}_2\pi^+{}_3K^-{}_4$ in the configuration of Fig. 1 is

$$\begin{aligned} A=\epsilon_{\mu\nu\rho\sigma}P_1{}^\mu P_2{}^\nu P_3{}^\rho P_4{}^\sigma &\times[B_5(\alpha_{12}-1,\,\alpha_{23}-1,\,\alpha_{34}-1,\,\alpha_{45}-1,\,\alpha_{15}-1) \\ &+B_5(\alpha_{23}-1,\,\alpha_{13}-1,\,\alpha_{14}-1,\,\alpha_{45}-1,\,\alpha_{25}-1) \\ &-B_5(\alpha_{12}-1,\,\alpha_{13}-1,\,\alpha_{34}-1,\,\alpha_{45}-1,\,\alpha_{25}-1)], \\ B_5(\alpha_{12}-1,\,\alpha_{23}-1,\,\alpha_{34}-1,\,\alpha_{45}-1,\,\alpha_{15}-1) &=\int_0^1 du_1\int_0^1 du_4\, u_1{}^{-\alpha_{12}}(1-u_1)^{-\alpha_{23}}u_4{}^{-\alpha_{45}} \\ &\quad\times(1-u_4)^{-\alpha_{34}}(1-u_1u_4)^{-\alpha_{15}+\alpha_{23}+\alpha_{34}-1}, \end{aligned} \tag{1}$$

where

$$\alpha_{ij}=\alpha_{ij}(s_{ij})=\alpha_{ij}{}^0+\alpha_{ij}{}'s_{ij} \quad\text{with}\quad \alpha_{ij}{}'=1/\mathrm{GeV}^2,$$
$$\alpha_{13}{}^0=\alpha_{12}{}^0=\alpha_{23}{}^0=\alpha_{45}{}^0=\alpha_\rho{}^0=\alpha_\omega{}^0,$$
$$\alpha_{34}{}^0=\alpha_{15}{}^0=\alpha_{14}{}^0=\alpha_{25}{}^0=\alpha_{K^*}{}^0.$$

Going to a pole in the 2-3 channel at $\alpha_{23}=N$ and taking s_{15} asymptotic,[5] we have

$$A=X(1,2,3,4,5)+X(1,3,2,5,4),$$

where

$$X(1,2,3,4,5)=\epsilon_{\mu\nu\rho\sigma}P_1{}^\mu P_2{}^\nu P_3{}^\rho P_4{}^\sigma \sum_{k=0}^{N-1}(-)^k\binom{z}{k} \times\frac{\Gamma(1-\alpha_{12})\Gamma(k+1-N)}{\Gamma(2+k-N-\alpha_{12})}\Gamma(k+1-\alpha_{45})(-\alpha_{15})^{\alpha_{45}-k-1},$$
$$z=s_{15}-s_{34}. \tag{2}$$

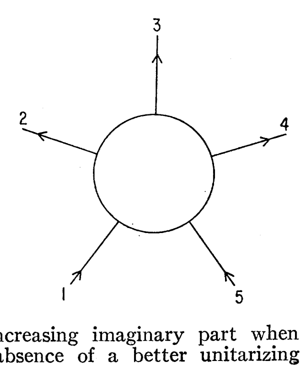

FIG. 1. Five-particle process where $s_{15}=(p_1+p_5)^2$, $s_{12}=(p_1-p_2)^2$, $s_{23}=(p_2+p_3)^2$, $s_{34}=(p_3+p_4)^2$, and $s_{45}=(p_4-p_5)^2$.

* Work supported in part by the National Science Foundation and the U. S. Atomic Energy Commission.

[1] See, for example, G. Veneziano, Nuovo Cimento **57A**, 190 (1968); K. Bardakci and H. Ruegg, Phys. Letters **28B**, 342 (1968); Chan Hong-Mo, *ibid.* **28B**, 425 (1969); Chan Hong-Mo and Tsou Sheung-Tsun, *ibid.* **28B**, 485 (1969).

[2] See, for example, D. Y. Wong, Phys. Rev. **183**, 1412 (1968); C. Lovelace, Phys. Letters **28B**, 264 (1968); J. Yellin and J. Shapiro, University of California Lawrence Radiation Laboratory Report No. UCRL-18500 (unpublished); J. Yellin, University of California Lawrence Radiation Laboratory Report No. UCRL-18637 (unpublished).

[3] K. Bardakci and H. Ruegg (hereafter BR), Phys. Letters **28B**, 671 (1969).

[4] The odd daughters of a resonance with spin N have spin $N-1$, $N-3$, $\cdots$.

[5] One gives α_{15} a linearly increasing imaginary part when taking s_{15} asymptotic in the absence of a better unitarizing procedure.

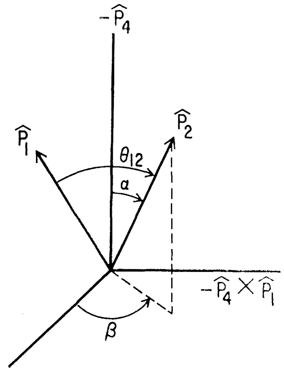

FIG. 2. Helicity frame where the z axis is given by the direction $-\hat{p}_4$, and where the y axis is perpendicular to the production plane.

Two reference frames are of use to us. We perform the angular momentum decomposition in the helicity frame, where we use the five variables s_{15}, s_{45}, s_{23}, α, and β (see Fig. 2), in terms of which one can express s_{12} and s_{34}. The angular distributions are given in the Jackson frame, where we use the five variables s_{15}, s_{45}, s_{23}, θ_{12}, and ϕ (see Fig. 3) and integrate over θ_{12} and ϕ alternatively.

Following the procedure outlined by Cook and Lee,[6] the helicity amplitudes for the production of a resonant state in the 2-3 channel with spin J, and z projection M, are given by

$$A_{JM}(s_{15},s_{45},s_{23})=\int d\cos\alpha d\beta Y_{JM}(\alpha,\beta) \times A(s_{15},s_{45},s_{23},\alpha,\beta)\,, \quad (3)$$

which becomes in the asymptotic limit $s_{15}\to\infty$,

$$A_{JM}(s_{15},s_{45},s_{23})=(1+(-)^J(-)^{\alpha_{45}})X_{JM}(s_{15},s_{45},s_{23}). \quad (4)$$

The residue at the pole is a polynomial of maximum degree N in the subenergies adjacent to the resonant channel. This ensures that angular momenta J greater than N will not be present in the resonant state.[7] The signature factor is expected on the basis of G-parity arguments applied for positive-integer values of α_{45}. There are similar signature factors for the other charge configurations we will speak of, but we shall usually ignore them and concern ourselves only with X_{JM}. X_{JM} also depends on the masses m_1, m_2, and m_3.

In Tables I(a) and I(b), we present the angular momentum composition of X for $m_1=m_2=m_3=0$, where

TABLE I. The relative probability for the production of angular momentum J for a resonance of given $\alpha_{23}=N$.

(a) $s_{45}=-0.01$

$J \backslash \bar{J}$	1	2	2.12	3.66	4.23	4.70
6						0.44
5					0.63	0
4				0.83	0	0.47
3			0.96	0	0.34	0
2		1	0	0.17	0	0.09
1	1	0	0.04	0	0.03	0
0	0	0	0	0	0	0
$N=$	1	2	3	4	5	6

(b) $s_{45}=-1.01$

$J \backslash \bar{J}$	1	2	2.74	3.30	3.75	4.16
6						0.27
5					0.43	0
4				0.65	0	0.55
3			0.87	0	0.51	0
2		1	0	0.35	0	0.18
1	1	0	0.13	0	0.06	0
0	0	0	0	0	0	0
$N=$	1	2	3	4	5	6

$\alpha_\rho{}^0=0.5$ and

$$\mathrm{Prob}(J)=\sum_{M=-J}^{+J}|X_{JM}|^2\Big/\sum_{J=0}^{N}\sum_{M=-J}^{+J}|X_{JM}|^2,$$

$$\bar{J}=\sum_{J=0}^{N} J\,\mathrm{Prob}(J)$$

for two values of s_{45}.[8] Prob(J) represents the relative probability that the resonance at $\alpha_{23}=N$ has spin J, and $\bar{J}$ is the average spin of the resonance. As expected, Prob(J) falls off for the higher values of J. It is also clear that as s_{45} becomes more negative there is a shift towards lower values of angular momentum. This is the case for all physical configurations one can study using the Bardakci-Ruegg (BR) representations. If, in the 2-3 c.m. system, one considers particles 4 and 5 as a single particle of $m^2=s_{45}$ and spin α_{45}, the crossed-channel t variable is s_{12}, and $d\cos\theta_{12}\propto ds_{12}/2k^2$, where

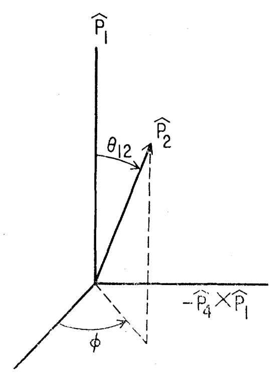

FIG. 3. Jackson frame, which is the same as the helicity frame except that the z axis is given by the direction $\hat{p}_1$.

[6] L. F. Cook and B. W. Lee, Phys. Rev. **127**, 283 (1962).

[7] Actually one should have a properly unitarized amplitude. However, as long as the exchanged masses in the 1-2 channel are reasonably large, the present polynomial is an adequate representation of the unitarized polynomial. However, the polynomial appearing at the pole in the BR representation is unable to represent a crossed channel 1-2 pole if the pole occurs at a small value of s_{12} such as the pion mass. The projection of such a pole term gives a much larger effective range than the BR representation is capable of reproducing. Thus, in such a case, one would expect substantial corrections due to proper unitarization. Approximate unitarization might proceed along the lines suggested by R. Z. Roskies [Phys. Rev. Letters **21**, 1851 (1968); **22**, 265(E) (1969)], who finds that for large energies many poles may contribute in the vicinity of a resonant energy.

[8] Note that $\bar{J}_A$ for the full amplitude A equals $\bar{J}$ despite the signature factor, if only even or odd J are present.

TABLE II. The relative probabilities for the production of angular momenta J for a resonance of given $\alpha_{23}=N$.

(a) $s_{45}=-0.01$

J \ $\bar{J}$	1	1.84	2.80	3.78	4.65	5.37
6						0.657
5					0.75	0.156
4				0.81	0.18	0.103
3			0.81	0.18	0.02	0.076
2		0.84	0.18	0.00	0.04	0.004
1	1	0.16	0.01	0.01	0.01	0.004
0	0	0	0	0	0	0
$N=$	1	2	3	4	5	6

(b) $s_{45}=-1.01$

J \ $\bar{J}$	1	1.94	2.89	3.76	4.51	5.17
6						0.585
5					0.72	0.096
4				0.83	0.11	0.241
3			0.90	0.11	0.13	0.068
2		0.94	0.09	0.05	0.04	0.005
1	1	0.06	0.01	0.01	0.00	0.005
0	0	0	0	0	0	0
$N=$	1	2	3	4	5	6

$k=\sqrt{(k_ik_f)}$.[9] Here k_i and k_f are the initial and final c.m. momenta in the 2-3 rest frame. One can then view the above shift in $\bar{J}$ as the combined result of two competing angular momentum barrier effects: (a) decreasing the effective spin of the 4-5 particle α_{45}, keeping s_{45} fixed, which causes $\bar{J}$ to decrease, and (b) increasing s_{45}, and hence the effective momentum k, keeping α_{45} fixed, which causes $\bar{J}$ to increase.[10] Were we to use somewhat unphysical values of the trajectory intercepts, the relative magnitude of the two effects can be reversed resulting in an increase of $\bar{J}$ as s_{45} becomes more negative. For very unphysical values of the intercepts, there is no clear pattern.

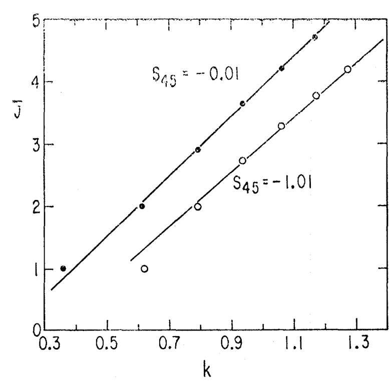

FIG. 4. $\bar{J}$ versus k for the cases given in Tables I(a) and I(b).

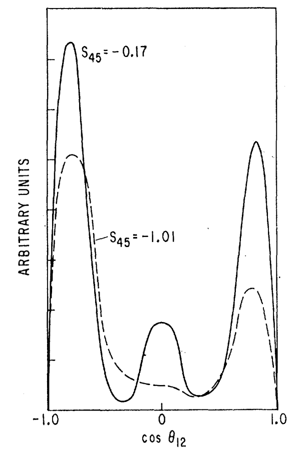

FIG. 5. Jackson-frame angular distribution in $\cos\theta_{12}$ for two values of s_{45}. The process is that of Table I, $\pi^0K^-\to\pi^-\pi^+K^-$.

From Tables I(a) and I(b) it should also be apparent that in the present case, for the value $\alpha_\rho{}^0=0.5$, there are no odd daughters with leading asymptotic behavior.[11] For the case where the 2-3 masses are equal, the condition for the vanishing of the odd daughters of a 2-3 resonance is

$$2\alpha_{12}{}^0+\alpha_{23}{}^0-\alpha_{45}{}^0-1+\alpha'(m_1{}^2+m_2{}^2+m_3{}^2)=0. \quad (5)$$

Rewritten it becomes, for the case where the same trajectory occurs in channels 1-3 and 2-3,

$$\alpha_{12}+\alpha_{23}+\alpha_{13}=1+\alpha_{45},$$

a sort of generalized Veneziano subsidiary condition. Expressing the intercepts in terms of the parent trajectory masses, measured in units of $1/\alpha'$, and demanding the vanishing of the odd daughters, we obtain from (5), for the case $\pi^0{}_1K^-{}_5\to\pi^-{}_2\pi^+{}_3K^-{}_4$,

$$3m_\rho{}^2-m_\omega{}^2-3m_\pi{}^2=1.$$

For the process of $K^-{}_1K^+{}_5\to\pi^-{}_2\pi^+{}_3\pi^0{}_4$, the condition (5) for the vanishing of odd daughters in the 2-3 channel ($\pi^-\pi^+$ channel) with

$$\alpha_{45}{}^0=\alpha_{12}{}^0=\alpha_{K^*}{}^0=1-m_{K^*}{}^2,\quad \alpha_{23}{}^0=\alpha_\rho{}^0=1-m_\rho{}^2$$

becomes

$$1-m_{K^*}{}^2-m_\rho{}^2+m_K{}^2+2m_\pi{}^2=0.$$

For the process $\pi^0{}_1\pi^-{}_5\to K^-{}_2K^+{}_3\pi^-{}_4$, the condition (5) reduces to a linear combination of the above two. For the process $K^-{}_1K^+{}_5\to K^-{}_2K^+{}_3\pi^0{}_4$ for which we use BR's $K\bar{K}\to K\bar{K}\pi$ representation with

$$\alpha_{12}{}^0=\alpha_\phi{}^0=1-m_\phi{}^2,\quad \alpha_{23}{}^0=\alpha_\rho{}^0=1-m_\rho{}^2,$$
$$\alpha_{45}{}^0=\alpha_{K^*}{}^0=1-m_{K^*}{}^2,$$

[9] The k of importance is that which occurs as the coefficient of $\cos\theta_{12}$ in $s_{12}=c-2k^2\cos\theta_{12}$. It is given by $k^2=k_ik_f=[\lambda(s_{23},s_{45},m_1{}^2)\times\lambda(s_{23},m_2{}^2,m_3{}^2)]^{1/2}/4s_{23}$, where $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc$. Notice that it is an increasing function of $(-s_{45})$ for s_{45} negative.

[10] In the BR representation, for which we have followed this procedure, these effects are quite dramatic as compared to smaller shifts in $\bar{J}$ resulting from changing s_{45} alone keeping the intercept constant. For instance, for the case of Table I, $N=6$:

$\bar{J}=4.69$ for $\alpha_{45}=0.49$, $s_{45}=-0.01$;
$\bar{J}=3.69$ for $\alpha_{45}=-0.51$, $s_{45}=-0.01$;
$\bar{J}=4.16$ for $\alpha_{45}=-0.51$, $s_{45}=-1.01$.

[11] In general, there are almost none. Note there are always odd daughter residues with nonleading asymptotic behavior $(s_{15})^{\alpha_{45}-1}$. There is never any spin zero in the present cases due to parity conservation.

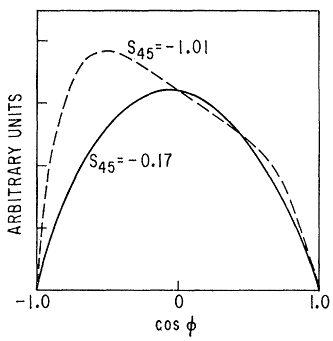

FIG. 6. Jackson-frame angular distribution in $\cos\phi$ for two values of s_{45}. The process is that of Table I.

condition (5) becomes

$$1=2m_\phi^2+m_\rho^2-m_{K^*}^2-3m_K^2.$$

In addition, their representation for this case requires ω-ρ degeneracy. Combining these equations, we have

$$m_\rho^2=m_\omega^2=\tfrac{1}{2}+\tfrac{3}{2}m_\pi^2\,, \tag{6a}$$

$$m_{K^*}^2=\tfrac{1}{2}+m_K^2+\tfrac{1}{2}m_\pi^2\,, \tag{6b}$$

$$m_\phi^2=2m_{K^*}^2-m_\rho^2. \tag{6c}$$

(6c) is the well-known Okubo mass formula, and (6a) and (6b) are the same as the formulas derived from the Veneziano representations in the soft-pion limit except for an extra $\frac{1}{2}m_\pi^2$.

Because of the success of these results, one might be tempted to take the vanishing of the odd daughters in the asymptotic limit as being fundamental. However, we need only look at the process $K^+{}_1\pi^0{}_5\rightarrow K^0{}_2\bar{K}^0{}_3K^+{}_4$ to see that this is not so. Indeed, the condition (5) becomes, using (6b) above, $M_\phi^2=4M_{K^*}^2-2m_\rho^2-\frac{1}{2}-\frac{3}{2}M\pi^2$, or, using (6c) above, $M_\phi^2\cong\frac{1}{2}$, which is clearly not satisfied. When the masses of particles 2 and 3 are not equal, then the condition for the vanishing of the odd daughters becomes quite complicated and is, in general, never satisfied. For example, we present in Tables II(a) and II(b) the angular momentum composition of X for the process $K^+{}_1\pi^-{}_5\rightarrow K^+{}_2\pi^0{}_3\pi^-{}_4$ for which there is no signature factor, i.e., $A=X$ with $\alpha_{23}{}^0=\alpha_{K^*}{}^0=0.25$, $\alpha_{12}{}^0=\alpha_{45}{}^0=\alpha_\rho{}^0=0.48$.[12] The odd daughters are clearly present though their leading asymptotic behavior becomes increasingly suppressed as s_{45} becomes more negative. It seems apparent that even when they do not vanish in the asymptotic limit, they are treated in a fashion fundamentally different from that in which the even daughters are treated. Experimentally then, it seems that by choosing the correct process one can hope to see pure resonant states for angular momenta as high as two, but that, in general, one should expect the full angular momentum content possible. This is particularly true at the present experimental energies which are not yet sufficiently large for the elimination of the odd daughters in this model to be complete. In fact, for the value $s_{15}=20$ GeV^2 the angular distributions in θ_{12} and ϕ are still fairly unsymmetric for the cases in which the odd daughters vanish asymptotically,[13] indicating the presence of substantial amounts of odd daughters.

Viewing particles 4 and 5 as a single particle, one might ideally expect that $\bar{J}$ would be a linear function of k. For the case of Tables I(a) and I(b), we plot $\bar{J}$ as a function of k in Fig. 4. It is very linear. Changing s_{45} will change the intercept of the graph but will not affect the slope very much. The effective range (i.e., the slope of the graph) turns out to be of the order of the inverse mass of the nearest singularity in the 1-2 channel.[7] The graphs for the cases of Tables II(a) and II(b) are very similar. This type of behavior seems to be a general feature of these representations.

In Figs. 5–8, we present the angular distributions for a resonance at $\alpha_{23}=3$ for the particle processes of Tables I and II, for two values of s_{45} (the smaller of which is near that value of s_{45} at which the number of events is maximum). In Figs. 7 and 8 notice that the distributions become more symmetric as s_{45} increases in magnitude corresponding to the suppression of odd daughters. As s_{45} becomes more negative, there is an increasing contribution to helicities other than $M=\pm1$ which must dominate in the forward direction.[14] Figures 5 and 6 demonstrate the asymmetry of which we spoke earlier at $s_{15}=20$, which increases as s_{45}

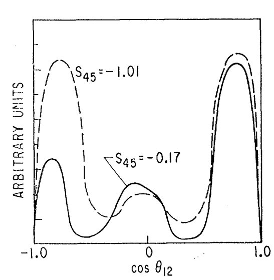

FIG. 7. Jackson-frame angular distribution in $\cos\theta_{12}$ for two values of s_{45}. The process is that of Table II, $K^+\pi^-\rightarrow K^+\pi^0\pi^-$.

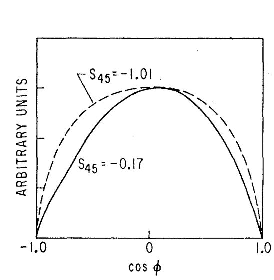

FIG. 8. Jackson-frame angular distribution in $\cos\phi$ for two values of s_{45}. The process is that of Table II.

[12] The calculation here was carried out with finite masses.

[13] Both distributions should be fully symmetric if only even or odd J are present. The factors causing odd daughter presence are of the order s_{23}/s_{15}, s_{45}/s_{15}, m_i^2/s_{15}, i.e., nonleading in s_{15}.

[14] Note that the density matrix $\rho_{M_{jk}M_{jk'}}$ defined in the Jackson frame, being given in terms of the density matrix $\rho_{MM'}$ defined in the helicity frame by $\rho_{M_{jk}M_{jk'}}=\sum_{MM'}d_{MM_{jk}}(\omega)\rho_{MM'}d_{M'M_{jk}}{}'(\omega)$, is dominated by ρ_{11}, ρ_{1-1}, and ρ_{-1-1} in the forward direction because ω (the t-channel crossing angle) becomes 0 or π, for which the d's are diagonal.

becomes larger in magnitude due to the increasing importance of factors such as s_{45}/s_{15}, which arise from the second term, which contributes to this process and gives rise to the signature factor for very large s_{15}. For greater values of α_{23}, the θ_{12} distributions take on more and more of a diffraction peak appearance, which will become less steep as s_{45} increases in magnitude because of the shift towards lower angular momenta.

In conclusion, it seems that resonance structure should, in general, be expected to be quite complicated with, certainly, even daughters present and most probably a substantial amount of odd daughters as well. The simplest, and best known, example of this is the ρ-ϵ interference in the $\pi^+\pi^-$ channel.[15] This seems to be a property of all presently existing crossing-symmetric amplitudes satisfying duality. To a certain extent, the manner in which the structure of a resonance in such an amplitude changes with the momentum transfer in the production process can be understood in terms of simple physical and intuitive arguments, involving angular momentum barrier effects. They explain, for instance, why the average angular momentum rises linearly with k even though the trajectory function, and hence the maximum angular momentum, is a linear function of s. In addition, we have seen that the BR representation for the processes considered has some unexpected features, which may reduce the complexity of the resonant structure under certain conditions. For instance, increasing the magnitude of s_{45} suppresses the odd daughters markedly, particularly at large laboratory energies. One may even find that in certain processes, the odd daughters are altogether absent at asymptotic energies, although this would seem to be the exception rather than the rule.

[15] P. B. Johnson *et al.*, Phys. Rev. **176**, 1651 (1968).

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Regge Trajectories and Nucleon Structure*

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Assuming linearly rising Regge trajectories and neglecting the ϕNN coupling constant, we consider a simple expression for the proton magnetic form factor $G_M{}^p(t)$ which has poles at positive-integer values of the degenerate trajectory functions $\alpha_\rho(t)$ and $\alpha_\omega(t)$.

In this paper, a simple model for the nucleon electromagnetic form factors is suggested and discussed. First, we must define the form factors of interest. The matrix element of the electromagnetic current operator between proton states is written

$$\begin{aligned}(4k_0'k_0)^{1/2}\langle p(k')|J_\mu(0)|p(k)\rangle &= e\bar{u}(k')[F_1{}^p(t)\gamma_\mu+F_2{}^p(t)(\mu_p-1)\\ &\qquad\times(2M)^{-1}i\sigma_{\mu\nu}q_\nu]u(k) \quad (1)\\ &= e(1-t/4M^2)^{-1}\bar{u}(k')[G_E{}^p(t)(2M)^{-1}P_\mu\\ &\qquad+G_M{}^p(t)\mu_p(8M^2)^{-1}(\boldsymbol{qP}\gamma_\mu-\gamma_\mu\boldsymbol{Pq})]u(k)\,, \quad (2)\end{aligned}$$

where $q=k-k'$, $P=k+k'$, $t=q^2$, $M=$nucleon mass, $e=$proton charge, μ_p is the total proton magnetic moment, and the normalization is

$$F_1{}^p(0)=F_2{}^p(0)=G_E{}^p(0)=G_M{}^p(0)=1\,. \quad (3)$$

The neutron electromagnetic form factors $F_1{}^n(t)$, $F_2{}^n(t)$, $G_E{}^n(t)$, and $G_M{}^n(t)$ are defined similarly. Using the algebra $\{\gamma_\mu,\gamma_\nu\}=2g_{\mu\nu}$, the Dirac equation and Eqs. (1) and (2) give

$$\mu_pG_M{}^p(t)=F_1{}^p(t)+(\mu_p-1)F_2{}^p(t)\,, \quad (4)$$

$$G_E{}^p(t)=F_1{}^p(t)+(t/4M^2)(\mu_p-1)F_2{}^p(t)\,, \quad (5)$$

so that there is the kinematic constraint at the proton-antiproton annihilation threshold

$$\mu_pG_M{}^p(4M^2)=G_E{}^p(4M^2)\,, \quad (6)$$

and similarly for the neutron.

There is no *a priori* reason to choose between the form factors of Eqs. (1) and (2) for theoretical analysis, but the experimental data indicate certain regularities for the form factors of Eq. (2). In terms of these Sachs form factors, the one-photon-exchange differential cross section for elastic electron-proton scattering in the laboratory frame is, apart from the radiative correction,

$$\frac{d\sigma}{d\Omega}=\frac{1}{t^2}\left[t^2\left(\frac{d\sigma}{d\Omega}\right)\right]_{t=0}\times\left[\left(\frac{1-t}{4M^2}\right)^{-1}\left([G_E{}^p(t)]^2-\frac{t}{4M^2}\mu_p{}^2[G_M{}^p(t)]^2\right)-\frac{t}{2M^2}\tan^2(\tfrac{1}{2}\theta_L)\,\mu_p{}^2[G_M{}^p(t)]^2\right], \quad (7)$$

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