Coherent Superposition of Charge States*

R. MIRMAN
Physics Department, Long Island University, The Brooklyn Center, Brooklyn, New York 11201
(Received 2 December 1969; revised manuscript received 17 June 1969)

Some *Gedankenexperimente* are considered to illustrate the analogy between coherent superposition of angular momentum states and coherent superposition of charge states.

THE question of the existence of a charge superselection rule and the interference between states of different charge has recently been discussed briefly, and the generally accepted view that it is impossible to coherently combine different charge states has been challenged by Aharonov and Susskind.[1] It would seem useful to consider this question in somewhat greater detail and in particular to see how far the analogy between angular momentum and charge can be pushed. This will provide further arguments for the Aharonov-Susskind view that states of different charge can be coherently superposed.

In order to do this we discuss several *Gedankenexperimente* to see how close we can make the similarity between the charge and spin cases.

We consider first an experiment for the charge case that is closely analogous to the Stern-Gerlach experiment for spin. Before doing this, we review the work of Aharonov and Susskind, who studied a system with a large number of identical setups, each consisting of two containers with charge coherently shared between them and with a proton beam entering the first. The beam emerges from the first container as a combination of neutron and proton beams. For some of the setups the relative intensity of the two particles in the beam emerging from the first container is measured. For the other setups the beam goes through the second container and the relative intensity in the beam emerging from the system is measured. They then show that the relative phase of the two particles between the containers can be determined from the knowledge of the two relative intensities.

Let us now consider what happens in the ordinary Stern-Gerlach experiment. The beam of particles (of spin $\frac{1}{2}$) travels through the first magnet, which produces a beam with a mixture of spin-up and spin-down states. It now enters the second magnet, where two things happen. First, a certain probability of spin flip is created. Second, the spin is measured by the magnet. Specifically, the magnet measures the spin by giving the particle a velocity parallel to the field of the magnet whose sign is a function of the sign of the spin.

Clearly we can do an experiment for spin analogous to the one just described for charge. We now wish to describe an experiment for charge analogous to the one just described for spin.

The basic difference between the two experiments just described is that Aharonov and Susskind discuss charge flip and not measurement of the charge, while the Stern-Gerlach experiment measures spin. We, therefore, need an apparatus which will measure charge.

To do this, we take the second container to be of triangular shape and assume that the mesons obey, e.g., the ideal gas law. Therefore, there will be more mesons in the upper, wider part of the container than in the lower part. A positive particle will be pulled up; a neutral one will be unaffected; or, to improve the analogy with the spin case, a negative one will be pushed down. Thus the charge will be measured in exactly the same way as the spin was, by giving the particle a component of velocity along an axis perpendicular to the line joining the two containers; the sign of this component of the velocity is a function of the sign of the charge.

Hence, we are able to get an experiment for charge that is analogous to that for spin. With this in mind let us consider further the question of coherence.

One intuitive meaning of the statement that two states are coherent is that there is, in some sense, "one state" which is equal to the sum of the two states. Thus, for angular momentum (spin $\frac{1}{2}$), the coherent superposition of spin up and spin down (along z) means that the spin is up along some axis z'. The second magnet of the Stern-Gerlach experiment can be rotated so that it is along the z' axis. In this case there will be only one line on the screen, that due to the spin-up particles. This is different from the case of incoherent spin up plus spin down. Thus we can say for the coherent case that there is only one state, representing a particle with spin pointing up in the z' direction.

It is possible to do something similar with charge? Is it possible to "rotate" the measuring container and vary the intensity of the different lines and, in fact, get a "position" for which the intensity of one line is zero?

First let us review the spin case. We take the Hamiltonian as

$$H=B_0S_z=\tfrac{1}{2}\omega\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{1}$$

* Work supported by a grant from Long Island University.
[1] Y. Aharonov and L. Susskind, Phys. Rev. **155**, 1428 (1967); W. Rolnick, Phys. Rev. Letters **19**, 717 (1967).

and the up and down states as

$$|u\rangle=\begin{pmatrix} e^{i\varphi/2}\cos\frac{1}{2}\theta \\ ie^{-i\varphi/2}\sin\frac{1}{2}\theta \end{pmatrix},\qquad |d\rangle=\begin{pmatrix} ie^{i\varphi/2}\sin\frac{1}{2}\theta \\ e^{-i\varphi/2}\cos\frac{1}{2}\theta \end{pmatrix}. \tag{2}$$

It follows that the wave function is

$$\psi=\alpha(t)\,|u\rangle+\beta(t)\,|d\rangle \tag{3}$$

and

$$id\psi/dt=-H\psi, \tag{4}$$

giving

$$id\alpha/dt=-\tfrac{1}{2}\omega(\alpha\cos\theta+i\beta\sin\theta), \tag{5}$$

$$id\beta/dt=\tfrac{1}{2}\omega(i\alpha\sin\theta+\beta\cos\theta). \tag{6}$$

For no transition, $\beta=0$ and $d\beta/dt=0$, so $\sin\theta=0$. Hence the requirement of no transition means that the spin must be along the z axis, which is the direction in which the magnet is aligned. Then the particle is in the state $\binom{1}{0}$ both before entering the magnet, and after leaving it. On the screen there is only one line.

Now for the charge case the Hamiltonian (following Aharonov and Susskind) is

$$H=g(\sigma^{+}a^{-}+\sigma^{-}a^{+}), \tag{7}$$

and the state, with the state of the containers suppressed, is given by

$$\psi=\mu(t)\,|p\rangle+\nu(t)\,|n\rangle. \tag{8}$$

This gives (Q and θ refer to the second container)

$$\begin{aligned} id\mu/dt&=-gQ^{1/2}e^{-i\theta}\nu, \\ id\nu/dt&=-gQ^{1/2}e^{i\theta}\mu, \end{aligned} \tag{9}$$

and, defining ρ and σ with coefficients A, B, C, and D to be determined,

$$\rho=\frac{D\mu+B\nu}{AD+BC},\quad \sigma=\frac{C\mu-A\nu}{AD+BC}, \tag{10}$$

we get the equations

$$i\frac{d\rho}{dt}=\frac{-gQ^{1/2}}{AD+BC}\big[(DCe^{-i\theta}+ABe^{i\theta})\rho+(B^2e^{i\theta}-D^2e^{-i\theta})\sigma\big], \tag{11}$$

$$i\frac{d\sigma}{dt}=\frac{gQ^{1/2}}{AD+BC}\big[-(C^2e^{-i\theta}-A^2e^{i\theta})\rho+(ABe^{i\theta}+CDe^{-i\theta})\sigma\big]. \tag{12}$$

Setting

$$\begin{aligned} -\cos\phi&=DCe^{-i\theta}+ABe^{i\theta} \\ -i\sin\phi&=B^2e^{i\theta}-L^2e^{i\theta} \\ &=-(C^2e^{-i\theta}-A^2e^{i\theta}), \end{aligned} \tag{13}$$

with

$$\tfrac{1}{2}\omega=-gQ^{1/2}/(AD+BC),$$

one obtains

$$i\frac{d\rho}{dt}=\tfrac{1}{2}\omega(\rho\cos\phi+i\sigma\sin\phi), \tag{14}$$

$$i\frac{d\sigma}{dt}=-\tfrac{1}{2}\omega(-i\rho\sin\phi+\sigma\cos\phi). \tag{15}$$

These expressions are exactly analogous to Eqs. (5) and (6). Again, for no transitions we have $\sigma=0$ (or $\rho=0$) and $\sin\phi=0$.

This gives the state vectors which remain invariant under this Hamiltonian as (the state of the containers is suppressed)

$$\psi_{\pm}=\tfrac{1}{2}(e^{i\theta}|p\rangle\pm|n\rangle). \tag{16}$$

This means that the first container must create a state with a definite relative phase between the p and n which is determined by the second container. So the relative phase between the two containers is determined by the requirement of no charge flip, just as the relative angle (zero) between the magnets is determined by the requirement of no spin flip.

For the spin case, when we found the angle (measured from the initial position of the second magnet) for which there were no transitions, we rotated the second magnet through that angle so that it became parallel to the spin. At this angle, there is only one line on the screen. Also note that we spatially separated the two states, spin up and spin down, by means of the Stern-Gerlach experiment. What is the analogy here? Can we separate the states ψ_+ and ψ_-, and how do we "rotate" the container?

First, let us note that the matrix elements of the interaction Hamiltonian between the two states are different. They are

$$\langle\psi_+|H|\psi_+\rangle=gQ^{1/2},\quad \langle\psi_-|H|\psi_-\rangle=-gQ^{1/2}, \tag{17}$$

so that the two states have different energy. Therefore, one way to separate the two states, with different phases between the positive and negative particles, is to send the beam through the second container and put on a strong nuclear field around the container. The two states will fall with different accelerations and will thus be separated.

In this experiment the separation depends on the mass difference, while the force is the same for the two states. In the spin Stern-Gerlach experiment the force differs for the two states. We now consider a somewhat more complicated apparatus which will give for the charge case a different force for the two states.

The energy depends on Q, which is the average charge of the container. We wish now to set up a container with a Q which varies in space (in the y direction, which is perpendicular to the direction of motion of the charges) and so obtain a space gradient of the energy: a force. The sign of the energy, and so of the force, will be different for the two states.

We shall use a container with a temperature gradient, and therefore also a density gradient. (Another way of getting a density gradient is by having a gas of mesons in a gravitational field.) Considering each little slice of the container as a separate container, we see that each such slice has a different average charge Q and, thus, Q varies with y.

The reason why the particle in the beam has an energy which is a function of y is that the number of mesons in its path, and so the number of mesons it interacts with, is a function of y.

We also put a charge distribution on the outside of the container, so chosen that the sum of the electric field owing to this distribution, plus the electric field due to the meson distribution, is zero. Thus, in the container, there is a density distribution but no potential gradient.

The reason that this is necessary is that the charge picks out one "direction" in the space of that phase which is conjugate to the charge; if there had been a field, there would have been a separation of a linear combination of states different from the linear combination desired. In other words, the n and p states would have been separated.

Finally we must consider how to "rotate" the container, that is, how to change its phase. This is done by sending it through an external potential. (The change in phase equals the potential times the time the container is in the potential.) For example, it can be sent through a square barrier potential. The phase change as a function of the width and height of the potential may be found in any quantum mechanics text.

Experimentally this can be realized by two infinite plane plates carrying equal and opposite charges and very close together. Some distance away is another, similar pair of plates, with charges reversed. The region between the two pairs forms the barrier. Moving the container through this region will give the required change of phase.

Thus, in exact analogy to the spin case, we are able here to alter the state of the second container and thus vary the relative intensities of the two lines made by the two states. And, in fact, we can get one pure state and one line on the screen by changing the phase of the second container by an amount determined by previous experimental results.

We now summarize the experiment. We send a beam of protons through the first container, choosing the time T that they spend in the container so that

$$\cos gTQ^{1/2} = \sin gTQ^{1/2} = 1/\sqrt{2} \tag{18}$$

and defining the phase of the first container so that $e^{i\theta} = -i$. The state of the particles leaving the first container is therefore [cf. Eq. (6) of Aharonov and Susskind]

$$\frac{|p) + |n)}{\sqrt{2}} \tag{19}$$

We now send the beam through the second container (with a temperature gradient as described above) and from there to a screen. For simplicity we choose $\sin gQ^{1/2}T = 1$ for the second container. Using Eq. (6) of Aharonov and Susskind, we find the relative intensity of the two lines on the screen to be $\tan^2\theta$, where θ is the phase of the second container relative to the first.

The second container is now removed and its phase is "rotated" by $-\theta$. The beam is then sent through again and on to the screen. We now find only one line on the screen. Hence the particles, before entering the second container, are in a single "pure" state in which the two charges are added coherently.

An interesting point has been raised which deserves some comment. It can be claimed that what is produced here is not a pure state of the particle and that, in our usage, only the particle and the containers has a state vector. It may also appear that what we mean by coherent state is different from what is usually meant. Many people would probably feel that the properties of a coherent state can be verified with an instrument that was not before in contact with the carrier of the state in question, and that this is not the case for the state produced by the thought experiment considered in this paper.

In discussing this point we first consider what is clearly the main idea of the above argument, which is that the properties of a coherent state can be verified with an instrument that was not before in contact with the carrier of the state in question. This view is (implicitly) disagreed with in this paper and it is worthwhile to discuss why.

Let us consider the case of angular momentum (the Stern-Gerlach experiment) where the analogous quantity to the phase difference between the containers is the angle between the two magnets. Can we measure the z component of the angular momentum of a particle with a magnet "that was not before in contact with the carrier of the state?" In the case in which it was not before in contact, the second magnet is completely uncorrelated with the first magnet (and therefore with the particle which is correlated with the first magnet, because the spin of the particle is in the direction of the field of the first magnet).

By the statement that the two magnets are uncorrelated, we mean (among other things) that it is impossible from a knowledge of the orientation of one magnet to make any prediction about the orientation of the other magnet.

Now if the particle is a coherent superposition of two states (assuming for concreteness spin $\frac{1}{2}$), then it is possible to rotate the second magnet so that it is oriented along some line z and find only one line on the screen (in a Stern-Gerlach experiment). But what is the direction of z? It is, of course, determined by the direction of the field of the first magnet. In other words, the two magnets are correlated.

Another way of saying this is to consider how we tell that a state is coherent. This is done by performing a large number of experiments on the state (or by performing one experiment on each of a large number of identical states). Now suppose the two magnets are uncorrelated so that for each experiment which is done to measure their relative orientation there is an equal chance of getting any angle. Then each time the second magnet measures the z component of the angular momentum of the particle, which is along the z axis as defined by the first magnet, there is an equal probability of the z axis having any orientation with respect to the second magnet. Clearly, in this case, it would be impossible to determine the direction in which the angular momentum was pointing, or even to show that it points in a definite direction (that is, that the particle is in a coherent superposition of states as defined with respect to some arbitrary direction).

If the magnets must be correlated, why has the common assumption developed that correlation is irrelevant?

The reason is that for angular momentum, which is the standard example for this type of discussion, the magnets are rigidly attached to a rigid body, the Earth, and are thus correlated without any thought being given to their correlation.

One might object that it is possible to consider the following experiment. The Sun emits particles quantized along its magnetic field, which are then intercepted by the Earth and measured by its field. Are the fields of the Sun and Earth correlated? The answer is yes for they are both rigidly attached to macroscopic bodies (in the sense that changes of orientation are very small in the time interval necessary for the emission or interception of enough particles required to do "a large number of experiments"). Clearly the spin axes of the Earth and Sun are correlated. The reason for this correlation is that, because of their mass, their orientation changes so slowly that it is possible for the Earth to intercept a large number of particles from the Sun and study their coherence before the angle between Earth and Sun changes enough to destroy the coherence.

As another example, let us suppose that the magnets were electrons. A particle would pass the first electron and its orientation would be determined. It would pass the second electron, and the orientation would be measured. Now, in order to see if the particle is in a coherent state, the experiment would have to be repeated with the second electron rotated with respect to the first, whose orientation must remain unchanged. This experiment would obviously be impossible.

Returning to the phase "not before in contact . . . ," we see that the experiment would be impossible if the magnets had an unknown relative angular momentum, and that they have to be in "contact" in order that their relative angular momentum be determined.

Let us finally consider the statement that what is produced is not a pure state of the particle, but that only the union of the particle and the containers has a state vector.

As we have shown above, from an operational point of view the only state vector is, indeed, one describing the union of the particles and the container. Yet in the formalism of quantum mechanics we do use state vectors which describe "a pure state of the particle." These vectors have no direct physical meaning, yet they clearly have a definite formalistic value and meaning.

Can these formal states be coherently superposed in the charge case? As we see from above, the analogy between the angular momentum case and the charge is very close. Thus, if we endow the formal angular momentum states with properties allowing them to be superposed, then for consistency we must also endow the formal charge states with these properties.

Our basic concern is not with the properties of any formal "pure states," but rather with the fact that there is no fundamental experimental difference, from the point of view of the superposition principle, between the charge case and other more familiar situations like angular momentum. Thus all these cases must be treated similarly.

All this has been stated, implicitly or explicitly, by Aharonov and Susskind.[1]

I would like to thank Dr. L. Susskind for several invaluable conversations.