An interesting offshoot of our investigation was the discovery of a three-hadron resonance which has the unusual property of moving to higher energy and becoming more prominent as the H-H coupling is increased. Pagels<sup>23</sup> has suggested that there might exist a three-pion resonance just above 0.42 BeV/ $c^2$ , with the quantum numbers of the pion. Although the absence of isospin in our model makes detailed comparison impossible, the analogy is suggestive, particularly since our *H*-*H* interaction, for moderate strengths, produces a phase shift closely resembling current ideas about

<sup>23</sup> H. Pagels, Phys. Rev. 179, 1337 (1969).

the S-wave  $\pi$ - $\pi$  interaction.<sup>24</sup> We are currently investigating the properties of this resonance in more detail.

# ACKNOWLEDGMENTS

We would like to thank Professor S. Frankel, Professor S. P. Rosen, and Professor H. Primakoff for many valuable conversations.

<sup>24</sup> That is, our phase shift resembles recent phenomenological analyses of the S-wave  $T=0 \pi - \pi$  phase shift [S. Marateck *et al.*, Phys. Rev. Letters **21**, 1613 (1968)]. It also has the small scatter-ing length required by current algebra [S. Weinberg, Phys. Rev. Letters **17**, 616 (1966)].

PHYSICAL REVIEW

#### VOLUME 185, NUMBER 5

25 SEPTEMBER 1969

# **Electromagnetic Simulation of Time-Reversal Violation in** Mirror Spin-<sup>3</sup> Beta Decays\*

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With time-reversal invariance, the  $\beta$ -decay correlation  $(\langle \mathbf{J} \rangle / J) \cdot \mathbf{p}_i \times \mathbf{p}_{\nu}$ , where  $\langle \mathbf{J} \rangle / J$  is the polarization of the decaying nucleus and  $\mathbf{p}_i$  ( $\mathbf{p}_i$ ) is the momentum of the electron (neutrino), can arise only through a final-state electromagnetic interaction. For allowed transitions with vector and axial-vector couplings, this effect is recoil-dependent. It was shown by Callan and Treiman that this effect, for the special class of spin- $\frac{1}{2}$  mirror transitions, and the assumption of the conserved-vector-current (CVC) hypothesis, is dominated by weak magnetism. A corresponding calculation for the class of spin-3 mirror transitions, of which the decay  $Ar^{35} \rightarrow Cl^{35} + e^+ + \nu_e$  is an example, shows a similar domination by weak magnetism. The magnitude of this effect is estimated for several mirror  $\beta$  transitions.

#### INTRODUCTION

NE of the classic tests of T invariance in nuclear  $\beta$ decay is the search for a possible correlation in the decay spectrum of the form  $(\langle \mathbf{J} \rangle / J) \cdot (\mathbf{p}_l \times \mathbf{p}_p)$ , where  $\langle \mathbf{J} \rangle / J$  is the polarization of the decaying nucleus, and  $\mathbf{p}_{l}$  ( $\mathbf{p}_{\nu}$ ) is the momentum of the electron (neutrino). In the absence of electromagnetic final-state interactions, this correlation is forbidden by time-reversal invariance. Experimental upper limits on the presence of such a correlation term have been obtained for the spin- $\frac{1}{2}$ mirror transitions  $n \to p e \bar{\nu}^{1}$  and  $\mathrm{Ne}^{19} \to \mathrm{F}^{19} e^{+} \nu^{.2}$  There is also possible experimental interest on this correlation in the spin- $\frac{3}{2}$  mirror transition Ar<sup>35</sup>  $\rightarrow$  Cl<sup>35</sup> $e^+\nu$ .<sup>3</sup>

The allowed  $\beta$  spectrum, summed over all final-spin polarizations, has the following form in the standard

<sup>8</sup> E. D. Commins (private communication via S. B. Treiman).

theory with vector and axial-vector couplings (with the neglect of nuclear recoil<sup>4</sup>):

$$d\omega(\langle \mathbf{J} \rangle | E_l, \Omega_l, \Omega_{\nu}) dE_l d\Omega_l d\Omega_{\nu}$$

$$= \frac{F(\mp Z, E_l)}{(2\pi)^5} p_l E_l (E_0 - E_l)^2 dE_l d\Omega_l d\Omega_{\nu} \xi \left\{ 1 + a \frac{\mathbf{p}_l \cdot \mathbf{p}_{\nu}}{E_l E_{\nu}} + c \left[ \frac{\mathbf{p}_l \cdot \mathbf{p}_{\nu}}{3E_l E_{\nu}} - \frac{(\mathbf{p}_l \cdot \hat{J})(\mathbf{p}_{\nu} \cdot \hat{J})}{E_l E_{\nu}} \right] \left[ \frac{J(J+1) - 3(\mathbf{J} \cdot \hat{J})^2}{J(2J-1)} \right] + \frac{\langle \mathbf{J} \rangle}{J} \cdot \left[ A \frac{\mathbf{p}_l}{E_l} + B \frac{\mathbf{p}_{\nu}}{E_{\nu}} + D \frac{\mathbf{p}_l \times \mathbf{p}_l}{E_l E_{\nu}} \right] \right\}, \quad (1)$$

where  $F(\mp Z, E_l)$  is the well-known Fermi function that accounts for the Coulomb modification of the electron (positron) spectrum;  $\xi$ , a, c, A, B, and D are simply related to the vector and axial-vector couplings which are, effectively, constants in the realm of  $\beta$  decay; and  $\hat{J}$  is a unit vector along **J**. Present experimental limits on D, the coefficient of the correlation effect  $(\langle \mathbf{J} \rangle / J)$ 

<sup>\*</sup> Work supported in part by the Atomic Energy Commission, under Contract No. AT(11-1) Gen. 10, P.A. 19.

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California, Irvine, Calif. 92664. <sup>1</sup> M. T. Burgy, V. E. Krohn, T. B. Novey, G. R. Ringo, and V. L. Telegdi, Phys. Rev. **120**, 1829 (1960); M. A. Clark and J. M. Robson, Can. J. Phys. **38**, 693 (1960); B. G. Erozolimsky, L. N. Bondarenko, Yu A. Mostovoy, B. A. Obinyakov, V. P. Zacharova, and V. A. Titov, Phys. Letters **27B**, 557 (1968). <sup>2</sup> F. P. Calaprice, E. D. Commins, H. M. Gibbs, and G. L. Wick, Phys. Rev. Letters **18**, 918 (1967). <sup>3</sup> E. D. Commins (private communication via S. B. Treiman).

<sup>&</sup>lt;sup>4</sup> J. D. Jackson, S. B. Treiman, and H. W. Wyld, Jr., Nucl. Phys. 4, 206 (1957); see also Phys. Rev. 106, 517 (1957).

 $(\mathbf{p}_l \times \mathbf{p}_{\nu})/E_l E_{\nu}$ , are for neutron  $\beta$  decay, D(n) = 0.01 $\pm 0.01$ , and for Ne<sup>19</sup> decay,<sup>2</sup>  $D(Ne^{19}) = 0.002 \pm 0.014$ .

There have been theoretical estimates on the magnitude of D based upon the assumption of time-reversal invariance, but with the inclusion of final-state electromagnetic interactions. It was found by Jackson, Treiman, and Wyld<sup>4</sup> that, for allowed transitions with vector and axial-vector couplings, and neglecting nuclear recoil, the coefficient D receives no contribution from the final-state electromagnetic interaction, i.e., D is not generated to lowest order in  $Z\alpha$ , where  $\alpha$  is the fine-structure constant  $\alpha \simeq 1/137$ . This is the basis for experimental interest in D for testing time-reversal invariance, since crude estimates then suggest that Dis of order  $Z\alpha E_l/M$  (M is the nuclear mass) or  $(Z\alpha)^2$ . The magnitude of D from the final-state interaction in these cases would not contaminate tests of T invariance for some time to come.

However, it was noticed by Callan and Treiman<sup>5</sup> that the nuclear-recoil effects can generate a much larger contribution to D. They considered spin- $\frac{1}{2}$  mirror transitions, but the argument is relevant to any mirror transition if one considers only the lowest relevant order in the nuclear recoil. Inspection of the recoil phenomena shows that there are two sources. One is electron (positron) scattering off the nuclear magnetic moment. It was shown that this contributes to D on the order  $Z\alpha E_l/M$ , as estimated. The other comes from recoil-dependent effects of the  $\beta$ -decay interaction. Assuming the existence only of "first class" currents as defined by Weinberg,<sup>6</sup> the  $\beta$ -decay interaction will have two contributions in addition to the normal vector and axial-vector terms. One is the induced pseudoscalar term,<sup>7</sup> but its contrubition is negligibly small. The other is the "weak magnetism"<sup>8</sup> term. The magnitude of this term can be estimated on the basis of the conservedvector-current (CVC) hypothesis<sup>8</sup> from the magnetic moments of the parent and daughter nucleus. It was shown that the contribution to D from this source is of order  $Z\alpha(E_l/M)(M/m)(\mu_f - \mu_i)$ , where *m* is the nucleon mass and  $\mu_f(\mu_i)$  is the magnetic moment of the daughter (parent) nucleus. It represents an increase by a factor of the order A (A = M/m), the atomic mass number of the nucleus). In the case of  $Ne^{19}\beta$  decay, it was found that D is about  $2 \times 10^{-4}$  for the most energetic positrons. The number is still very small. However, it is dominated by weak magnetism, and, in the absence of T violation in  $\beta$ decay, a measurement of D will constitute a new test of CVC. The hope is that for favorable allowed transitions, the magnitude of D may become of order  $10^{-3}$  and thus

be experimentally detectable in the near future. The calculation of D for spin- $\frac{3}{2}$  mirror  $\beta$  transitions became of interest due to the existence of higher-spin mirror  $\beta$  decays, and, in particular, the decay  $Ar^{35} \rightarrow Cl^{35}e^+\nu$ .

## DISCUSSION OF DETAILS

The calculation of *D* induced from an electromagnetic final-state interaction for the spin- $\frac{3}{2}$  mirror  $\beta$  transition is straightforward, but tedious, and it follows the general procedure outlined by Callan and Treiman.<sup>5</sup> We consider the invariant T-matrix element for the spin- $\frac{3}{2}$  mirror decay process  $(Z-1, A) \rightarrow (Z, A) + \beta^- + \bar{\nu}_e$ (using the Rarita-Schwinger<sup>9</sup> formalism for higher spins) in the absence of electromagnetism. It has the following structure, up to an over-all constant:

$$T_0^{\rm wk} = \bar{u}(l)\gamma_{\alpha}(1+\gamma_5)v(l^{\nu}) \times (V_{\alpha}+A_{\alpha}), \qquad (2)$$

where

$$V_{\alpha} = \bar{u}_{\mu}(p) \left[ \left( f_1 \delta_{\mu\nu} + f_3 \frac{q_{\mu}q_{\nu}}{4M^2} \right) \gamma_{\alpha} + \left( f_2 \delta_{\mu\nu} + f_4 \frac{q_{\mu}q_{\nu}}{4M^2} \right) \frac{q_{\beta}}{2M} \sigma_{\alpha\beta} \right] u_{\nu}(n), \quad (3)$$

$$A_{\alpha} = \vec{u}_{\mu}(p) \left[ \left( g_1 \delta_{\mu\nu} + g_3 \frac{q_{\mu}q_{\nu}}{4M^2} \right) \gamma_{\alpha} \gamma_5 + \left( g_2 \delta_{\mu\nu} + g_4 \frac{q_{\mu}q_{\nu}}{4M^2} \right) \frac{iq_{\alpha}}{2M} \gamma_5 \right] u_{\nu}(n), \quad (4)$$

and n, p, l, and  $l^{\nu}$  are the momenta of the parent nucleus. the daughter nucleus, the electron, and the neutrino, respectively;  $p^2 = n^2 = -M^2$ ,  $l^2 = -m_l^2$ ,  $(l^p)^2 = 0$ ,  $q_\alpha = n_\alpha - p_\alpha$  is the momentum transfer, and all  $f_i$ 's,  $g_i$ 's are functions of  $q^2$ , but are effectively constants in the energy region of  $\beta$  decay. They can be approximated by their respective values at  $q^2=0$ . The  $f_i$ 's and  $g_i$ 's are also relatively real if T invariance holds. The structure terms above, for both the vector and axial-vector current matrix elements, are those of Weinberg's6 "first class" type. The "second class" terms are ignored, despite possible large violation of charge symmetry. The complexity of the above is reduced by retaining terms only up to first order in nuclear recoil. The induced pseudoscalar term is also dropped. The invariant Tmatrix element then has the simpler form

$$T_{0}^{\mathrm{wk}} = \bar{u}(l)\gamma_{\alpha}(1+\gamma_{5})v(l^{\nu}) \\ \times \bar{u}_{\mu}(p)[\gamma_{\alpha}(1+g\gamma_{5})+f_{2}(q_{\beta}/2M)\sigma_{\alpha\beta}]u_{\mu}(n), \quad (5)$$

where the parameter g is related to the Fermi and Gamow-Teller matrix elements  $M_{\rm F}$  and  $M_{\rm GT}$ , respectively, by

$$g = -(g_{\rm A}/g_{\rm V})(\sqrt{\frac{3}{5}})(M_{\rm GT}/M_{\rm F}), \qquad (6)$$

<sup>&</sup>lt;sup>5</sup> C. G. Callan, Jr., and S. B. Treiman, Phys. Rev. **162**, 1494 (1967); also I. B. Khriplovich and L. B. Okun, Yadern Fiz. **6**, 1265 (1967) [English transl.: Soviet J. Nucl. Phys. **6**, 919

<sup>6, 1205 (1907) [</sup>Lungman and (1968)].
<sup>6</sup> S. Weinberg, Phys. Rev. 112, 375 (1958).
<sup>7</sup> L. Wolfenstein, Nuovo Cimento 8, 882 (1958); M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958); Y. Nambu, Phys. Rev. Letters 4, 380 (1960).
<sup>8</sup> R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958); M. Gell-Mann, *ibid.* 111, 362 (1958).

<sup>&</sup>lt;sup>9</sup> W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941); R. E. Behrends and C. Fronsdal, *ibid*. 106, 345 (1957).

and  $g_A/g_V = -1.23 \pm 0.02^{10}$  is the ratio of axial-vector to vector coupling constant in the elementary  $\beta$ -decay process. For the case of Ar<sup>35</sup>  $\rightarrow$  Cl<sup>35</sup> $e^+\nu$ ,  $g = 0.10 \pm 0.05$ .<sup>11</sup> In general,  $f_2$  is undetermined; but, on the basis of CVC, the parameter  $f_2$  has a definite relation to the magnetic moments of the parent and daughter nuclei, since these belong to a mirror pair. It is

$$1 + f_2 = (M/m)(\mu_+ - \mu_-) \equiv f_5, \qquad (7)$$

where M/m = A, the atomic mass number;  $\mu_{+}$  ( $\mu_{-}$ ) is the total magnetic moment of the  $I_3 = +\frac{1}{2} \left(-\frac{1}{2}\right)$  component of the nuclear isodoublet. These are given in units of the nucleon Bohr magneton. For the case of  $Ar^{35} \rightarrow Cl^{35}e^+\nu, \ \mu_+ = \mu_i = 0.632; \ \mu_- = \mu_f = 0.822, \ and \ thus$  $f_5 \simeq -6.7$ .

Assuming T invariance, a triple-product correlation in the decay spectrum can only be generated by the electromagnetic final-state interaction. This interaction modifies the structure of the  $\beta$ -decay matrix element both by changing the magnitude of the above coefficients and by giving rise to new types of terms. Because we are interested only in the lowest-order  $\alpha$  contribution to a triple-product correlation, it is sufficient to consider only the absorptive part of the decay amplitude generated by electromagnetism to lowest-order  $\alpha$ . This is given by the unitarity relation

Im
$$T_{fi}^{wk} = \frac{1}{2} (2\pi)^4 \sum_n T_{fn}^{\dagger} T_{ni},$$
 (8)

in which  $T_{ni}$  represents a decay process in the absence of electromagnetism, and  $T_{fn}^{\dagger}$  represents the electromagnetic scattering in the final state. In general, the set of intermediate states which are to be included above must contain excited states of the daughter nucleus that are also reached by the  $\beta$  interaction. However, the matrix elements to these states are, in general, smaller than those to the mirror state.<sup>12</sup> It is clear then that these states contribute little to the absorptive part above. The major contribution then comes from elastic electron-nucleus scattering in the final state.

The invariant T matrix for elastic electron-nucleus scattering is given, to lowest order in  $\alpha$ , by the onephoton exchange diagram. It has the structure

$$T^{\gamma} = \frac{Ze^{2}}{k^{2}} \bar{u}(l) \gamma_{\alpha} u(l') \bar{u}_{\mu}(p) \left[ \left( G_{1} \delta_{\mu\nu} + G_{3} \frac{k_{\mu} k_{\nu}}{4M^{2}} \right) \gamma_{\alpha} + \left( G_{2} \delta_{\mu\nu} + G_{4} \frac{k_{\mu} k_{\nu}}{4M^{2}} \right) \frac{k_{\beta}}{2M} \sigma_{\alpha\beta} \right] u_{\nu}(p'), \quad (9)$$

where p', l'(p,l) are the momenta of the nucleus and

electron before (after) the scattering, k = p' - p = l - l', and  $G_i$ , i=1-4, are form factors related to the charge, magnetic moment, etc. Again we make the approximation of keeping the two lowest-order terms in nuclear recoil, and evaluating the respective  $G_i(k^2)$ 's at  $k^2 = 0$ . The T matrix reduces to

$$T^{\gamma} = (Ze^{2}/k^{2})\bar{u}(l)\gamma_{\alpha}u(l') \\ \times \bar{u}_{\gamma}(p)[\gamma_{\alpha} + G_{2}(k_{\beta}/2M)\sigma_{\alpha\beta}]u_{\mu}(p'), \quad (10)$$

where  $G_1(0) = 1$  by normalization, and  $G_2(0) \equiv G_2$  is related to the magnetic moment by

$$1+G_2=(A/Z)\mu_f\equiv G_5,$$
 (11)

(12b)

and  $\mu_f$  is the magnetic moment of the daughter nucleus in units of the nucleon Bohr magneton. (In the case of interest,  $\mu_f = 0.822$  and  $G_5 = 1.7$ .) For ease of calculation, Eqs. (10) and (5) can be put into the respective forms

$$T^{\gamma} = (Ze^2/k^2) \,\bar{u}(l)\gamma_{\alpha}u(l')\bar{u}_{\mu}(p)G_{\alpha}(p,p')u_{\mu}(p') \,, \quad (12a)$$

with

and

$$G_{\alpha}(p,p')=G_{5}\gamma_{\alpha}+iG_{2}(p_{\alpha}/M),$$

$$T_0^{\text{wk}} = \bar{u}(l)\gamma_\beta(1+\gamma_5)v(l^\nu)\bar{u}_\nu(p)F_\beta(p,n)u_\nu(n), \quad (13a)$$

$$F_{\beta}(p,n) = f_5 \gamma_{\beta} + g \gamma_{\beta} \gamma_5 + i f_2 (n_{\beta} + p_{\beta})/2M. \quad (13b)$$

The unitarity equation (8) becomes, after summing over intermediate spin states,

$$\operatorname{Im} T^{\mathrm{wk}} = Z e^{2} \frac{(2\pi)^{4}}{2} \int \frac{d^{3} p' d^{3} l'}{(2\pi)^{6}} \frac{M}{P_{0}'} \frac{m_{e}}{l_{0}'} \delta(p + l - p' - l')$$

$$\times \frac{1}{(l' - l)^{2}} \bar{u}_{\mu}(p) G_{\alpha}(p, p') \Lambda_{\mu\nu}(p') F_{\beta}(p', n) u_{\nu}(n)$$

$$\times \bar{u}(l) \gamma_{\alpha} \Lambda(l') \gamma_{\beta}(1 + \gamma_{5}) v(l^{\nu}), \quad (14)$$

where  $\Lambda(l') = (-il' + m_l)/2m_l$  is the projection operator for an electron of momentum l'. Here  $\Lambda_{\mu\nu}(p')$  is the corresponding projection operator for a spin- $\frac{3}{2}$  nucleus of momentum p' and is given by

$$\Lambda_{\mu\nu}(p') = \left(\frac{-ip'+M}{2M}\right) \left[\delta_{\mu\nu} - \frac{1}{3}\gamma_{\mu}\gamma_{\nu} + \frac{i}{3M}(\gamma_{\mu}p_{\nu}' - \gamma_{\nu}p_{\mu}') + \frac{2p_{\mu}'p_{\nu}'}{3M^2}\right], \quad (15)$$

and which can be written in a more compact form

$$\Lambda_{\mu\nu}(p') = \begin{bmatrix} (-ip'+M)/2M \end{bmatrix} \\ \times \begin{bmatrix} M_{\mu\nu}(p') - \frac{1}{3}\gamma_{\alpha}\gamma_{\beta}M_{\mu\alpha}(p')M_{\nu\beta}(p') \end{bmatrix}, \quad (16a)$$
where

$$M_{\alpha\beta}(p') = \delta_{\alpha\beta} + p_{\alpha}' p_{\beta}' / M^2.$$
 (16b)

The above integration is straightforward, although extremely tedious. Minor simplifications arise as a result of our neglect of terms of second order and higher

 <sup>&</sup>lt;sup>10</sup> C. J. Christensen, A. Nielsen, A. Bahnsen, W. K. Brown, and B. M. Rustad, Phys. Letters 26B, 11 (1967).
 <sup>11</sup> F. P. Calaprice, E. D. Commins, and D. A. Dobson, Phys. Rev. 137, B1453 (1965).
 <sup>12</sup> H. H. Chen, Princeton University thesis, 1968 (unpublished).

The definition of the magnetic moment in the thesis for spin  $\frac{3}{2}$  is inconsistent with standard usage. In Chap. 3, spurious factors of 3 should be removed in order to be consistent.

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TABLE I. Numerical estimates of D [the coefficient of the triple-product correlation  $(\langle \mathbf{J} \rangle / J) \cdot (\mathbf{p}_l \times \mathbf{p}_r) / E_l E_r$ ] induced by the electromagnetic final-state interaction for several mirror  $\beta$  processes.

Mirror process	Nuclear spin	ft (sec)	$E_{oldsymbol{eta}}^{\mathrm{max}}$ (MeV)	${\mu_i}^{\rm a}$	$\mu_f^{\mathbf{a}}$	$g^{\mathrm{b}}$	$D^{\mathrm{m}\mathbf{a}\mathbf{x}}$
$C^{11} \rightarrow B^{11} e^+ \nu$	32	$3840 \pm 70$	1.49	-1.027	2.689	-0.6	0.7×10 <sup>-4</sup>
${ m N^{13}}  ightarrow { m C^{13}} e^+  u$	$\frac{1}{2}$	$4700 \pm 80$	1.73	-0.322	0.702	-0.3	$0.4 \times 10^{-4}$
$\mathrm{F^{17}} \rightarrow \mathrm{O^{17}}e^+ \nu$	$\frac{5}{2}$	$2330 \pm 80$	2.28	4.722	-1.894	1.1°	$-2.4 \times 10^{-4}$
${ m Ne^{19}} ightarrow { m F^{19}}e^+  u$	$\frac{1}{2}$	$1900 \pm 100$	2.75	-1.887	2.628	-1.0	$2.6  imes 10^{-4}$
${ m Ar^{35}}  ightarrow { m Cl^{35}} e^+  u$	$\frac{3}{2}$	$5680 \pm 400$	5.48	0.632	0.822	+0.1	$0.6  imes 10^{-4}$
${ m K}^{ m 37}  ightarrow { m Ar}^{ m 37} e^+  u$	3 2	$4250 \pm 500$	5	0.203	1.00	-0.5	$1.7 \times 10^{-4}$
$\mathrm{Sc}^{41} \rightarrow \mathrm{Ca}^{41} e^+ \nu$	$\frac{7}{2}$	$2560 \pm 160$	6	$(5.2)^{d}$	-1.59	1.0°	$13 \times 10^{-4}$

<sup>a</sup> We thank Dr. E. A. Phillips for providing these data.
 <sup>b</sup> The sign of g is estimated on the basis of the magnetic moments of both the parent and daughter nuclei [see E. J. Konopinski, *The Theory of Bela Radioactivity* (Clarendon Press, Oxford, 1966)], except for Ne<sup>10</sup> and Ar<sup>35</sup> where experimental information is available.
 <sup>a</sup> For F<sup>17</sup> and Sc<sup>41</sup>, we set g = 0 in order to estimate D<sup>max</sup> (see text).
 <sup>d</sup> Estimated (Dr. E. A. Phillips).

in the recoil parameter  $E_l/M$ . The paper work is also somewhat decreased because of our interest solely in the coefficient D of the triple-product correlation. This comes about because only the interference between  $T_0^{\text{wk}}$  and  $\text{Im}T^{\text{wk}}$  will contribute to D from  $|T^{\text{wk}}|^2 = |T_0^{\text{wk}} + i \,\text{Im}T^{\text{wk}}|^2$ . Terms in  $\text{Im}T^{\text{wk}}$  that are proportional to  $T_0^{wk}$  can be dropped, since an over-all change of phase for  $T_0^{wk}$  cannot contribute to D.

The net result of picking out all the contributions to D from  $|T^{wk}|^2$  is the expression

$$D(\beta^{\mp}) = \pm Z \alpha E_l^2 / \{4M p_l [1 + (5/3)g^2]\} \\ \times \{(1 \pm g)(f_5 \mp g) - G_5 (1 \mp g)(3 \pm 5g) + (m_l^2 / E_l^2) \\ \times [(3 \pm \frac{1}{3}g)(f_5 \mp g) + G_5 (1 \mp g)(3 \pm 5g)]\}, \quad (17)$$

where  $p_l(E_l)$  is the momentum (energy) of the electron,  $m_l$  is the electron mass, and M is the nuclear mass. g,  $G_5$ , and  $f_5$  are defined by Eqs. (6), (7), and (11). We note that the above is identical in form to the result calculated by Callan and Treiman for the case of the spin- $\frac{1}{2}$ mirror transition if one sets the axial-vector coupling g to zero.

The result for  $D(\beta^+)$  is obtained by noting that the coefficient of D is an even-parity correlation. Therefore, the application of charge conjugation to relate  $\beta^+$  to  $\beta^{-}$  decay can be used. The only changes which occur under charge conjugation are a change of sign between vector and axial-vector currents in the weak process and the over-all change of sign due to the charge of  $\beta^+$  relative to  $\beta^-$  in the final-state electromagnetic scattering.

For the particular spin- $\frac{3}{2}$  mirror transition  $Ar^{35} \rightarrow Cl^{35}e^+\nu$ , the various parameters have been given:

$$g \simeq 0.1, \quad G_5 \simeq 1.7, \quad f_5 \simeq -6.7.$$

 $f_5$  is somewhat smaller than expected. This is due to the

cancellation of the magnetic moments in (7). Since  $(E_l)_{\rm max}$  is 5.46 MeV, we ignore the positron mass, then the coefficient D has the value

$$D(\text{Ar}^{35}) \simeq +0.6 \times 10^{-4} E_l / (E_l)_{\text{max}}.$$
 (18)

This number is far smaller than present experimental errors.<sup>2</sup> This effect for Ar<sup>35</sup> is not likely to be measured in the near future, and measurements of D for Ar<sup>35</sup> will remain a clean test of T invariance for some time to come.

The list of mirror  $\beta$  decays is now examined. We calculate the value of D induced by electromagnetism for a few candidates. For the case of spin  $\frac{1}{2}$ , the result of Callan and Treiman is used. For the case of spin  $\frac{3}{2}$ , Eq. (17) is used. It is already noted that D has the same form for spin  $\frac{1}{2}$  and spin  $\frac{3}{2}$  in the absence of axial coupling (g=0). We hypothesize that this remains true for higher spins. In order to verify this and to find the dependence on g for the case of higher spins, explicit calculations will have to be done. We use the approximation g=0 for cases with spin> $\frac{3}{2}$ . The error involved in this approximation can be quite large and the numerical values are to be taken only as a rough estimate for these cases. The other approximation-viz., neglecting the contribution to  $\text{Im}T_{fi}^{\text{wk}}$  [Eq. (8)] from excited states of the daughter nucleus—is expected to be good, in general. The numerical results are tabulated in Table I. It is clear that for favorable cases, the value for Dhas been enhanced.

#### ACKNOWLEDGMENTS

We thank Professor S. B. Treiman for guidance and encouragement during the course of the calculation, and Dr. P. W. Coulter for a careful reading of the manuscript.