since it was seen that the anomaly in $G(p)$ affects the anomaly in $\Gamma^\mu(p,q)$, and also in the Compton amplitude. We must also make explicit mention of the fact that our results involving field commutators are gauge-dependent. In other gauges, no result could be given since the unrenormalized fields do not exist. Of course, the results for the Compton amplitude are gauge-invariant.

It should be noticed that the anomalous commutator algebra (4.1)–(4.3c) is nondistributive. That is, even if we accept the definition $j^i=\bar{\psi}(x+\frac{1}{2}\epsilon)\gamma^i\psi(x-\frac{1}{2}\epsilon)$, we cannot calculate the anomalous $[\psi,j^i]$ or the $[j^i,j^k]$ commutators from the anomalous $[\bar{\psi},\psi]_+$ commutator. This is analogous to the breakdown of the Jacobi identity discovered by Johnson and Low.[2]

ACKNOWLEDGMENT

It is a pleasure to acknowledge many helpful and stimulating conversations with Professor K. Johnson.

PHYSICAL REVIEW VOLUME 185, NUMBER 5 25 SEPTEMBER 1969

Sum Rules from Local Current Algebra

V. Gupta and G. Rajasekaran

Tata Institute of Fundamental Research, Bombay 5, India

(Received 20 March 1969; revised manuscript received 27 June 1969)

By combining the local current algebra with the commutation relations between the current densities and the Lorentz boost operator, we derive low-energy theorems as well as sum rules which involve neutrino and electron scattering form factors.

1. INTRODUCTION

A GREAT deal of recent activity has been based on the equal-time commutation relations proposed by Gell-Mann[1]:

$$[j_0^\alpha(x),j_0^\beta(y)]=if^{\alpha\beta\gamma}j_0^\gamma(x)\delta(\mathbf{x}-\mathbf{y}) \text{ at } x_0=y_0, \quad (1a)$$

$$[j_0^\alpha(x),j_{05}^\beta(y)]=if^{\alpha\beta\gamma}j_{05}^\gamma(x)\delta(\mathbf{x}-\mathbf{y}) \text{ at } x_0=y_0, \quad (1b)$$

$$[j_{05}^\alpha(x),j_{05}^\beta(y)]=if^{\alpha\beta\gamma}j_0^\gamma(x)\delta(\mathbf{x}-\mathbf{y}) \text{ at } x_0=y_0. \quad (1c)$$

Here j_0^α and j_{05}^α are the time components of the hadronic vector and axial current densities and α is the $SU(3)$ index.

Adler[2] has derived sum rules involving high-energy neutrino reactions which can provide tests of Eq. (1). A sum rule involving photoabsorption has been derived by Cabbibo and Radicati[3] and an inequality for high-energy electron scattering has been obtained by Bjorken.[4] Our purpose in this paper is to derive more sum rules from Eq. (1).

Our procedure is essentially the same as that of Adler[2] except for one crucial difference: We do not use Eq. (1) directly. Instead, we commute both sides of Eq. (1) with the Lorentz boost operator K_i ($i=1, 2, 3$) and use

$$i[K_i,j_0^\alpha(x)]=j_i^\alpha(x)-x_i\partial_0 j_0^\alpha(x) \quad \text{at} \quad x_0=0. \quad (2)$$

Defining

$$\tilde{j}_i^\alpha(x)\equiv j_i^\alpha(x)-x_i\partial_0 j_0^\alpha(x) \quad (\text{for all } x_0) \quad (3a)$$

$$\tilde{j}_{i5}^\alpha(x)\equiv j_{i5}^\alpha(x)-x_i\partial_0 j_0^\alpha(x) \quad (\text{for all } x_0), \quad (3b)$$

we find the following commutation relations:

$$[j_0^\alpha(x),\tilde{j}_i^\beta(y)]+[\tilde{j}_i^\alpha(x),j_0^\beta(y)] =if^{\alpha\beta\gamma}\tilde{j}_i^\gamma(x)\delta(\mathbf{x}-\mathbf{y}) \quad \text{at} \quad x_0=y_0=0, \quad (4a)$$

$$[j_0^\alpha(x),\tilde{j}_{i5}^\beta(y)]+[\tilde{j}_i^\alpha(x),j_{05}^\beta(y)] =if^{\alpha\beta\gamma}\tilde{j}_{i5}^\gamma\delta(\mathbf{x}-\mathbf{y}) \quad \text{at} \quad x_0=y_0=0, \quad (4b)$$

$$[j_{05}^\alpha(x),\tilde{j}_{i5}^\beta(y)]+[\tilde{j}_{i5}^\alpha(x),j_{05}^\beta(y)] =if^{\alpha\beta\gamma}\tilde{j}_i^\gamma\delta(\mathbf{x}-\mathbf{y}) \quad \text{at} \quad x_0=y_0=0. \quad (4c)$$

The equal-time commutation relations in Eq. (4) form the basis of the results to be obtained in the present paper. It may be pointed out that, in spite of the appearance of the space components of the currents, Eq. (4) is an exact consequence of Eq. (1) and Lorentz covariance. Moreover, Eq. (4) is free from the unknown Schwinger term which has to be present in the commutation relations between the time and space components of currents. Actually, Eq. (4) is a constraint on the commutation relations between the time and space components of the currents.[5]

In Sec. 2, we derive low-energy theorems based on Eq. (4). These are exact results following from the assumed local current commutation relations. We then convert these theorems into sum rules involving weak and elec-

[1] M. Gell-Mann, Physics **1**, 63 (1964).

[2] S. L. Adler, Phys. Rev. **143**, 1144 (1966).

[3] N. Cabibbo and L. A. Radicati, Phys. Letters **19**, 697 (1966).

[4] J. D. Bjorken, Phys. Rev. Letters **16**, 408 (1966).

[5] V. Gupta and G. Rajasekaran, Nucl. Phys. **B10**, 11 (1969).

tromagnetic processes by making the assumption that certain amplitudes satisfy unsubtracted dispersion relations in the energy variable for fixed q^2. This is presented in Sec. 2. The assumption of unsubtracted dispersion relations is a questionable one; however, all the derivations of sum rules from current algebra have made a similar or related assumption. This question is briefly discussed at the end of Sec. 3. In Sec. 4, we write down sum rules which are convergent on the basis of Regge-pole theory. These sum rules are, in fact, generalizations of the sum rules of Pagels and Harari[6] for nonzero q^2.

2. DERIVATION OF THE LOW-ENERGY THEOREM

We start with the basic identity

$$\int_0^\infty dx_0\, e^{iq_0x_0}\langle f|[\partial_0 A(x_0),B(x_0)]|i\rangle$$
$$=-\langle f|[A(0),B(0)]|i\rangle -iq_0\int_0^\infty dx_0\, e^{iq_0x_0}\langle f|[A(x_0),B(0)]|i\rangle, \quad (5)$$

which holds for all q_0 in the upper half of the complex plane. For $|i\rangle$ and $|f\rangle$ we shall take the single nucleon state at rest denoted by $|N\rangle$ and we shall further average over the nucleon spin.

Choose first

$$A_1(x_0)=\int d^3x\, e^{-i\mathbf{s}\cdot\mathbf{x}}j_0{}^\alpha(\mathbf{x},x_0),$$

$$B_1(x_0)=\tilde{j}_i{}^\beta(0,x_0),$$

and then

$$A_2(x_0)=\int d^3x\, e^{-i\mathbf{s}\cdot\mathbf{x}}\tilde{j}_i{}^\beta(\mathbf{x},x_0),$$

$$B_2(x_0)=j_0{}^\beta(0,x_0).$$

Now use Eq. (5) for these two cases and add. We next take the limit as $q_0\to 0$. The second term on the right-hand side of Eq. (5) vanishes in this limit for all $|\mathbf{s}|^2>0$, as can be seen by inserting a complete set of states. The first term on the right-hand side of Eq. (5) gives, on making use of Eq. (4a),

$$-if^{\alpha\beta\gamma}\bar{\sum_S}\langle N|\tilde{j}_i{}^\gamma(0)|N\rangle,$$

where $\bar{\sum}_S$ denotes the spin average. However, this is zero since the nucleon is at rest.

[6] H. Pagels, Phys. Rev. Letters **18**, 316 (1967); H. Harari, *ibid.* **18**, 319 (1967).

Thus, we have

$$\lim_{q_0\to 0}\int d^4x\,\exp(-i\hat{q}\cdot x)\,\theta(x_0)\bar{\sum_S}\langle N|[\partial_0 j_0{}^\alpha(x),\tilde{j}_i{}^\beta(0)] +[\partial_0\tilde{j}_i{}^\alpha(x),j_0{}^\beta(0)]|N\rangle=0, \quad (6)$$

where $\hat{q}=(iq_0,\mathbf{s})$. By straightforward manipulations, the integral in Eq. (6) can be rewritten in the form

$$\int d^4x\,\exp(-i\hat{q}\cdot x)\,\theta(x_0)$$
$$\times\bar{\sum_S}\langle N|[D^\alpha(x),j_i{}^\beta(0)]-[j_i{}^\alpha(x),D^\beta(0)]$$
$$-is_n\{[j_n{}^\alpha(x),j_i{}^\beta(0)]+[j_i{}^\alpha(x),j_n{}^\beta(0)]\}|\text{N}\rangle$$
$$+i\frac{\partial}{\partial s_i}\int d^4x\,\exp(-i\hat{q}\cdot x)\theta(x_0)\bar{\sum_S}\langle N|[D^\alpha(x),D^\beta(0)]$$
$$-is_n\{[j_n{}^\alpha(x),D^\beta(0)]-[D^\alpha(x),j_n{}^\beta(0)]\}$$
$$+s_ns_m[j_m{}^\alpha(x),j_n{}^\beta(0)]|\text{N}\rangle,$$

where we have introduced

$$D^\alpha(x)\equiv\partial_\mu j_\mu{}^\alpha(x).$$

We now define the amplitudes $d_V{}^{\alpha\beta}(q_0,q^2)$, $v_1{}^{\alpha\beta}(q_0,q^2)$, $v_2{}^{\alpha\beta}(q_0,q^2)$, and $i_V{}^{\alpha\beta}(q_0,q^2)$ through the equations

$$d_V{}^{\alpha\beta}(q_0,q^2)=\int d^4x\, e^{-iq\cdot x}\theta(x_0)\bar{\sum_S}\langle N|[D^\alpha(x),D^\beta(0)]|N\rangle,$$

$$v_1{}^{\alpha\beta}(q_0,q^2)\delta_{nm}+v_2{}^{\alpha\beta}(q_0,q^2)q_nq_m=\int d^4x\, e^{-iq\cdot x}\theta(x_0)\bar{\sum_S}\langle N|[j_n{}^\alpha(x),j_m{}^\beta(0)]|N\rangle, \quad (7)$$

$$iq_ni_V{}^{\alpha\beta}(q_0,q^2)=\int d^4x\, e^{-iq\cdot x}\theta(x_0)\bar{\sum_S}\langle N|[j_n{}^\alpha(x),D^\beta(0)] -[D^\alpha(x),j_n{}^\beta(0)]|\text{N}\rangle.$$

Using these functions, it can be verified that Eq. (6) reduces to the statement

$$\left(q^2\frac{\partial}{\partial q_2}(v_1{}^{\alpha\beta}+q^2v_2{}^{\alpha\beta}) +\frac{\partial}{\partial q_2}(d_V{}^{\alpha\beta}+q^2i_V{}^{\alpha\beta})-\tfrac{1}{2}i_V{}^{\alpha\beta}\right)\Bigg|_{q_0=0}=0, \quad (8a)$$

where we have replaced $|\mathbf{s}|^2$ by q^2. By repeating the same procedure with axial currents and using Eq. (4c), one arrives at

$$\left(q^2\frac{\partial}{\partial q^2}(a_1{}^{\alpha\beta}+q^2a_2{}^{\alpha\beta})+\frac{\partial}{\partial q^2}(d_A{}^{\alpha\beta}+q^2i_A{}^{\alpha\beta})-\tfrac{1}{2}i_A{}^{\alpha\beta}\right)\bigg|_{q_0=0}=0, \quad (8b)$$

where the amplitudes $d_A{}^{\alpha\beta}$, $a_1{}^{\alpha\beta}$, $a_2{}^{\alpha\beta}$, and $i_A{}^{\alpha\beta}$ are defined by equations similar to Eq. (7). These equations [(8a) and (8b)] are our low-energy theorems.

In the case of conserved currents (the vector currents for $\alpha=1$, 2, 3 and 8), $d_V=i_V=0$ and the low-energy theorem therefore assumes a very simple form:

$$(\partial/\partial q^2)(v_1{}^{\alpha\beta}+q^2v_2{}^{\alpha\beta})|_{q_0=0}=0. \quad (9)$$

One can derive the crossing-symmetry-type relations

$$d_V{}^{\alpha\beta}(q_0,q^2)=d_V{}^{\beta\alpha}(-q_0,q^2), \text{ etc.}, \quad (10)$$

for all the amplitudes entering in Eqs. (8) and (9). Consequently, if we take the antisymmetric part in α and β then Eqs. (8) and (9) are identically satisfied. Thus one will obtain low-energy theorems only when the part symmetric in α and β of Eqs. (8) and (9) is taken. Henceforth the symmetrization in the $SU(3)$ indices α and β is implicitly understood in all the results given below.

3. SUM RULES FOR NUCLEONS

We now convert the low-energy theorems into sum rules for neutrino reactions and electron scattering by using unsubtracted dispersion relations. We assume that the amplitude $\lambda_V{}^{\alpha\beta}(q_0,q^2)$, defined by

$$\lambda_V{}^{\alpha\beta}(q_0,q^2)=q^2\frac{\partial}{\partial q^2}(v_1{}^{\alpha\beta}+q^2v_2{}^{\alpha\beta})+\frac{\partial}{\partial q^2}(d_V{}^{\alpha\beta}+q^2i_V{}^{\alpha\beta})-\tfrac{1}{2}i_V{}^{\alpha\beta}, \quad (11)$$

satisfies the unsubtracted dispersion relation:

$$\lambda_V{}^{\alpha\beta}(q_0,q^2)=\frac{1}{\pi}\int_{-\infty}^{\infty}dq_0'\frac{\lambda_V{}'^{\alpha\beta}(q_0',q^2)}{q_0'-q_0}, \quad (12)$$

where $\lambda_V{}'^{\alpha\beta}$ is the absorptive part of $\lambda_V{}^{\alpha\beta}$. Putting $q_0=0$ and using Eq. (8a), we get

$$\int_0^\infty\frac{dq_0}{q_0}\left(q^2\frac{\partial}{\partial q^2}(v_1{}'^{\alpha\beta}+q^2v_2{}'^{\alpha\beta})+\frac{\partial}{\partial q^2}(d_V{}'^{\alpha\beta}+q^2i_V{}'^{\alpha\beta})-\tfrac{1}{2}i_V{}'^{\alpha\beta}\right)=0, \quad (13)$$

where $d_V{}'^{\alpha\beta}$, etc., are the absorptive parts defined by the equations

$$d_V{}'^{\alpha\beta}(q_0,q^2)=\frac{1}{2i}\int d^4x\, e^{-iq\cdot x}\overline{\sum_S}\langle N|[D^\alpha(x),D^\beta(0)]|\mathrm{N}\rangle,$$

$$v_1{}'^{\alpha\beta}(q_0,q^2)\delta_{nm}+v_2{}'^{\alpha\beta}(q_0,q^2)q_nq_m=\frac{1}{2i}\int d^4x\, e^{-iq\cdot x}\overline{\sum_S}\langle N|[j_n{}^\alpha(x),j_m{}^\beta(0)]|N\rangle, \quad (14)$$

$$iq_ni_V{}'^{\alpha\beta}(q_0,q^2)=\frac{1}{2i}\int d^4x\, e^{-iq\cdot x}\overline{\sum_S}\langle N|[j_n{}^\alpha(x),D^\beta(0)]-[D^\alpha(x),j_n{}^\beta(0)]|N\rangle.$$

In writing Eq. (13) we have made use of the fact that the absorptive parts $d_V{}'^{\alpha\beta}$, etc. (symmetric in α,β), are odd functions of q_0. We also have a similar equation for the axial case:

$$\int_0^\infty\frac{dq_0}{q_0}\left(q^2\frac{\partial}{\partial q^2}(a_1{}'^{\alpha\beta}+q^2a_2{}'^{\alpha\beta})+\frac{\partial}{\partial q^2}(d_A{}'^{\alpha\beta}+q^2i_A{}'^{\alpha\beta})-\tfrac{1}{2}i_A{}'^{\alpha\beta}\right)=0. \quad (13')$$

The absorptive parts $d_A{}'^{\alpha\beta}$, etc., are defined by equations similar to Eq. (14). For the conserved current case,

$$\int_0^\infty\frac{dq_0}{q_0}\frac{\partial}{\partial q^2}(v_1{}'^{\alpha\beta}+q^2v_2{}'^{\alpha\beta})=0. \quad (15)$$

Equations (13) and (15) are essentially the sum rules. However, to be able to test them experimentally one notes that the absorptive parts can be related to the elastic and inelastic form factors measured in neutrino or electron scattering experiments. This can be done in a straightforward manner and we shall give only the results.

A. Neutrino Reactions

Adler[2] has defined the form factors occurring in the differential cross section $d^2\sigma/dq_0dq^2$ for the neutrino reactions

$$\nu+N\to l+B, \quad (16)$$

$$\bar{\nu}+N\to \bar{l}+B, \quad (17)$$

where l is the final lepton and B is a system of hadrons. These are functions of q_0 and q^2, q being the lepton four-momentum transfer. We find that our sum rules involve only the two form factors $\beta(q_0,q^2)$ and $\epsilon(q_0,q^2)$. Let us denote these form factors for the reactions (16) and (17) by the superscripts + and −, respectively, and also let the subscripts p and n denote the proton and neutron target respectively. However, if B has strangeness $S=0$, then because of isospin invariance

$$\beta_p^{\pm}=\beta_n^{\mp}, \quad \epsilon_p^{\pm}=\epsilon_n^{\mp} \quad \text{(for the } \Delta S=0 \text{ case)}. \quad (18)$$

We first choose the $\Delta S=0$ currents in Eqs. (13) and (13′). Specifically, we put the $SU(3)$ indices $\alpha=\beta=1$ and then $\alpha=\beta=2$ and add. This calculation results in the sum rule

$$\int_0^\infty dq_0\left[\left(q_0\frac{\partial}{\partial q^2}\beta_p^{+}-\epsilon_p^{+}\right)+\left(q_0\frac{\partial}{\partial q^2}\beta_p^{-}-\epsilon_p^{-}\right)\right]=0 \quad (\Delta S=0). \quad (19)$$

Because of Eq. (18), by measuring $d^2\sigma/dq_0dq^2$ for the reactions (16) and (17) for both proton and neutron targets it is possible[7] to determine $\beta_p^{\pm}$ and $\epsilon_p^{\pm}$ in the $\Delta S=0$ case and thus test the sum rule Eq. (19).

By choosing the $\Delta S=\Delta Q=\pm 1$ currents (i.e., $\alpha=\beta=4$ and 5) we get two sum rules, one for the proton target and one for the neutron:

$$\int_0^\infty dq_0\left[\left(q_0\frac{\partial}{\partial q^2}\beta_{p,n}^{+}-\epsilon_{p,n}^{+}\right)+\left(q_0\frac{\partial}{\partial q^2}\beta_{p,n}^{-}-\epsilon_{p,n}^{-}\right)\right]=0 \quad (|\Delta S|=1). \quad (20)$$

In this case, relations analogous to Eq. (18) are not valid and so the functions $\epsilon_{p,n}^{\pm}$ cannot be determined from the differential cross-section measurements. Thus one cannot compare the sum rule directly with experiment. However, if one assumes $SU(3)$ symmetry, then a confrontation of Eq. (20) with experiment is possible.

B. Electron Scattering

We next take the electromagnetic current in Eq. (15) and get

$$\int_0^\infty dq_0 q_0\frac{\partial}{\partial q^2}\left(\frac{1}{q^2}\beta_{p,n}^{e}(q_0,q^2)\right)=0, \quad (21)$$

where $\beta_{p,n}^{e}(q_0,q^2)$ is a form factor[8] measured in the electron scattering process $e+N\rightarrow e+B$. Integrating with respect to q^2 and defining

$$\epsilon_{p,n}^{e}(q_0,q^2)=(q_0/q^2)[\beta_{p,n}^{e}(q_0,q^2)], \quad (22)$$

we can write Eq. (21) as

$$\int_0^\infty dq_0[\epsilon_{p,n}^{e}(q_0,q^2)-\epsilon_{p,n}^{e}(q_0,0)]=0. \quad (23)$$

This is our sum rule for electron scattering.

One may cast Eq. (23) into an alternate form involving the transverse and longitudinal cross sections $\sigma_{\text{trans}}(q_0,q^2)$ and $\sigma_{\text{long}}(q_0,q^2)$ for the fictitious process $\gamma+N\rightarrow B$, where γ has mass $=-q^2$. Using[8] the definition

$$\epsilon^{e}(q_0,q^2)=\frac{1}{4\pi^2\alpha}\frac{q_0}{(q_0^2+q^2)^{1/2}}[\sigma_{\text{trans}}(q_0,q^2)-\sigma_{\text{long}}(q_0,q^2)] \equiv\frac{1}{4\pi^2\alpha}\frac{q_0}{(q_0^2+q^2)^{1/2}}\sigma(q_0,q^2), \quad (24)$$

and separating the "pole term," we get

$$\frac{[G_E(q^2)]_{p,n}^2+(q^2/4M_N^2)[G_M(q^2)]_{p,n}^2}{2M_N(1+q^2/4M_N^2)}-\frac{[G_E(0)]_{p,n}^2}{2M_N} -\frac{1}{4\pi^2\alpha}\int_{q_0^t(0)}^{q_0^t(q^2)}dq_0\frac{q_0}{(q_0^2+q^2)^{1/2}}\sigma_{p,n}(q_0,q^2)+\frac{1}{4\pi^2\alpha}\int_{q_0^t(0)}^\infty dq_0\left(\frac{q_0}{(q_0^2+q^2)^{1/2}}\sigma_{p,n}(q_0,q^2)-\sigma_{p,n}(q_0,0)\right)=0, \quad (25)$$

where

$$q_0^t(q^2)=M_\pi+(q^2+M_\pi^2)/2M_N$$

and $[G_E]_{p,n}$ and $[G_M]_{p,n}$ are the electric and magnetic form factors for the proton and neutron.

The sum rule (23) or (25) was obtained by taking the electromagnetic current which is a sum of isovector and isoscalar currents. We could have taken the isovector (j_μ^3) or the isoscalar current (j_μ^8) separately and thus obtain independent sum rules for the isovector and isoscalar parts. Thus,

$$\int_0^\infty dq_0[\epsilon^S(q_0,q^2)-\epsilon^S(q_0,0)]=0, \qquad \int_0^\infty dq_0[\epsilon^V(q_0,q^2)-\epsilon^V(q_0,0)]=0, \quad (26)$$

where ϵ^S and ϵ^V are form factors corresponding to the isoscalar and isovector photons, respectively.

C. Convergence of the Sum Rules

We have exploited the low-energy theorems by converting them into sum rules through the assumption of

[7] It should be noted that the form factors $\pm$ can be obtained from differential cross sections only if the lepton mass m is not neglected.

[8] See, e.g., F. J. Gilman, Phys. Rev. **167**, 1365 (1968).

unsubtracted dispersion relations. The integrals in the sum rules may turn out to be divergent, in which case the assumption of unsubtracted dispersion relations would be wrong.

If we identify our amplitudes with the forward scattering amplitude of vector particles of mass $=-q^2$, then according to Regge-pole theory,

$$q_0\beta \xrightarrow[q_0\to\infty]{} q_0^{\alpha-1}, \quad \epsilon \xrightarrow[q_0\to\infty]{} q_0^{\alpha-1}, \tag{27}$$

where α is the zero-momentum-transfer intercept of the Regge trajectory for the crossed channel. Since $\alpha=1$ for the leading trajectory, the integrals $\int dq_0 q_0\beta$ and $\int dq_0\epsilon$ are predicted to be linearly divergent. However, our sum rules involve the differences of such integrals at two different q^2 values. If the asymptotic behavior is independent of the mass $-q^2$ of the vector particle, then our sum rules may be convergent.

Unfortunately, the above optimistic remark does not seem to be borne out by the presently available experimental data[9] for electron scattering on protons. Data indicate that the left-hand side of Eq. (25) is negative definite, which implies that the sum rule does not converge. However, to be really sure, one must await more complete experimental data, which separate the contributions of σ_{trans} and σ_{long}. In any case, even if sum rules like (25) are divergent, we can exploit the low-energy theorem in the form of the convergent sum rules given in the next section.

4. SUM RULES FOR UNSTABLE PARTICLE TARGETS

One can form suitable linear combinations of the amplitudes on various targets which lead to convergent integrals on the basis of Regge-pole theory. For example, starting from the electron scattering sum rule equation (23) for π^0 and π^+ as targets and also for Σ^0, Σ^+, and Σ^- as targets, one can write down the following sum rules:

$$\int_0^\infty dq_0\{[\epsilon_{\pi^0}{}^e(q_0,q^2)-\epsilon_{\pi^+}{}^e(q_0,q^2)] - [\epsilon_{\pi^0}{}^e(q_0,0)-\epsilon_{\pi^+}{}^e(q_0,0)]\}=0, \tag{28}$$

$$\int_0^\infty dq_0\{[2\epsilon_{\Sigma^0}{}^e(q_0,q^2)-\epsilon_{\Sigma^+}{}^e(q_0,q^2)-\epsilon_{\Sigma^-}{}^e(q_0,q^2)] - [2\epsilon_{\Sigma^0}{}^e(q_0,0)-\epsilon_{\Sigma^+}{}^e(q_0,0)-\epsilon_{\Sigma^-}{}^e(q_0,0)]\}=0. \tag{29}$$

The amplitudes corresponding to the sum rules (28) and (29) have isospin $I=2$ in the crossed channel. Since $\alpha<0$ for such Regge trajectories, the sum rules (28) and (29) are convergent.

Using the low-energy Thomson theorem for the real photons $q^2=0$, Pagels and Harari[6,10] have written down the sum rules

$$\int_0^\infty dq_0[\sigma_{\pi^0}(q_0,0)-\sigma_{\pi^+}(q_0,0)]=0, \tag{30}$$

$$\int_0^\infty dq_0[2\sigma_{\Sigma^0}(q_0,0)-\sigma_{\Sigma^+}(q_0,0)-\sigma_{\Sigma^-}(q_0,0)]=0, \tag{31}$$

where $\sigma_T(q_0,0)$ is the total absorption cross section of a photon on the target T. Noting that, by Eq. (24),

$$\epsilon^e(q_0,0)=(4\pi^2\alpha)^{-1}\sigma(q_0,0), \tag{32}$$

and using Eqs. (30) and (31), we find that the sum rules (28) and (29) can be rewritten as

$$\int_0^\infty dq_0[\epsilon_{\pi^0}{}^e(q_0,q^2)-\epsilon_{\pi^+}{}^e(q_0,q^2)]=0, \tag{33}$$

$$\int_0^\infty dq_0[2\epsilon_{\Sigma^0}{}^e(q_0,q^2)-\epsilon_{\Sigma^+}{}^e(q_0,q^2)-\epsilon_{\Sigma^-}{}^e(q_0,q^2)]=0. \tag{34}$$

Note that our sum rules (33) and (34) can be regarded as generalizations of the Compton scattering sum rules (30) and (31) to nonzero (but spacelike) values of q^2. By using quark-model arguments or by using $SU(3)$, Pagels and Harari[6] have written down many more sum rules of the type in Eqs. (30) and (31). We can extend all these sum rules[11] to nonzero q^2.

Of course, one cannot test these sum rules for unstable-particle targets directly. However, one can evaluate these sum rules by saturation with the various hadronic resonances and thus get information about the electromagnetic properties of these hadrons, as has been done[6] already for the $q^2=0$ case. In any case, these sum rules provide us with constraints on the dynamics of hadrons.

ACKNOWLEDGMENTS

We are grateful to Dr. J. Pasupathy and Dr. V. Singh for discussions.

[9] Rapporteur talk of W. K. H. Panofsky, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 23.

[10] To compare Eqs. (30) and (31) with Ref. 6, note that the left-hand sides of Eqs. (30) and (31) include the pole contributions, which are $-2\pi^2\alpha/M_\pi$ and $-4\pi^2\alpha/M_\Sigma$, respectively.

[11] Corresponding to each of these sum rules, we can write down an axial-vector sum rule also, which will be of the type in Eq. (19).