
COMMENTS AND ADDENDA

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Fermi-Liquid Transport Coefficients of Dilute Solutions of He³ in He⁴: An Addendum

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Using recent exact solutions of the quasiparticle transport equation, we have reexamined the consistency between Bardeen, Baym, and Pines's phenomenological theory of dilute solutions of He³ in He⁴ and measurements of the spin-diffusion and thermal-conductivity coefficients of two solutions in the degenerate Fermi-liquid regime. Previously, Baym and the author had used lowest-order variational solutions for this purpose. Discrepancies of 10–15% persist which, beyond experimental uncertainty, must be attributed to oversimplification in treating the He³ scattering amplitudes as being independent of spin, velocity, and concentration.

This paper is intended to bring up to date a recent paper by Baym and Ebner¹ (BE) in which it was demonstrated that the phenomenological theory of dilute solutions of He³ in He⁴ as given by Bardeen, Baym, and Pines,² is consistent with experimental determinations of the thermal-conductivity³ and spin-diffusion⁴ coefficients of 1.3 and 5.0% systems if simple variational solutions of the transport equation are used to relate the He³-He³ effective interaction to the transport coefficients. Previously, the approximate solutions of Abrikosov and Khalatnikov⁵ and of Hone⁶ had been used for this purpose.

More recently, exact analytical solutions of the transport equation have been determined by Brooker and Sykes⁷ and independently by Jensen and co-workers.⁸ Using the exact results of Brooker and Sykes, we have repeated the calculations reported in (BE). That is, we begin by assuming that $V(k)$, the Fourier transformed He³-He³ effective interaction, may be expanded in powers of k^2 with undetermined coefficients which are chosen by

attempting to fit the experimental transport coefficients. In so doing, $V(k)$ is allowed to be only reasonably rapidly varying. A typical result of this procedure is the interaction

$$V(k) = V_0(1 - 3.389y + 6.353y^2 - 9.576y^3 + 5.402y^4), \quad (1)$$

where $y = (k/2k_0)^2$; k_0 is the Fermi momentum of a 5.0% solution of He³ in He⁴, $k_0/\hbar = 0.318 \text{ \AA}^{-1}$; and $V_0 = -0.078 m_4 s^2/n_4$. The mass of a He⁴ atom is m_4 and the speed of first sound and the number density in pure He⁴ at $T=0$ are s and n_4 , respectively. Figure 1 shows this $V(k)$ and also the interaction found in BE as well as the original interaction of Bardeen, Baym, and Pines.² As can be seen, $V(0)$ is close to the original value of Ref. 2; it is also very close to the value deduced by Baym⁹ from thermodynamic arguments, which is $V(0) = \alpha^2 m_4 s^2/n_4 \cong -0.077 m_4 s^2/n_4$; α is the fractional excess molar volume of He³ in He⁴.

The present $V(k)$ shows different behavior from

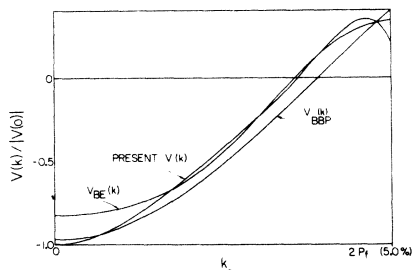


FIG. 1. The effective interaction $V(k)$, as given by Eq. (1) in the text, plotted in units of $|V(0)| = 0.078m_4s^2/n_4$. $V_{BE}(k)$ and $V_{BBP}(k)$ are the interactions of Refs. 1 and 2, respectively.

previous interactions at large k . This effect is very sensitive to the relative input values of the spin-diffusion and thermal-conductivity coefficients

at 5%, and may not be significant.

A more interesting implication, we feel, may be noted from Table I, which compares the fit obtained from the interaction (1) with that obtained from the interaction of BE. The fit of the earlier interaction, obtained using approximate solutions to the transport equation, is in fact somewhat better than the present fit. We conclude then that the 10–15% discrepancy between the phenomenological theory and experimental transport coefficients persists even given exact solutions of the transport equation. Beyond experimental uncertainty, this must be attributed to oversimplification in treating the quasiparticle scattering amplitude as being concentration- and velocity-independent.

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TABLE I. Experimental and calculated values of κT (erg/sec-cm) and DT^2 (cm²-°K²/sec) for 1.3% and 5.0% He³ concentrations. The calculated values denoted (BE) were determined in BE; the others, from the present $V(k)$, Eq. (1).

	κT (1.3%)	κT (5.0%)	DT^2 (1.3%)	DT^2 (5.0%)
Experiment	11	24	17.2×10^{-6}	90×10^{-6}
Calculated from $V_{BE}(k)$	10	26	18.6×10^{-6}	82×10^{-6}
Calculated from present $V(k)$	9.6	27	18.6×10^{-6}	80×10^{-6}

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