The modification of the electron spectrum by the weak-magnetism term can be shown explicitly from (2) to be the usual factor $[1+(8/3)E_lF]$, after ignoring the electron mass and dropping small constants. From CVC, the magnitude of F can be estimated[2] using the width of the 15.11-MeV level of C^{12} (the $I_3=0$ member of the $I=1$ triplet). It is $|F|=(2.40\pm0.25)/m$, where m is the nucleon mass. One test of CVC[2] is to search for the above modification in the electron spectrum. The experiment was carried out by Lee *et al.*[9] They found $F=(2.0\pm0.2)/m$ to be in good agreement with the CVC prediction.

The invariant T-matrix element for electron-nucleus scattering in the final state is given by the one-photon-exchange diagram in lowest-order α. Due to the spin-0 nature of C^{12} at low energies, it has the simple form

$$T_{fn}{}^{\gamma}=(Ze^2/k^2)[-i\bar{u}(l)\gamma_\alpha u(l')](p_\alpha+p_\alpha'), \qquad (3)$$

where p', l' (p,l) are the momenta of the nucleus and electron before (after) the scattering, and $k=p'-p=l-l'$.

Inserting (3) and (4) into the unitarity equation, one can now calculate the electromagnetic contribution to D. The result of the algebra is

$$D^{\gamma}(\beta^{\mp})=(Z\alpha E_l/4p_l)E_l(3+m_l^2/E_l^2)F. \qquad (4)$$

The above holds for both $B^{12}\to C^{12}e^-\bar{\nu}$ and $N^{12}\to C^{12}e^+\nu$, since, under charge conjugation, F and the charge of the electron both change sign.

[9] Y. K. Lee, L. W. Mo, and C. S. Wu, Phys. Rev. Letters **10**, 253 (1963).

DISCUSSION

Equation (4) shows that D, induced by the electromagnetic final-state interaction, is directly dependent upon F, the weak-magnetism factor. It is explicitly unencumbered by direct allowed processes. If one permits a violation of T invariance, its contribution to D will be

$$D^{\bar{T}}(\beta^{\mp})=\mp\sin\phi\, E_0|F|, \qquad (5)$$

where ϕ is the phase of F, $F=|F|e^{i\phi}$, and E_0 is the maximum energy of the electron. With the neglect of EFSI of higher order in α and assuming no T violation, a measurement of D, though extremely difficult, becomes a relatively clean check of the CVC hypothesis. The numerical value of $D^{\gamma}(\beta^{\mp})$ is estimated from the known value of F from B^{12} and N^{12} β decays[9]:

$$D^{\gamma}(\beta^{\mp})\simeq1.2\times10^{-3}E_l/E_0. \qquad (6)$$

This is still about an order of magnitude below present experimental capabilities,[4,5] although it is significantly larger than corresponding values estimated for mirror β decays.[1]

ACKNOWLEDGMENT

We thank Professor S. B. Treiman for stimulating and continued interest in the electromagnetic final-state problem.

PHYSICAL REVIEW VOLUME 185, NUMBER 5 25 SEPTEMBER 1969

Exchange Degeneracy and $SU(3)$†

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(Received 28 April 1969)

It is hypothesized that the imaginary parts of the contributions of exchange-degenerate Regge trajectories cancel in hadron-hadron states of internal quantum numbers for which no resonances exist. A recent work by the author on $SU(3)$-symmetric baryon trajectories in meson-baryon states is generalized to include all trajectories and all meson-meson, meson-baryon, and baryon-baryon states. Consistency is possible for these states. The difficulty concerning baryon-antibaryon states is discussed. The large, observed $\pi\pi N$ decay of the $j^P=\frac{3}{2}^-$ $\Delta^*(1670)$ is predicted from the consistency conditions. The magnitude of the coupling to baryons of any singlet meson is predicted in terms of the octet interactions. This result includes the prediction that φNN vertices of all spin dependences vanish. The modification imposed by the baryon exchange-degeneracy hypothesis on the Regge analysis of meson-baryon scattering is discussed. The relation of the results to the bootstrap hypothesis is discussed.

I. INTRODUCTION

The following "exchange-degeneracy hypothesis" has been useful recently in particle physics: in any two-hadron scattering amplitude of internal quantum numbers for which no resonances exist, the imaginary part of the sum of either the t- or u-channel Regge terms must vanish.[1,2] This usually requires cancellation from exchange-degenerate trajectories that

† Supported in part by the U. S. Atomic Energy Commission.

[1] See, e.g., A. Ahmadzadeh and C. H. Chan, Phys. Letters **22**, 692 (1966); V. Barger, D. Cline, and J. Matos, *ibid.* **29B**, 121 (1969); C. Schmid (unpublished report).
[2] C. B. Chiu and J. Finkelstein, Phys. Letters **27B**, 510 (1968).

correspond to physical particles with opposite parities and spins differing by one. In a recent paper,[3] this principle was applied to the baryon exchange (u-channel) contributions to the scattering of pseudoscalar mesons from spin-$\frac{1}{2}$ baryons, under the assumption of approximate $SU(3)$ symmetry. The purpose of this paper is to extend the investigation of R1 to all Regge trajectory contributions to all two-hadron $\rightarrow$ two-hadron reaction amplitudes.

One of the simplest examples of the principle involves the contributions of the ρ^0 and f^0 trajectories to π^+K^+ scattering. These contributions are of the form

$$A(s,t)=\beta_\rho(t)[1-e^{-i\pi\alpha_\rho(t)}](s/s_0)^{\alpha_\rho(t)}+\beta_f(t)[1+e^{-i\pi\alpha_f(t)}](s/s_0)^{\alpha_f(t)}. \quad (1)$$

The principle implies $\alpha_\rho(t)=\alpha_f(t)$ and $\beta_\rho(t)=\beta_f(t)$. This example is typical, except that in some cases there may be more than one trajectory of one parity, the sum of whose imaginary parts cancels that of one or more trajectories of the other parity.

The simplest argument that one may use to justify the hypothesis results from the duality assumption, that Regge expressions like Eq. (1) represent approximately the s-channel amplitudes in the intermediate- and high-energy regions $s^{1/2}>s^{1/2}(\text{threshold})+1$ BeV. Since either of the terms in Eq. (1) contains resonance loops in partial-wave states, cancellation must occur if no such loops appear. This implies the exchange-degeneracy conditions.[4]

The situation is slightly more complicated for baryon trajectories. Gribov has shown that if baryon trajectories are plotted in the u plane, each trajectory must coincide at $u=0$ with another trajectory that corresponds to physical particles of the opposite parity.[5] A popular model of baryon trajectories is that they are even functions of $u^{1/2}$, in which case the odd- and even-parity trajectories are symmetric around 0 in the $u^{1/2}$ plane. However, if the residue function has no zeros in this picture, MacDowell symmetry implies that baryons should appear as parity doublets.[6] Experimentally, the quantum numbers of observed $SU(3)$ baryon multiplets of one parity are quite different from those of the other. Hence the residue functions must contain zeros and are not at all symmetric in $u^{1/2}$. This destroys the attractiveness of the picture.[7] Therefore, although the detailed dependence of the trajectories on $u^{1/2}$ is not known, we assume an alternative picture in which one branch of each trajectory is relatively flat and does not correspond to physical particles. The exchange-degeneracy condition must apply to both branches. This asymmetric-trajectory picture is not necessary for our results, but enables us to use simple terminology, and to formulate the exchange-degeneracy conditions in the same way for baryons and for mesons.[8]

In Sec. II, the hypothesis is applied to meson and baryon trajectories of various hadron-hadron amplitudes. Some new conditions concerning decay partial widths are compared to experiment. The experimental consequences for scattering processes are discussed in Sec. III. The relation of the hypothesis to the bootstrap assumption is discussed in Sec. IV.

II. CONSISTENCY CONDITIONS

Experimentally, the only $SU(3)$ multiplets of resonances or stable states are singlets and octets of baryon number zero, and singlets, octets, and decuplets of baryon number one. We need not consider external singlets in applying the exchange-degeneracy hypothesis, since hadron-hadron states containing a singlet do not correspond to exotic (nonresonating) multiplets, except in the case of baryon-baryon states. Thus, the relevant external particle multiplets are baryon octets (B), baryon decuplets (D), and meson octets whose $Y=I_z=0$ members are even and odd under particle-antiparticle conjugation. We use pseudoscalar (P) and vector (V) octets as representatives of these meson classes.

In many hadron-hadron states the spins are large enough so that more than one type of coupling with a particular trajectory is possible. For example, there are helicity-flip and nonflip couplings of V trajectories with B particles. However, the exchange-degeneracy principle applies separately to each hadron-hadron spin state, so that the implied coupling ratios are the same for all spin dependences. Therefore, we will neglect spin indices throughout the paper, without further comment.

We consider the general process $a+b\rightarrow c+d$. Application of the exchange-degeneracy condition to the Regge trajectories in the crossed channel $\bar{c}+b\rightarrow \bar{a}+d$ leads to residue conditions of the type

$$\sum_j \beta(j^+,i)=\sum_j \beta(j^-,i)\,, \quad (2)$$

where $j^\pm$ denotes a trajectory whose physical particles are of parity $\pm$, and i denotes the $a+b\rightarrow c+d$ states (which must correspond to an exotic representation). Frequently, one or both of the sums will contain only one term. If we take the states to correspond to representations of $SU(3)$ and assume $SU(3)$ symmetry, the factorizability property allows us to write each residue in the form

$$\beta(j^\pm,i)=C_{ij}G(j^\pm,\bar{c}b)G(j^\pm,\bar{a}d)\,, \quad (3)$$

[3] R. H. Capps, Phys. Rev. Letters **22**, 215 (1969), hereafter referred to as R1.

[4] A recent discussion of implications of the duality assumption is given by Haim Harari, Phys. Rev. Letters **22**, 562 (1969). This paper contains a list of references in which self-consistency conditions are derived from the duality assumption.

[5] V. Gribov, Zh. Eksperim. i Teor. Fiz. **43**, 1529 (1963) [English transl.: Soviet Phys.—JETP **16**, 1080 (1963)].

[6] S. MacDowell, Phys. Rev. **116**, 774 (1960).

[7] Several arguments against the parity-symmetry picture are given by D. H. Lyth, Phys. Rev. Letters **20**, 641 (1968).

[8] The exact form of baryon-trajectory contributions to the scattering of pseudoscalar mesons from spin-$\frac{1}{2}$ baryons is given by Charles B. Chiu and John D. Stack, Phys. Rev. **153**, 1575 (1967).

where $G(j,\bar{a}b)$ is the coupling of the j trajectory to the $\bar{a}b$ state, and C_{ij} is the appropriate element of the $SU(3)$ crossing matrix.

A. Baryon Trajectory Contributions to Meson-Baryon Scattering

It was shown in R1 that the exchange-degeneracy hypothesis requires that the lightest baryon multiplets lie on two approximately exchange-degenerate trajectories, symbolized by

$$\mathbf{10}(\tfrac{3}{2}^+)_{1236} \to \mathbf{8}(\tfrac{5}{2}^-)_{1675}, \quad \text{upper trajectory}$$
$$\mathbf{8}(\tfrac{1}{2}^+)_{938} \to \begin{Bmatrix} \mathbf{8}(\tfrac{3}{2}^-)_{1515} \\ \mathbf{1}(\tfrac{3}{2}^-)+\text{a}\ \mathbf{10}(\tfrac{3}{2}^-)_{1670} \end{Bmatrix}, \quad \text{lower trajectory} \qquad (3')$$

where the quantities in parentheses are the spin and parity, and the subscripts are the masses of the nonstrange particles involved. The amplitude a is determined in R1 to be $(1/18)^{1/2}$. It is speculated in R1 that heavier baryons also lie on trajectories of one of these two types.

Since the meson-baryon vertices that are possible are not limited by the charge-conjugation quantum numbers of the mesons, we need consider only the processes $PB\to PB$, $PD\to PD$, and $PB\to PD$. The results generalize immediately to other octet mesons. For example the predicted F/D ratio for all couplings of the multiplet $\mathbf{8}(\frac{5}{2}^-)$ to V mesons is $-\frac{1}{3}$, the same as the P-meson ratio predicted in R1.

We consider first the contributions of the upper trajectory of Eq. (3′) to $PD\to PD$ processes. The nonresonating PD representations are **27** and **35**. The consistency condition of Eqs. (2) and (3) leads to the equations

$$C_{i,10}G^2(10^+,D)=C_{i,8}G^2(8_u^-,D)\,, \qquad (4)$$

where i refers to the multiplets **27** and **35**, D denotes PD states, and the subscript u denotes the $(j=\frac{5}{2})$ odd-parity octet on the upper trajectory. The coupling constants G^2 are sums over all u-channel PD states coupled to any member of the trajectory multiplet. Consistency requires the condition $C_{35,10}/C_{35,8}=C_{27,10}/C_{27,8}$. It may be seen from the octet-decuplet crossing matrix that these ratios are both equal to $\frac{5}{8}$, so consistency is possible, and the coupling ratio is predicted to be[9,10]

$$G^2(10^+,D)=(8/5)G^2(8_u^-,D)\,. \qquad (5)$$

We next consider PB states, where the exotic multiplets are **10*** and **27**. The couplings of the $j^P=\frac{5}{2}^-$ octet trajectory to D- and F-type PB states in the direct channel of the trajectory are denoted by the symbols $G(8_u^-,B)\cos\theta$ and $G(8_u^-,B)\sin\theta$, where $\tan\theta=(9/5)^{1/2}F/D$. The analysis of the $PB\to PB$ processes in R1 leads to the conclusions $\tan\theta=-(\frac{1}{5})^{1/2}$ and

$$G^2(10^+,B)=(8/3)G^2(8_u^-,B)\,. \qquad (6)$$

Only the multiplet **27** is forbidden in the inelastic process $PB\to PD$. The consistency condition for this process is

$$C_{27,10}G(10^+,D)G(10^+,B)=C_{27,8}G(8_u^-,D)G(8_u^-,B)\,, \qquad (7)$$

where $C_{27,8}=C_{27,8s}\cos\theta+C_{27,8a}\sin\theta$, and the C's are elements of the $\mathbf{8}+\mathbf{8}\to\mathbf{8}+\mathbf{10}$ crossing matrix.[11] For $\tan\theta=-(\frac{1}{5})^{1/2}$, $C_{27,8}/C_{27,10}=8/(15)^{1/2}$, so that Eq. (7) is consistent with the result obtained by multiplying Eqs. (5) and (6) and taking the square root. The reason for this consistency may be understood by considering the elastic and inelastic amplitudes involving K^+p and $K^+\Delta^+$ states. It was pointed out in R1 that $\theta=-\tan^{-1}[(\frac{1}{5})^{1/2}]$ is the angle for which the Λ^* is decoupled from $\bar{K}N$ states, so that only the Σ^{0*} states of the 10^+ and 8_u^- trajectories contribute to these amplitudes. Consistency for $PB\to PD$ then follows from the factorizability of the Σ^{0*} residues.

We now turn to the lower trajectory of Eq. (3′), where the octet and single-decuplet combinations both contribute in odd-parity states. Consistency with respect to elastic PD scattering may be obtained because of the proportionality relation discussed after Eq. (4), and the coupling constant condition is

$$G^2(8^+,D)=G^2(8_l^-,D)+\tfrac{5}{8}G^2(10^-,D)\,, \qquad (8)$$

where l denotes the $(j=\frac{3}{2})$ odd-parity octet of the lower trajectory of Eq. (3′).

The application to PB states is discussed in R1, under the assumption that $\tan\theta=(\frac{4}{5})^{1/2}$ for the 8^+ coupling. It is shown that $\tan\theta=5^{1/2}$ for the 8_l^-, and that $G^2(1^-,B)=18G^2(10^-,B)$. If this latter relation is used to eliminate the singlet coupling from the PB consistency relation, the result may be written

$$G^2(8^+,B)=(6/5)G^2(8_l^-,B)+9G^2(10^-,B)\,. \qquad (9)$$

Using the $\tan\theta$ values given above and the $\mathbf{8}+\mathbf{8}\to\mathbf{8}+\mathbf{10}$ crossing matrix, we next apply the consistency condition to the amplitude $PB\to PD$ in the multiplet **27**.[11] The result is

$$G(8^+,B)G(8^+,D)=-(6/5)^{1/2}G(8_l^-,B)G(8_l^-,D) +\tfrac{3}{2}(\tfrac{5}{2})^{1/2}G(10^-,B)G(10^-,D)\,. \qquad (10)$$

If this equation is squared and subtracted from the product of Eqs. (8) and (9), a quadratic equation for $G(8_l^-,B)G(10^-,D)/G(10^-,B)G(8_l^-,D)$ results, with only one (double) root. The root is given by

$$G(10^-,D)/G(8_l^-,D)=-(12)^{1/2}G(10^-,B)/G(8_l^-,B)\,. \qquad (11)$$

[9] The t- and u-channel octet-decuplet crossing matrices, and the octet-octet crossing matrix, are given by J. J. de Swart, Nuovo Cimento **31**, 420 (1964).

[10] The equality of these ratios is a special case of a theorem proved by R. H. Capps, Ann. Phys. (N. Y.) **43**, 428 (1967). The theorem states that in the $SU(n)$, u-channel crossing matrix for the scattering of the regular representation by a representation that corresponds to a rectangular Young tableau, all columns of the matrix $(C-1)$ are proportional.

[11] The $\mathbf{8}+\mathbf{8}\to\mathbf{8}+\mathbf{10}$ crossing matrix is given by K. Y. Lin and R. E. Cutkosky, Phys. Rev. **140**, B205 (1965).

This equation may be compared with experimental partial-width data, if it is assumed that ratios of couplings of vertices of the same spin structure are independent of u. We use the πN and $\pi\Delta$ decay widths of the $j^P=\frac{3}{2}^-$, $N^*(1515)$ and $\Delta^*(1670)$.[12] The partial-width formula is $\Gamma_i=(C_iG)^2\rho_i$, where C_i is the appropriate Clebsch-Gordan coefficient, and ρ_i is a phase-space factor. We take the PB coupling ratio from the data analysis of R1, i.e.,

$$G^2(10^-,B)/G^2(8_l^-,B)=2/9. \qquad (12)$$

(This ratio leads to a predicted 25-MeV $\Delta^*\to\pi N$ partial width, which compares favorably with the experimental value of $\sim$34 MeV.[13]) We assume the S-wave $\pi\Delta$ decays dominate the D waves, as is suggested by an approximately $SU(6)_W$-symmetric analysis of the decay data.[14] The S-wave phase-space factor is taken to be the decay momentum. We take $\Gamma(N^*\to\pi\Delta)=57$ MeV; this is the $\pi\pi N$ width, which is dominated by $\pi\Delta$. The prediction is then $\Gamma(\Delta^*\to\pi\Delta)=195$ MeV. The observed inelastic width of the Δ^* is about 190 MeV, so our prediction is that the $\pi\Delta$ mode dominates this $\pi\pi N$ decay. This experimental comparison is admittedly preliminary, but represents the first evidence for the approximate validity of the baryon exchange-degeneracy hypothesis that involves residues in other than PB states.

B. Impossibility of Consistency for Both DD and $D\bar{D}$ States

Rosner, without assuming $SU(3)$ symmetry, has shown that the exchange-degeneracy principle cannot be applied consistently to all DD and $D\bar{D}$ states.[15] In an $SU(3)$ framework, the reason for this is simple; the argument given below is a transcription from $SU(6)$ of an argument given previously by the author.[16] We consider the vector (V) and tensor (T) singlet and octet trajectories, and assume at least one of these couples to the decuplet. In order that the imaginary part of the Regge term cancels in all DD states, the two octet residues must be equal and the two singlet residues must be equal. Since the V and T terms behave oppositely under particle-antiparticle conjugation, it follows that the singlet and octet terms do not cancel separately in $D\bar{D}$ scattering. Furthermore, it is impossible that, in $D\bar{D}$ scattering, singlet and octet exchange contributions cancel one another to eliminate all exotic resonances. The impossibility that cancellation occurs in both the $D\bar{D}$ representations **27** and **64** follows from the crossing matrix inequality $C_{27,1}/C_{27,8}\neq C_{64,1}/C_{64,8}$.[17] It is seen that this argument does not depend on the details of the consistency conditions used. The conclusion is simply stated thusly: *In an $SU(3)$-symmetric theory, any nonzero effect in singlet or octet $D\bar{D}$ states that results from singlet and octet exchange in the t channel must be accompanied by an effect in the representation* **27** *or* **64**.

Since the exchange-degeneracy principle can be applied in the meson-meson scattering case (see Sec. II C), it is argued by Rosner that the exotic baryon-antibaryon resonances should not be coupled strongly to meson-meson states.[15] However, one cannot make this argument without considering the coupling of meson-meson and baryon-antibaryon states. We consider the $PP\to D\bar{D}$ amplitudes, since the PD coupling ratios of the baryonic Regge trajectories are determined in Sec. II A. It may be shown that if the 10^+ and 8_u^- trajectories [upper trajectory of Eq. (3′)] couple as is predicted in Eq. (5), their contributions to the 27-fold $PP\to D\bar{D}$ amplitude do not cancel, so such a resonance multiplet would couple to PP states.

Clearly, experimentalists should look for exotic resonances. However, one can argue that the exchange-degeneracy principle is not on very solid ground when applied to two-hadron states (like $D\bar{D}$ or $B\bar{B}$ states) that are directly coupled to much lighter two-hadron states. Hence, in the remainder of this work, we will not apply the principle to baryon-antibaryon states and will not discuss the question of whether or not exotic resonances coupled to these states exist.

C. Meson Trajectories

1. Baryon-Baryon Scattering

The exchange-degeneracy conditions for meson exchange may also be written in the form of Eqs. (2) and (3), provided that the $\bar{c}+b$ and $\bar{a}+d$ states are of baryon number zero. Since there are no resonance multiplets of baryon number two, application to BB, BD, and DD scattering implies that meson octet or singlet trajectories occur in exchange-degenerate pairs. This applies not only to vector and tensor mesons, but to pseudoscalar and axial-vector mesons, etc. In some cases, the spin dependence may be such that conspiracy is required; if so, our hypothesis implies that the conspiring sets occur in exchange-degenerate pairs.[18] Since the couplings to baryons of exchange-degenerate meson pairs must be the same, we suppress the meson spin and parity labels. The coupling constants analogous

[12] Experimental evidence for the $N^*(1670)$ appears (at about 1690 MeV) in the phase-shift analysis of A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968).

[13] The experimental numbers are taken from the compilation of N. Barash-Schmidt *et al.*, Rev. Mod. Phys. **41**, 109 (1969).

[14] R. H. Capps, Phys. Rev. **158**, 1433 (1967).

[15] Jonathan L. Rosner, Phys. Rev. Letters **21**, 950 (1968); **21**, 1468(E) (1968).

[16] R. H. Capps, Phys. Rev. **168**, 1731 (1968). The argument concerning baryon-baryon resonances is in Sec. VI.

[17] We are not aware of the presence of this crossing matrix in the literature. However, the inequality follows because the singlet-exchange terms are equal, the octet-exchange terms are proportional to the quantity Q_i-2Q_{10} (where the Q are the eigenvalues of the quadratic Casimir operator), and because $Q_{27}\neq Q_{64}$.

[18] The combination of exchange degeneracy and conspiracy is discussed by Akbar Ahmadzadeh, Phys. Rev. Letters **20**, 1125 (1968).

to the G's of Sec. II A are denoted by B_{8s}, B_{8a}, B_1, D_8, D_1, and $(BD)_8$, where (BD) refers to B-D-meson coupling, the subscript refers to the $SU(3)$ multiplet of the meson, and a and s denote antisymmetric and symmetric octet-octet-octet couplings. The angle θ of Sec. II A for such a coupling is given by $\tan\theta = B_{8a}/B_{8s}$. The normalization rule for the coupling constants is the same as was applied to baryon trajectories; e.g., D_8^2 is the sum of the squares of the couplings of one of the octet meson trajectory with all $\bar{D}\bar{D}$ states.

2. Meson-Meson Scattering

Since all the external mesons in exotic meson-meson representations are octet particles, the couplings at meson-meson-meson vertices may be denoted by M_{8s}, M_{8a}, and M_1, where again the index is the multiplet of the trajectory. In this case, the couplings of exchange-degenerate trajectories are not the same. If the symmetric (M_{8s} and M_1) couplings are allowed for a particular trajectory and particular external meson states, the antisymmetric M_{8a} coupling is allowed for the exchange-degenerate partner trajectory. We illustrate the self-consistency conditions by considering V and T trajectories coupled to PP and PV vertices. The tensor coupling constants are denoted with a prime.

There are two types of meson-meson scattering processes; in the first the symmetries of the couplings of a particular trajectory with the two vertices are the same. We consider the example $PP \to PP$. The exchange-degeneracy hypothesis has been applied to the representations $\mathbf{10} \oplus \mathbf{10^*}$ and $\mathbf{27}$ in the literature; the results are[2,19]

$$M_1'^2 = (16/5) M_{8s}'^2, \qquad (13)$$

$$M_{8s}'^2 = (5/9) M_{8a}^2. \qquad (14)$$

Similar conditions, with the roles of tensor and vector trajectories reversed, apply to the $PP \to VV$ process, i.e.,

$$M_{8s}^2 = (5/16) M_1^2 = (5/9) M_{8a}'^2. \qquad (15)$$

In the second type of meson-meson process, the symmetries of the couplings of a trajectory are opposite, so only $8s \to 8a$ type t-channel amplitudes are involved. An example is the $PP \to PV$ process. It is seen from the octet-octet crossing matrix that this type of coupling does not contribute to the representation $\mathbf{27}$, while cancellation in the representation $\mathbf{10} \oplus \mathbf{10^*}$ implies the condition

$$M_{8s} M_{8a} = M_{8a}' M_{8s}'. \qquad (16)$$

Clearly, if Eqs. (13)–(15) are valid for all processes of the first type, the signs of the couplings may be chosen so that Eq. (16) is true also.

The condition of Eq. (13) is the same as is obtained from a simple nonet theory of symmetric, meson coupling. In the physical region of the PP decays of tensor mesons, this condition has been tested, but the uncertainty concerning f-f' mixing and the experimental inaccuracy are such that these tests are not very meaningful.[20]

3. Meson-Baryon Scattering

The only external mesons we need consider in meson-baryon scattering amplitudes are P mesons. (The case where the external mesons are a P and V may be obtained by reversing the roles of the vector and tensor trajectories.) We make use of the octet-octet and $\mathbf{8}+\mathbf{8} \to \mathbf{8}+\mathbf{10}$ crossing matrices, and the matrix relating $\mathbf{8}+\mathbf{8} \to \mathbf{10}+\mathbf{10^*}$ trajectories to the amplitudes $\mathbf{8}+\mathbf{10} \to \mathbf{8}+\mathbf{10}$.[9,11] The sign conventions used in the literature for such matrices differ, since one may change the sign of a whole column without altering the basic properties of the matrices. We adopt the following two conventions: (a) Positive values of the octet coupling constants B_{8s}, B_{8a}, D_8, $(BD)_8$, M_{8s}, and M_{8a} correspond to positive couplings of the ρ^0 and $(A_2)^0$ trajectories to the vertices p-p, $\Delta^{++}-\Delta^{++}$, $p-\Delta^+$, and K^+-K^+. (b) Positive singlet coupling constants B_1, D_1, and M_1 correspond to positive couplings to the individual particles. With these conventions, all crossing-matrix elements in singlet and octet columns corresponding to the largest meson-baryon representations are positive (except for the zero elements in the $8s \to 8a$ column in the octet-octet case).

We consider the $PD \to PD$ process first. Application of the consistency condition, Eqs. (2) and (3), to the representations $\mathbf{27}$ and $\mathbf{35}$ leads to the equations[21]

$$M_1' D_1 - 6(\tfrac{2}{5})^{1/2} M_{8s}' D_8 = -\tfrac{2}{3}(2)^{1/2} M_{8a} D_8, \qquad (17)$$

$$M_1' D_1 + 2(\tfrac{2}{5})^{1/2} M_{8s}' D_8 = 2(2)^{1/2} M_{8a} D_8. \qquad (18)$$

These equations imply

$$M_{8s}' = (5/9)^{1/2} M_{8a}, \qquad (19)$$

and, if $M_1'^2/M_{8s}'^2$ is taken from Eq. (13), they also imply

$$D_1^2 = 2 D_8^2. \qquad (20)$$

The first of these, Eq. (19), is consistent with Eq. (14).

In the $PB \to PB$ case, application of the condition to the representations $\mathbf{10^*}$ and $\mathbf{27}$ leads to the equations

$$\tfrac{1}{8} M_1' B_1 - \tfrac{2}{5} M_{8s}' B_{8s} - (\tfrac{1}{5})^{1/2} M_{8s}' B_{8a} = -(\tfrac{1}{5})^{1/2} M_{8a} B_{8s}, \qquad (21)$$

$$\tfrac{1}{8} M_1' B_1 + \tfrac{1}{5} M_{8s}' B_{8s} = \tfrac{1}{3} M_{8a}' B_{8a}. \qquad (22)$$

One may use Eq. (13) to reduce these equations to Eq. (19) and the equation

$$B_1^2 = [2B_{8a} - (\tfrac{4}{5})^{1/2} B_{8s}]^2. \qquad (23)$$

[19] The conditions of Eq. (12) of Ref. 16, are equivalent to Eqs. (13)–(15) of the present paper.

[20] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965); R. H. Capps, Phys. Rev. **165**, 1899 (1968).

[21] The B_{8a}/B_{8s} ratio is related to F/D by $B_{8a}/B_{8s} = (9/5)^{1/2} F/D$.

We also consider the inelastic process $PB \to PD$. It may be shown that application of the condition to the representation **27** again leads to Eq. (19).

The fact that a fixed F/D ratio for meson couplings to baryons is not predicted is fortunate, since this ratio appears to be very different for P and V couplings. If $B_{8a}/B_{8s} \sim (\frac{4}{5})^{1/2}(F/D \sim \frac{2}{3})$ for P mesons, Eq. (23) implies that the coupling of the singlet η' (958) to the baryon octet is small.

III. COMPARISON WITH SCATTERING EXPERIMENTS

One type of experimental comparison of the exchange-degeneracy predictions involves the ratios of decay amplitudes. Such comparisons are discussed in R1 and in Sec. II of this paper. In this section, we discuss the predictions concerning Regge-pole analyses of scattering data, starting with meson trajectories.

The predicted ratio of vector and tensor octet-trajectory couplings in PB scattering is consistent with Regge analyses of the experimental data.[22] The well-known prediction that the F/D ratio for the VBB and TBB couplings of the same spin structure are the same has been tested several times. It is in excellent agreement with the 1966 data analysis of Barger, Olsson, and Sarma,[23] and in fair agreement with the recent analysis of Reeder and Sarma.[24] Equations (4.27) of Reeder and Sarma are equivalent to the ratios $B_{8s}/B_{8a}(V) = -0.11$ and $B_{8s}/B_{8a}(T) = -0.43$ for the helicity-nonflip parts of the V and T trajectory couplings, and to the ratios -2.1 and -0.42 for the helicity-flip parts.[21]

We next discuss the predicted singlet/octet ratios of the V and T trajectory residues. If B_{8s} for the helicity-non-flip V coupling were zero, Eqs. (20) and (23) would coincide with the prediction of the quark model for these coupling ratios, a prediction that includes "universality" of V coupling. Furthermore, if the φ meson is taken as the quark-model combination, $\varphi = (\frac{1}{3})^{1/2}(\sqrt{2}\omega_8 - \omega_1)$, then Eq. (23) corresponds to the decoupling of the φ from nucleons for any value of B_{8s}, provided the sign of B_1/B_{8a} is taken correctly. The smallness of the φNN coupling is a well-known effect.

Since the observed D/F ratios are negative and not nearly zero, an interesting question is whether or not the B_1/B_{8a} ratios are greater than 2, as predicted by Eq. (23). The couplings to baryons of the tensor singlet are better known than those of the vector singlet, since the former contribute to $PB \to PB$ amplitudes. The analysis of tensor couplings of Barger, Olsson, and Sarma, normalized by the condition $F+D=1$, yields $F = 2.0 \pm 0.6$ and $\delta = 2.3 \pm 1.1$, where the condition $\delta = 2F - 1$ is equivalent to our Eq. (23).[21,23] Thus, these data are not yet accurate enough to answer the question.

[22] Jane C. Jackson, Phys. Rev. **174**, 2098 (1968).

[23] V. Barger, M. Olsson, and K. V. L. Sarma, Phys. Rev. **147**, 1115 (1966).

[24] D. D. Reeder and K. V. L. Sarma, Phys. Rev. **172**, 1566 (1968).

Recently, Banerjee and Levinson have made a study of various ω and ρ interactions.[25] They assume ω and ρ universality, and conclude that the coupling ratio g_ω^2/g_ρ^2 is ~ 1.9 times the simple quark-model prediction [the prediction is $B_1 = 2B_{8a}$ in the notation of Eq. (23)]. Their input includes other data besides Regge-pole fits to N-N scattering; nevertheless, their result is tentative evidence in favor of our speculation that $B_1/B_{8a} > 2$.

We now turn to baryon-exchange Regge poles. The experimental implications of the exchange-degeneracy hypothesis are not well known for baryon poles. Odd-parity baryon trajectories are frequently omitted from analyses of meson-baryon scattering near the backward direction. However, the coupling-ratio predictions of R1 and Sec. II A lead to definite predictions for the ratio of odd-parity to even-parity baryon-trajectory contributions of meson-baryon scattering. Hence, one may include the odd-parity contributions in an analysis without adding extra undetermined parameters.

In order to facilitate testing the baryon exchange-degeneracy hypothesis, we have calculated the predicted ratios of odd-parity to even-parity trajectory contributions to some PB amplitudes. The baryon Regge terms may be written in the form[8]

$$A = \sum_i \{[\beta_i(\sqrt{u})/\sqrt{u}](1+\eta_i e^{-i\pi[\alpha_i(\sqrt{u})-1/2]}) \times (s/s_0)^{\alpha_i(\sqrt{u})-1/2-n} - [\beta_i(-\sqrt{u})/\sqrt{u}] \times (1+\eta_i e^{-i\pi[\alpha_i(-\sqrt{u})-1/2]})(s/s_0)^{\alpha_i(-\sqrt{u})-1/2-n}\}, \quad (24)$$

where n is an integer that depends on the spin structure of the amplitude, and η is ± 1; the values of η are opposite for exchange-degenerate partners. The η values are 1 and -1, respectively, for the B and D trajectories.

Our hypothesis requires that the residue ratio for exchange-degenerate partners be a constant at both positive and negative $\sqrt{u}$. The values of these ratios are given for certain amplitudes in Table I. For convenience, we have listed in columns 1 and 3 the relative amplitudes for the even-parity decuplet and octet

TABLE I. Relative contributions of exchange-degenerate baryon partners to some PB amplitudes.

Amplitude	1 $C(10^+)$	2 R_{10^+}	3 $C(8^+)$	4 $R_{8^+}{}^a$	5 $R_{8^+}{}^b$
$\pi^- p \to \pi^0 n$	$\frac{1}{3}(\frac{1}{2})^{1/2}$	$-\frac{1}{8}$	$-(25/54)(\frac{1}{2})^{1/2}$	16/25	$-2/25$
$\pi^- p \to \bar{K}^0 \Lambda$	$\frac{1}{2}(\frac{1}{6})^{1/2}$	$-\frac{1}{2}$	$-\frac{1}{9}(\frac{1}{6})^{1/2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$\pi^- p \to K^+ \Sigma^-$	$\frac{1}{12}$	$-\frac{1}{2}$	$-7/54$	1	-2
$K^- p \to \pi^0 \Lambda$	No contribution		$-(5/12)(\frac{1}{3})^{1/2}$	$\frac{4}{5}$	0
$K^- p \to \pi^0 \Sigma^0$	$-\frac{1}{3}$	$-\frac{1}{8}$	5/108	$-\frac{4}{5}$	$-\frac{4}{5}$
$K^- p \to \pi^+ \Sigma^-$	$-\frac{1}{6}$	$-\frac{1}{2}$	5/54	$-\frac{4}{5}$	$-\frac{1}{5}$
$\pi^+ p \to \pi^+ p$	$\frac{1}{6}$	$\frac{1}{4}$	25/54	16/25	1/25
$\pi^+ p \to K^+ \Sigma^+$	$-\frac{1}{12}$	$-\frac{1}{2}$	$-11/54$	2/11	$-13/11$

[25] M. K. Banerjee and C. A. Levinson, Phys. Rev. **176** 2140 (1968).

exchanges, respectively. Column 2 shows the ratio $R_{10^+}=\beta_{8^-u}/\beta_{10^+}$, where β_{8^-u} is the residue of the $j^P=\frac{5}{2}^-$ octet [upper trajectory of Eq. (3′)]. This ratio is one for PB states of the representations **27** and **10***. The ratios of columns 4 and 5 are $R_{8^+}{}^a=\beta_{8^-l}/\beta_{8^+}$, $R_{8^+}{}^b=\beta_{(10+1)}/\beta_{8^+}$; these refer to the cases where the entire contribution of the odd-parity baryons to the lower trajectory results from the $j^P=\frac{3}{2}^-$ octet trajectory ($R_{8^+}{}^a$), and singlet-decuplet trajectory ($R_{8^+}{}^b$). The actual (odd-parity/even-parity) residue ratio for this lower trajectory is the following mixture:

$$R_{8^+}=\frac{2G^2(8_l^-,B)R_{8^+}{}^a+15G^2(10^-,B)R_{8^+}{}^b}{2G^2(8_l^-,B)+15G^2(10^-,B)}\,, \qquad (25)$$

where the G^2 are the coupling constants of Sec. II A. If the ratio of the G^2 is taken from Eq. (12), definite values of R_{8^+} result.

We illustrate the use of the table by considering the process $\pi^-p\to\pi^0n$. If the exchange-degeneracy assumption is made, the contributions of the Δ and $N^*(\frac{5}{2}^-$ octet) trajectories may be combined, so that the first term in the sum of Eq. (24) becomes

$$(\beta_\Delta/\sqrt{u})[(1-e^{-i\pi(\alpha-1/2)}) -\tfrac{1}{8}(1+e^{-i\pi(\alpha-1/2)})](s/s_0)^{\alpha-1/2-\pi}.$$

The factor $-\frac{1}{8}$ is taken from the second column of the table. We have suppressed the $\sqrt{u}$ in the arguments of α and β. The second term of Eq. (24) is modified in the same way. Similarly, the contributions of the $N^*(\frac{3}{2}^-$ octet) and $\Delta^*(\frac{3}{2}^-$ decuplet) may be combined with that of the N. In this case, the signature factor is different and the factor in front of the odd-parity term is R_{8^+}, rather than $-\frac{1}{8}$. The value of R_{8^+} that results from Eqs. (25) and (12) and the first line of the table is 0.19.

It is seen from Table I and Eqs. (24) and (25) that for many PB processes, the odd-parity trajectories are expected to make a large effect on the magnitude and phase of the baryon Regge term.

The experimental tests concerning decay amplitudes, discussed in Sec. II and in R1, refer to the ratios of couplings of the same spin structure. However, the ratios of residues of trajectories of opposite parity are also predicted by the basic hypothesis; these too may be checked with decay data if a particular model of the t (or u) dependence of residues is used.

IV. REMARKS ON SELF-CONSISTENCY CONDITIONS

The exotic (nonresonating) meson-meson, meson-baryon, and baryon-baryon representations may be characterized by a simple quark-counting criterion, they contain states in which no quark occurs together with its own antiquark. Several years ago the author used this quark-counting picture to prove that if the MMM and MBB vertices are $SU(6)_W$-symmetric, the spin-dependent (tensor) forces resulting from P- and V-meson exchange vanish in these exotic states.[26] The present work and R1 show that exchange degeneracy is necessary if all meson and baryon exchange forces are to vanish in the exotic states. Rosner and Harari have shown that some exchange-degeneracy predictions concerning scattering are obtained simply if the quark-counting argument is generalized to two processes related by crossing.[4,27] The situation seems confusing because these related arguments start with different premises. However, all these results have been obtained from some sort of self-consistency requirements, so it is possible that they all originate in a general bootstrap requirement.

In order to illustrate the connection between different bootstrap models, we compare the present work with a previous work by the author.[16] In Ref. 16, application of a self-consistency requirement to hadron-hadron scattering in the foward and backward directions led to the prediction of an $SU(n)_W$ vertex symmetry and to baryons and mesons of both parties. If the symmetry group is $SU(6)$, and the positive-parity baryons are associated with the representation **56**, the odd-parity baryons could be associated with the **70** (as observed experimentally), or with the **56**. On the other hand, the exchange-degeneracy requirement, coupled with the bootstrap hypothesis that both meson and baryon exchange forces are necessary to produce the existing hadrons, rules out the possibility that the odd- and even-parity multiplets are the same, since cancellation of trajectory terms would then occur in all MB representations, rather than just the exotic ones. This difference in predictions arises because the self-consistency requirements of the two models are complementary. In Ref. 16, states of all internal quantum numbers are considered, but only two scattering angles are involved. The exchange-degeneracy conditions involve all momentum transfers but only internal states of large quantum numbers. It is quite possible that a complete bootstrap model would lead to the prediction both of $SU(6)_W$ symmetry and of different representations for the even- and odd-parity baryons.

The mesons of different parities, in contrast to the baryons, *must* correspond to the same representations of $SU(3)$ [or $SU(6)$] in order that no BB or DD resonances occur. This does not lead to cancellation of the imaginary parts of the meson trajectories in all MM, MB, and MD representations, because particle-antiparticle conjugation invariance requires that if one trajectory interacts with mesons with an F-type interaction, its exchange-degenerate partner must interact with the different, D-type, interaction.

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[26] R. H. Capps, Phys. Rev. Letters **16**, 1066 (1966).
[27] J. L. Rosner, Phys. Rev. Letters **22**, 689 (1969).