(*d*, *t*) and (*d*, He^3) Reactions on the Calcium Isotopes*

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The $Ca^{42}(d,t)Ca^{41}$, $Ca^{43}(d,t)Ca^{42}$, $Ca^{44}(d,t)Ca^{43}$, $Ca^{46}(d,t)Ca^{45}$, $Ca^{48}(d,t)Ca^{47}$, $Ca^{42}(d,He^3)K^{41}$, $Ca^{43}(d,He^3)K^{42}$, and $Ca^{44}(d,He^3)K^{43}$ reactions have been investigated at an incident deuteron energy of 22 MeV. The angular distributions have been compared with distorted-wave calculations and the spectroscopic strength has been extracted. The larger fraction of the $d_{3/2}$ strength was found in the $\frac{3}{2}^+$ states at 2.02, 0.99, 1.89, and 2.60 MeV in Ca^{41}, Ca^{43}, Ca^{45}, and Ca^{47}, respectively. Strong *s*-wave transitions were observed to levels at 1.96, 2.40, and 2.60 MeV in Ca^{43}, Ca^{45}, and Ca^{47}, respectively. The $\frac{1}{2}^+$ hole state at 2.67 MeV in Ca^{41} was found to be less pure than the other strong *s*- and *d*-hole states. Similarly, a pronounced fractionation of the *s*-hole configuration among the levels at 0.982, 1.273, 1.595, and 2.73 MeV was found in K^{41}—in contrast with the other even-parity hole states in K^{41} and K^{43} (including the $\frac{1}{2}^+$ level at 0.56 MeV), which are rather pure. This splitting is attributed in part to the interaction of another $\frac{1}{2}^+$ state with the *s*-proton hole state. Some $2p_{3/2}$ admixture was observed in each of the neutron pickup reactions and a $2p_{1/2}$ admixture was seen in Ca^{48}. Comparison between the relative intensities of the transitions to the $\frac{3}{2}^-$ levels in Ca^{41} and Ca^{43} and those of the (d,p) reactions to those levels suggest the admixture of core-excited configurations in the ground state of Ca^{42} and Ca^{44}. The upper limit for the $f_{7/2}$-proton admixture in the ground state of Ca^{42} was found to be $C^2S \leq 0.4$. The second 0^+ state in Ca^{42} is excited with about 10% of the strength of the first 0^+ state. The strength of the transitions to the 0^+ and 2^+ states together with those of the 4^+ and 6^+ states is significantly smaller than expected from simple shell-model considerations and suggests that other levels with $(f_{7/2})^2$ configurations exist in Ca^{42} at higher energies. The $l=0$ transitions to states in Ca^{42} rarely mix with the $l=2$ transitions. There is a fairly good correspondence between the *s*-hole states and the 3^- states. Application of the sum rule and comparison with the other reactions suggest that the total spectroscopic factor for $s_{1/2}$, $T=1$ pickup is considerably smaller than would be expected from a closed 2*s* shell. The four lowest levels in K^{42} have $l=2$ transitions and the strengths are in agreement with a 2^-, 3^-, 4^-, 5^- sequence. In addition, *s*-proton hole states in K^{42} were observed at 1.2, 2.01, and 2.13 MeV. A possible $d_{5/2}$-hole state was found in K^{41} at an excitation energy of 3.566 MeV.

I. INTRODUCTION

NUCLEON pickup reactions have been used successfully for a determination of the components of the ground-state wave functions of the target nuclei. They also provide information on the spin, parity, and some wave-function components of the final nuclei. The investigation of the Ca isotopes other than Ca^{40} is of interest, since these nuclei in the simple shell-model picture should have an $f_{7/2}$ neutron configuration outside the doubly closed *sd* shell. It is well known that the simple shell model is an oversimplification and that the admixtures of excited configurations in the ground-state wave functions are the rule rather than the exception. The simultaneous investigation of the five $Ca(d,t)$ reactions and the three $Ca(d,He^3)$ by use of the same techniques of target-thickness determination and distorted-wave Born-approximation (DWBA) analysis reduces the relative errors in a comparison of the spectroscopic factors and allows a somewhat more reliable analysis of doublets and mixed transitions.

The $Ca^{42}(p,d)Ca^{41}$ and $Ca^{44}(p,d)Ca^{43}$ reactions were investigated at $E_p=26.5$ MeV by Smith, Bernstein, and Rickey,[1] the $Ca^{43}(d,t)Ca^{42}$ reaction at $E_d=8.522$ MeV incident energy by Bjerregaard, Blieden, Hansen, Sidenius, and Satchler,[2] the $Ca^{48}(d,p)$ reaction at $E_d=18$ MeV by Peterson,[3] and the $Ca^{42}(He^3,\alpha)Ca^{41}$ reaction by Lynen, Bock, Santo, and Strock.[4] Earlier work on the $Ca^{44}(p,d)Ca^{43}$ and $Ca^{48}(p,d)Ca^{47}$ was reported by Conlon, Bayman, and Kashy.[5]

Recently, it has been shown that the spectroscopic factors obtained in the $Ca^{40}(d,p)$ reaction depend on the energy of the incident deuteron.[6] Since the incident deuteron energy of the $Ca^{43}(d,t)Ca^{42}$ reaction of Ref. 2 was quite low, differences between the results of that experiment and the present work are to be expected. In general, the results for strong transitions in the (p,d) and (d,t) experiments should be in good agreement. In a number of weak transitions some discrepancy in the assignment of *l* values or spectroscopic factors is likely to occur. In the (p,d) and (He^3,α) reactions, contributions from processes other than a direct reaction appear to be more probable than in the (d,t) reaction. Furthermore, the characteristic differences between the angular distributions associated with different *l* values are less pronounced in one reaction than the other, and therefore

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[1] S. M. Smith, A. M. Bernstein, and M. E. Rickey, Nucl. Phys. **A113**, 303 (1968).

[2] J. H. Bjerregaard, H. R. Blieden, O. Hansen, G. Sidenius, and G. R. Satchler, Phys. Rev. **136**, B1348 (1964).

[3] R. J. Peterson, Phys. Rev. **170**, 1003 (1968).

[4] U. Lynen, R. Bock, R. Santo, and R. Strock, Phys. Letters **22**, 9 (1967).

[5] T. W. Conlon, B. F. Bayman, and E. Kashy, Phys. Rev. **144**, 941 (1966).

[6] A. Denning, J. G. B. Haigh, and G. Brown, Phys. Letters **27B**, 159 (1968).

errors in assignment for weak transitions, which are not always fitted well by DWBA curves, cannot be ruled out. Information on the neutron-hole states can also be obtained from the $Ca^{40,42,44,46}(d,p)Ca^{41,43,45,47}$ reactions,[7–10] the $K^{39}(He^3,p)Ca^{41}$ reaction,[11] and the $Ca^{44}(p,t)Ca^{42}$ reaction.[12]

No information on the proton-hole states of K^{40}, K^{42}, and K^{43} has been published in the literature. However, the Ca^{41} and Ca^{43} states that are the analogs of the proton-hole states of K^{41} and K^{43} have been reported.[1,4]

II. EXPERIMENTAL PROCEDURE

The experiment was performed in the 60-in. scattering chamber[13] at the Argonne cyclotron. After magnetic analysis, the energy spread in the incident beam of 21.4-MeV deuterons was approximately 30 keV. Some runs were taken before the magnetic analysis became available and the $Ca^{46}(d,t)Ca^{45}$ reaction has not yet been repeated. A number of runs were taken after the energy of the incident beam was increased to 22.6 MeV. The scattered particles were detected with a $(dE/dx)-E$ telescope of surface-barrier detectors. In a number of runs on Ca^{42}, Ca^{43}, and Ca^{44}, the thickness of the dE/dx detector was chosen to permit simultaneous observation of the (d,He^3) reactions. Two pulse-multiplier circuits, one for each reaction, were used for particle identification. The targets were prepared by evaporation of isotopically enriched $CaCO_3$ onto a Formvar backing. The target thickness was measured by comparing the experimentally measured angular distribution and the theoretical angular distribution predicted from the optical-model-potential parameters used in the DWBA calculations. These parameters fit the elastic deuteron scattering on Ca^{40} at 21.6 MeV.[14] Since the targets contain an appreciable amount of O and C, the elastic scattering measurements were made from 30° to 80°. At smaller angles the deuterons from the oxygen were not completely separated from the deuterons scattered by Ca. A typical comparison of experimental and predicted angular distributions is given in Fig. 1. We have rather arbitrarily assigned an uncertainty of 10% to the target thickness determined by this process. The target thickness was then corrected for the isotopic composition of the target. The only targets with enrichments less than 98% in the enriched isotope were the Ca^{43} target with 81.1% and Ca^{46} with 31%.

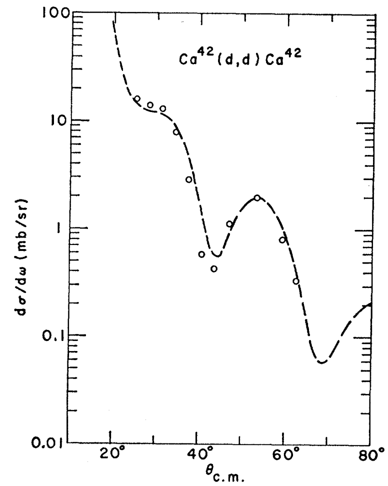

FIG. 1. Comparison between the angular distribution from the $Ca^{42}(d,d)$ reaction and the prediction of the optical-model potential of Table I.

It is well known that the necessary spectroscopic information for the four l values ($l=0, 1, 2, 3$) that occur in this range of atomic number and incident deuteron energy can be obtained between 12° and 30°. The secondary maximum of the $l=1$ angular distribution occurs near 27° and of the $l=0$ angular distribution near 21°. The primary maximum of $l=2$ occurs near 16° and for $l=3$ near 22°. In a few cases, angular distributions were obtained at angles larger than 30°, but these points have not contributed additional information. The optical-model-potential parameters used in the JULIE calculations[15] are listed in Table I. The He^3 and t parameters were suggested by Bassel. The zero-range local (ZR/L) approximation was used in the analysis without a lower cutoff (LCO) of the radial integrals. The justification for using the ZR/L approximation, with or without LCO, was that the finite-range nonlocal (FR/NL) approximation does not appear to be theoretically more valid in the case of deuteron pickup reactions. The attenuation of contributions from the interior of the nucleus, as produced by the FR/NL calculations, is undoubtedly necessary in (d,p) reactions, but there is still considerable uncertainty in the theoretical formulation. Since the (d,He^3) and (d,t) reactions are less known, it appeared at this point adequate to use the ZR/L approximation and allow for the possibility of errors in the comparison of the strength of transitions with different l values. Such an uncertainty is present anyway, because the strengths of the $l=0$ and $l=1$ transitions are deduced from the secondary maximum while those of $l=2$ and $l=3$ transitions are taken at the primary maximum.

[7] T. A. Belote, A. Sperduto, and W. W. Buechner, Phys. Rev. **139**, B80 (1965).
[8] W. E. Dorenbusch, T. A. Belote, and O. Hansen, Phys. Rev. **146**, 734 (1966).
[9] J. Rapaport, W. E. Dorenbusch, and T. A. Belote, Phys. Rev. **156**, 125 (1967).
[10] J. H. Bjerregaard, O. Hansen, and G. Sidenius, Phys. Rev. **138**, 1067 (1965).
[11] T. A. Belote, Fu Tak Dao, W. E. Dorenbusch, J. Kuperus, and J. Rapaport, Phys. Letters **23**, 480 (1966).
[12] S. M. Smith and A. M. Bernstein, Nucl. Phys. **A125**, 339 (1969).
[13] J. L. Yntema and H. W. Ostrander, Nucl. Instr. Methods **16**, 69 (1962).
[14] J. L. Yntema and G. R. Satchler, Phys. Rev. **134**, B976 (1964).
[15] The code JULIE was made available by Dr. R. M. Drisko.

Table I. Parameters of the optical-model potential.

Target+particle	V (MeV)	r_0 (F)	a_0 (F)	W (MeV)	W' (MeV)	a' (F)	r' (F)	V_{so} (MeV)	r_c (F)
Ca+d	105.0	1.02	0.86	···	60.0	0.65	1.42	6.0	1.3
$Ca^{42}+t$	178.5	1.139	0.86	14.1	···	0.769	1.675	···	1.4
$Ca^{43}+t$	176.9	1.139	0.86	14.7	···	0.769	1.662	···	1.4
$Ca^{44}+t$	175.3	1.139	0.86	15.3	···	0.769	1.65	···	1.4
$Ca^{46}+t$	172.3	1.139	0.86	16.5	···	0.769	1.624	···	1.4
$Ca^{48}+t$	169.4	1.139	0.86	17.7	···	0.769	1.60	···	1.4
$Ca^{42}+He^3$	180.9	1.139	0.86	14.1	···	0.769	1.675	···	1.4
$Ca^{43}+He^3$	180.4	1.139	0.86	14.7	···	0.769	1.662	···	1.4
$Ca^{44}+He^3$	179.5	1.139	0.86	15.3	···	0.769	1.624	···	1.4

In comparing the theoretical and experimental curves, the normalization constant 3 was used as suggested by Bassel.[16] The fact that the theoretical results of the normalization for the (d, He^3) reaction and the (d, t) reaction, which are expected to be the same, differ about 10% from the average theoretical results for the two reactions gives an indication of the uncertainty in the normalization constant.

The resolution width of the detection system varied from 70 to 130 keV full width at half-maximum (FWHM). The resolution depended on the target thickness as well as on the particular set of detectors used in a given run. The energy calibration was generally accurate to ±40 keV. The energy-resolution width for the Ca $^{6}(d, t)Ca^{45}$ experiment was about 250 keV.

III. EXPERIMENTAL RESULTS

A. $Ca^{42}(d, t)Ca^{41}$ Reaction

A spectrum of the $Ca^{42}(d, t)Ca^{41}$ reaction is shown in Fig. 2. The angular distribution of the group with energy near 2.0 MeV is shown in Fig. 3. The theoretical curve for an $l=2$ transition differs markedly from the experimental points near 13° and 27°; other $l=2$ transitions in this mass and energy region, e.g., the $Ca^{44}(d, t)Ca^{43}$ transition to the 0.99-MeV level, are generally in excellent agreement with the theoretical curves from 12° to 27°. Figure 3(b) is a plot of the residue when the theoretical curve was subtracted from the experimental points. The difference points are fairly well fitted with an $l=1$ theoretical curve if one shifts the theoretical curve by about 1.5° toward smaller angles. The need for such a shift has been found for several known $l=1$ transitions in the other Ca isotopes. This $l=1$ contribution is due to the excitation to the $\frac{3}{2}^-$ state at 1.94 MeV; the computed strength is $C^2S=0.3$. The ratio of intensities of .he $Ca^{40}(d, p)Ca^{41}$ reaction to the levels at 1.94 and 2.462 MeV is about 3:1. The region in which the transition to the 2.462-MeV level occurs was carefully investigated in the (d, t) reaction and the upper limit for the strength of the transiton to this level is 5% of the strength observed for the 1.94-MeV level. The $l=2$ strength of the transition to the $\frac{3}{2}^+$ level at 2.01 MeV is 2.4. This is to be compared with a maximum strength of 2.67 for transitions to $\frac{3}{2}^+$ levels with $T=\frac{1}{2}$ if one assumes that the $d_{3/2}$ level is completely filled.

The angular distributions of the other observed groups and of the ground-state transition are shown in

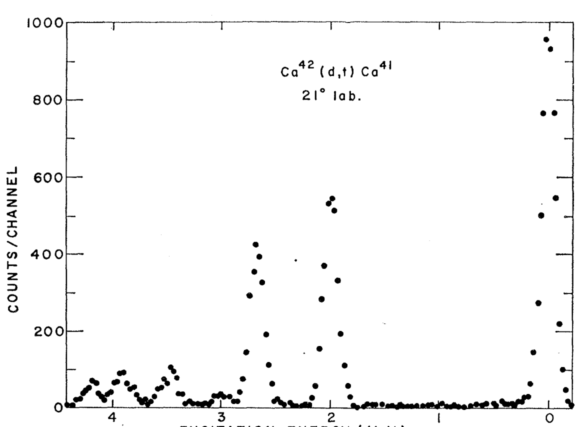

Fig. 2. Spectrum of the $Ca^{42}(d, t)Ca^{41}$ reaction.

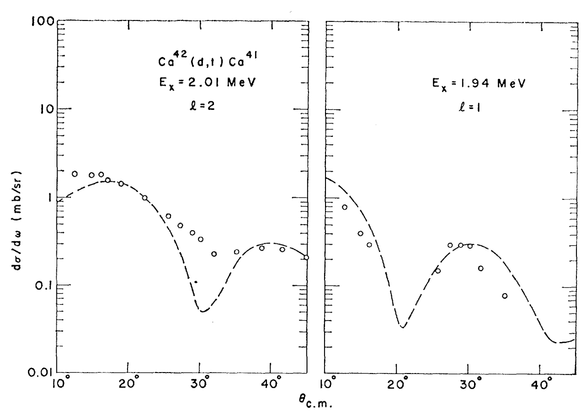

Fig. 3. Experimental angular distributions of the 2-MeV group in the $Ca^{42}(d, t)Ca^{41}$ reaction. At the left, the experimental points are compared with an $l=2$ distorted-wave curve (dashed). At the right, the points obtained by subtracting the theoretical $l=2$ values from the experimental points are plotted along with an $l=1$ calculated angular distribution (dashed).

[16] R. H. Bassel, Phys. Rev. 149, 791 (1966).

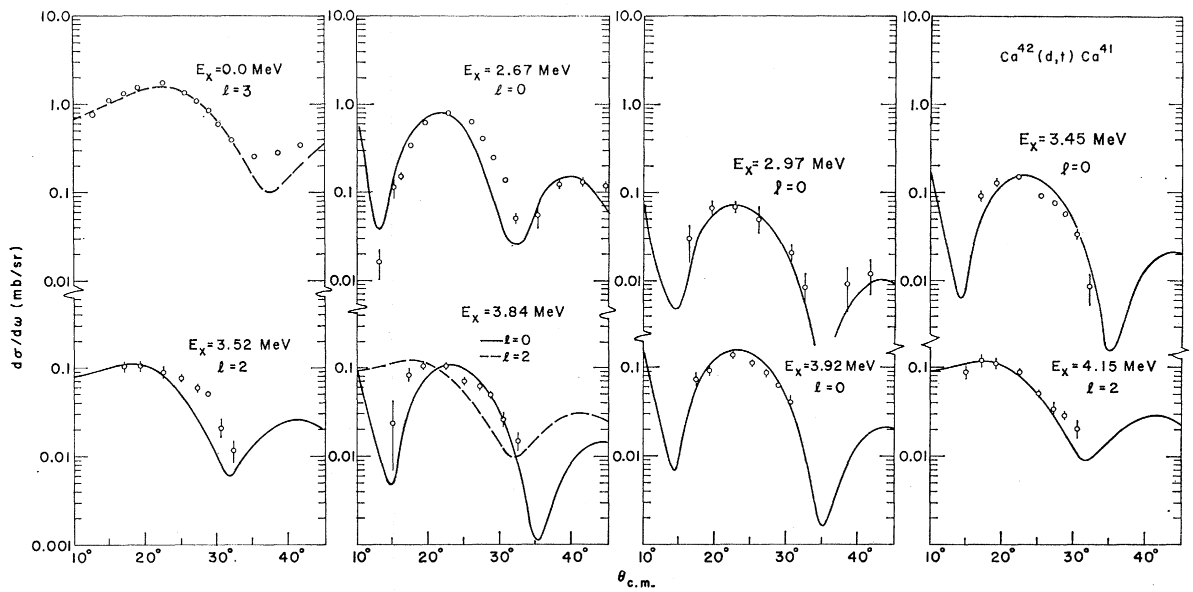

FIG. 4. Experimental angular distributions of groups in the $Ca^{42}(d, t)Ca^{41}$ reaction together with the calculated distorted-wave curves.

Fig. 4 together with the appropriate theoretical curves. The strength of the ground-state transition is 2. The excitation of the $\frac{1}{2}^+$ level at 2.67 MeV has a strength $C^2S=0.46$, compared with an expectation value 1.33 for a transition to a $\frac{1}{2}^+$ neutron-hole state with $T=\frac{1}{2}$. A level observed near 2.97 MeV is fitted quite well by an $l=0$ curve. The statistical errors do not permit one to rule out the possibility that this is an $l=3$ transition even though the fit of the $l=3$ curve both at small and larger angles is poor. The $\frac{1}{2}^+$ level at 3.45 MeV with $C^2S=0.12$ corresponds to the one found at 3.40 MeV in the $Ca^{40}(d, p)Ca^{41}$ reaction. The calculation of the excitation energy was affected by the nearby incompletely resolved resolved level at 3.52 MeV, which has a distinct $l=2$ characteristic and a strength of 0.30 if one assumes it to be a $\frac{3}{2}$ level. The level at 3.73 MeV does not appear to be excited with more than 30% of the intensity of the 3.52-MeV level. The $\frac{1}{2}^+$ level at 3.84 MeV is fitted with an $l=0$ curve with $C^2S=0.12$. The fit is not quite as good as for other $l=0$ transitions. The 3.92-MeV level has an $l=0$ angular distribution with $C^2S=0.12$. And, finally, a group observed near 4.15 MeV is fitted well by an $l=2d_{5/2}$ curve with $C^2S=0.46$. This transition cannot be fitted with an $l=3$ curve.

B. $Ca^{43}(d, t)Ca^{42}$ Reaction

A $Ca^{43}(d, t)Ca^{43}$ spectrum is shown in Fig. 5. The spectrum shows clearly that the 0^+ state at 1.84 MeV is excited in the reaction.

The angular distributions for the $Ca^{43}(d, t)Ca^{42}$ reaction to the ground state and second 0^+ state at 1.84 MeV in Ca^{42} are shown in Fig. 6 together with the calculated $l=3$ curves. The experimental and theoretical angular distributions are in excellent agreement. The computed strength of the ground-state transition is

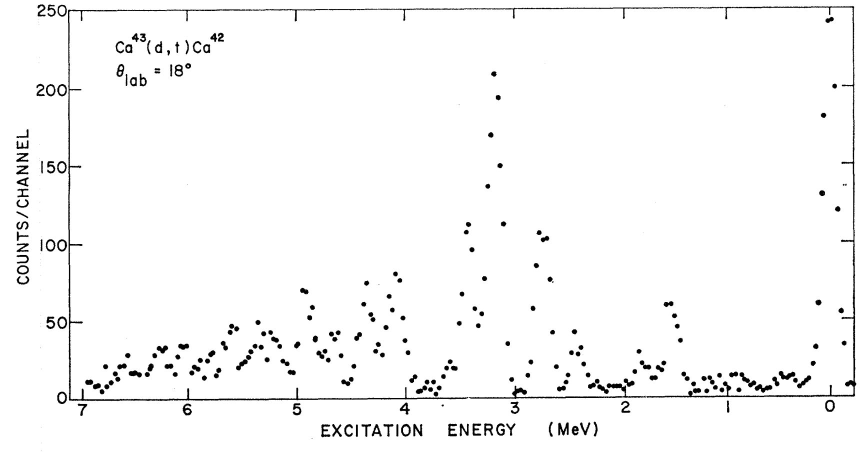

FIG. 5. Spectrum of the $Ca^{43}(d, t)Ca^{42}$ reaction.

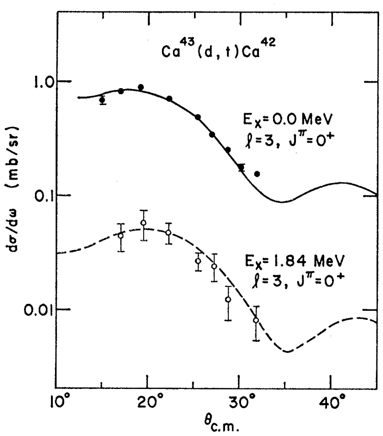

FIG. 6. Angular distributions of the $Ca^{43}(d, t)Ca^{42}$ transitions to 0^+ states of Ca^{42}.

$C^2S=0.50$, and that for the second excited state is $C^2S=0.05$.

The angular distributions to the 2^+ states at 1.52 and 2.43 MeV are shown in Fig. 7 together with the theoretical curves. The agreement between theoretical and experimental angular distributions is quite good; in particular, the fairly rapid variation in the shape of the experimental $l=3$ angular distributions with l values is reproduced quite well in the theoretical curves. The strengths of the transitions to these 2^+ states are $C^2S=0.16$ and $C^2S=0.11$, respectively.

The transitions to the 4^+ level at 2.75 MeV ($C^2S=0.47$) and the 6^+ state at 3.18 MeV ($C^2S=1.0$) are shown in Fig. 8. The angular distribution of the strong $l=0$ transition to the level at 3.44 MeV is shown in Fig. 9. This limits the spin of the state to 3^- or 4^-.

The angular distributions that could be analyzed in this experiment are shown in Fig. 10. The levels at 3.58, 4.72, and 5.67 MeV are in good agreement with the calculated $l=0$ curves. The strengths of these four levels are 0.09, 0.03, 0.04, and 0.05, respectively. The transition to the level at 3.78 MeV shows the deep minimum near 20° that is characteristic of an $l=1$ transition. The fit of the theoretical curve is similar to that for known $l=1$ transitions. The strength for this transition is 0.01. The

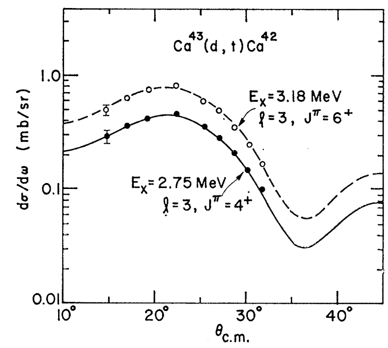

FIG. 8. Experimental angular distributions to the 2.75- and 3.19-MeV levels of Ca^{42} together with the calculated curves.

levels at 3.92, 4.02, 4.23, and 4.31 MeV are fitted quite well by the $l=2$ curve and have strengths of 0.12, 0.25, 0.14, and 0.15, respectively. The 4.43- and 4.97-MeV levels are probably $l=2$ transitions. The experimental angular distributions do not show quite as rapid a decrease in cross section near 30° as the others. The strengths of these two transitions are 0.23 and 0.29, respectively. The 4.56-MeV level is fitted reasonably well by an $l=3$ curve, and it has $C^2S=0.08$. The probability that this level has an angular distribution corresponding to a mixed $l=0$, $l=2$ transition cannot be ruled out. A typical case of a mixed $l=0$, $l=2$ transition is the 5.37-MeV group. For comparison purposes, the calculated $l=3$ angular distribution is also shown. The estimated $l=2$ strength is 0.17 and the $l=0$ strength is 0.04. The other transition for which an angular distribution could be extracted is the 5.87-MeV level with $l=0$, $C^2S=0.04$. An admixture of $l=2$ in this transition cannot be ruled out.

C. $Ca^{44}(d, t)Ca^{43}$ Reaction

A typical spectrum of the $Ca^{44}(d, t)Ca^{43}$ reaction at 18° lab is shown in Fig. 11. The experimental angular distributions and theoretical curves are shown in Fig. 12. The ground-state transition is fitted well by the $l=3$ curve and has $C^2S=4$. A weak group (not shown) was

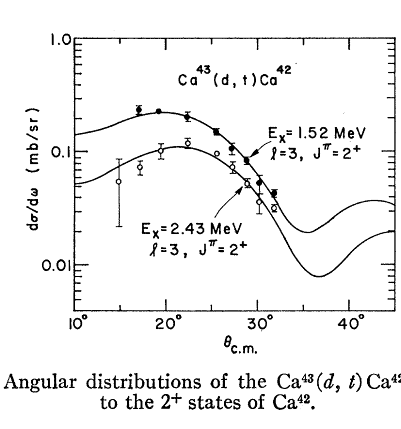

FIG. 7. Angular distributions of the $Ca^{43}(d, t)Ca^{42}$ reaction to the 2^+ states of Ca^{42}.

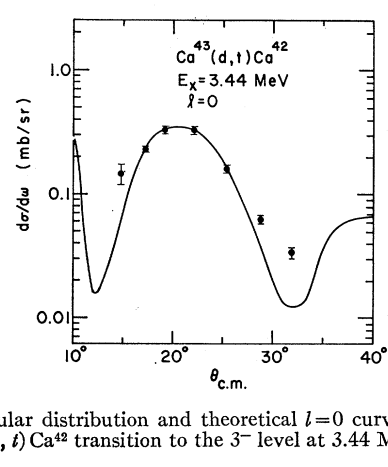

FIG. 9. Angular distribution and theoretical $l=0$ curves for the $Ca^{43}(d, t)Ca^{42}$ transition to the 3^- level at 3.44 MeV.

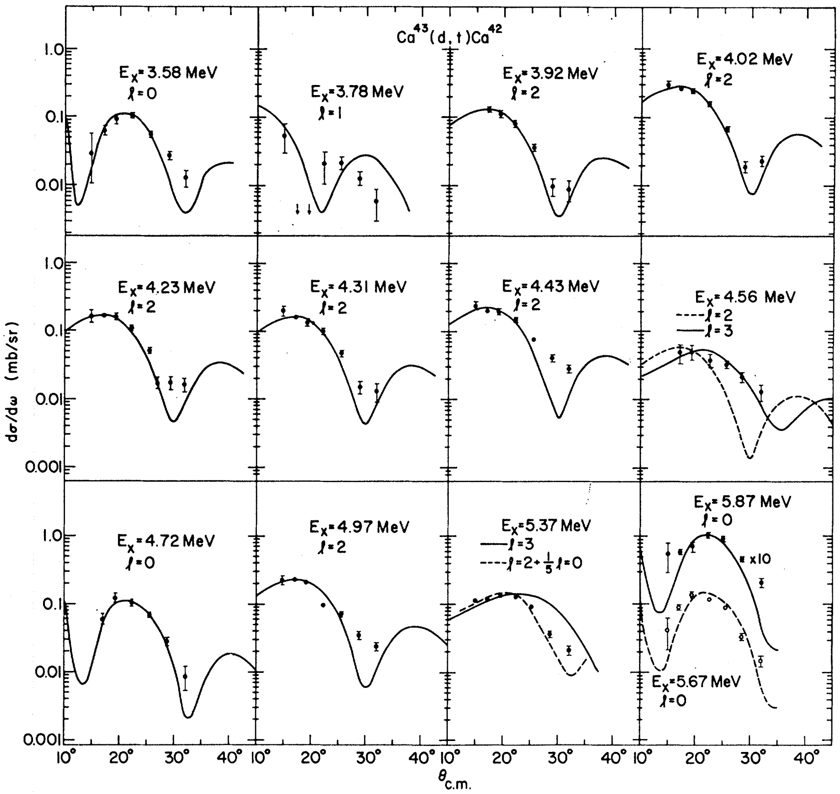

FIG. 10. Experimental angular distributions of groups observed in the $Ca^{43}(d, t)Ca^{42}$ reaction together with the calculated curves.

observed in the vicinity of the $\frac{5}{2}^-$ level at 0.374 MeV with $C^2S=0.12$ for an $f_{5/2}$ pickup. The $l=1$ transition to the $\frac{3}{2}^-$ level at 0.59 MeV has a strength of about 0.18. Agreement between the positions of the secondary maximum in the experimental and theoretical curves would require about the same shift as was needed for the $l=1$ curve found for the $Ca^{42}(d, t)Ca^{41}$ reaction to the 1.94-MeV level (the curve obtained by the subtraction procedure in Fig. 3). The strength of the $d_{3/2}$ transition to the 0.99-MeV level is 2.2, whereas the maximum value for $d_{3/2}$ transitions to $T=\frac{3}{2}$ levels is 3.2. The transition to the 1.389-MeV level is quite weak. If one assumes this to be a $\frac{3}{2}^+$ level, its strength is $C^2S=0.06$. The strength of the $2s_{1/2}$ level at 1.96 MeV is 0.9, as compared with a theoretical maximum of 1.6 for the transitions of $T=\frac{3}{2}$ states. The fit to the theoretical curve at small and large angles is not as good as in the case of the $l=0$ transition to the 2.67-MeV level of Ca^{41} and for several of the $l=0$ transitions in the $Ca^{43}(d, t)Ca^{42}$ reaction. This deviation is attributed to contributions from the transitions to the $\frac{3}{2}^-$ level at 2.048 MeV. If one subtracts the $l=0$ curve from the experimental points (much the same as was done for the $\frac{3}{2}^+$ group in Ca^{41}), one obtains a strength of 0.2 for the $l=1$ transition. In addition to these levels, there are four levels with $l=2$ angular distributions at 2.30, 2.74, 2.91, and 3.33 MeV with strengths of 0.2, 0.3, 0.22, and 0.4 MeV, respectively. However, the 3.33-MeV level is fitted rather poorly by an $l=2$ curve and it is equally justifiable to fit this transition with an $l=3$ curve. The only $l=0$ transition observed besides the strong level at 1.96 MeV is the state at 3.15 MeV with $C^2S=0.3$.

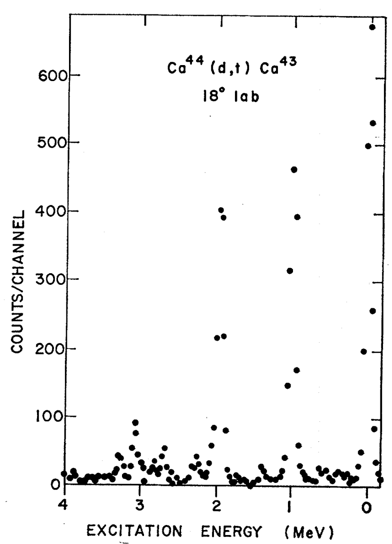

FIG. 11. Spectrum of the $Ca^{44}(d, t)Ca^{43}$ reaction.

D. $Ca^{46}(d, t)Ca^{45}$ Reaction

This reaction was studied before the magnetically analyzed beam at the cyclotron became available. The groups observed were the ground state ($l=3$, $C^2S=6$),

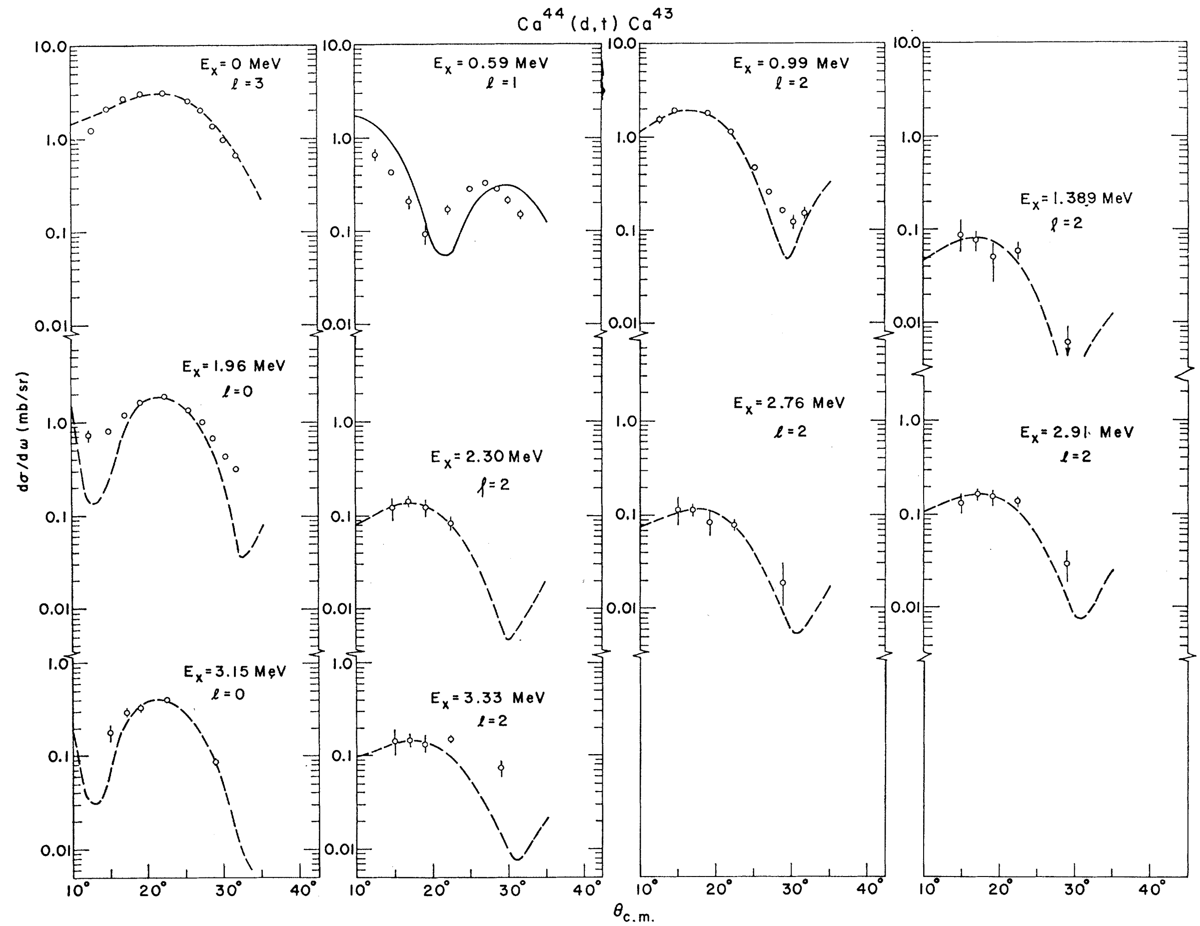

FIG. 12. Experimental angular distributions of transitions observed in the $Ca^{44}(d, t)Ca^{43}$ reaction together with the distorted-wave curves.

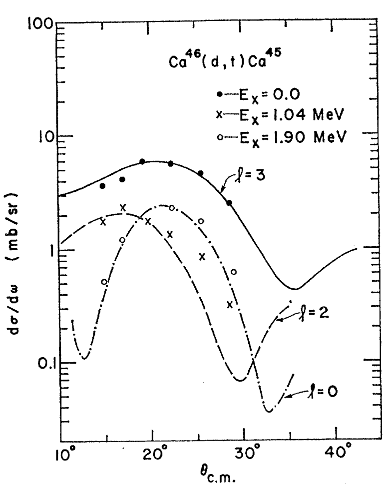

FIG. 13. Angular distributions of groups observed in the $Ca^{46}(d, t)Ca^{45}$ reaction.

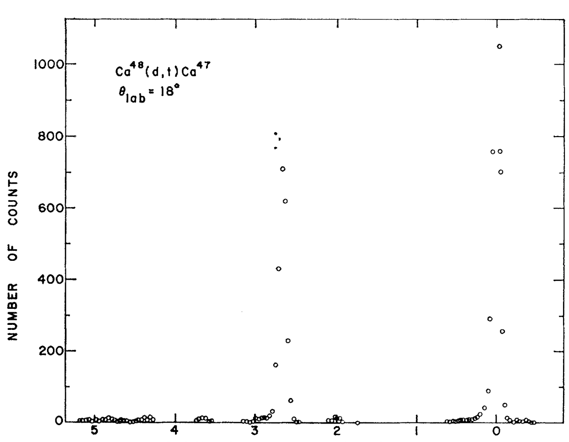

FIG. 14. Spectrum of the $Ca^{48}(d, t)Ca^{47}$ reaction at 18° lab.

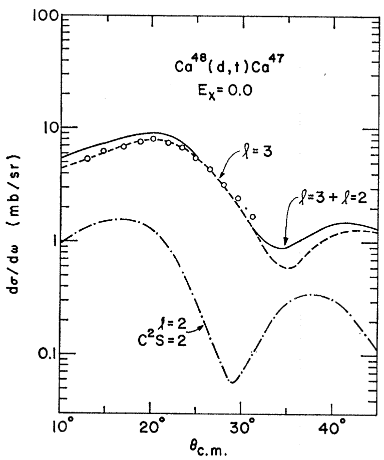

FIG. 15. Angular distribution of the $Ca^{48}(d,t)Ca^{47}$ ground-state reaction. The experimental points are compared with both the $l=3$ calculated curve (dashed) and the $l=3$, $C^2S=7$ curve, to which an $l=2$, $C^2S=2.0$ distribution has been added (solid).

the $\frac{3}{2}^+$ level at 1.89 MeV ($C^2S=2.3$), and a group at 2.40 MeV with $l=0$, $C^2S=1$. The angular distributions are shown in Fig. 13.

E. $Ca^{48}(d, t)Ca^{47}$ Reaction

A typical spectrum of the $Ca^{48}(d,t)Ca^{47}$ reaction at 18° is shown in Fig. 14. The angular distribution of the transition to the ground state of Ca^{47} is shown in Fig. 15. The experimental angular distribution is quite adequately fitted by an $l=3$ curve with $C^2S=7$. Bassel and French[17] have predicted that the $\frac{3}{2}^+$ level is very low in excitation and hence may not be resolved from the ground-state transitions. To show the effect of a low-lying $\frac{3}{2}^+$ state on the theoretical angular distribution, an $l=2$ theoretical distribution with $C^2S=2$ has been added to the $l=3$ curve. The experimental data do not rule out the possibility of some $l=2$ contributions but the theoretical curve without the addition of an $l=2$ contribution fits somewhat better.

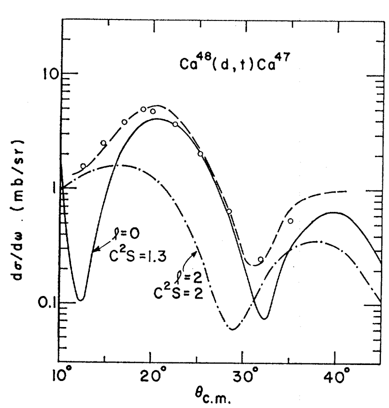

FIG. 16. Angular distribution of the $Ca^{48}(d,t)Ca^{47}$ reactions to the doublet at 2.60 MeV. The calculated curves for an $l=0$, $C^2S=1.3$ and $l=2$, $C^2S=2.0$ transition are shown as well as the sum of these two curves.

[17] R. K. Bassel and J. B. French, Phys. Letters 11, 143 (1963).

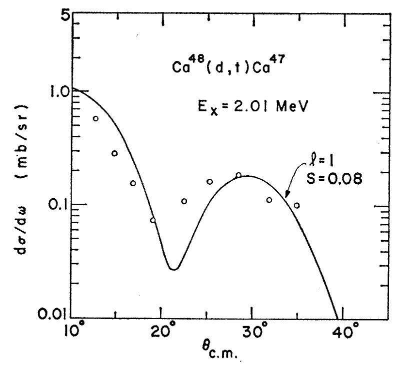

FIG. 17. Angular distribution of the transition to the $\frac{3}{2}^-$ level at 2.01 MeV in Ca^{47} and the theoretical curve for $2p_{3/2}$ pickup ($C^2S=0.08$).

The experimental angular distribution of the transition to the doublet at 2.60 MeV is shown in Fig. 16. The empirical systematics of $l=0$ transitions in the Ca isotopes indicates a definite need for an $l=0$ and $l=2$ admixture. The curves that have been added together to fit the data are $l=0$, $C^2S=1.3$, and $l=2$, $C^2S=2.0$. Empirically we expect the fit at the larger angles to be somewhat poorer than the one obtained with this combination; this suggests that the $l=0$ strength is somewhat overestimated and the $l=2$ strength underestimated. Somewhat better over-all agreement with the experimental data is obtained if the $l=0$ strength is reduced to 1.2 and the $l=2$ strength is increased to 2.5. The maximum $l=2$ strength that could be used is less than $C^2S=3$, with $C^2S=1.17$ for $l=0$. The angular distribution of the transition to the $\frac{3}{2}^-$ state at 2.01 MeV is shown in Fig. 17. The theoretical curve corresponds to a strength of 0.08 and exhibits a shift similar to the one observed for the other $l=1$ transtions.

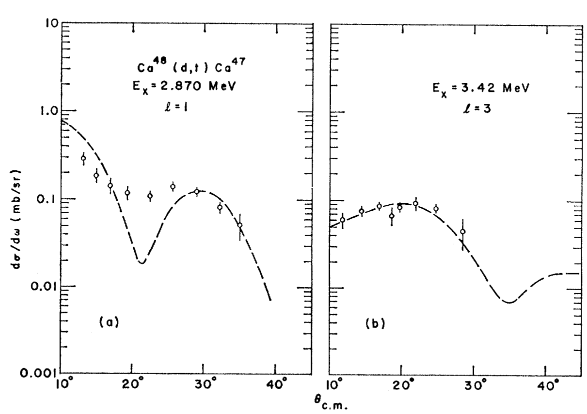

FIG. 18. Angular distributions for groups at 2.87 and 3.42 MeV in the $Ca^{48}(d,t)Ca^{47}$ reaction. The calculated curves are shown for comparison.

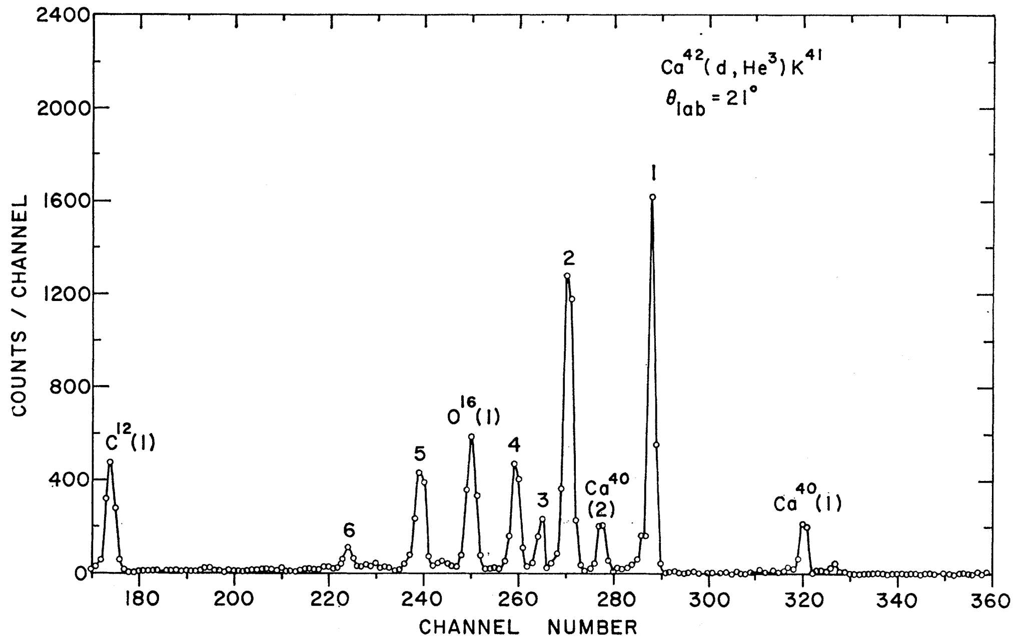

FIG. 19. Spectrum of the $Ca^{42}(d, He^3)K^{41}$ reaction.

Another $l=1$ transition is found at 2.870 MeV. The experimental and theoretical angular distributions are shown in Fig. 18(a) with $C^2S=0.05$. If there is an $l=3$ component in the angular distribution to this group, the upper limit for its strength is 0.03. The transition to the level near 3.42 MeV, shown in Fig. 18(b), has an angular distribution that may be fitted by an $l=3$ curve with $C^2S=0.09$ if one assumes that this is an $f_{7/2}$ pickup. The weak states at higher energies as well as the state near 3.30 MeV do not exhibit distinct direct-reaction patterns.

F. $Ca^{42}(d, He^3)K^{41}$ Reaction

A spectrum of the $Ca^{42}(d, He^3)K^{41}$ reaction is given in Fig. 19. There is a strong peak from the $O^{16}(d, He^3)N^{15}$ ground-state transition. The transitions to the ground

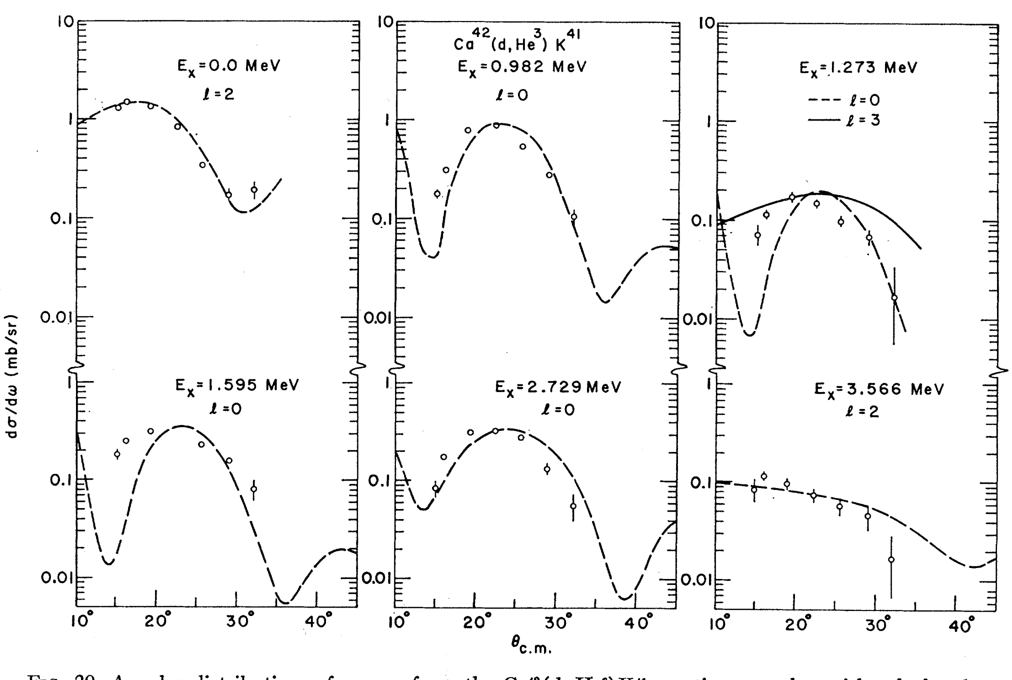

FIG. 20. Angular distributions of groups from the $Ca^{42}(d, He^3)K^{41}$ reaction, together with calculated curves.

state and first excited state of K^{39} from the Ca^{40} component in the target material were a convenient reference for energy calibration.

The angular distributions of the six groups from the $Ca^{42}(d, He^3)K^{41}$ reaction are shown in Fig. 20. The $l=2$ transition to the K^{41} ground state is fitted with an $l=2$ curve with $C^2S=3.3$. The next four states all contain fragments of the $2s_{1/2}$-hole states. There is a consistent shift of $\sim 1°$ between the calculated and experimental angular distributions. The strengths are $C^2S=0.9$ for the 0.98-MeV level, 0.17 for the 1.27-MeV, 0.4 for the 1.595-MeV, and 0.65 for the 2.73-MeV. There are several levels near 1.59 MeV, and it is possible that a weak $l=2$ transition is present, but it cannot be extracted from the data with any degree of certainty. The angular distribution for the 1.27-MeV level is fitted fairly well by the $l=0$ curve but quite badly by the $l=3$ curve that would be expected for a transition to the $\frac{7}{2}^-$ level at 1.29 MeV. The latter yields $C^2S=1.0$. If one were to add an $l=3$ with $C^2S=0.36$, the resulting curve would be acceptable. However, a stronger $l=3$ addition would cause the fit to deteriorate rapidly. Just as the $l=3$ curve for the $Ca^{48}(d, t)Ca^{47}$ ground-state reaction is modified only slightly by the addition of an $l=2$ component, the $l=0$ curve is only modified slightly by additions of $l=3$ components even when one adds as much as 3 times the value of C^2S given for the $l=0$. This indicates that for weak transitions, admixtures of considerable $l=3$ strength are not readily detected in the presence of an $l=0$ transition. The level at 3.566 MeV is shown together with the $d_{5/2}$ angular distribution calculated with $C^2S=1.0$.

G. $Ca^{43}(d, He^3)K^{42}$ Reaction

The spectrum of the $Ca^{43}(d, He^3)K^{42}$ reaction is shown in Fig. 21. The experimental angular distributions of nine states, along with the corresponding DWBA curves, are shown in Fig. 22. The three lowest levels were not clearly separated with the available resolution. The ground-state group was separated into three components by use of a computer program which used the shape of the transition to the 5^- state as a standard curve. The curves drawn through the distributions for the ground state and the 0.104-, 0.252-, and 0.68-MeV levels yield $C^2S=0.36$, 0.70, 0.92, and 1.07, respectively. Three levels which appear to correspond to $l=0$ transitions are observed at 1.20, 2.01, and 2.13 MeV and have $C^2S=0.53$, 0.27, and 0.27, respectively. Complete angular distributions for levels at 1.46 and 1.63 MeV could not be obtained since the $O^{16}(d, He^3)N^{15}$ ground-state transition occurs in this region. Part of the angular distribution of the 1.46-MeV level is shown in Fig. 22. For the level at 2.70 MeV, both the $l=2$ and the $l=0$ curves are shown.

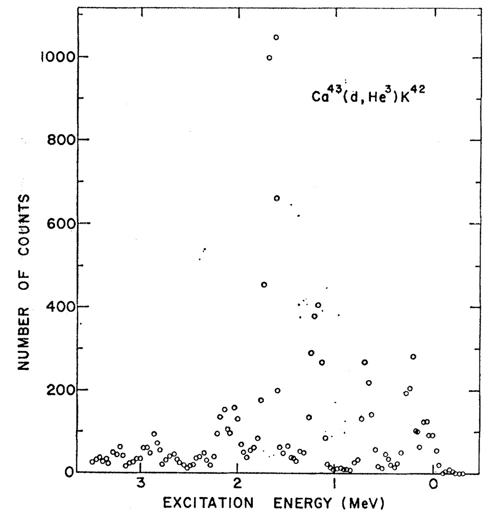

FIG. 21. Spectrum of the $Ca^{43}(d, He^3)K^{42}$ reaction.

TABLE II. Levels in Ca^{41} from the $Ca^{42}(d, t)Ca^{41}$ reaction.

E_x (MeV)	J^π	l	Spectroscopic factor C^2S (d, t)	(p, d)[a]	(He^3, α)[b]	$(2J+1)S$ (d, p)[c]
0	$\frac{7}{2}^-$	3	2	1.6	1.6	8
1.94	$\frac{3}{2}^-$	1	0.3	...	0.15	3.8
2.01	$\frac{3}{2}^+$	2	2.4	2.7	2.0	0.78
2.47	$\frac{3}{2}^-$	1	>0.015	...	0.01	1.11
2.67	$\frac{1}{2}^+$	0	0.46	0.34	0.65	0.035
2.97[d]	$(\frac{1}{2}^+)$	0	0.026			
2.97	$(\frac{5}{2}^-, \frac{7}{2}^-)$	3	0.13	0.17	0.14	
3.41	$\frac{1}{2}^+$	0	0.12	0.049	0.12	0.031
3.52	$(\frac{5}{2}^+)$	2	0.30[e]	0.31[e]	0.11[f]	
3.74	$(\frac{3}{2}^+)$	2	>0.1	0.28[e]	0.26	0.28
3.84	$\frac{1}{2}^+$	0	0.12	0.11	0.19	0.01
3.92	$(\frac{1}{2}^+)$	0	0.12	...[g]	0.03[h]	
4.15	$(\frac{5}{2}^+)$	2	0.46[i]	0.49[i]	0.13[g]	

[a] C^2S values were taken from Ref. 1 with the FR/NL approximation.
[b] C^2S values from Ref. 4.
[c] $(2J+1)S$ values from Ref. 7.
[d] This level could be fitted by $l=3$.
[e] C^2S corresponds to $1d_{3/2}$ pickup.
[f] Computed for an $l=3$ transition.
[g] Only three experimental points were available.
[h] Computed for an $l=1$ transition.
[i] Computed for $1d_{5/2}$ pickup.

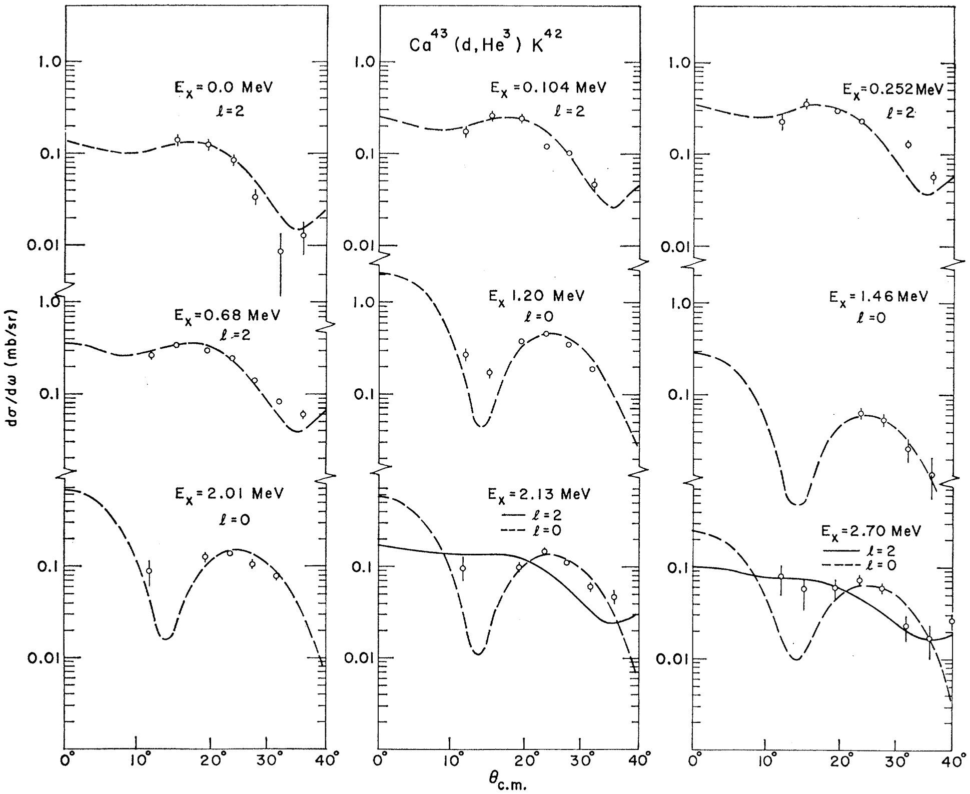

FIG. 22. Angular distributions of the groups observed in the $Ca^{43}(d, He^3)K^{42}$ reaction, together with calculated theoretical curves.

H. $Ca^{44}(d, He^3)K^{43}$ Reaction

A spectrum of the $Ca^{44}(d, He^3)K^{43}$ reaction at 21° lab is shown in Fig. 23. The $O^{16}(d, He^3)N^{15}$ ground-state transition appears between the ground state and first excited state of K^{43}. This made it impossible to obtain reliable data for the ground-state transition at angles smaller than 18°. The groups observed at excitation energies of 0.74 and 0.958 MeV were too weak to give useful angular distributions over a sufficient angular range. The experimental angular distributions and the theoertical curves for the ground state and the 0.56-MeV level are shown in Fig. 24. The ground state gives $l=2$, $C^2S=4.5$ with, of course, considerable uncertainty. The 0.56-MeV state is well fitted by $l=0$, $C^2S=2$.

IV. DISCUSSION OF RESULTS

A. $Ca^{42}(d, t)Ca^{41}$ Reaction

A summary of the results from this reaction is given in Table II, together with the results on the same levels as observed in the (p, d) reaction (Ref. 1), the (He^3, α) reaction (Ref. 4), and the $Ca^{40}(d, p)Ca^{41}$ reaction (Ref. 7). In general, the results of the present work are in good agreement with those of the (p, d) reaction. Our assignment of an $l=2$ transition to the levels at 3.52 and 4.15 MeV agrees with the (p, d) work but not with the $l=3$ assignment of the (He^3, α) work. The 3.74-MeV level, excited in both of the other experiments, is much weaker in the present experiment and could not be analyzed. The upper limit on the strength of this transition is $C^2S=0.10$. The transition to the 2.97-MeV level is fitted better by an $l=0$ theoretical curve than by an $l=3$ in the present experiment but the experimental angular distribution is not unique.

In contrast with the (p, d) experiment, which is not as sensitive to $l=1$ admixtures to a strong $l=2$ transition as is the (d, t) reaction, a pronounced deformation of the 2-MeV angular distribution by contributions from the nearby $\frac{3}{2}^-$ level was observed in the present

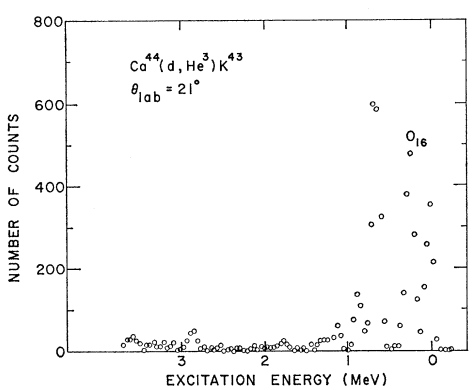

FIG. 23. Spectrum of the $Ca^{44}(d, He^3)K^{43}$ reaction.

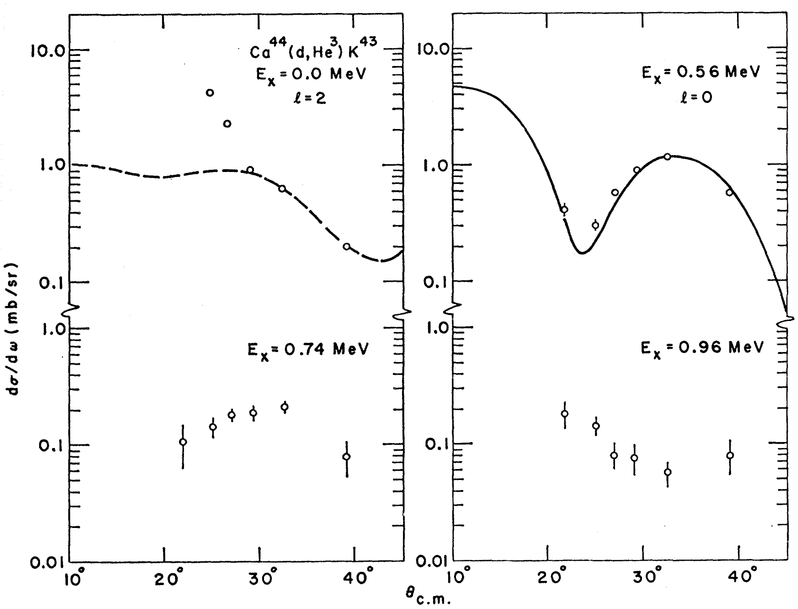

FIG. 24. Angular distributions and theoretical curves for the transitions to the ground state and first excited state in the $Ca^{44}(d, He^3)K^{43}$ reaction.

experiment. The ratio of 20:1 between the population of the 1.943-MeV level and that of the 2.47-MeV level is in good agreement with the 15:1 ratio found in the (He^3, α) experiment. The discrepancy between this ratio and the ~3.5:1 found in the $Ca^{40}(d, p)Ca^{41}$ reaction is consistent with the model used by Gerace and Green,[18] which assumes the presence of core-excited configurations in the Ca^{42} ground state. Such an assumption implies that the $2s_{1/2}$ and $1d_{3/2}$ also should be split.

The major part of the $d_{3/2}$-hole strength is concentrated in the 2.01-MeV level. One would not expect to observe the $d_{5/2}$-hole states in Ca^{41} in either

TABLE III. Results of the $Ca^{43}(d, t)Ca^{42}$ reaction.

E_x (MeV)	J^π	l	C^2S
0.0	0^+	3	0.50
1.53	2^+	3	0.16
1.84	0^+	3	0.05
2.43	2^+	3	0.11
2.75	4^+	3	0.47
3.18	6^+	3	1.0
3.44	3^-	0	0.09
3.58		0	0.03
3.78		1	0.01
3.92		2	0.12
4.02		2	0.25
4.23		2	0.14
4.31		2	0.15
4.43		2	0.23
4.56		(3)	0.08
4.72	(3^-)	0	0.04
4.97	(3^-)	2	0.29
5.37		{2	0.17
		{0	0.04
5.67	(3^-)	0	0.05
5.87	(0^+)	(0)	0.04

[18] W. J. Gerace and A. M. Green, Nucl. Phys. A93, 292 (1967).

the $Ca^{40}(d, p)Ca^{41}$ or the $K^{39}(He^3, p)Ca^{41}$ reactions[11] with an appreciable strength. Therefore, the 3.52- and 4.15-MeV transitions appear more likely to correspond to $d_{5/2}$-hole states.[19]

The 3.74-MeV level, which in the present experiments was not excited with sufficient intensity to allow the extraction of a reliable angular distribution, would be a $\frac{3}{2}^+$ level. The $\frac{1}{2}^+$ level at 2.68 MeV contains less than half of the expected $2s_{1/2}$ strength for $T=\frac{1}{2}$ hole states. The level at 3.40 MeV, which has a much stronger transition here than was reported in the (p, d) experiment, and the 3.86-MeV level have together about half the strength of the 2.68-MeV level. Less than 70% of the total $2s_{1/2}$ strength to $T=\frac{1}{2}$ states which would be expected from a filled $2s$ shell was observed. Another $\frac{1}{2}^+$ state was reported near 6 MeV in the (He^3, α) experiment. If the relative strength is correctly given, it follows that either the $2s$ shell is less than 75% filled or the summed $l=0$ strength is too low. The latter possibility is enhanced by the fact that the strength is extracted from the secondary maximum.

B. $Ca^{43}(d, t)Ca^{42}$ Reaction

The results of the present experiment are summarized in Table III together with those of the lower-energy (d, t) experiment.[2] The present result differs from the lower-energy data both in the relative intensities of the $l=3$ transitions and in the presence of a great number of rather strongly excited states in the region between 3 and 5-MeV excitation. Absolute spectroscopic factors were not obtained in Ref. 2. The theoretically expected sum of $f_{7/2}$ strengths is not obtained. Since the $l=3$

[19] D. S. Gemmell, L. Meyer-Schützmeister, H. Ohnuma, and N. G. Puttaswamy, Bull. Am. Phys. Soc. 12, 1183 (1967).

strength in Ca^{42}, Ca^{44}, and Ca^{48} is equal to the expected strength, it appears probable that in the case of Ca^{43} a number of weaker $l=3$ transitions to higher levels were not observed. As discussed in Sec. III F, a failure to detect even fairly strong $f_{7/2}$ transitions is quite probable when the levels are not well separated. The summed strength of the two 0^+ transitions is 0.55, as compared with an expectation value of 0.75. The excitation of the second 0^+ state was not observed in Ref. 2. In the $Ca^{44}(p,t)Ca^{42}$ reaction,[12] the 1.84-MeV level was weakly excited. It has been suggested[18] that these states are primarily 4p-2h states arising from the excitation of protons from the Ca^{40} core. Gerace and Green predict 19% of the 0_2^+ state in Ca^{42} to have a 2p-0h configuration with $1f_{7/2}$,[2] as compared with 66% for the ground state. Flowers and Skouras[20] predict 12 and 78%, respectively. The present result, which gives a ratio of 10:1 for the $0_1^+:0_2^+$ transitions, appears not to be inconsistent with this interpretation. The sum of the transitions to the 2_1^+ and 2_2^+ states is 0.27, about 65% of the theoretically expected 5/12. The intensity ratio of about 1.5:1 between the 2_1^+ and 2_2^+ transitions is larger than the 1:1 ratio reported in Ref. 2. The statistical errors is that case were, however, quite large.

In the $Ca^{44}(p,t)Ca^{42}$ reaction the relative intensity of the 2_2^+ transition was only 10% of that of the 2_0^+ transition. The strength of the transition to the 4^+ state is about 60% of the expected 9/12 and the transition to the 6^+ level has approximately the expected strength 12/13. The 6^+ state is close[21] to states at 3.25 and 3.30 MeV with possible 4^+ and 2^+ assignments. If there are transitions to those levels, they could not possibly be separated from the transition to the 6^+ level.

The $l=0$ transitions can proceed to either 3^- or 4^- states. These levels can also be reached by either $d_{3/2}$ or $d_{5/2}$ pickup. It is therefore surprising that the $l=0$ transitions to the levels at 3.44, 3.58, 4.72, and 5.67 MeV show no measurable admixture of $l=2$ transitions. The 3.44-MeV level presumably corresponds to the 3^- level at 3.44 MeV, and the 4.71- and 5.67-MeV levels appear to correspond to the 3^- levels at 4.75 and 5.69 MeV, which were observed in inelastic α scattering.[22] The 3^- level at 4.97 MeV is close to the $l=2$ transition at 4.98 MeV; the 3^- level at 5.51 MeV was not observed. The 5^- level reported at 4.11 MeV cannot be excited by an $l=0$ transition but could be excited by either $d_{3/2}$ or $d_{5/2}$ pickup. The 4^+ level reported at 4.45 MeV is fairly close to the 4.56-MeV level which was assigned an $l=3$ curve and which therefore should have positive parity. The only other positive-parity state observed in the present experiment is the 3.78-MeV level. It is obvious that a great many weaker states in this range of excitation energies could not be analyzed and these probably carry the missing strength for the $l=0$ and $l=2$ transitions.

C. $Ca^{44}(d,t)Ca^{43}$ Reaction

The experimental results are summarized in Table IV together with the corresponding information from the $Ca^{44}(p,d)Ca^{43}$ experiment.[1] The only discrepancy occurs for the 3.33-MeV level, which, in the (d,t) experiment, can be either an $l=2$ or $l=3$ transition and which was assigned $l=3$ in the (p,d) experiment.

The first excited state, which is presumably $\frac{5}{2}^-$ with a predominant $(f_{7/2})_{5/2-}{}^3$ configuration, is weakly excited. Since this state has approximately the same Q value as the $Ca^{42}(d,t)Ca^{41}$ ground-state transition, one must subtract the contribution from this reaction which is present because of the Ca^{42} component in the target. Even though the material from which the target is prepared contains presumably only a small amount of Ca^{42}, the possibility of contamination during target preparation and the inaccuracy inherent in the quantitative determination of a small component make an accurate quantitative statement about the strength of the $f_{5/2}$ transition impossible. The $C^2S=0.12$ given in Sec. III C is an upper limit. The ratio of the population of the $(\frac{3}{2}^-)_1$ state to that of the $(\frac{3}{2}^-)_2$ is 0.9—somewhat larger than the 0.56 reported in the (p,d) reaction. In view of the uncertainties involved in the extraction of spectroscopic factors of weak transitions, this can be considered to be a reasonably good agreement. In the $Ca^{42}(d,p)Ca^{43}$ reaction, the ratio is 0.07.

The 0.99-MeV level has only 70% of the expected strength for the $d_{3/2}$ transition to $\frac{3}{2}^+$, $T=\frac{3}{2}$ states, while in Ca^{41} the 2.01-MeV state almost exhausted the available $d_{3/2}$ strength. On the other hand, the 1.96-MeV level has a greater percentage of the available $\frac{1}{2}^+$ strength than the 2.67-MeV level has in Ca^{41}. The two $\frac{1}{2}^+$ levels observed in this experiment have about 75%

TABLE IV. Levels in Ca^{43} from the $Ca^{44}(d,t)Ca^{43}$ reaction.

E_x (MeV)	J^π	l	C^2S (d,t)	C^2S (p,d)[a]	$(2J+1)S$ (d,p)[b]
0.0	$\frac{7}{2}^-$	3	4.0	3.8	5.5
0.373	$\frac{5}{2}^-$	3	c	0.15	
0.59	$\frac{3}{2}^-$	1	0.18	0.10	0.21
0.99	$\frac{3}{2}^+$	2	2.2	2.5	0.52
1.40	$(\frac{3}{2}^+)$	2	0.06	0.16	(0.12)
1.96	$\frac{1}{2}^+$	0	0.9	0.62	0.1
2.05	$\frac{3}{2}^-$	1	0.2	0.18	3.0
2.30	$(\frac{3}{2}^+)$[d]	2	0.2	0.28	
2.74	$(\frac{3}{2}^+)$[d]	2	0.22	0.36	
2.90	$(\frac{3}{2}^+)$[d]	2	0.3	0.37	
3.15	$\frac{1}{2}^+$	0	0.3	0.22	
3.33	$(\frac{3}{2}^+)$[d]	(2)[e]	(0.4)	0.28[f]	

[a] Values for C^2S for the (p,d) reaction are taken from Ref. 1 and are obtained with a FR/NL approximation.

[b] Results from Ref. 8.

[c] Only an upper limit could be extracted.

[d] Values listed are those obtained on the assumption that the transition corresponds to $d_{3/2}$ pickup.

[e] This transition could also be fitted by an $l=3$ curve.

[f] C^2S computed for $f_{7/2}$ pickup.

[20] B. H. Flowers and L. D. Skouras, Nucl. Phys. **A116**, 529 (1968).

[21] P. M. Endt and C. Vander Leun, Nucl. Phys. **A105**, 1 (1967).

[22] E. P. Lippincott and A. M. Bernstein, Phys. Rev. **93**, 110 (1967).

of the available strength to $T=\frac{3}{2}$ states. The $l=0$ level at 2.753 MeV, reported in the $Ca^{42}(d, p)Ca^{43}$ work, was not excited in the present experiment, but a level with $l=2$ was seen at 2.74 MeV. Even a very small $l=0$ contribution is readily observable and, therefore, the strength of the (d, t) transition to the $\frac{1}{2}^+$ level at 2.75 MeV is much smaller than one would predict from the ratio of the transition strength to the 0.99-MeV level to that to the 2.75-MeV level for the case of the (d, p) reaction. Excitation of the 1.68-MeV level was not observed in the present experiment. The summed strength $\sum C^2S=2.95$ for the $l=2$ transitions (not including the possible $l=2$ transition to the 3.33-MeV level) almost exhausts the available strength for $d_{3/2}$, $T=\frac{3}{2}$ transitions.

D. $Ca^{46}(d, t)Ca^{45}$ Reactions

The results are summarized in Table V. There is a strong indication that the known $\frac{3}{2}^-$ level contributes to the angular distribution of the $\frac{3}{2}^+$ state near 1.90 MeV.

E. $Ca^{48}(d, t)Ca^{47}$ Reaction

The results are summarized in Table VI. The spectrum shown in Fig. 14 shows no strong $l=0$ or $l=2$ transitions in the region of excitation between 3 and 5 MeV. The doublet at 2.60 MeV contains the major part of the $d_{3/2}$- and $s_{1/2}$-hole strength. It is apparent that the (d, t) reaction permits a better guess at the relative contributions of $l=0$ and $l=2$ to the experimental angular distributions than does the (p, d) reaction.

The transition to the 2.870-MeV level is fitted by an $l=1$ angular distribution with $C^2S=0.05$. The $Ca^{46}(d, p)Ca^{47}$ reaction[10] shows two levels in this region at 2.849 and 2.874 MeV, both assigned $\frac{1}{2}^-$. The latter is much more strongly excited than the former. In his study of the $Ca^{48}(p, d)Ca^{47}$ reaction, Peterson[3] also found that this region included a group which appeared to have an $l=3$ transition. His result does not seem to preclude the $l=1$ transition with the strength found in the present experiment. In the case of $l=1$ transitions, however, fairly weak $l=3$ transitions fill up the minimum in the angular distribution quite quickly. It is estimated that an $l=3$ component in this angular distribution could not have a strength in excess of $C^2S=0.03$. The transition at 2.01 MeV, which is far stronger than this transition, is fitted rather poorly by the theoretical curve; and, since the present result already precludes an $l=3$ transition with more than 30% of the value obtained from the (p, d) reaction, it seems superfluous to postulate a $\frac{7}{2}^-$ or $\frac{5}{2}^-$ state in this region.

TABLE V. Levels in Ca^{45} from the $Ca^{46}(d, t)Ca^{45}$ reaction.

E_x (MeV)	J^π	l	C^2S
0.0	$\frac{7}{2}^-$	3	6
1.89	$\frac{3}{2}^+$	2	2.3
2.40	$\frac{1}{2}^+$	0	1

TABLE VI. Levels in Ca^{47} from the $Ca^{48}(d, t)Ca^{47}$ reaction.

E_x (MeV)	J^π	l	C^2S (d, t)	C^2S (p, d)[a]
0	$\frac{7}{2}^-$	3	7.0	6.5
2.01	$\frac{3}{2}^-$	1	0.08	0.05
2.60	$\frac{1}{2}^+$	0	1.2	
2.60	$\frac{3}{2}^+$	2	2.5	
2.87	$(\frac{1}{2}^-)$	1	0.05	0.08[b]
3.42	$(\frac{7}{2}^-)$	3	0.09	0.08

[a] Results of the (p, d) reaction are taken from Ref. 3. The values are those in which the well depth was adopted to give the separation energy.
[b] Calculated for $l=3$.

The state near 3.296 MeV is quite weakly excited and does not show a stripping pattern. The spectroscopic factor is less than 0.02, in contradiction to the (p, d) results, which gave a strength greater than that observed for the 2.01-MeV level. The (p, d) and (d, t) experiments agree quite well on the 3.42-MeV level. In contrast, the weak states at higher energies do not exhibit angular distributions with sufficiently developed stripping patterns to permit meaningful comparisons with distorted-wave calculations.

F. $Ca^{42}(d, He^3)K^{41}$ Reaction

The results of the $Ca^{42}(d, He^3)K^{41}$ reaction are summarized in Table VII. The ground-state transition has $C^2S=3.3$, as compared with an expectation value of 4. In a simple picture, it is expected that the strength of the $2s_{1/2}$-hole state should be concentrated in a single level as is the case in K^{43}. In K^{41} the strength appears to be split among four levels with a calculated centroid at 1.65 MeV. This is fairly close to the excitation energy of the 2^+ state in Ca^{42}. It is then possible to make a $\frac{1}{2}^+$ state by coupling a $d_{3/2}$ hole to the Ca^{42} 2_1^+ level which might be in the vicinity of the s^{-1} state and therefore might mix strongly with it. It appears likely that the strong $l=2$ transition to the 3.566-MeV level corresponds to part of the $d_{5/2}$-hole state.

Some of the analog levels in Ca^{41} have been reported.[1,4] Tbe analog to the ground state $(C^2S=2.3)$ and the transition to analogs of a 2.44-MeV level $(C^2S=0.26)$ are listed as $d_{3/2}$ pickup reactions. An $l=3$ transition to the analog of the $\frac{7}{2}^-$ level at 1.29 MeV $(C^2S=0.37)$ and two $l=0$ transitions to the analogs of the 0.98-MeV level $(C^2S=0.54)$ and 2.67-MeV level $(C^2S=0.40)$ are also listed. These values of C^2S are well known to be overestimated; a corrected value for the transition to the $\frac{7}{2}^-$ level is $C^2S=0.2$. In Ref. 1, the FR/NL values of C^2S for the first three analogs in K^{41} are given as 1.7, 0.85, and 0.20.

In general, the strength of the isobaric analog in Ca^{41} should be $\frac{1}{3}$ of the strength observed in the (d, He^3) reaction. This indicates that the strength of the isobaric analog to the ground state is overestimated by about a factor of 2 in the case of the (He^3, α) reaction and at

TABLE VII. Levels in K^{41} from the $Ca^{42}(d, He^3)K^{41}$ reaction.

E_x (MeV)	J^π	l	Spectroscopic factor C^2S (d, He^3)	(He^3, α)[a]	(p, d)[b]
0	$\frac{3}{2}+$	2	3.3	2.3	1.7
0.98	$\frac{1}{2}+$	0	0.9	0.54	0.93
1.27	$\frac{1}{2}+$	0	0.12	···	···
1.29	$\frac{7}{2}-$	(3)	(0.36)[c]	0.37	0.20
1.59	$\frac{1}{2}+$	0	0.4	···	···
2.73	$\frac{1}{2}+$	0	0.65	0.40	···
3.57	$(\frac{5}{2}+)$	2	1.0	0.67[d]	···

[a] C^2S for the $Ca^{42}(He, \alpha)Ca^{41}$ reaction (Ref. 4) to isobaric analog states in Ca^{41}. These numbers would be expected to be $\frac{1}{3}$ the C^2S of the $Ca^{42}(d, He)K^{41}$ reaction.

[b] C^2S for the $Ca^{42}(p, d)Ca^{41}$ reaction (Ref. 1) to isobaric analog states in Ca^{41}. These numbers are also expected to be $\frac{1}{3}$ of the C^2S values from the $Ca^{42}(d, He)K^{41}$ reaction and should be the same as the ones in the previous column. The values are derived from a FR/NL calculation.

[c] Upper limit allowed by the experimental data. The value 0 is equally permissible.

[d] Computed for $d_{3/2}$ neutron capture.

least 25% for the (p, d) reaction. The strength of the transition to the isobaric analog of the 0.98-MeV level is again overestimated by a factor of 2 for the (He^3, α) reaction but by almost a factor of 3 for the (p, d) reaction. From this it follows that the spectroscopic information derived from isobaric analog states is somewhat uncertain. From the discussion in Sec. III. F, it follows that the value $C^2S=0.4$ for an $l=3$ transition to the 1.29-MeV level is the upper limit on the value the experimental angular distribution allows for the transition to the $\frac{7}{2}-$ level in K^{41}.

G. $Ca^{43}(d, He^3)K^{42}$ Reaction

The results for the $Ca^{43}(d, He^3)K^{42}$ reaction are summarized in Table VIII. On the basis of the simple model, the strengths of the 2^-, 3^-, 4^-, and 5^- levels should be proportional to $2J+1$ and this appears to hold in this case. The angular distributions of the first and second excited states are somewhat affected by the transition to the $\frac{1}{2}+$ level of K^{39}, which appears because of the Ca^{40} component of the target. The transition to the ground state of N^{15} from the oxygen in the target made it impossible to obtain an angular distribution for the 1.63-MeV level. It is probable that the 1.46-MeV level corresponds to an $l=0$ strength. The summed $d_{3/2}$ strength for the four lowest states is $\sum C^2S=3.07$. It appears likely that there are additional $d_{3/2}$ and $s_{1/2}$ pickup transitions which were not observed in the present experiment.

V. SUM RULES

On the basis of the shell model, the sum of the strengths is expected to be equal to the nucleon occupation number. Since the strengths are derived from the comparison between the absolute differential cross sections (which are uncertain by 10%) and a distorted-wave calculation (which involves a normalization factor having a uncertainty of 10% and which makes a number of approximations), it is clear that an accurate determination of the nucleon occupation number by nucleon pickup reactions is in practice impossible. In the present series of experiments, target thicknesses were determined by identical techniques and the results were analyzed by the same DWBA program. Hence the error in the relative values of the occupation numbers for the same l value on the different targets should be much smaller than that in the absolute individual numbers.

The values of $\sum C^2S$ are listed in Table IX together with the expectation values for $l=0$, 2, and 3 and the observed percentage of the expectation value. The expectation value is based on the assumption of a pure $(f_{7/2})^n$ neutron configuration outside a doubly closed s-d shell. The expectation value of the $l=3$ transition is thus simply n. For the neutron transitions with $l=0$ and $l=2$, only the $T-\frac{1}{2}$ states are observable and the expectation value given is therefore the total strength to the $T-\frac{1}{2}$ states of four $d_{3/2}$ particles and two $s_{1/2}$ particles. In the case of proton pickup, a fully closed d-s proton shell is assumed and it follows that $\sum C^2S=4$ for the $d_{3/2}$ pickup and $\sum C^2S=2$ for $2s_{1/2}$ pickup in all these isotopes.

For the (d, t) reaction on the even-A isotopes, the table shows that the expectation value of the $l=3$ transition is obtained; the value of $\sum C^2S$ for $l=0$ transitions is about 70% and $\sum C^2S$ for $l=2$ transitions averages near 80%. In contrast, these three l values for Ca^{43} are consistently low. It appears probable that the remaining $l=3$ strength is scattered among fairly weak states, and this is also likely for part of the $l=2$ strength. The strength of the $l=0$ transitions is quite low. States with only a small fraction of $s_{1/2}$-hole-state configurations are readily detected, since the (d, t) reaction is rather sensitive to $l=0$ transitions. Therefore it seems likely that the greater part of the $l=0$ transition proceeds to states with higher excitation than 6 MeV or that $\sum C^2S$ for $l=0$ transitions is indeed much smaller for Ca^{43} than for either Ca^{42} or Ca^{44}. The latter is probable because the separation of the centroids of the $f_{7/2}{}^{n-1}$ and $s_{1/2}{}^{-1}$ configurations on the basis of the present results is much larger for Ca^{43} than for either Ca^{42} or Ca^{44}, while one would expect it to be smaller than the larger of the two. The admixture of $2p_{3/2}$ configurations in the neutron wave functions of all Ca isotopes has

TABLE VIII. Levels in K^{42} from the $Ca^{43}(d, He^3)K^{42}$ reaction.

E_x (MeV)	J^π	l	C^2S (d, He^3)
0.0	2^-	2	0.36
0.104	(3^-)	2	0.70
0.252	(4^-)	2	0.92
0.68	(5)	2	1.07
1.2	$(3^-, 4^-)$	0	0.53
1.46	···	(0)	(0.08)
1.63	···	···	···
2.01	$(3^-, 4^-)$	0	0.27
2.13	$(3^-, 4^-)$	0	0.27
2.70	···	···	···

TABLE IX. Sum rules for Ca isotopes.

Reaction	$\Sigma\, C^2S_{l=0}$ Expt.	$\Sigma\, C^2S_{l=0}$ Theory	%	$\Sigma\, C^2S_{l=1}$ Expt.	$\Sigma\, C^2S_{l=2}$ Expt.	$\Sigma\, C^2S_{l=2}$ Theory	%	$\Sigma\, C^2S_{l=3}$ Expt.	$\Sigma\, C^2S_{l=3}$ Theory
$Ca^{42}(d,t)Ca^{41}$	0.85	1.33	66	0.3	2.4	2.7	90	2	2
$Ca^{43}(d,t)Ca^{42}$	0.32	1.5	21	0.01	1.35	3.0	45	2.33	3
$Ca^{44}(d,t)Ca^{43}$	1.2	1.6	75	0.38	3.18	3.2	100	4	4
$Ca^{46}(d,t)Ca^{45}$	1.0	1.72	60		2.3	3.43	66	6	6
$Ca^{48}(d,t)Ca^{47}$	1.2	1.78	68	0.13	2.5	3.56	70	7	8
$Ca^{42}(d,He^3)K^{41}$	2.07	2.00	100		3.3	4.0	82	(0.4)	0
$Ca^{43}(d,He^3)K^{42}$	1.15	2.00	58		3.05	4.0	75		
$Ca^{44}(d,He^3)K^{43}$	2.0	2.00	100		4.5	4.0	110		

been observed. It seems that the admixture is significantly smaller for Ca^{48} than for either Ca^{42} or Ca^{44}.

In the (d, He^3) reactions, the value $\sum C^2S=4.5$ for the $1d_{3/2}$ transition in $Ca^{44}(d, He^3)K^{43}$ is obviously quite uncertain. Somewhat smaller values of $\sum C^2S$ are to be expected for Ca^{43} than for the two even-A isotopes. The fairly good agreement of the ground-state strength indicates that the low values for C^2S in the $Ca^{43}(d, t)Ca^{42}$ are not due to normalization errors. Comparison of the (d, t) and (p, d) results indicated a fairly good agreement on the $l=3$ transitions. The $l=2$ transitions give a somewhat lower sum for the (d, t) reaction, in somewhat better agreement with the expectation values; while for $l=0$ the (d, t) sum is about 50% higher than for the (p, d) results and also in better agreement with the expectation values. Such differences between different l values are to be expected in the present state of the art and indicate the degree of uncertainty in the extraction of spectroscopic factors from these experiments. The summation over the same levels yields fairly good agreement between the (He^3, α) and (d, t) results, but such agreement can be somewhat fortuitous since the (He^3, α) reaction usually requires the use of an arbitrary unhappiness factor.

VI. CONCLUSIONS

The comparison between different neutron pickup reactions at a sufficiently high energy yields generally comparable results for strong transitions. The sensitivities of these reactions to unresolved or mixed angular distributions are quite different. In the (He^3, α) experiments it is difficult to distinguish between $l=2$ and $l=3$ reactions. In the (d, p) reactions, neither $l=1$ and $l=0$ nor $l=2$ and $l=0$ are readily separated, while in the (d, t) reaction fairly strong $l=3$ transitions in the vicinity of $l=0$ transitions are almost completely masked.

Sensitivities, even in the energy region in which direct reactions are quite predominant, may be different at different energies. Recently the j dependence in the (d, He^3) reaction[23] was shown to be energy-sensitive.

In (p, d) reactions at low outgoing deuteron energy, contributions from other than direct reactions tend to obscure the result from the direct-reaction process. A marked discrepancy, presumably also due to the contributions from other reaction processes at low incident deuteron energies, is found between the (d, t) reactions at different energies.

The spectroscopic factors extracted for transitions to isobaric analog states in the (p, d) and (He^3, α) reactions are not in good agreement with the results from the (d, He^3) reaction. This implies that conclusions on the proton configuration of the target nucleus cannot be reliably drawn from the excitation of isobaric analog states.

Comparison between the relative intensities of neutron pickup reactions to $\frac{3}{2}^-$ states and those obtained from the (d, p) reactions implies that core-excited admixtures are present in both the Ca^{42} and Ca^{44} ground-state wave functions. The $d_{3/2}$- and $s_{1/2}$-hole states appear to be split into several components in both Ca^{41} and Ca^{43}, with the most pronounced splitting apparent in the $s_{1/2}^{-1}$ states in Ca^{41}. The single-particle proton-hole states for the $d_{3/2}^{-1}$ configuration in K^{41} and K^{43} and the $s_{1/2}^{-1}$ configuration in K^{43} appear rather pure, while the $s_{1/2}^{-1}$ configuration in K^{41} shows rather pronounced fractionation. This latter effect can possibly be explained by the interaction of the s^{-1} state with another $\frac{1}{2}^+$ state, e.g., the one corresponding mainly to an $[(f_{7/2})_{2^+}^{2}]_\nu[d_{3/2}^{-1}]_\pi$ configuration which might occur at almost the same energy in K^{41}. The most pronounced splitting of d-s neutron-hole states occurs for the s^{-1}-hole states in Ca^{41} and a similar mixing with a $(f_{7/2})_{2}{}^{2}(d_{3/2})^{-1}$ configuration could account partially for such splitting.

The transition to the 0_2^+ level in Ca^{42} was observed with 10% of the intensity of the ground-state transition.

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The transition to the 2_1^+ level of Ca^{42} is about 50% more intense than the transition to 2_2^+ level. It is suggested that the transitions to the lowest six states do not exhaust the available $f_{7/2}$ strength. The spectroscopic factor for observed $l=0$ transitions in $Ca^{43}(d, t)Ca^{42}$ strongly suggests that the spectroscopic factor for this transition is significantly smaller than would be expected on the basis of the simple shell model. The $Ca^{48}(d, t)Ca^{47}$ result rules out the possibility of a small separation of the centroids of the $f_{7/2}^{-1}$ and $d_{3/2}^{-1}$ holes in Ca^{47}. The greatest part of the s^{-1} and $d_{3/2}^{-1}$ strength is found in the 2.60-MeV doublet. The $2p_{3/2}$-neutron admixture in the ground-state wave function of Ca^{48} is smaller than in those of Ca^{44} and Ca^{42}, but a significant $p_{1/2}$-neutron admixture was observed which did not appear present in the other two nuclei.

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Positron Decays of ^{42}Ti and ^{38}Ca

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The positron decay of ^{42}Ti has been investigated using the $^{40}Ca(h, n)$ reaction with a natural calcium target. A strong 610.7±0.5-keV γ ray with a half-life $T_{1/2}=0.202\pm0.005$ sec is attributed to a positron branch to the 611-keV level of ^{42}Sc. A weak 2222±2-keV γ ray has been observed, which corresponds to a positron branch to the 2223-keV level of ^{42}Sc. Using this measured half-life and the β-decay energies, the branching ratios are calculated from β-decay systematics to be $r_{\beta 0}{}^+=(44.1\pm1.2)\%$, $r_{\beta 1}{}^+=(55.2\pm1.2)\%$, and $r_{\beta 9}{}^+=(0.7\pm0.3)\%$. The log$ft$ values 3.179±0.015 and $4.36_{-0.24}^{+0.16}$ establish $J^\pi=1^+$ for both the 611- and the 2223-keV states of ^{42}Sc. The positron decay of ^{38}Ca has been investigated using the $^{36}Ar(h, n)$ reaction. The half-life 0.439±0.012 sec has been measured for this decay. In addition to the strong Gamow-Teller transition feeding the 1695-keV level of ^{38}K, a weak 3210±2-keV γ-ray line has been observed which is tentatively attributed to a transition from the 3337-keV level in ^{38}K fed by a (0.6±0.2)% β^+ branch of ^{38}Ca. Its logft ($3.90_{-0.13}^{+0.10}$) then establishes the $J^\pi=1^+$ assignment for this state.

I. INTRODUCTION

THE isotope ^{42}Ti has a mass 6983±8 keV greater than ^{42}Sc, and must therefore decay by superallowed β^+ emission to its isobaric analog ($J^\pi=0^+$, $T=1$) ground state of ^{42}Sc.[1,2] Using the above value for the total decay energy and the comparative half-life[3] for the superallowed 0^+ to 0^+ transitions, $ft=3123\pm31$ sec, one finds the corresponding half-life for ^{42}Ti to be 0.458±0.005 sec.

The first reported evidence for the β^+ decay of ^{42}Ti has been that of Oberholtzer,[4] who used the $^{40}Ca(h, n)$ reaction (the notation h for helion,[5] is used for ^{3}He) and observed, with a plastic scintillator, positrons with energy 6.0±0.6 MeV and a half-life of 0.25±0.04 sec, which he attributed to ^{42}Ti.

The present study of ^{42}Ti was undertaken to measure more accurately the half-life of ^{42}Ti and to search for γ rays following Gamow-Teller transitions from ^{42}Ti to $J^\pi=1^+$ states of the self-conjugate nucleus ^{42}Sc. Such strong transitions have already been observed in the $A=4N+2$ series of nuclei up to ^{30}P and in ^{38}Ca.[6–8]

The isotope ^{38}Ca decays by superallowed β^+ emission to its isobaric analog level ($J^\pi=0^+$, $T=1$) at an excitation of 127 keV and by a strong Gamow-Teller transition (logft=3.41±0.09) to the 1695-keV state in ^{38}K.[8] On the basis of a recently proposed sum rule[9] for the reduced matrix elements for the Gamow-Teller transitions in the $A=4N+2$ nuclei, one expects an additional strength of at least 0.1 in $||\int\sigma||^2$ to other levels in ^{38}K. The study of ^{38}Ca was undertaken to search for such additional transitions to $J^\pi=1^+$ states in ^{38}K.

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