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Transmission Spectrum of a Metal Slab in the Presence of a Normal Magnetic Field*

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A theory is developed to calculate the electromagnetic transmission spectrum of a metal slab in the presence of a normal static magnetic field. The boundaries are explicitly included in the development. The transmitted signal is shown to be composed of contributions from the zeros of the infinite-medium dispersion relation (that is, helicon modes), and from the branch points of the conductivity which give the Gantmakher-Kaner oscillations. Results of several numerical calculations are given as examples.

I. INTRODUCTION

THE propagation of electromagnetic waves in a metal or a semiconductor, in the presence of an external magnetic field approximately in the direction of propagation, was predicted by Aigrain and by Konstantinov and Perel,[1] was detected by Bowers *et al.*,[2] and has been investigated by a large number of authors for a number of years.[3] In metals or doped semiconductors, these waves are commonly referred to as "helicons" and have been used to investigate certain features of the Fermi surface of metals, for instance, the Gaussian curvature of the Fermi surface in the direction of the magnetic field[4,5] and properties such as carrier density and the sign of the carriers in semiconductors.[6]

Gantmakher and Kaner[7] observed rapid oscillations in the surface impedance of a slab sample of tin in the "helicon" geometry, that is, when the external magnetic field is normal to the surface of the slab. They attributed these oscillations to "ineffective electrons" with extremal values of the average velocity along the magnetic field direction. Since $m_c\bar{v}_z \sim \partial A/\partial k_z$, where m_c is the cyclotron mass and $\bar{v}_z$ the average velocity component along $\mathbf{B}_0$, the electrons concerned are those which lie near an extremum in $\partial A/\partial k_z$, the derivative of the cross-sectional area of the Fermi surface in the direction of the applied static field $\mathbf{B}_0$. Extremal values of $\partial A/\partial k_z$ occur at elliptic limiting points of the Fermi surface, or for a nonellipsoidal Fermi surface, they may occur in a region of the Fermi surface with a finite transverse dimension of the electron orbit. An extremum in $\partial A/\partial k_z$ implies that a large group of electrons will follow a helical path through the specimen with an effective wave vector $q_c = \omega_c/\bar{v}_z$, where $\omega_c = eB_0/m_c c$ is the cyclotron frequency, and will arrive with a phase shift with respect to a reference signal at the other side of the slab.

Weisbuch and Libchaber[8,9] have also seen these "Gantmakher-Kaner" oscillations. They have seen these oscillations in two directions in Cu, but only in conjunction with the beginning of helicon oscillations, i.e., a little above and somewhat below what would appear to be the conventional absorption edge. Their

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[1] P. Aigrain, in *Proceedings of the International Conference on Semiconductor Physics, Prague, 1960* (Academic Press Inc., New York, 1961), p. 224; O. V. Konstantinov and V. I. Perel, Zh. Eksperim. i Teor. Fiz. 38, 161 (1960) [English transl.: Soviet Phys.—JETP 11, 117 (1960)].

[2] R. Bowers, C. Legéndy, and F. Rose, Phys. Rev. Letters 7, 339 (1961).

[3] For a list of references see S. J. Buchsbaum and R. Bowers in *Proceedings of the Symposium on Plasma Effects in Solids, Paris, 1964*, edited by J. Bok (Academic Press Inc., New York, 1965); D. P. Morgan, Phys. Status Solidi 24, 9 (1967).

[4] E. A. Stern, Phys. Rev. Letters 10, 91 (1963).

[5] J. L. Stanford and E. A. Stern, Phys. Rev. 144, 534 (1966).

[6] J. K. Furdyna, Phys. Rev. Letters 16, 646 (1966).

[7] V. F. Gantmakher and E. A. Kaner, Zh. Eksperim. i Teor. Fiz. 48, 1572 (1965) [English transl.: Soviet Phys.—JETP 21, 1053 (1965)].

[8] G. Weisbuch and A. Libchaber, Phys. Rev. Letters 19, 498 (1967).

[9] When this paper was essentially finished, I learned that B. Perrin and A. Libchaber (to be published) have observed Gantmakher-Kaner oscillations in three directions in copper, and presumably one of these is due to an elliptic limiting point.

interpretation was in terms of three waves present in the metal: a helicon wave, a Gantmakher-Kaner wave, and a damped electromagnetic wave which excites the Gantmakher-Kaner wave. It is not quite clear what this damped wave represents, or, from the analysis, why the Gantmakher-Kaner waves exist only over a limited region of magnetic field. For further discussion of this paper see Antoniewicz *et al.*[10]

Theoretical work, in which the presence of a boundary or a surface was explicitly considered, has been concentrated principally on the surface impedance of a semi-infinite slab.[11–13] McGroddy *et al.*[13] investigated the surface impedance of metals with Fermi surfaces which exhibit an orbit edge, that is, a helicon absorption edge due to electrons with extremal values of velocity, and showed a qualitative difference between this result and the free-electron result.

The theory for the transmission geometry, that is, including the second boundary of a slab, was first done by Platzman and Buchsbaum,[14] where they considered a free-electron metal with specular reflection of the electrons at the boundaries of an infinite slab. However, they investigated the transmitted power at frequencies near the cyclotron frequency. Legéndy[15] investigated the effect of boundaries on the observed spectra, but his investigation was for a local conductivity (independent of the wave vector), and therefore none of the nonlocal effects appear in his results. Finally, Baraff[16] has investigated the transmission at higher frequencies through a free-electron metal slab with diffuse reflection of electrons at the boundary.

The author[17] has pointed out that, for a non-free-electron model of the Fermi surface, it is possible for a damped helicon mode to exist below the usual absorption edge if there is an extremum in $\partial A/\partial k_z$. The above paper considered the infinite-medium dispersion relation. The author, Wood, and Gavenda[10] interpreted experimental results in the [110] direction in copper as the enhancement of Gantmakher-Kaner oscillations because of the damped helicon below the absorption edge. The interpretation was made on the basis of a model calculation containing the essential features of the copper Fermi surface and having the boundaries explicitly included. The qualitative agreement of this model calculation with experimental results was very good. The present paper gives the basis on which the conclusions in the above letter were reached and includes some further numerical results.

In this paper, the transmission of an electromagnetic field through a slab of metal in the presence of an external magnetic field normal to the slab surface and propagating along the magnetic field is investigated. The effect of the boundaries on the transmission is explicitly taken into account, and the experimental situation is mimicked by retaining the transmitted electric field instead of the power. This is important, since experimentally one detects the interference of the transmitted electric field and a leakage field around the specimen. Specular reflection at the boundaries is assumed for simplicity. It will be shown that in this case the transmitted spectrum depends on the zeros and branch points of the infinite-medium dispersion relation. Each zero and branch point contribute an oscillatory term, as a function of the external magnetic field, to the spectrum.

The dispersion relation for a circularly polarized wave in a metal may be written as

$$q^2=(4\pi\omega/c^2)(\mathrm{Im}\sigma_\pm - i\,\mathrm{Re}\sigma_\pm)\,, \qquad (1)$$

where q is the complex wave vector, ω is the circular frequency, $\sigma_\pm=\sigma_x\mp i\sigma_y$ the conductivity, and c is the speed of light. A static magnetic field is assumed along the z axis. It should be noted that, if for some reason the real part of the conductivity is much smaller than the imaginary part, a wave is propagated with little attenuation. In contrast, however, below the absorption edge for a spherical Fermi surface, the real part of the conductivity for a helicon is approximately the same magnitude as the imaginary part, and, therefore, the helicon does not exist.

The conductivity tensor for a surface that is cylindrically symmetric about the magnetic field is[12]

$$\sigma_\pm=\frac{e^2}{4\pi^2\hbar^2}\int_{-k_z\,\mathrm{max}}^{k_z\,\mathrm{max}}\frac{m_c\tau v_\perp^2dk_z}{1+i(\omega\mp\omega_c-qv_z)\tau}\,, \qquad (2)$$

where τ is the relaxation time (assumed constant on the surface), $v_\perp(k_z)$ is the velocity in the k_x-k_y plane, and v_z is related to the geometry of the Fermi surface through the derivative of the cross-sectional area of the Fermi surface (for a closed orbit), that is, $m_c\bar{v}_z = -(\hbar/2\pi)(\partial A/\partial k_z)$.

The dispersion relation, Eq. (1), is satisfied by all of the modes in the electromagnetic spectrum with the proper polarization. It should be emphasized that if the conductivity depends on the wave vector $\mathbf{q}$, then the dispersion relation may have other solutions or branches besides the helicon. The nonlocal conductivity arises from the motion of the electrons; they sample the electric field of the electromagnetic waves at different positions in the metal.

II. THEORY

Let us take a metallic slab extending to infinity in the x-y directions and assume that there is specular

[10] P. R. Antoniewicz, L. T. Wood, and J. D. Gavenda, Phys. Rev. Letters **21**, 998 (1968).
[11] P. B. Miller and R. R. Haering, Phys. Rev. **128**, 126 (1962).
[12] A. W. Overhauser and Sergio Rodriguez, Phys. Rev. **141**, 431 (1966).
[13] J. C. McGroddy, J. L. Stanford, and E. A. Stern, Phys. Rev. **141**, 437 (1966).
[14] P. M. Platzman and S. J. Buchsbaum, Phys. Rev. **132**, 2 (1963).
[15] C. R. Legéndy, Phys. Rev. **135**, A1713 (1964).
[16] G. A. Baraff, Phys. Rev. **167**, 625 (1968).
[17] P. R. Antoniewicz, Phys. Letters **24A**, 83 (1967).

reflection at the boundaries. Then, an equivalent model, consisting of the slab repeated to infinity both in the positive and negative z directions, may be investigated.[14,18] That is, the problem is treated as in the case of an infinite solid, except that the electric field and its derivative are forced to assume given values at the "boundaries" of the slabs.

The equations of motion of the system are the macroscopic Maxwell's equations describing the self-consistent electromagnetic field in the metal and a Boltzmann transport equation describing the motion of the electrons in this field. The conductivity tensor then represents the motion of the electrons.[12]

It is assumed that all quantities except the static magnetic field vary as $e^{i\omega t}$. Since a slab geometry is considered, where the x and y dimensions are infinite, all amplitudes will be functions of the z coordinate only. Consider propagation in a direction with at least a threefold symmetry. In terms of circularly polarized coordinates, one can then write Maxwell's equations as

$$\partial^2 E_\pm/\partial z^2+(\omega^2/c^2)D_\pm=(4\pi i\omega/c^2)J_\pm\,,$$

where

$$E_\pm=E_x\pm iE_y\,,\quad \text{etc.}$$

The current in an infinite sample is

$$\mathbf{J}(z,\omega)=\int_{-\infty}^{\infty}\boldsymbol{\sigma}(z,z',\omega)\cdot\mathbf{E}(z')dz'$$

and

$$J_\pm(z,\omega)=\int_{-\infty}^{\infty}\sigma_\pm(z,z',\omega)E_\pm(z')dz'.$$

Then, assuming specular reflection at the boundaries, the material may be considered homogeneous and the effect of the boundaries is included by stipulating the electric field and its derivative at the boundary,[14,18,19] that is,

$$J_\pm(z)=\int_{-\infty}^{\infty}\sigma_\pm(z-z')E_\pm(z')dz'$$

and

$$\partial^2E_\pm(z)/\partial z^2+(\omega^2K/c^2)E_\pm(z)$$
$$=(4\pi i\omega/c^2)J_\pm(z)+2\sum_{n=-\infty}^{\infty}[E'(0)\delta(z-2nL)$$
$$+E'(L)\delta(z-(2n+1)L)]\,,\quad(3)$$

where $E'(0)$ and $E'(L)$ are the derivatives of the electric field with respect to the z direction and are evaluated at $z=0$ and $z=L$, and where K is the ionic dielectric constant. The fields are symmetric with respect to $z=0$; therefore,

$$E_\pm(z)=\sum_{n=0}^{\infty}E_n^\pm\cos k_nz\,,$$

where $k_n=n\pi/L$, and L is the thickness of the slab. Expanding all other relevant quantities in similar series (see Appendix), one gets

$$E_n^\pm=-2b_n[k_n^2-(\omega^2K/c^2)+(4\pi i\omega/c^2)\sigma_n^\pm]^{-1}\,,$$

where

$$b_n=(1/L)[E'(0)+E'(L)\cos k_nL]\,,$$
$$b_0=(1/2L)[E'(0)+E'(L)]\,,\quad(4)$$

and

$$\sigma_n^\pm=\int_{-\infty}^{\infty}\sigma_\pm(y)\cos(k_ny)dy.$$

The electric field finally becomes

$$E_\pm(z)=-\sum_{n=0}^{\infty}\frac{(2-\delta_{n0})\cos(k_nz)b_n}{k_n^2-(\omega^2/c^2)[K+(4\pi/i\omega)\sigma_n^\pm]}.\quad(5)$$

Let us define the quantities

$$A_\pm(z)=-\frac{1}{L}\sum_{n=0}^{\infty}\frac{(2-\delta_{n0})\cos(k_nz)}{k_n^2-(\omega^2/c^2)[K+(4\pi/i\omega)\sigma_n^\pm]}\,,$$
$$B_\pm(z)=-\frac{1}{L}\sum_{n=0}^{\infty}\frac{(2-\delta_{n0})(-1)^n\cos k_nz}{k_n^2-(\omega^2/c^2)[K+(4\pi/i\omega)\sigma_n^\pm]}\,;\quad(6)$$

then we have

$$E_\pm(z)=A_\pm(z)E_\pm'(0)+B_\pm(z)E_\pm'(L).\quad(7)$$

Evaluating the above at $z=0$ and $z=L$, and noting that $B(L)=A(0)$ and $A(L)=B(0)$, we obtain for the fields immediately inside the boundaries

$$E_\pm(0)=B_\pm(L)E_\pm'(0)+A_\pm(L)E_\pm'(L)\,,$$
$$E_\pm(L)=A_\pm(L)E_\pm'(0)+B_\pm(L)E_\pm'(L).\quad(8)$$

Consider the case of an incident electromagnetic wave propagating in vacuum from the left onto a metal slab, partially entering the slab, and partially reflected. On the other side of the slab, a wave is emitted and continues propagating to the right. Since only plane waves at normal incidence are being considered, one can consider the tangential fields only. The tangential component of the electric field is continuous across a boundary, and since nonmagnetic metals are being investigated and there are no surface currents, the components of the magnetic field are also continuous. Since the tangential components of the magnetic fields are continuous, it follows that the derivatives of the tangential components of the electric fields with respect to z are also continuous across the boundary. Finally, all of the above comments apply to circularly polarized waves. Therefore, the relations between the

[18] G. L. Flint, Jr., M.S. thesis, University of Texas, 1967 (unpublished).
[19] C. Kittel, *Quantum Theory of Solids* (J. Wiley & Sons, Inc., New York, 1963), p. 313.

electric fields at the left boundary are

$$E_{\pm i}+E_{\mp r}=E_{\pm}(0),$$
$$E_{\pm i}'+E_{\mp r}'=-ik(E_{\pm i}-E_{\mp r})=E_{\pm}'(0),$$

where k is the free-space wave vector. At the right-hand boundary one gets

$$E_{\pm}(L)=E_{\pm t},\quad E_{\pm}'(L)=E_{\pm t}'=-ikE_{\pm t},$$

where the subscripts i, r, and t refer to the incident, reflected, and transmitted waves, respectively.

Substituting these relations between the electric fields into Eq. (8) and eliminating $E_{\mp r}$ from them, one finally obtains

$$\frac{E_{\pm t}}{E_{\pm i}}=\frac{-i2kA_{\pm}(L)}{k^2B_{\pm}^2(L)+k^2A_{\pm}(L)+1}, \tag{9}$$

the ratio of the transmitted electric field to the incident wave electric field. The ratio of the electric fields, $E_{\pm t}/E_{\pm i}$, represents both the ratio of the magnitudes of the two circularly polarized fields and also the phase relation between them. The detected signal, which consists of the sum of the transmitted and the leakage signals, is proportional to the real part of $E_{\pm t}/E_{\pm i}$.

For situations under consideration, namely, the transmission through a metal slab, the quantities $k^2A_{\pm}^2(L)$ and $k^2B_{\pm}^2(L)$ are small compared to 1 and consequently one may consider only the numerator in the expression for $E_{\pm t}/E_{\pm i}$, that is,

$$E_{\pm t}/E_{\pm i}\simeq -i2kA_{\pm}(L). \tag{10}$$

The subsequent numerical calculations, however, were done with the full expression as a check.

In order to see, qualitatively, the contributions to the transmitted field, one has to investigate the quantity

$$A_{\pm}(L)=-\frac{1}{L}\sum_{n=0}^{\infty}\frac{(2-\delta_{n0})(-1)^n}{k_n^2-(\omega^2/c^2)(K+(4\pi/i\omega)\sigma_n{}^{\pm})}. \tag{11}$$

The above sum may be expressed in a closed form. Since $\sigma_n{}^{\pm}$ is symmetric in n (i.e., $\sigma_n{}^{\pm}=\sigma_{-n}{}^{\pm}$), we have

$$A_{\pm}(L)=\sum_{n=-\infty}^{\infty}(-1)^n f(n),$$

where

$$f(n)=-\frac{1}{L}\left[k_n^2-\frac{\omega^2}{c^2}\left(K+\frac{4\pi}{i\omega}\sigma_n{}^{\pm}\right)\right]^{-1}.$$

A device found in Morse and Feshbach[20] is then used: One finds the sum by a contour integration of an auxiliary function. One requires a function which has simple poles at $z=n$ and residues $(-1)^n$. Such a function is $\pi\csc\pi z\, f(z)$. It is also bounded at $|z|\to\infty$. The contour integral of the above function has to be found:

$$F=\oint \pi\csc\pi z\, f(z)dz.$$

It should be noted that $z\csc\pi z\, f(z)$ is bounded and goes to zero as $|z|\to\infty$, and, therefore, one can take the contour to infinity. One can not in fact take the contour to infinity at all points if the function $f(z)$, or equivalently the conductivity $\sigma(z)$, has a branch point. Then F is not zero but its value is equal to the integrals along the branch cuts. Now

$$F=\sum_{n=-\infty}^{\infty}(-1)^n f(n)$$
$$+\sum[\text{residues of } \pi\csc\pi z\, f(z) \text{ at the poles of } f(z)].$$

Therefore,

$$A(L)=F-\sum[\text{residues of } \pi\csc\pi z\, f(z) \text{ at the poles of } f(z)].$$

The poles of $f(z)$ are the solutions of the infinite-medium dispersion relation. That is, the number of poles will correspond to the number of branches of the infinite-medium dispersion relation and, consequently, each branch will contribute to the transmitted electric field essentially via the periodic function $\csc\pi z_0$. The amplitude of this contribution will depend on the residue of the function $f(z)$ at the pole z_0 because of that branch, and also on the imaginary part of z_0 through $\csc\pi z_0$. The transmitted signal is ocillatory as a function of some parameter—for instance, the static magnetic field. The periodicity of this signal will depend on the infinite-medium dispersion relation of this particular branch as a function of the variable parameter.

The other term in the result, F, is due to the branch points of the conductivity, $\sigma(z)$, in the complex wave-vector plane. It was shown by Gantmakher and Kaner[7] that the branch points of the $\sigma(z)$ arise from elliptic limiting points and other extrema in $\partial A/\partial k_z$ of the Fermi surface under consideration. The contributions to F are due to the integrals around the branch points of $\sigma(z)$ and running parallel to the imaginary axis. Then,

$$F_j=\int_{\text{along cut}}\pi f(z)\csc\pi z dz$$

is the contribution due to a branch point at z_j. Make a change of variable $w=z-z_j$ which gives

$$F_j=\pi\int_{i\infty}^{0} f(w+z_j)\csc\pi(w+z_j)dw$$
$$+\int_{0}^{i\infty} f(w+z_j)\csc\pi(w+z_j)dw,$$

$$F_j=i\pi\int_0^{\infty}[f(re^{-i3\pi/2}+z_j)-f(re^{+i\pi/2}+z_j)]$$
$$\times\csc\pi(ir+z_j)dr.$$

[20] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Co., New York, 1953), p. 413.

In general, in terms of the wave vector, one has $z_j = (L/\pi)(\omega_{cj} \pm i/\tau)/\bar{v}_{zj}$, where $\bar{v}_{zj}$ is the average Fermi velocity at the limiting point j and ω_{cj} the cyclotron frequency at that point. Now in the identity $\csc x = 2i/(e^{ix} - e^{-ix})$, for large L (compared to electron translation along the field during a mean free time) one or the other of the exponentials will dominate, so that one gets

$$F_j = 2\pi e^{i\pi z_j} \int_0^\infty [f(re^{-i3\pi/2} + z_j) - f(re^{i\pi/2} + z_j)] e^{-\pi r} dr.$$

One sees, therefore, that each branch point and cut contributes an oscillatory term to the transmitted signal. The period of the oscillation of a given term is given by

$$\Delta B = (\pi c/eL) m_{cj} \bar{v}_{zj} = -(\hbar c/2eL)(\partial A/\partial k_z)_j,$$

where $m_{cj}\bar{v}_{zj}$ is evaluated at the branch point. In this simple conjecture the effect due to the change of the integral as a function of the magnetic field has not been taken into account.

Finally, F is the sum over all of the branch points of $\sigma(z)$ so that $A_\pm(L)$ is a sum of a number of oscillatory contributions, both from the branches of the infinite dispersion relation and from the branch points of the conductivity $\sigma(z)$. Each of these contributions, however, occurs with a different amplitude; this determines whether it will or will not be seen in a given experiment. The relative amplitudes of the contributions of the various branches and branch points depends on the residues and on the positions of the poles of the function $f(z)$ and must be investigated for each individual case.

If the conductivity is local, that is, if it is independent of the wave vector, there is only one branch of the infinite dispersion relation, namely, the helicon branch, and the conductivity has no branch points. The transmitted signal will then consist of the pure helicon transmission. However, if the conductivity is nonlocal, then there may be more than one solution to the infinite-medium dispersion equation, and also there will be branch points of $f(z)$, so that a more complicated spectrum results.

III. NUMERICAL RESULTS

Several numerical computations were made on various models of the Fermi surface in order to gain some qualitative appreciation of the behavior of the transmitted signal for various situations. First of all, a spherical Fermi surface was used. The attempt was to see whether Gantmakher-Kaner oscillations which are due to the elliptical edge of the Fermi surface are present, and what is their qualitative behavior. The numerical parameters used were frequency $\omega = 10^6$/sec, relaxation time $\tau = 10^{-9}$ sec, electron concentration $n = 1.4 \times 10^{22}/\text{cm}^3$, and, as in each of the following cases, the sample thickness was 1 mm and a free-electron effective mass was used. The numerical results show that there are Gantmakher-Kaner oscillations below the helicon absorption edge, but they are very small in amplitude compared to the helicon oscillations and this prompted us[10] to doubt whether they could be seen experimentally.[9]

A model with an orbit edge was investigated next. A dumbbell surface [see Fig. 3(a)] with the ends cut off where $\partial A/\partial k_z = 0$ [see Fig. 1(a)] is used as a model of a Fermi surface with an orbit edge. In an extended zone scheme, it looks like an undulating cylinder. A more detailed description of this surface is given below. The results using this surface, as seen in Fig. 1(b), show that the Gantmakher-Kaner oscillations are somewhat larger, but still appear small. One can see the magnitude of the undulations below the absorption edge as compared to the steepness of the edge itself, that is, the start of the first helicon oscillations. The parameters used in the computation are the same as the ones for the spherical Fermi surface.

In order to simulate the situation in copper in the [111] direction, a model surface was used whose electron surface was an undulating cylinder with an electron concentration $8.5 \times 10^{22}/\text{cm}^3$ and whose hole surface was a sphere with a hole concentration of 8.415 $\times 10^{22}/\text{cm}^3$. The other parameters are the same as used above. No attempt was made to reproduce the copper Fermi surface except in the two general aspects mentioned. Figure 2 shows the Gantmakher-Kaner oscillations appearing on a background of a slowly varying helicon wave. Qualitatively, this resembles the experimentally obtained results.

The most striking results were obtained using a dumbbell model of a Fermi surface. The model, proposed by Eckstein,[21] has a Fermi energy $E_F = (\hbar^2/2m) \times (k_\perp^2 - k_z^2 + k_z^4/2k_0^2)$, where $k_\perp$ and k_z are the electron

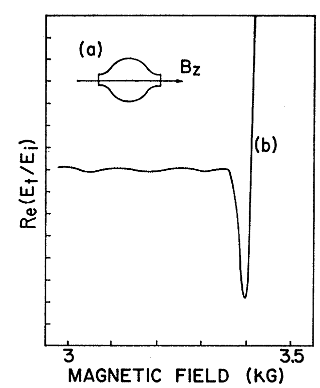

FIG. 1. Real part of the ratio of the transmitted to the incident wave electric fields, $\text{Re}(E_t/E_i)$, for a Fermi surface, which is an undulating cylinder (a). This surface displays an orbit edge. The parameters used in the computation were frequency $\omega = 10^6$/sec, relaxation time $\tau = 10^{-9}$ sec, and an electron density $n = 1.4 \times 10^{22}/\text{cm}^3$. One can see very weak Gantmakher-Kaner oscillations below the absorption edge for helicons.

[21] S. G. Eckstein, Phys. Rev. Letters 16, 611 (1966).

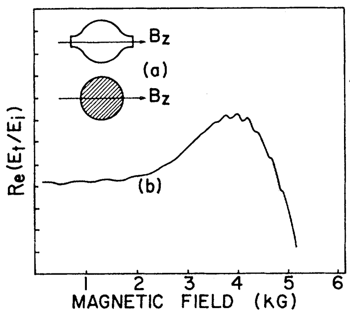

FIG. 2. Re(E_t/E_i) for a Fermi surface consisting of two pieces: one an undulating cylinder with electron concentration of 8.5 $\times10^{22}$/cm^3 and the other a spherical hole surface with hole concentration of 8.415$\times10^{22}$/cm^3. A frequency of $\omega=10^4$/sec was used and a relaxation time of $\tau=10^{-9}$ sec. Gantmakher-Kaner oscillations appear against the background of a slowly varying helicon signal.

wave vectors perpendicular and parallel to the magnetic field and where k_0^2 is chosen to be $k_0^2=12mE_F$. This model has the following features: It has a large density of electron states with the same value of the z component of the Fermi velocity (that is, along the magnetic field), and some electrons with a larger value of the z component of the Fermi velocity which are responsible for the primary absorption edge for helicons, but there is a smaller number of these electrons. The presence of this large extremum was used by the author to predict a "helicon window"[17] occurring below the primary absorption edge. That is, the real part of the wave vector of the dispersion relation in the window region is larger than the imaginary part, and a damped helicon wave should then propagate and be detected. However, this large maximum is also responsible for the generation of strong Gantmakher-Kaner oscillations. The calculations, as shown in Fig. 3, bear this picture out. The damped helicon enhances the Gantmakher-Kaner oscillations, and these in turn are visible in the "window" region, i.e., above the primary absorption edge (see Fig. 3). This agrees very well with what is observed experimentally in copper in the [110] direction.[8,10]

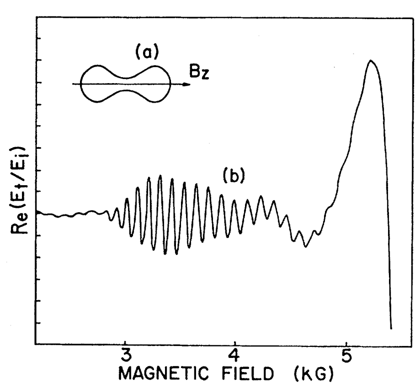

FIG. 3. Re(E_t/E_i) for a dumbbell Fermi surface shown in (a) with parameters: frequency $\omega=10^6$/sec, relaxation time $\tau=10^{-9}$ sec, and electron density $n=1.4\times10^{22}$/cm^3. Enhanced Gantmakher oscillations occur in the "window" region, and may be compared with Gantmakher-Kaner oscillations below the "window" region.

IV. CONCLUSION

In this paper we have investigated the effect of boundaries, or explicitly, the transmitted signal through a metal-slab sample. We show the relation between the infinite-medium dispersion relation, the conductivity tensor, and the transmitted signal. We have derived an expression for the ratio of the transmitted to the impinging electric fields of the experiment. It is pointed out that one should retain the field in the calculation, since it is the interference of the transmitted field with a reference field which gives the detected oscillatory signal.

We find that the relation between the incident electric field and the transmitted field is proportional to the sum of one over the infinite-medium dispersion relation over all wave vectors which have an integer or half integer wavelength in the sample. The numerical results were computed using the summed version of the results.

It is also shown that the results may be cast into an analytical form which then makes transparent the various contributions to the transmitted signal. It clearly shows that the various branches of the infinite-medium dispersion relation contribute separately to the transmitted signal as oscillatory functions of the external magnetic field. It also shows that branch points of the conductivity also make an oscillatory contribution and give the so-called Gantmakher-Kaner oscillations. Finally, although the possibility is not pursued further in this paper, it can be seen that the analytical form of the results can be used as a starting point in investigating the amplitudes of the contributions from the various branches and branch points to the transmitted field.

In a previous paper the author predicted a damped helicon existing in a region below the conventional absorption edge for certain Fermi-surface geometries. This was referred to as the helicon "window." This prediction was made on the basis of an infinite-medium dispersion relation. A numerical calculation of a model Fermi surface shows that the helicon window manifests itself in a transmission experiment as the enhancement of Gantmakher-Kaner oscillations in its region of existence.

Numerical results for a spherical Fermi surface and for a Fermi surface with an orbit edge show that the Gantmakher-Kaner oscillations for these geometries are much smaller than the Gantmakher-Kaner oscillations for the window region. This also seems to be borne out by experiment.

Finally, a result of a calculation using a composite Fermi surface, including both an orbit edge surface and a hole surface is shown. This was done to mimic the situation occurring in Cu in the [111] direction and gives qualitative agreement with experiment.

Further work on this subject should include calculations using the real Fermi surface of a metal to compare directly with experimental data, and the use of the analytical expression for the transmitted signal to study the amplitudes of the various branches.

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APPENDIX

The equation of motion for the electric field in the slab is

$$\partial^2 E_\pm(z)/\partial z^2+(\omega^2 K/c^2)E_\pm(z)$$
$$=(4\pi i\omega/c^2)J_\pm(z)+2\sum_{n=-\infty}^{\infty}[E_\pm'(0)\delta(z-2nL)+E_\pm'(L)\delta(z-(2n+1)L)]. \quad \text{(A1)}$$

The field is symmetric with respect to the plane $z=0$, since one is constructing a mirror image in z 0 of what is in $z>0$. Therefore, one may write

$$E_\pm(z)=\sum_{n=0}^{\infty}E_n^\pm \cos k_n z,$$

where $k_n=n\pi/L$ and L is the thickness of the slab. Since the nonlocal conductivity depends only on the distance between the applied field and the resultant current and not on their absolute positions (this is the reason for using specular reflection and the infinitely repeating slabs), one may write

$$J_\pm(z)=\int_{-\infty}^{\infty}\sigma_\pm(z'-z)E_\pm(z')dz'=\sum_{n=0}^{\infty}\sigma_n^\pm E_n^\pm \cos k_n z,$$

where

$$\sigma_n^\pm=\int_{-\infty}^{\infty}\sigma_\pm(y)\cos(k_n y)dy.$$

In order to find the coefficients of expansion of the sum over the δ functions, one has to be a little more careful. Let

$$\delta(z)=\sum_{n=-\infty}^{\infty}[E_\pm'(0)\delta(z-2nL)+E_\pm'(L)\delta(z-(2n+1)L)]$$
$$=\sum_{m=0}^{\infty}b_m^\pm \cos k_m z.$$

In order to find b_m, multiply the above equation by $\cos k_l z$ and integrate over z between the symmetric limits $-NL$ and NL, where N is some large number; then finally take the limit $N\to\infty$. Now,

$$\int_{-NL}^{NL}\cos k_l z\,\delta(z)dz=\sum_{m=0}^{\infty}b_m^\pm\int_{-NL}^{NL}\cos k_l z\cos k_m z dz$$
$$=b_l^\pm NL \quad \text{or} \quad b_0^\pm 2NL;$$

thus,

$$b_l^\pm=\lim_{N\to\infty}\left\{\frac{1}{NL}\sum_{n=-\infty}^{\infty}\left[E_\pm'(0)\int_{-NL}^{NL}\cos k_l z\,\delta(z-2nL)dz+E_\pm'(L)\int_{-NL}^{NL}\cos k_l z\,\delta(z-(2n+1)L)dz\right]\right\}$$

and

$$\sum_{n=-\infty}^{\infty}\int_{-NL}^{NL}\cos k_l z\,\delta(z-2nL)dz=N-\alpha,$$

where α is a number $\lesssim 3$, and depends on whether N is odd or even and on whether the argument of the δ function coincides with a limit of integration. Similarly,

$$\sum_{n=-\infty}^{\infty}\int_{-NL}^{NL}\cos k_l z\,\delta(z-(2n+1)L)dz=(N-\beta)\cos k_l L, \quad \text{where} \quad \beta\lesssim 3;$$

therefore,

$$b_l^\pm=[E_\pm'(0)+E_\pm'(L)\cos k_l L]/L$$

and

$$b_0^\pm=[E_\pm'(0)+E_\pm'(L)]/2L.$$

Substituting these values in Eq. (A1), collecting coefficients of each cosine term, and setting them equal to zero, one has

$$E_n^\pm=\frac{-2b_n^\pm}{k_n^2-(\omega^2K/c^2)+(4\pi i\omega/c^2)\sigma_n^\pm}.$$