PHYSICAL REVIEW VOLUME 185, NUMBER 5 25 SEPTEMBER 1969

S-Wave Baryon-Baryon Scattering in the Quark Model

C. V. SASTRY* AND C. SUBRAHMANYAM
Regional Engineering College, Rourkela-8, Orissa, India
(Received 22 January 1969)

S-wave baryon-baryon scattering is considered in the quark model with a $T^3{}_3$ breaking term in the quark space. Thirteen sum rules for various baryon-baryon amplitudes for scattering in the $^1S_0$ state and 10 sum rules for scattering in the $^3S_1$ state are obtained.

I. INTRODUCTION

THE quark model[1] has been applied with some success to various processes[2] including baryon-baryon scattering.[3] In these models one usually seeks to obtain agreement at high energies, and corresponding assumptions about amplitudes are made. It may be desirable to make the same analysis at low energies. Harari[4] has pointed out that, while the predictions of exact $SU(3)$ symmetry are in many cases incompatible with experiment, a symmetry-breaking interaction of the octet type leads always to relations which are well obeyed by the data. Gupta and Pande[5] have considered S-wave baryon-baryon scattering under a broken $SU(3)$-symmetry scheme and obtained many sum rules for the scattering amplitudes, which, however, cannot be subjected to experimental tests at present. Earlier[6] attempts have also been made to obtain relations among the scattering lengths for baryon-baryon scattering under different symmetry schemes. While nothing definite can be said about the experimental status of the $SU(3)$ predictions,[7] some of the results of higher-symmetry schemes[6] such as $SU(6)$ and $U(12)$ are definitely in contradiction with experiment. Gupta and Mitra[8] have also considered baryon-baryon processes in the quark model and have obtained a number of sum rules for the spin-flip and non-spin-flip amplitudes. However, these are valid only at high energies.

In an earlier paper,[9] several mass relations for baryons and mesons were obtained by using the quark model with a $T^3{}_3$ violation of the $SU(3)$ symmetry in the quark space. It has been observed that this symmetry breaking in quark space reproduces many of the $SU(6)$ results that are experimentally well satisfied, and does not reproduce the others. In this paper we consider S-wave baryon-baryon scattering in the same model. It is assumed that only two-body quark-quark interactions are responsible for the elastic, as well as the inelastic, baryon-baryon scattering processes. For two non-strange quarks, the interaction is characterized[9] by the amplitudes V_{dd}, V_{de}, V_{ed}, and V_{ee}, where the first index d (e) stands for spin-nonflip (spin-flip) amplitude, and the second index d (e) stands for unitary-spin-nonflip (unitary-spin-flip) amplitude. Further, the amplitudes $V_{ij}{}^{(1)}$ and $V_i{}^{(2)}$, respectively (i and j standing for d or e), describe the interaction when one or two strange quarks participate in the interaction. We may write V in terms of A, B, C, and D defined in Ref. 9, from which it follows for both spin-flip and spin-nonflip amplitudes that

$$V_i{}^{(2)}+V_{id}+V_{ie}=2V_{id}{}^{(1)}+2V_{ie}{}^{(1)}. \qquad (1)$$

When we evaluate these amplitudes for the quark wave functions, we have eight independent constants to describe the baryon-baryon scattering.

II. SUM RULES FOR THE $^1S_0$ STATE

For the $^1S_0$ state $SU(2)_I$ invariance gives the number of independent amplitudes as 20, whereas the scattering in the triplet state is described by 17 amplitudes only. Also, we find that the baryon-baryon scattering in the S wave is described effectively by seven constants only. Therefore, we obtain 13 sum rules for the amplitudes in the $^1S_0$ state and 10 sum rules for the amplitudes in the $^3S_1$ state.

We illustrate our procedure by calculating the amplitude $A(pn\to pn)$ in the $^1S_0$ state. The proton and neutron states are given by[10]

$$\begin{aligned} p_{1/2}&=(1/3\sqrt{2})[2s(\mathcal{P}_+\mathcal{P}_+\mathcal{N}_-)-s(\mathcal{P}_+\mathcal{P}_-\mathcal{N}_+)],\\ n_{1/2}&=(1/3\sqrt{2})[s(\mathcal{N}_+\mathcal{N}_-\mathcal{P}_+)-2s(\mathcal{N}_+\mathcal{N}_+\mathcal{P}_-)]. \end{aligned} \qquad (2)$$

* On leave of absence from Rourkela Science College, Rourkela.
[1] M. Gell-Mann, Phys. Letters **8**, 214 (1964); G. Zweig, CERN Report (unpublished).
[2] H. J. Lipkin and F. Scheck, Phys. Rev. Letters **16**, 71 (1966); R. H. Dalitz, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967); H. J. Lipkin, Phys. Rev. Letters **16**, 1015 (1966); J. J. Kokkedee and L. Van Hove, Nuovo Cimento **43**, 711 (1966); C. Becchi and G. Morpurgo, Phys. Rev. **B140**, 687 (1965); R. Van Royen and V. F. Weisskopf, Nuovo Cimento **50A**, 617 (1967); P. A. Cook, *ibid.* **48A**, 570 (1967); A. N. Mitra and M. Ross, Phys. Rev. **158**, 1630 (1967); S. P. Misra and C. V. Sastry, *ibid.* **172**, 1402 (1968).
[3] E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu **2**, 105 (1965) [English transl.: Soviet Phys.—JETP Letters **2**, 65 (1965)]; S. D. Gupta and A. N. Mitra, Phys. Rev. **159**, 1285 (1967).
[4] H. Harari, in *High Energy Physics and Elementary Particles* (International Centre for Theoretical Physics, Trieste, 1965), p. 353.
[5] S. D. Gupta and L. K. Pande, Phys. Rev. **143**, 1190 (1966).
[6] V. Barger and M. H. Rubin, Phys. Rev. **140**, B1336 (1965); D. A. Akyeampong and R. Delbourgo, *ibid.* **140**, B1013 (1965).
[7] P. D. DeSouza, G. A. Snow, and S. Meshkov, Phys. Rev. **135**, B565 (1964).
[8] S. D. Gupta and A. N. Mitra, Ref. 3.
[9] S. P. Misra and C. V. Sastry, Ref. 2.
[10] J. L. Friar and J. S. Trefil, Nuovo Cimento **49**, 642 (1967).

The $^1S_0$ state of the proton and neutron is given by

$$|pn\rangle_s=\tfrac{1}{2}(|p_+n_-\rangle-|p_-n_+\rangle+|n_+p_-\rangle-|n_-p_+\rangle). \quad (3)$$

We have

$$V|p_+n_-\rangle=9V_{dd}|p_+n_-\rangle+V_{de}(4|p_+n_-\rangle+|n_+p_-\rangle)+V_{ed}(4|p_+n_-\rangle+|p_-n_+\rangle)+V_{ee}[(22/9)|p_+n_-\rangle-(8/9)|p_-n_+\rangle-(8/9)|n_+p_-\rangle+(25/9)|n_-p_+\rangle], \quad (4)$$

and, therefore,

$$V|p_+n_-\rangle_s=9V_{dd}|p_+n_-\rangle_s+5V_{de}|p_+n_-\rangle_s+3V_{ed}|p_+n_-\rangle_s-\tfrac{1}{3}V_{ee}|p_+n_-\rangle_s. \quad (5)$$

Now taking the scalar product of Eq. (5) with ${}_s\langle p_+n_-|$, we obtain

$$A(pn\to pn)={}_s\langle pn|V|pn\rangle_s=9V_{dd}+5V_{de}+3V_{ed}-\tfrac{1}{3}V_{ee}. \quad (6)$$

All 20 reactions of the $^1S_0$ state can be dealt with similarly. We observe that, of these 20 reactions, 10 involve only nucleon targets; therefore, there will be three sum rules among the nucleon-target reactions. These are, for example,

$$\sqrt{3}A(p\Xi^-\to\Lambda\Sigma^0)+3A(p\Xi^-\to\Lambda\Lambda)-3A(p\Xi^-\to\Sigma^0\Sigma^0)=0, \quad (7)$$

$$(6\sqrt{6})A(p\Sigma^-\to n\Lambda)+5A(p\Xi^-\to\Lambda\Lambda)+9A(p\Xi^-\to\Sigma^0\Sigma^0)-12A(p\Xi^-\to n\Xi^0)+6\sqrt{2}A(p\Sigma^-\to n\Sigma^0)=0, \quad (8)$$

$$2A(p\Sigma^+\to p\Sigma^+)-4A(pn\to pn)-4A(p\Xi^0\to p\Xi^0)+6A(p\Lambda\to p\Lambda)-12A(p\Xi^-\to\Lambda\Lambda)+12A(p\Xi^-\to\Sigma^0\Sigma^0)+\sqrt{2}A(p\Sigma^-\to n\Sigma^0)=0. \quad (9)$$

From these sum rules we can obtain three cross-section inequalities which are convenient for experimental verification (the experimental cross sections are to be corrected by a kinematical factor[4] before comparing these inequalities with experiment):

$$[3\sigma(p\Xi^-\to\Lambda\Sigma^0)]^{1/2}\leqslant 3[\sigma(p\Xi^-\to\Lambda\Lambda)]^{1/2}+3[\sigma(p\Xi^-\to\Sigma^0\Sigma^0)]^{1/2}, \quad (7')$$

$$6[6\sigma(p\Sigma^-\to n\Lambda)]^{1/2}\leqslant 5[\sigma(p\Xi^-\to\Lambda\Lambda)]^{1/2}+9[\sigma(p\Xi^-\to\Sigma^0\Sigma^0)]^{1/2}+12[\sigma(p\Xi^-\to n\Xi^0)]^{1/2}+6[2\sigma(p\Sigma^-\to n\Sigma^0)]^{1/2}, \quad (8')$$

$$2[\sigma(p\Sigma^+\to p\Sigma^+)]^{1/2}\leqslant 4[\sigma(pn\to pn)]^{1/2}+4[\sigma(p\Xi^0\to p\Xi^0)]^{1/2}+6[\sigma(p\Lambda\to p\Lambda)]^{1/2}+12[\sigma(p\Xi^-\to\Lambda\Lambda)]^{1/2}+12[\sigma(p\Xi^-\to\Sigma^0\Sigma^0)]^{1/2}+[2\sigma(p\Sigma^-\to n\Sigma^0)]^{1/2}. \quad (9')$$

The remaining sum rules, for which we take independent amplitudes rather than $SU_I(2)$-invariant amplitudes, are

$$2A(p\Sigma^+\to p\Sigma^+)-A(pn\to pn)-A(\Sigma^+\Sigma^+\to\Sigma^+\Sigma^+)=0, \quad (10)$$

$$A(\Sigma^+\Xi^0\to\Sigma^+\Xi^0)-A(p\Sigma^+\to p\Sigma^+)-A(p\Xi^0\to p\Xi^0)+A(pn\to pn)-3A(p\Xi^-\to\Lambda\Lambda)+A(p\Xi'\to\Sigma^0\Sigma^0)+A(p\Xi^-\to n\Xi^0)=0, \quad (11)$$

$$30A(\Sigma^0\Sigma^0\to\Lambda\Lambda)+4A(p\Xi^-\to n\Xi^0)-2\sqrt{2}A(p\Sigma^-\to n\Sigma^0)+A(p\Xi^-\to\Sigma^0\Sigma^0)+A(p\Xi^-\to\Lambda\Lambda)=0, \quad (12)$$

$$15A(p\Sigma^+\to p\Sigma^+)-15A(p\Lambda\to p\Lambda)-51A(p\Xi^-\to\Lambda\Lambda)+49A(p\Xi^-\to\Sigma^0\Sigma^0)+16A(p\Xi^-\to n\Xi^0)-\tfrac{1}{2}\sqrt{2}A(p\Sigma^-\to n\Sigma^0)-30A(p\Xi^-\to n\Xi^0)+30A(\Lambda\Lambda\to\Lambda\Lambda)=0, \quad (13)$$

$$6A(\Lambda\Sigma^0\to\Lambda\Sigma^0)-6A(p\Xi^0\to p\Xi^0)-6A(p\Xi^-\to\Lambda\Lambda)-10A(p\Xi^-\to\Sigma^0\Sigma^0)+8A(p\Xi^-\to n\Xi^0)+\tfrac{1}{2}\sqrt{2}A(p\Sigma^-\to n\Sigma^0)-3A(p\Sigma^+\to p\Sigma^+)+3A(p\Lambda\to p\Lambda)=0, \quad (14)$$

$$A(\Xi^-\Xi^-\to\Xi^-\Xi^-)-2A(p\Xi^0\to p\Xi^0)+A(pn\to pn)-6A(p\Xi^-\to\Lambda\Lambda)+2A(p\Xi^-\to\Sigma^0\Sigma^0)+2A(p\Xi^-\to n\Xi^0)=0, \quad (15)$$

$$10A(\Sigma^+\Sigma^-\to\Sigma^+\Sigma^-)-10A(p\Xi^0\to p\Xi^0)-15A(p\Sigma^+\to p\Sigma^+)+15A(p\Lambda\to p\Lambda)-36A(p\Xi^-\to\Lambda\Lambda)+24A(p\Xi^-\to\Sigma^0\Sigma^0)+16A(p\Xi^-\to n\Xi^0)+(29/2)\sqrt{2}A(p\Sigma^-\to n\Sigma^0)=0, \quad (16)$$

$$30A(\Lambda\Xi^0\to\Lambda\Xi^0)-30A(\Sigma^+\Xi^0\to\Sigma^+\Xi^0)+30A(p\Sigma^+\to p\Sigma^+)-30A(p\Lambda\to p\Lambda)-7A(p\Xi^-\to\Lambda\Lambda)-13A(p\Xi^-\to\Sigma^0\Sigma^0)+8A(p\Xi^-\to n\Xi^0)-4\sqrt{2}A(p\Sigma^-\to n\Sigma^0)=0, \quad (17)$$

$$5A(\Sigma^+\Xi^-\to\Sigma^-\Xi^-)-5A(\Sigma^+\Xi^0\to\Sigma^+\Xi^0)+12A(p\Xi^-\to\Lambda\Lambda)-28A(p\Xi^-\to\Sigma^0\Sigma^0)+8A(p\Xi^-\to n\Xi^0)+\sqrt{2}A(p\Sigma^-\to n\Sigma^0)=0, \quad (18)$$

$$(5\sqrt{6})A(\Lambda\Xi^0\to\Sigma^+\Xi^-)+12A(p\Xi^-\to\Lambda\Lambda)-8A(p\Xi^-\to\Sigma^0\Sigma^0)-2A(p\Xi^-\to n\Xi^0)+\sqrt{2}A(p\Sigma^-\to n\Sigma^0)=0. \quad (19)$$

We note that Eqs. (10) and (18) of Ref. 5 are also true in our model.

III. SUM RULES FOR THE $^3S_1$ STATE

As has been noted earlier, there will be 17 independent amplitudes for the $^3S_1$ state, and seven constants to describe them. There will be nine reactions with nucleon targets only, and therefore two sum rules result for them:

$$A(p\Xi^0\to p\Xi^0)-A(p\Xi^-\to p\Xi^-)-\tfrac{1}{2}\sqrt{2}A(p\Sigma^-\to n\Sigma^0)-A(p\Xi^-\to\Sigma^+\Sigma^-)-\sqrt{3}A(p\Lambda\to p\Sigma^0)=0, \quad (20)$$

$$2A(pn\to pn)-A(p\Sigma^+\to p\Sigma^+)+2A(p\Xi^0\to p\Xi^0)-3A(p\Lambda\to p\Lambda)+2\sqrt{3}A(p\Xi^-\to\Lambda\Sigma^0)-2A(p\Xi^-\to\Sigma^+\Sigma^-)+\tfrac{3}{2}\sqrt{2}A(p\Sigma^-\to n\Sigma^0)-4\sqrt{3}A(p\Lambda\to p\Sigma^0)=0. \quad (21)$$

The cross-section inequalities are

$$[\sigma(p\Xi^0\to p\Xi^0)]^{1/2}\leqslant[\sigma(p\Xi^-\to p\Xi^-)]^{1/2}+[\tfrac{1}{2}\sigma(p\Sigma^-\to n\Sigma^0)]^{1/2}+[\sigma(p\Xi^-\to\Sigma^+\Sigma^-)]^{1/2}+[3\sigma(p\Lambda\to p\Sigma^0)]^{1/2}, \quad (20')$$

$$2[\sigma(pn\to pn)]^{1/2}\leqslant[\sigma(p\Sigma^+\to p\Sigma^+)]^{1/2}+2[\sigma(p\Xi^0\to p\Xi^0)]^{1/2}+3[\sigma(p\Lambda\to p\Lambda)]^{1/2}+2[3\sigma(p\Xi^-\to\Lambda\Sigma^0)]^{1/2}+2[\sigma(p\Xi^-\to\Sigma^+\Sigma^-)]^{1/2}+3[\tfrac{1}{2}\sigma(p\Sigma^-\to n\Sigma^0)]^{1/2}+4[3\sigma(p\Lambda\to p\Sigma^0)]^{1/2}. \quad (21')$$

The remaining sum rules are

$$10A(p\Sigma^+\to p\Sigma^+)-5A(\Sigma^+\Sigma^-\to\Sigma^+\Sigma^-)-5A(pn\to pn)-15\sqrt{2}A(p\Sigma^-\to n\Sigma^0)-2A(p\Xi^-\to\Sigma^+\Sigma^-)+2\sqrt{3}A(p\Lambda\to p\Sigma^0)-\sqrt{3}A(p\Xi^-\to\Lambda\Sigma^0)=0, \quad (22)$$

$$5A(\Sigma^+\Xi^-\to\Sigma^+\Xi^-)+5A(pn\to pn)-5A(p\Sigma^+\to p\Sigma^+)-5A(p\Xi^0\to p\Xi^0)+3\sqrt{3}A(p\Lambda\to p\Sigma^0)+17A(p\Xi^-\to\Sigma^+\Sigma^-)+\sqrt{3}A(p\Xi^-\to\Lambda\Sigma^0)+(25/2)\sqrt{2}A(p\Sigma^-\to n\Sigma^0)=0, \quad (23)$$

$$12A(\Lambda\Xi^-\to\Lambda\Xi^-)-12A(\Sigma^+\Xi^-\to\Sigma^+\Xi^-)-8A(pn\to pn)+16A(p\Sigma^+\to p\Sigma^+)-8A(p\Xi^0\to p\Xi^0)-18\sqrt{2}A(p\Sigma^-\to n\Sigma^0)-16A(p\Xi^-\to\Sigma^+\Sigma^-)+12\sqrt{3}A(p\Xi^-\to\Lambda\Sigma^0)=0, \quad (24)$$

$$5A(\Lambda\Xi^-\to\Sigma^0\Xi^-)-A(p\Lambda\to p\Sigma^0)+3A(p\Xi^-\to\Lambda\Sigma^0)+2\sqrt{3}A(p\Xi^-\to\Sigma^+\Sigma^-)=0, \quad (25)$$

$$5A(\Sigma^+\Xi^0\to\Sigma^+\Xi^0)-5A(\Sigma^+\Xi^-\to\Sigma^+\Xi^-)-6\sqrt{3}A(p\Xi^-\to\Lambda\Sigma^0)-12A(p\Xi^-\to\Sigma^+\Sigma^-)-8\sqrt{3}A(p\Lambda\to p\Sigma^0)-5\sqrt{2}A(p\Sigma^-\to n\Sigma^0)=0, \quad (26)$$

$$5A(\Lambda\Sigma^-\to\Sigma^0\Sigma^-)+8A(p\Lambda\to p\Sigma^0)+4\sqrt{3}A(p\Xi^-\to\Sigma^+\Sigma^-)+A(p\Xi^-\to\Lambda\Sigma^0)=0, \quad (27)$$

$$15A(\Lambda\Sigma^-\to\Lambda\Sigma^-)-10A(p\Xi^0\to p\Xi^0)+5A(pn\to pn)-10A(p\Sigma^+\to p\Sigma^+)+4\sqrt{3}A(p\Xi^-\to\Lambda\Sigma^0)+58A(p\Xi^-\to\Sigma^+\Sigma^-)+22\sqrt{3}A(p\Lambda\to p\Sigma^0)+\sqrt{2}A(p\Sigma^-\to n\Sigma^0)=0, \quad (28)$$

$$5A(\Xi^0\Xi^-\to\Xi^0\Xi^-)+5A(pn\to pn)-10A(p\Xi^0\to p\Xi^0)+34A(p\Xi^-\to\Sigma^+\Sigma^-)+12\sqrt{3}A(p\Xi^-\to\Lambda\Sigma^0)+6\sqrt{3}A(p\Lambda\to p\Sigma^0)+15\sqrt{2}A(p\Sigma^-\to n\Sigma^0)=0. \quad (29)$$

Not all the sum rules obtained here can be experimentally verified at the present stage, but many of the unknown amplitudes can be obtained in terms of the known ones.

IV. DISCUSSION

As usual, the sum rules are derived under the assumption that only two-body quark-quark interactions are to be considered. The assumptions of the quark model do not bear a direct relationship to the higher-symmetry schemes, although they give many good results of $SU(6)$ without giving the bad ones.[9] We have the $SU(2)_I$ and $SU(2)_J$ symmetry, which in the static approximation gives $I=0$, 1 and $J=0$, 1 contributions only. The $T^3{}_3$ violation here has the structure of $\Delta I=0$, $\Delta Y=0$ octet tensor operator in quark space. In the three-particle space the symmetry-violating term will also have $\Delta I=0$, $\Delta Y=0$, but will no longer belong to the octet representation only. In the crossed channel also this scheme no longer gives the Gell-Mann–Okubo mass relation for mesons, and further, it gives a mixing of the octet and singlet mesons which is experimentally justified.[9]

Thus in the space of physical particles, the model will necessarily give terms like

$$\langle\mu''I''I_3''Y''|V^{(\mu)}|\mu'I'I_3'Y'\rangle=\langle\mu''\|V^{(\mu)}\|\mu'\rangle\delta_{I'I''}\delta_{y'y''}\delta_{I_3'I_3''},$$

as generated by the $T^3{}_3$ symmetry violation in the quark space. This bears an analogy with, e.g., Weinberg and Treiman's[11] approach, where, for electromagnetic multiplets, $\Delta I_3=0$ contributions from different tensor operators of $SU(2)_I$ were considered. But whereas in that work these operators were arbitrary, and assumed to be successively smaller, here we have all of them generated from the octet $\Delta I_3=0$, $\Delta Y=0$ $SU(3)$ tensor operator of the quark space.

As before,[9] we have considered the composite-particle wave functions to be the same for all the particles, and have assumed that these do not give rise to further corrections. The structure of the wave functions[12] is a problem which we feel is beyond the scope of the present techniques.

ACKNOWLEDGMENTS

We are very thankful to Professor S. P. Misra for his constant guidance and encouragement during the course of this work. We are also thankful to the Ministry of Education of the Government of India for financial help.

[11] S. Weinberg and S. B. Treiman, Phys. Rev. **116**, 465 (1959).
[12] R. Van Royen and V. F. Weisskopf, Nuovo Cimento **50A**, 617 (1967).