Measurement of the Ratio of the τ' Partial Decay Rates: $\Gamma(K^- \to \pi^-\pi^0\pi^0)/\Gamma(K^+ \to \pi^+\pi^0\pi^0)$*

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A scintillation-counter–optical-spark-chamber arrangement has been used with incident 3.5-BeV/c positive and negative beams at the Argonne National Laboratory Zero-Gradient Synchrotron to study $\tau'(K^\pm \to \pi^\pm\pi^0\pi^0)$ decays. A difference between the τ'^- and τ'^+ partial decay rates would indicate the presence of a CP-violating $\Delta I \geq \frac{5}{2}$ amplitude in the weak Hamiltonian for these decays. Furthermore, in the absence of electromagnetic interactions, CPT invariance requires that $\Gamma(\tau^-)+\Gamma(\tau'^-)=\Gamma(\tau^+)+\Gamma(\tau'^+)$, where τ represents $K^\pm \to \pi^\pm\pi^+\pi^-$ decays. The result found after scanning the film and making small corrections for differences between the detection efficiencies for the two beam polarities was that $\Gamma(\tau'^-)/\Gamma(\tau'^+)=1.011 \pm 0.018$. Also, by combining our result with the results of previous experiments on $K \to 3\pi$ decays, it was found that $[\Gamma(\tau^-)+\Gamma(\tau'^-)]/[\Gamma(\tau^+)+\Gamma(\tau'^+)]-1=0.0029\pm0.0045$.

I. INTRODUCTION

ALTHOUGH equal lifetimes for K^+ and K^- mesons are required by CPT invariance[1,2] and have been verified by experiment,[3] CPT invariance does not guarantee equal $K^+ \to \pi^+\pi^0\pi^0(\tau'^+)$ and $K^- \to \pi^-\pi^0\pi^0(\tau'^-)$ partial decay rates. However, CP invariance does guarantee such an equality,[4] and hence a deviation from unity for the τ'^-/τ'^+ ratio would imply that the Hamiltonian responsible for these decays is CP-noninvariant. Theoretical models involving such CP violations have been suggested by Barrett and Truong[5] and by Kenny.[6] At the time they published, the experimental evidence permitted as much as a 10% asymmetry in the $\tau'^\pm$ rates arising from an $I=3$ ($\Delta I=\frac{5}{2}$ and/or $\Delta I=\frac{7}{2}$) CP-violating amplitude.

Specifically, one can show that if Bose statistics are applied and the decay matrix element is assumed to be linearly dependent on the energy of the odd pion,[7] then the $\tau'^\pm$ decay rates divided by phase space are given by[8]

$$\gamma(\tau'^\pm)=\frac{\Gamma(\tau'^\pm)}{\Phi}\approx\frac{|A_1|^2}{15\times 2!}\times\left[1-4\frac{|A_3|}{|A_1|}\cos(\delta_{31}\pm\phi_{31})+\frac{5(1.35)^2}{4}\left(\frac{|A_1'|}{|A_1|}\right)^2\right], \quad (1)$$

where Φ is the phase space factor, A_1 and A_3 are transition amplitudes to totally symmetric $I=1$ and $I=3$ isospin states, A_1' is the transition amplitude to the $I=1$ state of mixed isospin symmetry, δ_{31} is the difference between the $I=3$ and $I=1$ three-pion final-state strong-interaction phase shifts, and ϕ_{31} is the difference between the $I=3$ and $I=1$ CP-violating phases.

The equivalent expression for τ decays ($K^\pm \to \pi^\pm\pi^+\pi^-$)

$$\gamma(\tau^\pm)=\frac{\Gamma(\tau^\pm)}{\Phi}\approx\frac{4|A_1|^2}{15\times 2!}\times\left[1+\frac{|A_3|}{|A_1|}\cos(\delta_{31}\pm\phi_{31})+\frac{5(1.22)^2}{16}\left(\frac{|A_1'|}{|A_1|}\right)^2\right] \quad (2)$$

shows that they may be CP-noninvariant in the same manner as τ' decays. However, because of different Clebsch-Gordan coefficients in (1) and (2), the τ' decay rates are four times more sensitive to the possible CP-violating A_3 amplitude.

Earlier experimental studies[9,10] undertaken to detect a difference between the τ^- and τ^+ decay rates have found no such difference. In the experiment described herein, we have measured the ratio of the τ'^- to τ'^+ decay rates. Our result is 1.011 ± 0.018. Hence, within the accuracy obtained, we also find no evidence for a CP violation in $K\to 3\pi$ decays.

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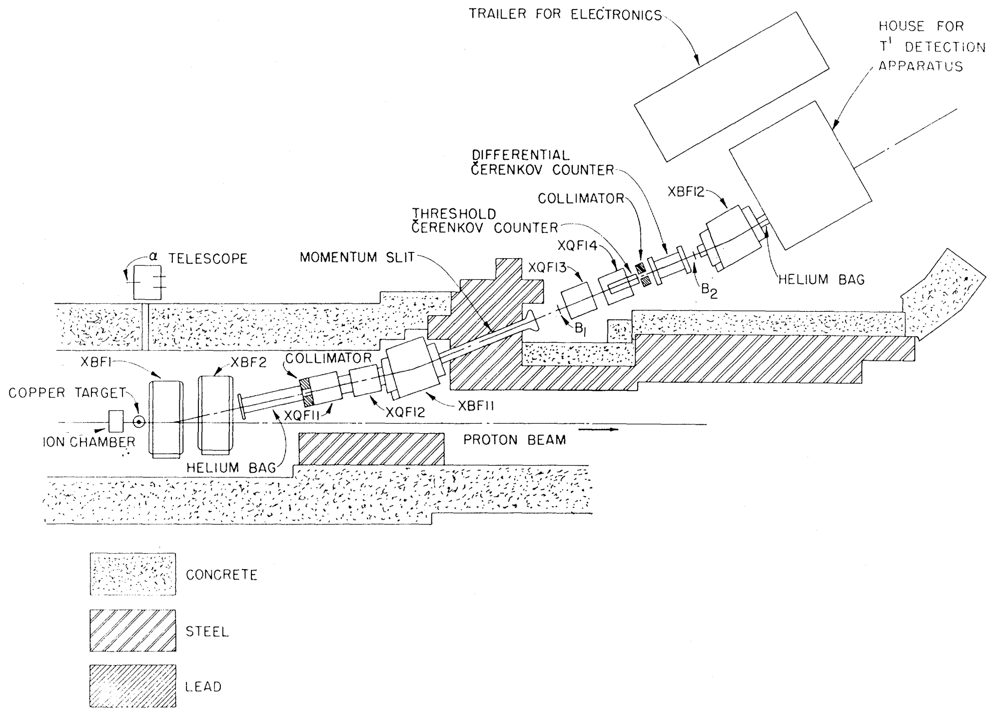

FIG. 1. Meson beam layout. XBF1, XBF2, XBF11, and XBF12 are bending magnets. XQF11, XQF12, XQF13, and XQF14 are quadrupole magnets. B_1 and B_2 are scintillation counters. The α telescope is a combination of three scintillators.

II. EXPERIMENTAL METHOD AND PROCEDURES

The determination of the τ'^-/τ'^+ ratio has been accomplished by the measurement of two quantities for each polarity of the incident beam—the number $N(K^\pm)$ of $K^\pm$ mesons incident on the τ' detection apparatus and the number $N(\tau'^\pm)$ of detected $\tau'^\pm$ decays. The K mesons were identified by Čerenkov counters and the τ' decays were identified by the tracks of their products in spark chambers. Provided equal detection efficiencies for both beam polarities are maintained, the desired ratio is given by the equation

$$\frac{\Gamma(K^-\rightarrow\pi^-\pi^0\pi^0)}{\Gamma(K^+\rightarrow\pi^+\pi^0\pi^0)}=\frac{N(\tau'^-)/N(K^-)}{N(\tau'^+)/N(K^+)}. \qquad (3)$$

The procedures that were followed to assure the validity of (3) are described below, and small corrections due to experimental asymmetries are detailed in Sec. IV.

A. Beam

The experiment was run at the Argonne Zero-Gradient Synchrotron (ZGS) with an unseparated meson beam that has been described in detail elsewhere.[11] Protons of 12-BeV energy were extracted from the main ring of the ZGS and focused onto a $0.4\times0.4\times5$-in.[3] copper target. The beam optics shown in Fig. 1 selected 3.5-BeV/c kaons produced at 2° and guided them to the τ' detection apparatus located about one kaon decay length downstream. A 0.375-in.-wide slit restricted the momentum spread to $\pm1.5\%$. The central momentum was maintained to an accuracy of 2×10^{-3} by monitoring the magnet currents every hour, by checking the magnetic fields with fixed Hall probes every 4 h, and by checks with a nuclear-magnetic-resonance probe at the beginning of each running period.

A CO_2-gas different Čerenkov counter[12] identified the kaons and vetoed lighter particles. A 20-cm-diam ring of six phototubes viewed a 10° cone of light produced by 3.5-BeV/c kaons, while a larger ring viewed light from the faster 3.5-BeV/c pions, muons, and electrons. The electronic logic used with this Čerenkov counter required coincident signals from at least four of the six phototubes in the inner ring. In addition, a veto was produced by signals from any two of the six phototubes in the outer ring.

A Freon-13 threshold Čerenkov counter[12] provided additional rejection of pions, muons, and electrons. As shown in Fig. 2, the combination of the differential and threshold counters restricted the pion and proton contaminations to less than 5×10^{-4} of the kaon beam intensity.

In addition to the two Čerenkov counters, the electronic logic made use of several plastic scintillation counters to determine $N(K)$. Two circular scintillators

[11] W. V. Hassenzahl, C. Mencuccini, E. Beier, R. Edwards, D. Herzo, L. J. Koester, and A. Wattenberg, University of Illinois Technical Report No. 160(COO-1195-96), 1967 (unpublished).

[12] E. Beier, Ph.D. thesis, University of Illinois, 1966 (unpublished).

B1 and B2 were located along the beam as shown in Fig. 1. A 3-in.-square array of four scintillators C1–C4 was centered on the beam axis between the small spark chamber and the vacuum tank, and two L-shaped veto counters L1 and L2 were joined to form a 13×24-in.[2] rectangle with a 3×3-in.[2] hole for the C1–C4 array (Fig. 3). The logic required coincident signals from B1 and B2, one and only one signal from the C1–C4 array, and no signal from L1 or L2. In this manner upstream decays and interactions were discriminated against.

B. τ' Detection Apparatus

Multigap spark chambers exhibited τ' decays

$$K^{\pm} \rightarrow \pi^{\pm} + \pi^0 + \pi^0, \quad \pi^0 \searrow 2\gamma, \quad \pi^0 \searrow 2\gamma \tag{4}$$

that occurred mainly in a 4-ft-long vacuum tank (Fig. 3). The incoming kaon track appeared in a small thin-foil spark chamber. A large thin-foil chamber following the vacuum tank showed the charged pion. The continuation of the charged pion and the development of showers from the π^0-decay photons were exhibited in four $3\times4\times1$-ft^3 aluminum-plate (shower) spark chambers representing 3.2 radiation lengths. The events were photographed by two cameras to give 90° stereo pictures.

Downstream of the vacuum tank, scintillation-counter arrays detected the τ' decays and triggered the spark chambers. The charged pion was first detected in a tenfold counter array called the pi array; it consisted of a 4-in.-square array of four scintillators D1–D4 in the region of the beam and of six larger scintillators E1–E6 that surrounded the D's to cover a 20×24-in.[2] area. Located behind the second shower spark chamber was a 3×4-ft[2] array of scintillators P11–P18 called the τ' array; this τ' array detected the continuation of the charged pion and the development of the showers from the π^0-decay photons.

Kaons that did not decay were vetoed by twofold coincidences of corresponding members of the upstream beam-veto (C1–C4) and downstream beam-veto (D1–D4) counter arrays. Decays having more than one charged product were vetoed by coincidences of two or more counters of the pi array (D1–D4, E1–E6). Beam interactions in the shower chambers were greatly reduced by a 5.5-in.-diam hole in the center of the plates of the first three chambers.

Specifically, the τ' coincidence logic required the absence of a beam-veto signal, one and only one signal from the pi array, and at least four signals from the τ' array in coincidence with the incident kaon. An additional requirement on the τ' counters was that at least one should register above, below, to the left, and to the right of the beam axis. The resultant triggerings were found to be three- and four-shower τ' events about 1/15 of the time.

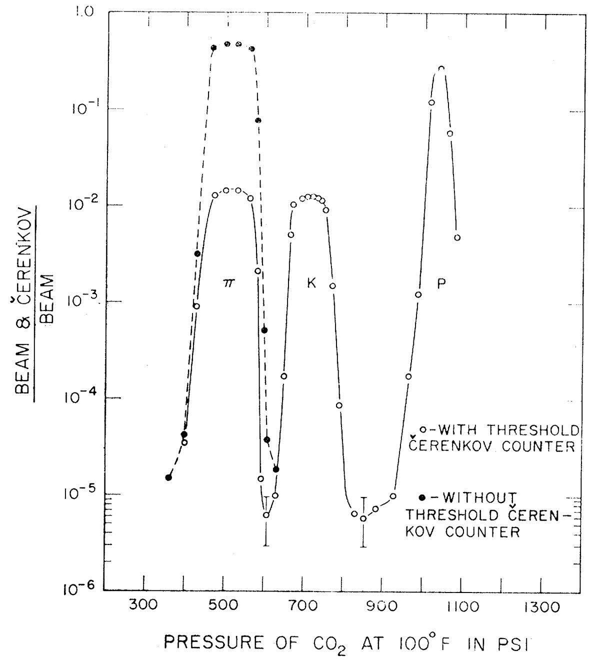

FIG. 2. Resolution curves for the differential Čerenkov counter with and without the threshold Čerenkov counter. The ordinate represents the fraction of beam particles that satisfy the digital logic requirements associated with the Čerenkov counters. The abscissa represents the pressure of the gas in the differential Čerenkov counter.

C. Experimental Procedures

The data for each run consisted of a record of the scaler reading for the number of incident kaons $N(K)$ and a pair of 100-ft rolls of film each containing about 750 frames. About 200 runs with each beam polarity were taken during three separate running periods. To avoid asymmetries which could result from drifts in efficiencies, the beam polarity was always reversed after four runs (4–5 h) of one polarity. The momentum-defining magnets were recycled after each polarity reversal to avoid hysteresis effects.

The over-all positive- and negative-beam intensities at the upstream end of the vacuum tank were restricted to 2×10^5 particles/sec and kept equal to minimize and equalize any intensity-dependent effects. The spatial distribution of the beam was monitored by coincidences of corresponding upstream (C1–C4) and downstream (D1–D4) beam-veto counters in each quadrant of the beam.

Coincidence rates involving the scintillator hodoscopes were monitored continuously to check the efficiencies of all vetoes. To minimize unwanted triggers, the vetoes in the incident kaon logic and the τ' logic were run in a deadtimeless mode. Before and after each of the three running periods, efficiency checks on each counter were made by use of standard Co^{60} sources, and during the runs they were made by recording single and coincidence counting rates.

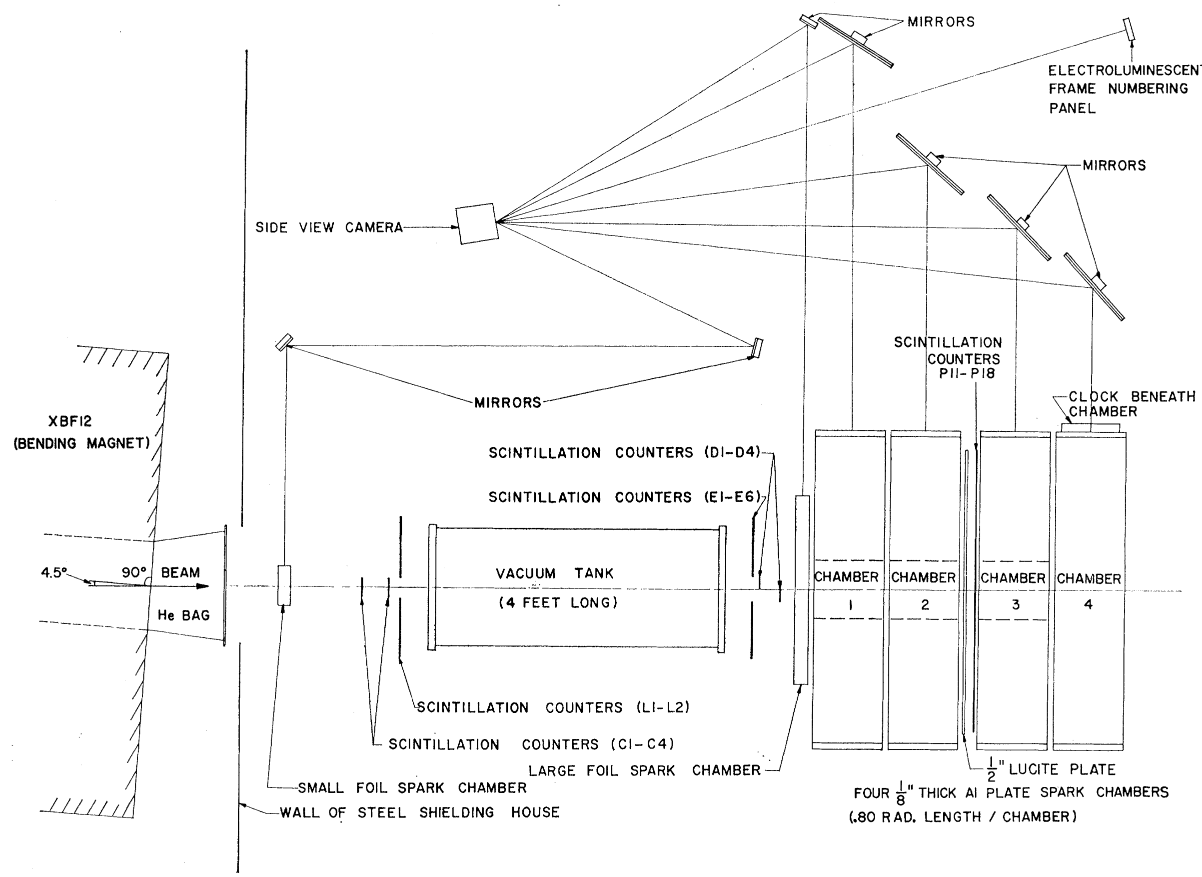

FIG. 3. Overhead view of the τ' detection apparatus.

III. DATA REDUCTION

A. Scanning

The τ' decays were identified solely by scanning, although about 20% of the film was measured to serve as a check on the scanning. An acceptable τ' event was a frame containing a nonshowering track entering the shower chambers through the large foil chamber, plus three or four showering tracks originating within the first three shower chambers (Fig. 4). The requirement

TABLE I. Scanning results.

Running period		1	2	3
Type of data			τ' decays	
No. of rolls		25	17	116
$N(\tau'^-)/N(K^-)$ / $N(\tau'^+)/N(K^+)$	1st scan	1.043±0.039	1.058±0.048	1.000±0.018
	2nd scan	0.988±0.038	1.049±0.051	0.987±0.019
	Collation	1.067±0.045	1.047±0.051	0.979±0.018
Scanning efficiencies	1st scan τ'^-	0.738±0.024	0.787±0.029	0.835±0.012
	τ'^+	0.759±0.025	0.741±0.029	0.834±0.012
	2nd scan τ'^-	0.884±0.026	0.837±0.031	0.849±0.012
	τ'^+	0.879±0.026	0.830±0.031	0.852±0.012
	Collation τ'^-	0.973±0.027	0.962±0.033	0.978±0.013
	τ'^+	0.972±0.028	0.952±0.033	0.977±0.013
Type of data			$K_{\pi 2}$ decays	
No. of rolls		24	17	38
$N(K_{\pi 2}^-)/N(K^-)$ / $N(K_{\pi 2}^+)/N(K^+)$		0.986±0.016	0.998±0.019	0.961±0.012

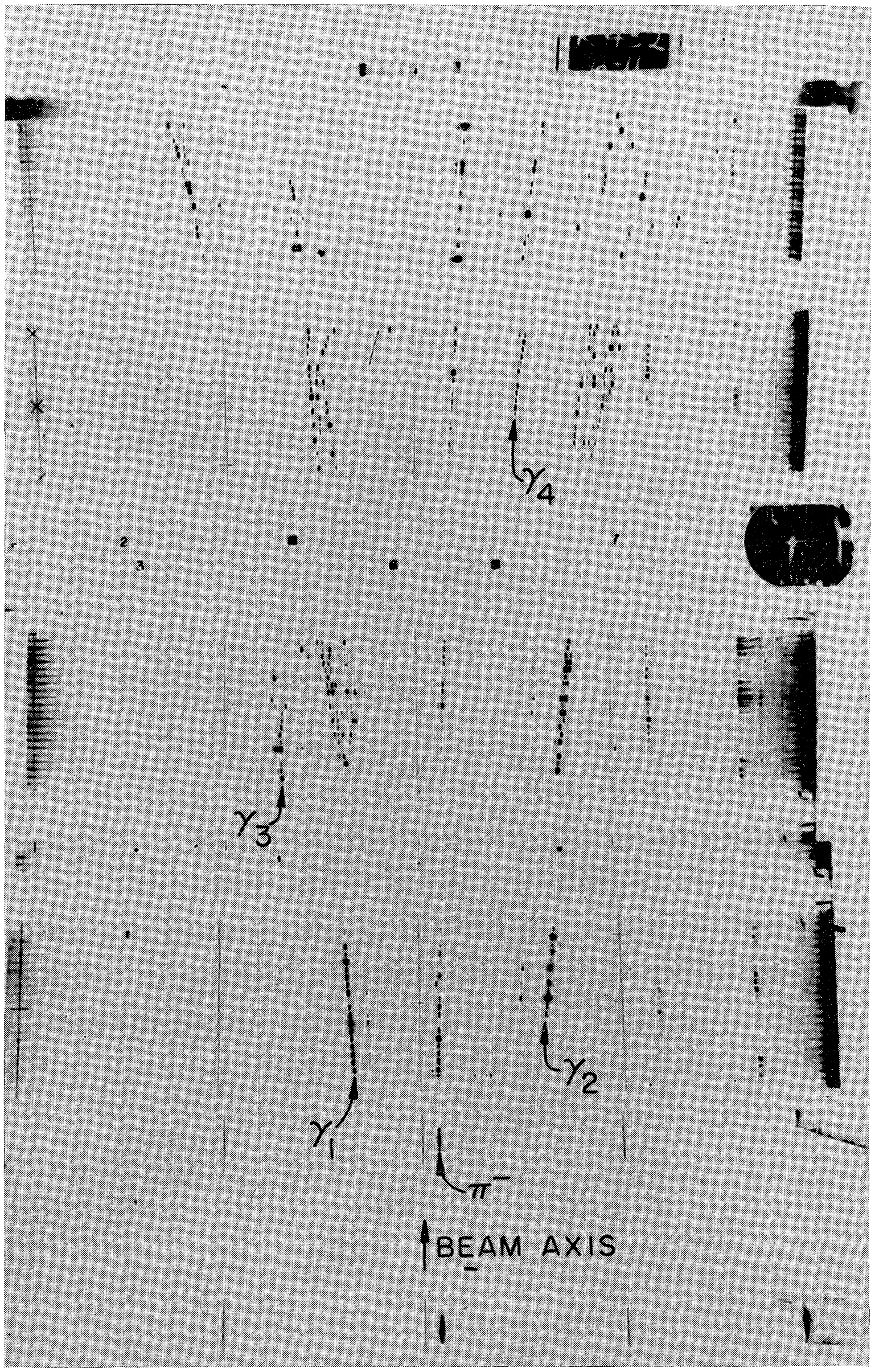

FIG. 4. Top view of a four-shower τ' event.

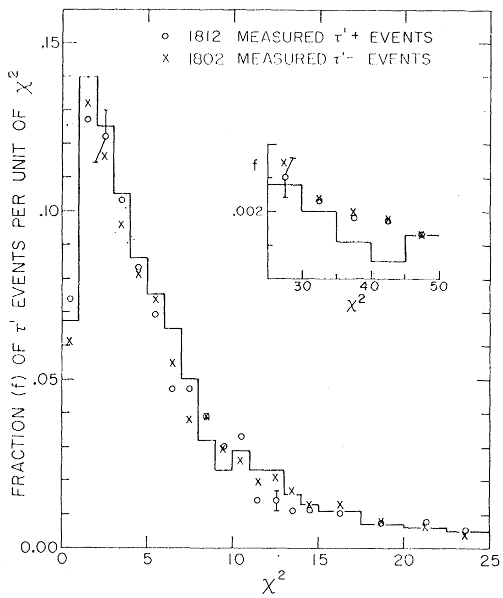

FIG. 5. χ^2 distributions of fitted vertices for measured τ'^+ and τ'^- events and for 1972 τ' decays from a Monte Carlo detection-efficiency program (solid-line histogram). χ^2 values were obtained for the Monte Carlo τ' decays by shifting the directions of the charged-pion and -photon showers according to their expected multiple scattering and then processing the results with the same vertex-fitting routine that was used for the measured events.

of three or more photon showers eliminated $K_{\pi 2}$ $(K^\pm \to \pi^\pm\pi^0)$, $K_{\mu 3}$ $(K^\pm \to \mu^\pm\pi^0\nu)$, and K_{e3} $(K^\pm \to e^\pm\pi^0\nu)$ decays from our τ' sample. Types of background events not eliminated by these criteria are discussed in Sec. IV A.

The scanners alternately scanned a roll of positive and a roll of negative τ' film to prevent differing scanning efficiencies from producing an asymmetry in the results. Two independent scans of the film were made, and a collation of the findings was carried out mainly by physicists. The term "collation" is used here to refer to an examination of all τ' candidates found in the two scans and a subsequent decision as to their acceptability based on the scanning rules. About 10 000 acceptable τ' events of each sign were found in this manner. However, about 20% of the data did not satisfy the above procedures and were therefore eliminated. The results for the good data are presented in Table I.

In addition to scanning for τ' decays, the scanners looked for $K_{\pi 2}$ decays. These decays comprised ~40% of the film because of the high probability of individual showers spreading sufficiently to trigger more than one of the τ' counters (P11–P18). The scanning rules for $K_{\pi 2}$ decays were the same as those used for identifying τ' events, except that the number of showers required was two. In about one-half of the useful film, 25 000 two-shower events were found for each beam polarity. They are estimated to contain a 15% background of $K_{\mu 3}$ decays and an 8% background of τ' decays. These data, also presented in Table I, provided a check of high statistical precision on the analysis of the τ' data because they were treated in the same manner as the τ' data and are expected to produce a $K_{\pi 2}^-/K_{\pi 2}^+$ ratio of 1.

In order to check that there were no biases in the scanning efficiencies, calculations of these efficiencies were made. Assuming that the laws of probability hold for the efficiencies, the inefficiency of the final result is the product of the inefficiencies for each of the two scans. Symbolically, this statement is representable by the equation

$$(1-N_c/N_t)=(1-N_1/N_t)(1-N_2/N_t)\,, \qquad (5)$$

where N_1 (N_2) is the number of τ' events found in the first (second) scan and confirmed by the collator, N_c is the total number of τ' events confirmed by the collator, and N_t is the total number of τ' events actually on the film. It is then straightforward to show that

$$N_c/N_t = \text{collation efficiency} = N_c(N_1+N_2-N_c)/N_1N_2 \qquad (6)$$

and

$$N_i/N_t = i\text{th single-scan efficiency} = (N_1+N_2-N_c)/N_j\,, \qquad (7)$$

where $i=1$ and $j=2$ or $i=2$ and $j=1$. The results, given in Table I, show that the collation efficiencies averaged ~97%, with no significant differences between the τ'^+ and τ'^- runs.

From the collation scanning results given in Table I, an average τ'^-/τ'^+ ratio was computed by weighting the ratio for each running period inversely as the square of its statistical uncertainty. This average, which is denoted by $R^{-+}(\tau')$, is given by

$$R^{-+}(\tau') \equiv \langle [N(\tau'^-)/N(K^-)]/[N(\tau'^+)/N(K^+)] \rangle_{\text{av}} = 0.998 \pm 0.016\,. \qquad (8)$$

In a similar manner, an average $K_{\pi 2}^-/K_{\pi 2}^+$ ratio was

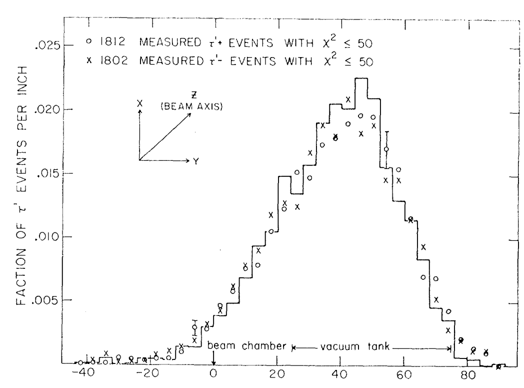

FIG. 6. Vertex distributions along the beam (z) axis for measured τ'^+ and τ'^- events and for 1972 Monte Carlo τ' decays (solid-line histogram). All distances are measured from the small foil chamber located at $z=0$ in. The vacuum tank, which extends from $z=25$ in. to $z=75$ in., is seen to contain ~75% of the observable τ' decays. The bin size used is 4 in.

computed from the first scan results. It is denoted by $R^{-+}(K_{\pi2})$ and is given by

$$R^{-+}(K_{\pi2})\equiv\langle[N(K_{\pi2}^-)/N(K^-)]/[N(K_{\pi2}^+)/N(K^+)]\rangle_{\text{av}} =0.976\pm0.009. \quad (9)$$

In Sec. IV are discussed small corrections to these two results that are necessary due to experimental asymmetries.

B. Measurements

In order to verify that the events recorded by the scanners were τ' decays, and in order to look for possible background differences between the τ'^- and τ'^+ data, about 20% of the τ' events were measured. A χ^2 fitting routine determined the vertices of these events and the laboratory angles of the outgoing particles.

The χ^2 distributions (Fig. 5) have longer tails than expected for our data, which are comprised of 76% three-shower events (5 degrees of freedom) and 24% four-shower events (7 degrees of freedom). This circumstance arises from the χ^2 program's assumption of the same rms uncertainty in the direction of every shower. This uncertainty actually varied over a large range because of energy-dependent fluctuations in the showering process. However, since a determination of the uncertainty for the direction of each track for every measured event would have greatly increased the measuring time, and since the τ'^- and τ'^+ vertex distributions could be obtained without such determinations, they were not made.

To verify the correctness of the above explanation for the long χ^2 tail, τ' decays were generated by a Monte Carlo program (called MCDEP and described in Sec. III C) and run through a subroutine called SHIFT. This routine shifted the angles of the $\pi^\pm$ and showers randomly according to their expected multiple scattering in the shower chambers. The χ^2 vertex-fitting routine

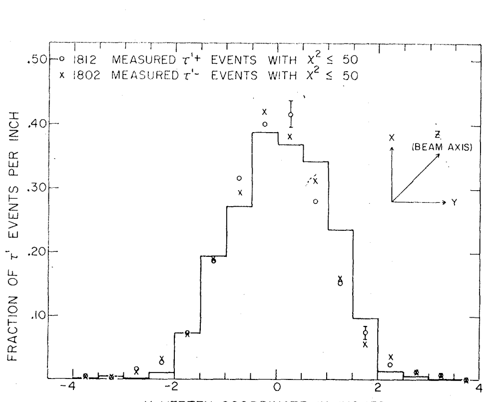

FIG. 7. Vertex distributions along the vertical (x) axis perpendicular to the beam axis for measured τ'^+ and τ'^- events and for 1972 Monte Carlo τ' decays (solid-line histogram). The Monte Carlo program generated a parallel beam whose spatial distribution approximated the results of a counter experiment. The bin size used is 0.5 in.

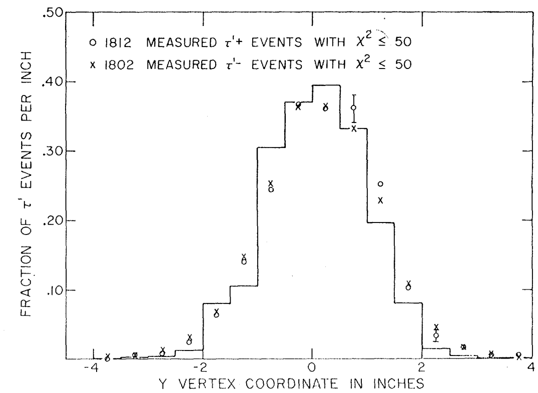

FIG. 8. Vertex distributions along the horizontal (y) axis perpendicular to the beam axis for measured τ'^+ and τ'^- events and for 1972 Monte Carlo τ' decays (solid-line histogram). The Monte Carlo program generated a parallel beam whose spatial distribution approximated the results of a counter experiment. The bin size used is 0.5 in.

was applied to these "shifted" events with the result shown by the histogram in Fig. 5. The good agreement with distributions for the measured events indicates that the events found by the scanners do have definite vertices.

In order to estimate the amount of background in these data, the χ^2 tails of the MCDEP-SHIFT-χ^2 and measured events were compared. An excess of 5%, mainly in the $\chi^2\geq 50$ region, was found for the measured events. However, the difference between the τ'^- and τ'^+ data in the $\chi^2\geq 50$ region is negligible, namely, (0.2 ± 0.7)%. Thus, no correction to the observed τ'^-/τ'^+ ratio is necessary, but the estimated error for the ratio is appropriately increased.

Further evidence that the scanning results represent τ' decays is given by the good agreement between the detailed shapes of the vertex distributions of the measured events and the corresponding histograms of MCDEP-SHIFT-χ^2 events (Figs. 6–8). In particular, 75% of the events are found to have vertices occurring in the vacuum tank (Fig. 6), indicating that they are indeed decays.

The angular distributions of the π^+ and π^- shown in Fig. 9 indicate the presence of events with pion angles greater than kinematically allowed for τ' decays ($\theta_\pi > 0.15$ rad). Such events are attributable to $K^\pm\to\pi^\pm\pi^0\gamma$ and $K^\pm\to\mu^\pm\pi^0\nu\gamma$ decays, kaon interactions, and scattering of the $\pi^\pm$ in the pi counters (D1–D4, E1–E6). A study of $K^\pm\to\pi^\pm\pi^0\gamma$ decays in our data is presently being made by measuring all events with $\theta_\pi\gtrsim 0.15$ rad.

C. τ' Detection Efficiency

The Monte Carlo detection-efficiency program (MCDEP) mentioned above generated τ' decays according to the exponential-decay law, with a matrix element linearly dependent on the energy of the charged pion. All of the geometric requirements imposed by the scin-

TABLE II. Experimental asymmetries ($\Delta\Gamma^{-+}/\Gamma$) in the determination of the τ'^-/τ'^+ ratio. Corrections to the τ'^-/τ'^+ ratio are made for those asymmetries that appear in 8, 9, and 12.

Source of asymmetry	$\Delta\Gamma^-/\Gamma$ (%)	$\Delta\Gamma^+/\Gamma$ (%)	$\Delta\Gamma^{-+}/\Gamma = \Delta\Gamma^-/\Gamma - \Delta\Gamma^+/\Gamma$ (%)
1. Other decays that simulate τ' decays	$\lesssim \|-1\|$	$\lesssim \|-1\|$	0.0±0.1
2. Interactions that simulate τ' decays	$\lesssim \|-0.8\|$	~ -0.5	0.0±0.3
3. Two-shower decays with one or more additional showers from an interaction of another beam particle	~ -0.5	~ -0.5	0.0±0.1
4. Masking of τ' decays by spark-chamber breakdowns	0.31±0.02	0.29±0.02	0.02±0.03
5. Difference between K^+ and K^- momenta			0.0±0.2
6. Difference between K^+ and K^- beam locations			0.0±0.1
7. Absorption of charged π's (from τ' decays) in the scintillators upstream of the vacuum tank			0.00±0.02
8. Absorption of charged π's (from τ' decays) in the material between the vacuum tank and the scintillator array (P11–P18) behind the 2nd shower chamber	5.8±0.1	4.9±0.1	0.9±0.2
9. Multiprong interactions of charged π's (from τ' decays) in the material between the vacuum tank and P11–P18 scintillator array	−1.5±0.1	−1.8±0.1	0.3±0.1
10. Counter-discriminator deadtimes	$\lesssim 0.1$	$\lesssim 0.1$	0.00±0.01
11. Accidentals in the counting of useful kaons	−0.0030±0.0002	−0.0260±0.0002	0.023±0.002
12. Absorption of $K^\pm$ mesons in the scintillators upstream of the vacuum tank	−0.68	−0.54	0.14±0.01

tillator arrays were put into the program, and an energy-dependent pair-production cross section was used to generate the showers. Corrections were made for several effects such as the absorption and scattering of $\pi^\pm$'s in the shower chambers and the triggering of two τ' counters (P11–P18) by the products from a single-photon shower. The results of this program are that the τ' detection efficiency should be 7.1% and the decay region should be as indicated by the Z vertex distributions of Fig. 6. On the other hand, a comparison of the scanning results with the known τ' partial decay rate and $K^\pm$ lifetime[14] indicates a τ' detection efficiency of 5.2%. This 27% "loss" of efficiency can be explained by two effects .The main effect, possibly as large as 25%, is associated with γ rays that produce showers in the wall of the vacuum tank in such a way as to trigger a second pi counter (D1–D4,E1–E6) and veto the event. The second effect, of unknown magnitude, is the difficulty for a scanner to recognize a shower that originates within or close to a previous shower in both views on the film.

IV. EXPERIMENTAL ASYMMETRIES

It is very important in this experiment to understand all processes which affect the number of observed $\tau'^\pm$ decays, $N(\tau'^\pm)$, and the number of incident $K^\pm$ mesons, $N(K^\pm)$. All such processes which may require unequal corrections to $N(\tau'^-)/N(K^-)$ and $N(\tau'^+)/N(K^+)$ are listed in Table II and are discussed below.[13] The corrections to the $K_{\pi2}^-/K_{\pi2}^+$ ratio are listed in Table III.

A. τ'-like Decay and Interactions

The decays that simulate τ' decays (row 1 in Table II) are $K^\pm \to \pi^\pm\pi^0\gamma$ and $K^\pm \to \mu^\pm\pi^0\nu\gamma$ decays. Monte Carlo calculations indicate that our $\pi\pi\gamma$ detection efficiency was $\sim\frac{2}{5}$ of our τ' detection efficiency. Thus, with a $\pi\pi\gamma$ branching ratio of 2.2×10^{-4} and a τ' branching ratio of 1.7×10^{-2},[14] the $\pi\pi\gamma$ decays are expected to represent $\sim$0.5% of the τ' data. Rough estimates for the $\mu\pi\nu\gamma:\tau'$ efficiencies and decay branching ratio indicate a similar size $\mu\pi\nu\gamma$ contribution to the background. Consequently radiative $K_{\pi2}$ and $K_{\mu3}$ decays are predicted to comprise a net background of $\sim$1%. This means that even a substantial asymmetry in these decays (10%) would have a negligible effect (0.1%) on the τ'^-/τ'^+ ratio.

[13] For a more detailed discussion of these corrections see D. Herzo's Ph.D. thesis, University of Illinois, 1969 (unpublished).

[14] N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, A. H. Rosenfeld, P. Söding, C. G. Wohl, M. Roos, and G. Conforto, Rev. Mod. Phys. **41**, 109 (1969).

TABLE III. Experimental asymmetries ($\Delta\Gamma^{-+}/\Gamma$) in the determination of the $K_{\pi 2}^{-}/K_{\pi 2}^{+}$ ratio. Background of two-shower $K_{\mu 3}$ and τ' decays is taken into account.

Source of asymmetry	$\Delta\Gamma^{-}/\Gamma$ (%)	$\Delta\Gamma^{+}/\Gamma$ (%)	$\Delta\Gamma^{-+}/\Gamma = \Delta\Gamma^{-}/\Gamma - \Delta\Gamma^{+}/\Gamma$ (%)
1. Absorption of charged π's (from $K_{\pi 2}$ decays) in the material between the vacuum tank and scintillator array (P11–P18) behind the second shower chamber	8.2±0.1	6.9±0.1	1.3±0.2
2. Multiprong interactions of charged π's (from $K_{\pi 2}$ decays) in the material between the vacuum tank and P11–P18 scintillator array	−6.9±0.1	−7.1±0.1	0.2±0.1
3. Absorption of $K^{\pm}$ mesons in the scintillators upstream of the vacuum tank	−0.68	−0.54	0.14±0.01

Interactions that appear like τ' decays (row 2 in Table II) may occur in the upstream beam-veto counters (C1–C4). Interactions in the downstream beam-veto counters (D1–D4) are no problem because of the beam-veto logic. A rough calculation based on the known $K^{+}p$ interactions at or near 3.5 BeV/c[15] yields an interaction background of ~0.5% in the τ'^{+} data. The difference between the τ'^{-}- and τ'^{+}-like interaction backgrounds is estimated to be $\lesssim 50\%$ on the basis of the difference between the $K^{-}p$ and $K^{+}p$ total cross sections at 3.5 BeV/c.[16] Thus the asymmetry in the τ'^{-}/τ'^{+} ratio due to $\tau'^{\pm}$-like interactions may be $\lesssim 0.25\%$.

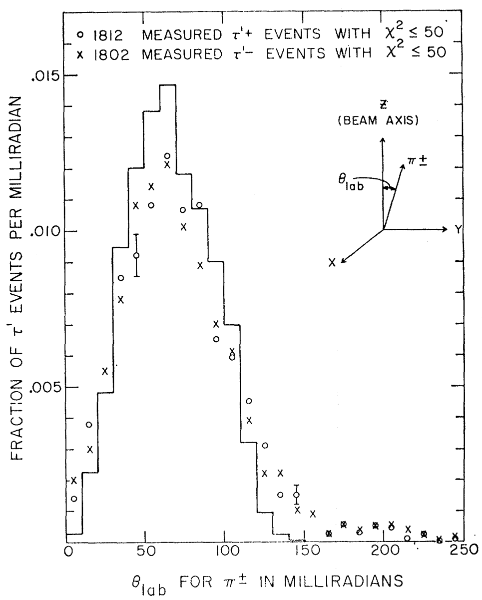

FIG. 9. Lab angular distributions with respect to the beam axis of charged pions from measured τ'^{+} and τ'^{-} events and from 1972 Monte Carlo τ' decays (solid-line hisotgram). The fact that a parallel beam was used by the Monte Carlo program should not be detectable since the 8-mrad uncertainty in the measured direction of the charged pion is approximately equal to the divergence of the beam. The measured events having charged-pion angles greater than ~150 mrad are expected to be $K^{\pm}\to\pi^{\pm}\pi^{0}\gamma$ and $K^{\pm}\to\mu^{\pm}\pi^{0}\nu\gamma$ decays, kaon interactions, and τ' decays with a large-angle scattering of the $\pi^{\pm}$.

The above estimate of the interaction background can be checked by an examination of the τ' vertex distributions along the beam axis. In the region of the upstream beam-veto counters ($12\leq Z\leq 18$ in. in Fig. 6), no significant difference between the measured events and the Monte Carlo events is found. Furthermore, the difference between the τ'^{-} and τ'^{+} measured events in this region, namely, (0.4±0.7)%, is also not significant. Therefore, no correction is made to the τ'^{-}/τ'^{+} ratio for the interaction background.

A type of τ'-like background that is not characterized by a single vertex (row 3 in Table II) is the combination of a two-shower decay with one or more additional showers resulting from a beam interaction in an upstream or downstream beam-veto counter. Such events are not true coincidences, since the interaction producing the extra shower(s) may occur at any time within the 1-μsec sensitive time of the spark chambers. On the basis of the known cross sections for π^{0}- (and consequently photon-) producing $\pi^{-}p$ interactions at 3.5 BeV/c,[17] this τ'-like background is predicted to comprise (0.5±0.1)% of the τ' data. A rough calculation of the asymmetry between the τ'^{-}- and τ'^{+}-like background based on the absorption cross sections for[18] $\pi^{\pm}p$ and[19] pp interactions yields a negligible difference of

[15] R. George, Y. Goldschmidt-Clermont, V. P. Henri, B. Jongejans, M. Krammer, F. Muller, J. M. Perrau, W. De Baere, J. Debaisieux, P. Dufour, F. Grard, J. Henghebaert, L. Pape, P. Peeters, F. Verbeure, and R. Windmolders, Nuovo Cimento **49A**, 9 (1967); P. Sällström, G. Otter, and G. Ekspong, *ibid.* **49A**, 348 (1967).

[16] W. Baker, R. Cool, E. Jenkins, T. Kycia, R. Phillips, and A. Read, Phys. Rev. **129**, 2285 (1963).

[17] L. Bondár, K. Bongartz, M. Deutschmann, E. Keppel, G. Kraus, H. Weber, D. C. Colley, W. P. Dodd, J. Simmons, B. Tallini, A. M. Freire-Endler, B. Nellen, G. Winter, D. Cords, Ch. Dehne, E. Lohrmann, P. Söding, M. Teucher, G. Wolf, J. M. Brownlee, I. Butterworth, F. I. Campayne, M. Ibbotson, N. N. Biswas, I. Derado, K. Gottstein, D. Lüers, G. Lütjens, and N. Schmitz, Nuovo Cimento **31**, 729 (1964); M. Feldman, W. Frati, J. Halpern, A. Kanofsky, M. Nussbaum, S. Richert, and P. Yamin, *ibid.* **50A**, 89 (1967).

[18] A. Citron, W. Galbraith, T. F. Kycia, B. A. Leontic, R. H. Phillips, A. Rousset, and P. H. Sharp, Phys. Rev. **144**, 1101 (1966).

[19] D. V. Bugg, D. C. Slater, G. H. Stafford, R. F. Goerge, K. F. Riley, and R. J. Tapper, Phys. Rev. **146**, 980 (1966).

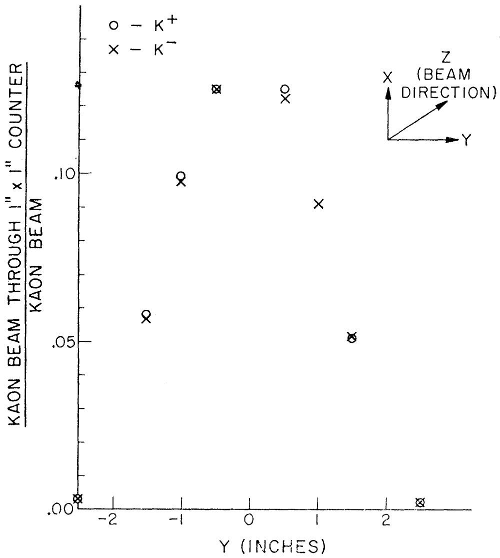

FIG. 10. Horizontal profiles of the K^+ and K^- beams. The kaon beam is defined by the incident kaon logic described in the text. The 1×1-in.^{2} counter was located between the vacuum tank and the first shower chamber. Every point except the two extreme ones are based on 10 000 kaons traversing the 1×1-in.^{2} counter.

$\sim -0.001\%$ between the backgrounds of the two polarities.

B. Detection Asymmetries

The effective τ'^+ and τ'^- detection efficiencies are reduced by spontaneous breakdowns of the spark chambers (row 4 in Table II), which cause spurious sparks to obscure possible τ' events on the film. The scanners were instructed to list all such frames. The results are a loss of $(0.31\pm0.02)\%$ for the τ'^- data and $(0.29\pm0.02)\%$ for the τ'^+ data. Hence the asymmetry caused by this effect is only $(0.02\pm0.03)\%$, and so no correction to the τ'^-/τ'^+ ratio is needed.

An asymmetry between the observed number of τ'^+ and τ'^- decays could be produced by anything that causes unequal solid-angle efficiencies for the detection of these decays. One such cause is a difference in the central momenta for the K^+ and K^- beams (row 5 in Table II). However, a change in the momentum of the K beam is accompanied by a change in the decay rate in such a direction as to tend to cancel the effect of the change in the solid-angle efficiency. The actual result found by the use of Monte Carlo events was that a shift in momentum of 0.3% (and consequently a 0.3% shift in the decay rate) would produce a τ' solid-angle efficiency change of $(0.12\pm0.06)\%$ in the opposite direction. Thus the possible asymmetry in the τ'^-/τ'^+ ratio is only $(0.0\pm0.2)\%$.

Another possible cause of a change in the solid-angle efficiency is a shift in the position of the beam (row 6 in Table II). A horizontal profile of the kaon beam for both polarities was taken by the use of a 1×1-in.^{2} counter located just downstream of the vacuum tank. No statistically significant difference between the locations of the two beams was found (Fig. 10). Another check on the beam locations comes from the vertex distributions for the measured events shown in Figs. 7 and 8. Good agreement for both polarities is found here also.

In order to determine whether the horizontal profile results were of sufficient precision to guarantee a negligible asymmetry in the solid-angle efficiencies, Monte Carlo τ' events were run through the computer with small shifts made in the vertices of the events. A 1% shift (1 standard deviation in the horizontal profile results) was found to cause only a $(0.04\pm0.10)\%$ change in the solid-angle efficiency.

Decays which occur before the upstream beam-veto counters (C1–C4) can be lost by absorption of the charged pion in these counters (row 7 in Table II). An asymmetry in the $\tau'^\pm$ detection efficiencies may result because the $\pi^+p(n)$ cross section is not equal to the $\pi^-p(n)$ cross section. A simple calculation based on the total pion-nucleon cross sections indicates that the asymmetry is no more than 0.02%.

Two corrections are made to the numbers of observed τ'^- and τ'^+ decays as a result of the unequal probabilities for π^+ and π^- interactions in the shower chambers. The larger correction is for the $\pi^\pm$ being either absorbed or scattered out of the shower chambers without traversing the τ' counter array located behind the second chamber (row 8 in Table II). A separate counter experiment was performed with π^+ and π^- beams of several different momenta, and the results were integrated over the $\pi^\pm$ momentum spectrum. The resulting correction of $(0.9\pm0.2)\%$ to the τ'^-/τ'^+ ratio for absorbed and scattered $\pi^\pm$'s has a compensating factor for those events in which the triggering requirement on the τ' counter array was satisfied by four shower events or by three shower events with one shower traversing two counters. The smaller correction to the τ'^-/τ'^+ ratio accounts for the unequal enhancement of the τ'^+ and τ'^- detection efficiencies resulting from $\pi^\pm$ multiprong interactions in the shower chambers (row 9 in Table II). This enhancement is due to those events in which the showers triggered only two counters and the $\pi^\pm$ interaction products triggered two other counters. This correction, also determined by the $\pi^\pm$ beam-counter experiment, is $(0.3\pm0.1)\%$.

An asymmetry could result from unequal deadtimes in the counter discriminators for the two polarities (row 10 in Table II). The counters with the highest rates were dead less than 0.1% of the time, and the difference in the counting rates for the two polarities was less than 10%. Hence, this possible asymmetry between the positive and negative runs is less than 0.01%.

A significant accidental rate in the incident kaon coincidence could create an asymmetry in the counting of K^+ and K^- mesons (row 11 in Table II). These accidentals are coincidences between kaons that are lost after triggering the differential Čerenkov counter and particles that trigger one of the upstream beam-veto counters (C1–C4) without being vetoed by the Čerenkov counters. The accidentals are expected to be more numerous in the positive beam because of the presence of a large number of protons ($\sim 46\%$ of the beam) which are not vetoed by the Čerenkov counters. Appropriate coincidence rates were measured to determine the accidental rates, and they were found to be 0.026% of the K^+ coincidences and 0.003% of the K^- coincidences. Thus the accidentals cause a negligible asymmetry in the τ'^-/τ'^+ ratio.

A small correction to the counted numbers of K^+ and K^- mesons arises from the difference in the interactions of K^+ and K^- mesons in the upstream beam-veto countters (row 12 in Table II). Kaons that interact in these counters are no longer available as a source of τ' decays and hence should not be counted as part of $N(K)$. The results of a counter-transmission experiment, as well as a rough calculation based on the absorption cross sections for K^+p and K^-p interactions,[16,20] show that there is a net asymmetry of 0.1% in the τ'^-/τ'^+ ratio because of the unequal interactions of the K^+ and K^- mesons.

C. Summary of Corrections

The situation regarding corrections has been discussed above and summarized in Tables II and III for the τ' and $K_{\pi 2}$ data, respectively. The only corrections found to be necessary are for $\pi^\pm$ interactions in the shower chambers and for $K^\pm$ interactions in the upstream beam-veto counters. If each correction is represented by a value of $\Delta\Gamma^{-+}/\Gamma$ as it is in Tables II and III, then the net corrections are given by the sums

$$\sum(\Delta\Gamma^{-+}/\Gamma)=(1.3\pm0.2)\% \quad \text{for } \tau' \text{ data} \quad (10)$$

and

$$\sum(\Delta\Gamma^{-+}/\Gamma)=(1.6\pm0.2)\% \quad \text{for } K_{\pi 2} \text{ data}. \quad (11)$$

These corrections are applied to the average ratios $R^{-+}(\tau')$ and $R^{-+}(K_{\pi 2})$, which are given in Eqs. (8) and (9), by use of the relation

$$\Gamma(\alpha^-)/\Gamma(\alpha^+)=R^{-+}(\alpha)[1+\sum(\Delta\Gamma^{-+}/\Gamma)], \quad (12)$$

with $\alpha=\tau'$ or $\alpha=K_{\pi 2}$. The difference between the corrections to the τ' data and the $K_{\pi 2}$ data arises from the difference in the $\pi^\pm$ momentum spectra for these two decay modes. This follows from the fact that the $\pi^\pm$ interaction probabilities are dependent on the momentum of the $\pi^\pm$.

V. RESULTS

When the collation scanning result given by $R^{-+}(\tau')=0.998\pm0.016$ [Eq. (8)] is combined with the net asymmetry correction given by $\sum(\Delta\Gamma^{-+}/\Gamma)=0.013\pm0.002$ [Eq. (10)] in Eq. (12), we find that

$$\Gamma(K^-\to\pi^-\pi^0\pi^0)/\Gamma(K^+\to\pi^+\pi^0\pi^0)=1.011\pm0.018. \quad (13)$$

In addition to the statistical and experimental errors quoted above, the net uncertainty in the τ'^-/τ'^+ ratio of ±0.018 includes on a rms basis the 0.7% uncertainty in the χ^2 test for a possibly asymmetric background in the data (see Sec. II B.). In a similar manner with $R^{-+}(K_{\pi 2})=0.976\pm0.009$ [Eq. (9)] and $\sum(\Delta\Gamma^{-+}/\Gamma)=0.016\pm0.002$ [Eq. (11)], we find that

$$\Gamma(K^-\to\pi^-\pi^0)/\Gamma(K^+\to\pi^+\pi^0)=0.992\pm0.012. \quad (14)$$

A. *CP* Invariance and $|\Delta I|=\frac{1}{2}$ Rule

Since our result for the τ'^-/τ'^+ ratio is consistent with unity and therefore with *CP* invariance, we can only set an upper limit on the possible *CP*-violating A_3 ($|\Delta I|=\frac{5}{2}$ and/or $|\Delta I|=\frac{7}{2}$) amplitude. To do this, we make use of the relation

$$1-\Gamma(\tau'^-)/\Gamma(\tau'^+)\simeq 8(|A_3|/|A_1|)\sin\delta_{31}\sin\varphi_{31}, \quad (15)$$

which is derivable from Eq. (1) if $(|A_3|/|A_1|)^2$ and higher-order terms are neglected. We assume that reasonable values of the 3π final-state strong-interaction phase shifts are bounded by $10°\leq\delta_1-\delta_3\leq30°$, with $\delta_3\simeq0°$ [5] and $\delta_1=10°$ [21] or $\delta_1=30°$.[5] We also assume that A_3 is completely *CP*-violating and hence is imaginary relative to A_1 ($\varphi_{31}=90°$). Then our experimental uncertainty implies that with a 2-standard-deviation effect

$$|A_3|/|A_1|\leq0.009 \quad \text{with } \delta_{31}=-30°, \quad \varphi_{31}=90° \quad (16)$$

or

$$|A_3|/|A_1|\leq0.026 \quad \text{with } \delta_{31}=-10°, \quad \varphi_{31}=90°. \quad (17)$$

A relation similar to Eq. (15) can be derived for the more precisely known τ^-/τ^+ ratio[10] to obtain upper limits for $|A_3|/|A_1|$ which are about two times smaller than those given in (16) and (17). It is to be stressed that these limits are for an A_3 amplitude that is imaginary relative to A_1 and is consequently *CP*-violating. The upper limit for an A_3 amplitude that is real relative to A_1 and is therefore *CP*-invariant is obtainable from the present experimental uncertainty in the ratio of the τ to τ' partial decay rates. For a 2-standard-deviation effect, $|A_3|/|A_1|\leq0.014$ for $\delta_1-\delta_3\leq30°$ and $\varphi_{31}=0°$.[22]

B. *CPT* Invariance

One well-known consequence of *CPT* invariance is equal lifetimes for K^+ and K^- mesons.[1,2] This has been

[20] J. Gordan, Phys. Letters **21**, 117 (1966); W. De Baere, J. Debaisieux, P. Defour, F. Grard, J. Heughebaert, L. Pape, P. Peeters, F. Verbeure, and F. Windmolders, Nuovo Cimento **45A**, 885 (1966).

[21] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

[22] Neglecting $(|A_1'|/|A_1|)^2$, $(|A_3|/|A_1|)^2$, and higher-order terms, and assuming $\varphi_{31}=0°$, Eqs. (1) and (2) may be combined to yield the relation $(|A_3|/|A_1|)\cos\delta_{31}\cong\frac{1}{5}[\gamma(\tau^+)/4\gamma(\tau'^+)-1]$. It then follows that with $\gamma(\tau^+)/4\gamma(\tau'^+)=0.99\pm0.03$ according to a recent compilation of the relevant experiments in Ref. 14, a 2-standard-deviation effect allows an $|A_3|/|A_1|$ ratio of 0.014 provided that $|\delta_{31}|\leq30°$.

verified to one part in 10^3 by Lobkowicz *et al.*,[3] who found that

$$\Gamma_{\text{tot}}(K^-)/\Gamma_{\text{tot}}(K^+)=1-0.00049\pm0.00097. \quad (18)$$

Another consequence of CPT invariance[1,4,23] is that

$$[\Gamma(\tau^-)+\Gamma(\tau'^-)]/[(\Gamma\tau^+)+\Gamma(\tau'^+)]=1, \quad (19)$$

neglecting radiative decay modes which could possibly introduce an asymmetry of no more than the order of 10^{-3}.[24] If one then assumes that a CP and a CPT violation may occur in the 3π decay channels only, the result of Lobkowicz *et al.* sets an upper limit of ±0.028 on the possible asymmetry of (19) provided there is a 2-standard-deviation effect. On the other hand, we find that, by combining our result for the τ'^-/τ'^+ ratio with the already known τ^-/τ^+ ratio[9,10] and the τ and τ' partial decay rates,[14]

$$[\Gamma(\tau^-)+\Gamma(\tau'^-)]/[\Gamma(\tau^+)+\Gamma(\tau'^+)]-1 = 0.0029\pm0.0045. \quad (20)$$

Thus, at the present time, (20) is the most accurate direct check on the CPT theorem for a case where two partial decay modes are involved.[25]

C. $K_{\pi 2}$ Decays

Since only an isospin-2 final state is allowed for $K^\pm \rightarrow \pi^\pm\pi^0$ decays, a CP violation in the weak Hamiltonian cannot manifest itself as an asymmetry in the $K_{\pi 2}^-/K_{\pi 2}^+$ ratio. Furthermore, if there were a CP violation in the electromagnetic interaction, the expected asymmetry in the $K_{\pi 2}^-/K_{\pi 2}^+$ ratio would at most be of the order of the fine-structure constant (1/137),[26] and hence it would not be detectable by our result, which is uncertain by 1.2%. Nevertheless, the $K_{\pi 2}^-/K_{\pi 2}^+$ ratio is useful in that it gives a good indication that all significant aysmmetries in our experiment were taken into account.

ACKNOWLEDGMENTS

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[23] D. Cline, Nuovo Cimento **36**, 1055 (1965).

[24] To lowest order in the electromagnetic and weak interactions, CPT invariance requires that (assuming parity conservation in the electromagnetic interaction)

$$\Gamma(\tau^-)+\Gamma(\tau'^-)+\Gamma(\tau^-\gamma)+\Gamma(\tau'^-\gamma)+\Gamma_d(\pi^-\pi^0\gamma;M1) = \Gamma(\tau^+)+\Gamma(\tau'^+)+\Gamma(\tau^+\gamma)+\Gamma(\tau'^+\gamma)+\Gamma_d(\pi^+\pi^0\gamma;M1),$$

where $\Gamma_d(\pi^\pm\pi^0\gamma;M1)$ represents the rate for the lowest-order multipole radiation (magnetic dipole) in the direct emission process which may couple to the 3π decay modes [see Refs. 1, 23, and J. D. Good, Phys. Rev. **113**, 352 (1959)]. It is known that $\Gamma(\tau^+\gamma)\Gamma(\tau^+)=(1.7\pm0.7)\times10^{-3}$ according to P. Stamer, T. Heutter, E. L. Koller, S. Taylor, and J. Grauman, Phys. Rev. **138**, 440 (1965). Also, $\Gamma_d(\pi^+\pi^0\gamma;M1)/\Gamma$ (all K^+ modes) $<\Gamma(\pi^+\pi^0\gamma)/\Gamma$ (all K^+ modes) $=(2.2\pm0.7)\times10^{-4}$ according to D. Cline and W. Fry, Phys. Rev. Letters **13**, 101 (1964). Thus, even if the radiative decay rates are neglected, the resulting relation [Eq. (19) in the text] is still accurate to at least the order of 10^{-3}.

[25] For a recent summary of the situation regarding CPT invariance, see L. B. Okun, Comments Nucl. Particle Phys. **2**, 116 (1968).

[26] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, 1650 (1965).

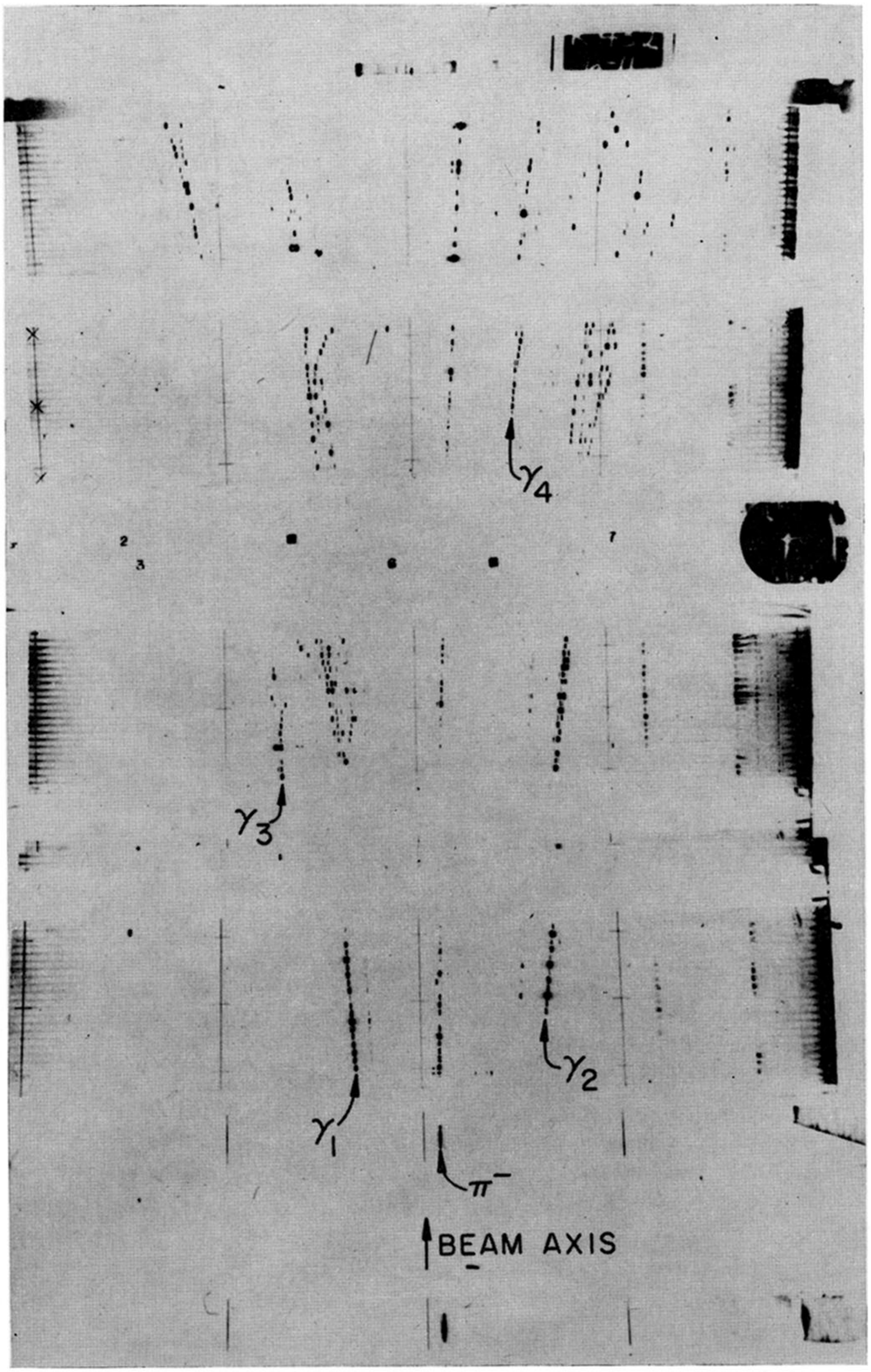

FIG. 4. Top view of a four-shower τ' event.