We have

$$F_k(k-x)=\sum_{n=k}^{\infty} c_n^{(k)}\frac{\Gamma(n-x)}{\Gamma(k-x)}$$

$$=\sum_{n'=0}^{\infty} c_{n'+k}^{(k)}\frac{\Gamma(n'+k-x)}{\Gamma(k-x)}. \quad \text{(B11)}$$

This gives us, as in (28),

$$c_n^{(k)}=(-1)^{n-k}\Delta^{(n-k)}F_k(0)/(n-k)!. \quad \text{(B12)}$$

The conditions necessary for the convergence of (B11) and all other series in this Appendix are given in detail in Sec. V.

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Boson Spectra, F_K/F_π, and Sum Rules

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Present experiments on semileptonic decays indicate $F_K/F_\pi \simeq 1.22$. However, in contrast with the apparent success of many other sum rules, asymptotic $SU(3)$ symmetry and chiral $SU(3)\otimes SU(3)$ algebra tend to predict $F_K=F_\pi$. This problem is studied in the approach which utilizes the $SU(3)\otimes SU(3)$ charge-charge density algebra and a form of asymptotic $SU(3)$ symmetry called the "$SU(3)$ approximation." This approximation is assumed for the $SU(3)$ raising and lowering operator V_K only in the zero-momentum-transfer limit. It is suggested that the problem may reflect the fact that we have not yet taken into account the full contribution of the boson spectrum. As one of the simplest and most interesting ways of introducing a more complicated boson spectrum, we study mainly the contribution of the exotic mesons with abnormal charge-conjugation parities. It is shown that the introduction of exotic channels removes the difficulty of $F_K=F_\pi$, while the essential features of other sum rules remain unaffected. The value $F_K/F_\pi \simeq 1.22$ can be obtained in our approach if there exists the $I=\frac{1}{2}$ $0^+\kappa$ meson with mass around 1.1 BeV. This value of the κ mass can also be predicted from our intermultiplet mass formulas which are also based on the $SU(3)$ approximation.

I. INTRODUCTION AND SUMMARY

ONE of the important problems of particle physics is whether we have two Cabibbo angles, θ_A and θ_V, or one angle. Recent developments in both the experiments and the theory of semileptonic interactions seem to favor one angle. If one assumes for the $F_+(s)$ form factor of K_{e3}^+ decay a form $F_+(s)=m_{K^*}^2(m_{K^*}^2+s)^{-1}F_+(0)$, in agreement with present experiments, and assumes the value $F_+(0)=1$, neglecting the second-order $SU(3)$-breaking effect,[1] the present[2] experimental rate $\Gamma(K_{e3}^+)=(3.87\pm0.07)\times10^6$ sec^{-1} gives[3] a value $\sin\theta_V=0.218\pm0.002$ when compared with the rate of the μ-decay mode. On the other hand, comparison of the $K\rightarrow\mu+\nu$ and $\pi\rightarrow\mu+\nu$ decays gives[4] $\sin\theta_A=0.2655\pm0.0006$ if we assume exact $SU(3)$ symmetry, $F_K=F_\pi$, for these decay amplitudes. However, this value of θ_A may not reflect the true value if the first-order $SU(3)$-breaking effect on the F_K/F_π is large. Let us turn to the baryon leptonic decays. For the vector couplings at zero momentum transfer, we usually adopt the $SU(3)$ values as in the case of $F_+(0)$, since the renormalization is again of second order in the $SU(3)$-breaking interaction. However, for the axial-vector couplings, the renormalization problem is more acute and the use of exact $SU(3)$ couplings appears to be more problematical. Recently we have proposed[5] to work with the broken-$SU(3)$ sum rules for the axial-vector coupling constants which can be obtained by using an approximation[6] called the "$SU(3)$ approximation" and a chiral $SU(3)\otimes SU(3)$ algebra. This approximation is certainly a weaker assumption than exact $SU(3)$ symmetry and

* Supported in part by the National Science Foundation under Grant No. NSF GP 6036.

[1] M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964).

[2] For a recent review, see, e.g., J. W. Cronin, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberg (CERN, Geneva, 1968), p. 281.

[3] S. Oneda and J. Sucher, Phys. Rev. Letters 15, 927 (1965); 15, 1049(E) (1965). The effect of electromagnetic η^0-π^0 mixing increases the value of $\sin\theta$ by 1.7%. See Ref. 5.

[4] N. Brene, M. Roos, and A. Sirlin, Nucl. Phys. 131, 255 (1968).

[5] S. Matsuda, S. Oneda, and P. Desai, Phys. Rev. 178, 2129 (1969).

[6] S. Matsuda and S. Oneda, Phys. Rev. 174, 1992 (1968). For the early references and the summary, see S. Matsuda and S. Oneda, Nucl. Phys. B9, 55 (1969).

assumes, roughly speaking, that the $SU(3)$ raising and lowering operator V_K still acts as an $SU(3)$ generator in broken symmetry but only at the zero-momentum-transfer limit. In the computation, this limit can be realized *only* by taking an appropriate infinite-momentum limit.

It may be claimed[5] that the accuracy of the sum rules is of the same order as that of the Gell-Mann–Okubo mass formulas, since one can derive these mass formulas on the same footing, utilizing the same algebra and the $SU(3)$ approximation. Based on these sum rules for the physical coupling constants and on some of the recent experiments on the rates and the V-A interference terms of the $\Lambda \to p$, $\Sigma \to \Lambda$, and $n \to p$ β decays, we have derived[5] $\sin\theta_V = 0.225 \pm 0.002$ and $\sin\theta_A = 0.225 \pm 0.005$. In agreement with this observation, Cronin[2] reported a value $\sin\theta = 0.227 \pm 0.006$, which is deduced from the world-average hyperon β-decay rates by fitting with one Cabibbo angle θ. Therefore, though we certainly need more accurate experiments, there is at present no compelling reason to assume two Cabibbo angles. If this is the case, the value of F_K/F_π seems to deviate from unity rather significantly. For example, if we tentatively assume $\sin\theta = 0.22$, then $F_K/F_\pi = 1.22$. Various attempts have been made to derive the value of F_K/F_π. The first attempts[7] based on Weinberg's sum rules have had difficulty, since they are based on the second spectral-function sum rule which disagrees with experiment. The model of broken chiral symmetry due to Glashow and Weinberg[8] gives $F_K/F_\pi = 1.08$, and favors a mass $m_\kappa \lesssim 670$ MeV for the so-called κ meson. So far, such a κ meson has not been observed. In order to remedy the inconsistency of the second spectral-function sum rule which gives the exact $SU(3)$ result, it has been proposed to patch up the difficulty by replacing this sum rule by the one which takes into account explicitly the first-order $SU(3)$ violation.[9,10] However, it was also realized that these procedures rather tend to predict $F_K = F_\pi$.[11–13] The purpose of this paper is to study this problem from a different approach which we have been pursuing recently.[6] The approach is as follows.[14] Instead of utilizing the spectral functions, we deal directly with the chiral $SU(3) \otimes SU(3)$ charge-charge density algebra. Furthermore, we use the aforementioned $SU(3)$ approximation for V_K instead of the so-called asymptotic $SU(3)$ condition imposed upon the spectral functions. In this approach, the spins of the intermediate states are also restricted to either *zero* or *one*, and the dominance of the single-particle resonance approximation is adopted as in the spectral-function approach. In the spirit of the hypothesis of partially conserved axial-vector current (PCAC), we also assume the gentleness condition for the divergence of the partially conserved vector and axial-vector currents. In this approach, we have derived[14] sum rules which include the result essentially equivalent to the first spectral-function sum rule. However, in Sec. II we show that we shall also encounter the problem of $F_K = F_\pi$ in other places, if there is only one K meson and one κ meson in the spectra of $I = \frac{1}{2}$ 0^- and 0^+ mesons, respectively.

Therefore, the simplest and most natural way (to us) to obtain over-all consistency is to assume that we have not yet taken into account the full spectra of the spin-zero bosons. This possibility does not seem unrealistic, since in baryon spectra we have already observed the recurrence of the multiplets of the same spin and parity. Furthermore, one may also imagine the existence of exotic spin-zero mesons with abnormal charge-conjugation parities. Although such exotic bosons may not be constructed in the simple $\bar{q}q$ quark model, there is no *a priori* reason to reject them.[15] For example, the need for daughter parities has been brought out by the Regge-pole analysis. Experimentally,[16] the recently observed $A_{2L}(1270)$ meson might be a candidate for the $I = 1$ 1^{-+} meson, i.e., the abnormal ρ meson [or the daughter of the 2^{++} meson $A_{2H}(1315)$]. For the sake of theoretical interest (and also to remove the $F_K = F_\pi$), we assume the existence of such exotic mesons in this paper. In Sec. III we discuss the modification of our sum rules in the presence of such bosons and derive the value of the F_K/F_π in terms of the mass of the κ meson which belongs to the 0^{++} multiplet in the symmetry limit. If the mass of another κ meson, κ', is not very close to the mass of the K meson, then the mass of the κ meson is predicted to be around 1.1 BeV. In Sec. IV we remark that one can also predict a κ mass around 1.1 BeV from the recently established[16] $I = 1$ 0^+ δ meson ($m_\delta \simeq 960$ MeV) or the $\pi_N(1016)$ meson by using the intermultiplet mass formula obtained from the $SU(3)$ approximation. Therefore, the over-all

[7] S. L. Glashow, H. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 139 (1968); H. T. Neih, *ibid.* **19**, 43 (1968).

[8] S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968).

[9] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **19**, 470 (1967).

[10] R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters **19**, 1266 (1967).

[11] This was first observed by K. Kawarabayashi and W. W. Wada, Phys. Rev. Letters **19**, 1193 (1967); C. S. Lai, *ibid.* **20**, 509 (1968); R. J. Oakes, *ibid.* **20**, 513 (1968); Riazuddin and Fayyazuddin, Phys. Rev. **172**, 1737 (1968); J. Dooher, *ibid.* **179**, 1530 (1969).

[12] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968). If the K_A mass is around 1230 MeV, $F_K \simeq F_\pi$ is also obtained by P. K. Mitter and L. J. Swank, Nucl. Phys. **B8**, 205 (1968).

[13] For a review of extensive literatures on the related problems, see S. Weinberg, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberg (CERN, Geneva, 1968), p. 253.

[14] S. Matsuda and S. Oneda, Phys. Rev. **171**, 1743 (1968).

[15] For example, there is the Gell-Mann–Zweig model that is discussed by H. Harari, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberg (CERN, Geneva, 1968), p. 195.

[16] N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, A. H. Rosenfeld, P. Söding, C. G. Wohl, M. Roos, and G. Conforto, Rev. Mod. Phys. **41**, 109 (1969). The $\delta(962)$ might belong to the 0^{+-} octet. However, the $\delta(962)$ and $\pi_N(1016)$ could also be related.

consistency of the result is rather good. In Sec. V, a comment is made on the sum rules for $V \to l+\bar{l}$ decays.

II. PROBLEM OF $F_K = F_\pi$ AND THE BOSON SPECTRUM

Consider the charge-charge density commutator

$$[V_0^{\pi^+}(x), V_{K^0}] = V_0^{K^+}(x). \tag{1}$$

In the following, we write, for example, the isovector vector and axial-vector currents as $V_\mu^{\pi^+}(x)$ and $A_\mu^{\pi^+}(x)$, respectively, and denote the vector and axial-vector charges as V_K and A_π, etc. Let us sandwich Eq. (1) between the vacuum state and the state of the $I=\frac{1}{2}$ 0^+ meson, called the κ meson, with infinite momentum. Then with the $SU(3)$ approximation for the charge V_{K^0} [which implies that the V_K acts as an $SU(3)$ generator in this infinite-momentum limit], and neglecting many-particle continuum states, we obtain

$$\lim_{|\mathbf{p}|\to\infty} [\langle 0|V_0^{\pi^+}(x)|\delta^-\rangle\langle\delta^-|V_{K^0}|\kappa^-(\mathbf{p})\rangle - \langle 0|V_{K^0}|\bar{\kappa}^0\rangle\langle\bar{\kappa}^0|V_0^{\pi^+}(x)|\kappa^-(\mathbf{p})\rangle] = \lim_{|\mathbf{p}|\to\infty} \langle 0|V_0^{K^+}(x)|\kappa^-(\mathbf{p})\rangle. \tag{2}$$

Here δ is the $I=1$ counterpart[17] of the κ meson and we have assumed that there is only one kind of 0^+ multiplet. Since $\langle 0|V_0^{\pi^+}(x)|\delta^-\rangle = 0$, because of the conservation of the current $V_\mu^{\pi^+}(x)$, we obtain the following from Eq. (2):

$$F_\kappa f_\kappa(\infty) = F_\kappa. \tag{3}$$

Here F_κ is the amplitude for the process $\kappa \to e+\nu$ through the current $V_\mu^K(x)$, defined by $\langle 0|V_\mu^{K^+}(0)|\kappa^-(\mathbf{p})\rangle = (2p_0)^{-1/2} F_\kappa p_\mu$. The function $f_\kappa(s)$ is the form factor for the vertex, defined by

$$\langle\bar{\kappa}^0(\mathbf{p}')|V_\mu^{\pi^+}(0)|\kappa^-(\mathbf{p})\rangle = (2p_0 2p_0')^{-1/2} f_\kappa(s)(p+p')_\mu,$$

where $s = -(p-p')^2$.

In this paper, along with the commutators of the type (1), we always consider the following types of commutators:

$$[V_0^{\pi^+}(x), \dot{V}_{K^0}] = \partial_\mu V_\mu^{K^+}(x). \tag{4}$$

This commutator gives information for quantities like $f_\kappa(\infty)$ which involve the high-energy behavior of certain vertex functions which appear in the computation. The commutator (4), which can be derived from Eq. (1) by a term-wise differentiation, will be valid if all the quantities involved behave in a reasonable fashion. Although the validity of this commutator might be less secure than Eq. (1) under special circumstances, we here assume its validity. If we sandwich Eq. (4) again between the same states as in Eq. (2), we obtain, corresponding to Eq. (2),

$$-\lim_{|\mathbf{p}|\to\infty} \langle 0|\dot{V}_{K^0}|\bar{\kappa}^0\rangle\langle\bar{\kappa}^0|V_0^{\pi^+}(x)|\kappa^-(\mathbf{p})\rangle = \lim_{|\mathbf{p}|\to\infty} \langle 0|\partial_\mu V_\mu^{K^+}(x)|\kappa^-(\mathbf{p})\rangle.$$

If we take the usual gentleness condition[18] for the divergence of the current $V_\mu^{K^+}(x)$ [i.e., the form factor of the matrix elements of the current $\partial_\mu V_\mu^K(x)$ vanishes at infinity], the right-hand side of the above equation vanishes, and we obtain

$$F_\kappa f_\kappa(\infty) = 0. \tag{5}$$

Therefore, Eq. (3), together with Eq. (5), implies $F_\kappa = 0$. In broken symmetry this does not seem correct, and in fact, as will be shown below, it leads also to the result $F_K = F_\pi$.

Consider now the commutator

$$[A_0^{\pi^+}(x), V_{K^0}] = A_0^{K^+}(x), \tag{6}$$

and sandwich it between the states $\langle 0|$ and $|K(\mathbf{p})\rangle$, with $|\mathbf{p}| = \infty$. Similar to Eq. (2), we obtain

$$\lim_{|\mathbf{p}|\to\infty} [\langle 0|A_0^{\pi^+}(x)|\pi^-\rangle\langle\pi^-|V_{K^0}|K^-(\mathbf{p})\rangle - \langle 0|V_{K^0}|\kappa^0\rangle\langle\kappa^0|A_0^{\pi^+}(x)|K^-(\mathbf{p})\rangle] = \lim_{|\mathbf{p}|\to\infty} \langle 0|A_0^{K^+}(x)|K^-(\mathbf{p})\rangle. \tag{7}$$

Since $F_\kappa = 0$, the second term on the left-hand side of Eq. (7) vanishes. One can also show this, $\lim_{|\mathbf{p}|\to\infty} \langle 0|V_{K^0}|\kappa^0\rangle\langle\kappa^0|A_0^{\pi^+}(x)|K^-(\mathbf{p})\rangle = 0$, directly if we consider the commutator $\partial_\mu[A_\mu^{\pi^+}(x), V_{K^0}] = \partial_\mu A_\mu^{K^+}(x)$ inserted between the same states as in Eq. (7) and use the gentleness condition for the matrix elements of the $\partial_\mu A_\mu^{\pi^+}(x)$ and $\partial_\mu A_\mu^{K^+}(x)$. Then Eq. (7) leads to the equation $F_K = F_\pi$. Except for the gentleness conditions for the divergence of the vector and axial-vector currents, we have essentially made two approximations: the single-particle resonance approximation and the $SU(3)$ approximation. The value of F_K/F_π seems to deviate from unity by $\simeq 20\%$. The $SU(3)$ approximation seems to give a better accuracy judging from the Gell-Mann–Okubo mass formula for the pseudoscalar mesons,[19] and also from the value of $F_+(0)$. Therefore, we are tempted to suspect that we have not yet taken fully into account the spectra of 0^+ resonance states in the above computation. As mentioned in Sec. I, one may introduce another κ meson, κ', which belongs (say) to the $J^{PC} = 0^{+-}$ octet, in addition to the usual κ meson belonging to the normal 0^{++} octet. (One can,

[17] For simplicity we assume that the κ meson belongs to the $SU(3)$ octet.

[18] Y. Nambu, Phys. Rev. Letters 4, 380 (1960); see also, e.g., J. Bernstein and S. Weinberg, *ibid.* 5, 481 (1960), especially Eq. (6). In our particular case

$$\lim_{|\mathbf{p}|\to\infty} \langle 0|\partial_\mu V_\mu^{K^+}(0)|\kappa^-(\mathbf{p})|\kappa^-(\mathbf{p})\rangle \propto \lim_{|\mathbf{p}|\to\infty} m_\kappa^2 F_\kappa (2p_0)^{-1/2} \to 0.$$

[19] Using the commutator $[V_{K^0}, \dot{V}_{K^0}] = 0$, the $SU(3)$ approximation $[F_+(0) = 1]$ gives the usual value 10.4° for the η' mixing angle, which brings the pseudoscalar-meson mass formula into perfect agreement with experiment. See Ref. 6.

of course, achieve the same goal by introducing another κ meson which belongs to the 0^{++} multiplet but has a larger mass.) If we have two κ mesons, Eq. (3) will now be replaced by

$$F_\kappa f_\kappa(\infty)+F_{\kappa'}f_+^{\kappa\kappa'}(\infty)=F_\kappa. \tag{8}$$

$F_{\kappa'}$ is defined analogously to F_κ. Here $f_+^{\kappa\kappa'}(s)$ is defined by

$$\langle\bar{\kappa}'^0(\mathbf{p}')|V_\mu^{\pi^+}(x)|\kappa^-(\mathbf{p})\rangle=(2p_0 2p_0')^{-1/2}\times[f_+^{\kappa\kappa'}(s)(p+p')_\mu+f_-^{\kappa\kappa'}(s)(p-p')_\mu].$$

Since $\partial_\mu V_\mu^{\pi^+}(x)=0$, it follows that

$$f_-^{\kappa\kappa'}(s)=\frac{m_{\kappa'}^2-m_\kappa^2}{s}f_+^{\kappa\kappa'}(s).$$

Therefore, Eq. (3) will now become

$$F_\kappa f_\kappa(\infty)+F_{\kappa'}f_+^{\kappa\kappa'}(\infty)\left(1+\frac{m_{\kappa'}^2-m_\kappa^2}{s}\right)_{s\to\infty}=F_\kappa.$$

Thus we obtain Eq. (8). Equation (5) is now replaced by

$$m_\kappa F_\kappa f_\kappa(\infty)+m_{\kappa'}F_{\kappa'}f_+^{\kappa\kappa'}(\infty)=0. \tag{9}$$

Equations (8) and (9) now allow us to have nonzero solutions for F_κ and $F_{\kappa'}$ if $m_\kappa\neq m_{\kappa'}$. In this case, Eq. (7) will no longer[20] lead to $F_K=F_\pi$. In the Appendix, inserting the commutator $[A_{K^0},V_0^{\pi^0}(x)]=\frac{1}{2}A_0^{K^0}(x)$ between the states $\langle 0|$ and $|\bar{K}^0(\mathbf{p})\rangle$, with $|\mathbf{p}|=\infty$, and using again the same gentleness condition, we show that $F_K=F_\pi$ also follows if there is only one kind of 0^- K meson. In summary, we have argued that the problem of $F_K=F_\pi$ may be related to our ignorance about the boson spectrum.

III. F_K/F_π AND κ-MESON MASS

Previously we have derived[14] the following relation for the F_K/F_π, assuming the existence of only one kind of κ meson:

$$F_K/F_\pi=1+F_\kappa G_{\kappa K\pi}/(m_\kappa^2-m_K^2). \tag{10}$$

The essential assumptions involved were the $SU(3)$ approximation and the pion PCAC which only involve the mass-shell extrapolation $m_\pi^2\to 0$. Thus the $\kappa\to K\pi$ coupling $G_{\kappa K\pi}$ in Eq. (10) is defined with a pion off the mass shell. Equation (10) was derived by using the charge-charge density commutators

$$[V_0^{K^0}(x),A_{\pi^+}]=-A_0^{K^+}(x) \tag{11}$$

and

$$\partial_\mu[V_\mu^{K^0}(x),A_{\pi^+}]=-\partial_\mu A_\mu^{K^+}(x). \tag{12}$$

The constraint equation (12) led to the condition $F_+(\infty)-F_-(\infty)=0$ for the K_{l3}-decay form factors $F_+(s)$ and $F_-(s)$,[21] when we used the gentleness condition on the maxtrix elements of $\partial_\mu V_\mu^K(x)$ and $\partial_\mu A_\mu^K(x)$ at infinity.

It is interesting to notice that Eq. (10) can be also derived in a more indirect way by utilizing the information obtained by the soft-pion result.[22] Namely, if one assumes unsubtracted dispersion relations for both $F_+(s)$ and $F_-(s)$ [or the once-subtracted form for $F_+(s)$] with $F_+(0)\simeq 1$ (such dispersion relations are dominated by the K^* and the κ meson), and then imposes the soft-pion constraint $F_+(m_K^2)+F_-(m_K^2)=F_K/F_\pi$ at $s=m_K^2$, then one also arrives at Eq. (10).

We now combine Eq. (10) with the relation obtained by using the partially conserved vector current (PCVC) or the gentleness condition for $\partial_\mu V_\mu^K(x)$. That is, if the form factors of $\langle\pi^0(\mathbf{p}')|\partial_\mu V_\mu^K(x)|K^+(\mathbf{p}')\rangle$ vanish at infinity,[18] one obtains with $F_+(0)=1$ [which follows from the $SU(3)$ approximation]

$$F_\kappa G_{\kappa^0 K^+\pi^-}=(m_K^2-m_\pi^2). \tag{13}$$

In the same spirit, this can also be derived[23] if we compute $\langle\pi^0(\mathbf{p})|V_{K^-}|K^+(\mathbf{p})\rangle$ with $|\mathbf{p}|\to\infty$ using PCVC, $\partial_\mu V_\mu^K(x)=F_\kappa m_\kappa^2\phi_\kappa(x)$. Equations (10) and (13) lead to

$$F_K/F_\pi=1+(m_K^2-m_\pi^2)/(m_\kappa^2-m_K^2), \tag{14}$$

or, neglecting m_π^2,

$$m_\kappa\simeq m_K(1-F_\pi/F_K)^{-1/2}. \tag{15}$$

If the value of $F_K/F_\pi\simeq 1.22$ is used, the mass of the κ meson is then predicted to be around 1.1 BeV. This was noticed by us[24] some time ago and later also by various authors[24] using different methods. However, at that time we also became aware of the fact that consistency of the theory in other places leads to $F_K=F_\pi$. as discussed in Sec. II, if we bar the rather unusual circumstance that the commutator, Eq. (4), or the gentleness conditions for $\partial_\mu A_\mu^K(x)$ or $\partial_\mu V_\mu^K(x)$ are not valid. In Sec. II, however, we argued that the problem of $F_K=F_\pi$ will disappear if there exist more than one κ-like object. Our present task is to study how the result given by Eqs. (14) or (15) is modified in such a case.

Because of its theoretical and experimental interest, we discuss in the following a rather general case where one has, in addition to the usual $P(J^{PC}=0^{-+})$, $V(1^{--})$, and $S(0^{++})$ mesons, the exotic mesons denoted by $P'(0^{--})$, $V'(1^{-+})$, and $S'(0^{+-})$ which will contribute

[20] Equation (7) now gives including the κ' contribution $F_\pi=F_K-(F_\kappa/2)f_+^{K\kappa}(\infty)-(F_{\kappa'}/2)f_+^{K\kappa'}(\infty)$, with the constraint $m_\kappa F_\kappa f_+^{K\kappa}(\infty)+m_{\kappa'}F_{\kappa'}f_+^{K\kappa'}(\infty)=0$.

[21] This condition is the same as that imposed by H. T. Nieh, Phys. Rev. Letters **21**, 116 (1968).

[22] We thank Professor S. Okubo for pointing out this to us. For a thorough study along this line, see, e.g., J. C. Pati and K. J. Sebastian, Phys. Rev. **174**, 2033 (1968). In this paper a similar prediction on the mass of the κ meson can be made if $F_+(0)=1$ is assumed.

[23] As in the case of PCAC, the $G_{\kappa K\pi}$ will be defined with the κ meson off the mass shell. However, in Ref. 22 it has been shown that the result is not sensitive to this extrapolation.

[24] S. Matsuda, University of Maryland Technical Report No. 768, 1967, p. 17 (unpublished); L. N. Chang and Y. C. Leung, Phys. Rev. Letters **21**, 122 (1968). See also Ref. 22.

to the exotic channels in the computation. However, the introduction of exotic mesons is not an absolute necessity. The result will not be changed much even if we introduce higher-lying normal (0^{-+}, 1^{--}, and 0^{++}) mesons instead of the abnormal ones. Once we introduce the abnormal mesons, mixing possibilities take place between the $I=\frac{1}{2}$ states of normal and abnormal octets of the same J^P.

We shall show below that Eqs. (14) or (15) may not be modified drastically by the inclusion of exotic channels. We also discuss some of the interesting sum rules which will be modified by the contribution of exotic mesons and have some experimental interest. We now sandwich Eq. (11) between the states $\langle 0|$ and $|K^-(\mathbf{p})\rangle$ with $|\mathbf{p}|=\infty$. We obtain (κ' and $K^{*\prime}$ belong to 0^{+-} and 1^{-+} octets, respectively)

$$\begin{aligned}&\langle 0|V_0^{K^0}(x)|\bar{K}^*(\mathbf{p})\rangle\langle\bar{K}^*(\mathbf{p})|A_{\pi^+}|K^-(\mathbf{p})\rangle\\&\quad+\langle 0|V_0^{K^0}(x)|\bar{K}^{*\prime}(\mathbf{p})\rangle\langle\bar{K}^{*\prime}(\mathbf{p})|A_{\pi^+}|K^-(\mathbf{p})\rangle\\&\quad+\langle 0|V_0^{K^0}(x)|\bar{\kappa}(\mathbf{p})\rangle\langle\bar{\kappa}(\mathbf{p})|A_{\pi^+}|K^-(\mathbf{p})\rangle\\&\quad+\langle 0|V_0^{K^0}(x)|\bar{\kappa}'(\mathbf{p})\rangle\langle\bar{\kappa}'(\mathbf{p})|A_{\pi^+}|K^-(\mathbf{p})\rangle\\&\quad-\langle 0|A_{\pi^+}|\pi^-(\mathbf{p}')\rangle\langle\pi^-(\mathbf{p}')|V_0^{K^0}(x)|K^-(\mathbf{p})\rangle\\&\qquad=-\langle 0|A_0^{K^+}(x)|K^-(\mathbf{p})\rangle.\end{aligned}$$

For the A_{π^+} we use pion PCAC. The use of the commutator (12) sandwiched between the same states again leads to the condition $F_+(\infty)-F_-(\infty)=0$. Then the last term on the left-hand side of the above equation drops out to give

$$\frac{G_{K^*}G_{K^*\pi^-\pi^+}}{m_{K^*}^2}+\frac{G_{K^{*\prime}}G_{K^{*\prime}\pi^-\pi^+}}{m_{K^{*\prime}}^2}=\frac{F_K}{F_\pi}-\frac{F_\kappa G_{\kappa K^-\pi^+}}{m_\kappa^2-m_K^2}-\frac{F_{\kappa'}G_{\kappa' K^-\pi^+}}{m_{\kappa'}^2-m_K^2}.\quad(16)$$

Here G_{K^*} and $G_{K^{*\prime}}$ are defined by $\langle 0|V_\mu^K(0)|K^*(\mathbf{p})\rangle=(2p_0)^{-1/2}G_{K^*}\epsilon_\mu^{K^*}$ and $\langle 0|V_\mu^K(0)|K^{*\prime}(\mathbf{p})\rangle=(2p_0)^{-1/2}\times G_{K^{*\prime}}\epsilon_\mu^{K^{*\prime}}$. The coupling constants involving the pion are defined with the pion off the mass shell because of the use of PCAC. We now consider the matrix element $\langle K^{*0}(\mathbf{p})|[V_{K^0},A_{\pi^-}]|\pi^+(\mathbf{p})\rangle=0$ with $|\mathbf{p}|=\infty$ and use the $SU(3)$ approximation for V_{K^0} and PCAC for A_{π^-}. We notice that matrix elements such as $\langle K^{*0\prime}(\mathbf{p})|V_{K^0}|\rho(\mathbf{p})\rangle$ and $\langle K'(\mathbf{p})|V_{K^0}|\pi^+(\mathbf{p})\rangle$ at $|\mathbf{p}|\to\infty$ are not zero, since there exist K^*-$K^{*\prime}$ and K-K' mixing. Denoting as usual these mixing angles by $\theta_{K^*K^{*\prime}}$ and $\theta_{KK'}$, we obtain

$$\begin{aligned}&\cos\theta_{KK'}G_{K^{*+}K^-\pi^0}-\sin\theta_{KK'}G_{K^{*+}K'^-\pi^0}\\&\qquad=\tfrac{1}{2}(m_{K^*}/m_\rho)\cos\theta_{K^*K^{*\prime}}G_{\rho^0\pi^+\pi^-}.\quad(17)\end{aligned}$$

If we take $\theta_{K^*K^{*\prime}}=\theta_{KK'}=0$, we recover the previously[6] obtained result, $G_{K^{*+}K^-\pi^0}=(2)^{-1}(m_{K^*}/m_\rho)G_{\rho^0\pi^+\pi^-}$.[25] In a similar way, from the $\langle K^{*0\prime}(\mathbf{p})|[V_{K^0},A_{\pi^-}]|\pi^+(\mathbf{p})\rangle=0$ with $|\mathbf{p}|=\infty$, we obtain

$$\begin{aligned}&\cos\theta_{KK'}G_{K^{*\prime}K^+K^-}-\sin\theta_{KK'}G_{K^{*\prime}K'^+\pi^-}\\&\qquad=-\tfrac{1}{2}\sqrt{2}(m_{K^*}/m_\rho)\sin\theta_{K^*K^{*\prime}}G_{\rho\pi^+\pi^-}.\quad(18)\end{aligned}$$

If there are no mixings, $G_{K^{*\prime}K^+\pi^-}=0$. This reflects the fact that we cannot construct $V'PP$ coupling in the $SU(3)$-symmetry limit, and under the $SU(3)$ approximation $G_{K^{*\prime}K\pi}$ coupling becomes nonvanishing only through mixing.

Next we study the sum rules for vector meson $\to l+\bar{l}$ decays. We sandwich Eq. (1) between the states $\langle 0|$ and $|K^*(\mathbf{p})\rangle$ with $|\mathbf{p}|\to\infty$ and obtain

$$\begin{aligned}&-\langle 0|V_{K^0}|\bar{\kappa}\rangle\langle\bar{\kappa}|V_0^{\pi^+}(x)|\bar{K}^*(\mathbf{p})\rangle\\&\quad-\langle 0|V_{K^0}|\bar{\kappa}'\rangle\langle\bar{\kappa}'|V_0^{\pi^+}(x)|\bar{K}^*(\mathbf{p})\rangle\\&\quad+\langle 0|V_0^{\pi^+}(x)|\rho\rangle\langle\rho|V_{K^0}|\bar{K}^*(\mathbf{p})\rangle\\&\qquad=\langle 0|V_0^{K^+}(x)|\bar{K}^*(\mathbf{p})\rangle.\quad(19)\end{aligned}$$

Again by considering the constraint given by the commutator $[V_0^{\pi^+}(x),\dot{V}_{K^0}]=\partial_\mu V_\mu^{K^+}(x)$ taken between the same states, one can obtain information on the terms $\langle\kappa^-(\mathbf{p}'=0)|V_0^{\pi^+}(x)|K^{-*}(\mathbf{p})\rangle$ and $\langle\kappa'(\mathbf{p}'=0)|V_0^{\pi^+}(x)|K^{-*}(\mathbf{p})\rangle$ at $|\mathbf{p}|\to\infty$. If there is no κ' meson, the constraint requires $\langle\kappa^-(\mathbf{p}'=0)|V_0^{\pi^+}(x)|K^{-*}(\mathbf{p})\rangle=0$ at $|\mathbf{p}|\to\infty$.[14] However, even in the presence of the κ' meson the most natural way to satisfy the constraint is to assume that both $\langle\kappa^-(\mathbf{p}'=0)|V_0^{\pi^+}(x)|K^*(\mathbf{p})\rangle$ and $\langle\kappa'^-(\mathbf{p}'=0)|V_0^{\pi^+}(x)|K^*(\mathbf{p})\rangle$ separately vanish in the limit $|\mathbf{p}|\to\infty$.[26] Then Eq. (19) gives

$$G_{K^*}=(m_{K^*}/m_\rho)G_\rho\cos\theta_{K^*K^{*\prime}}.\quad(20)$$

In the absence of $K^{*\prime}$, Eq. (20) reduces to the usual one. Similarly one also obtains the following from Eq. (1):

$$G_{K^{*\prime}}=-(m_{K^{*\prime}}/m_\rho)\sin\theta_{K^*K^{*\prime}}G_\rho.\quad(21)$$

By using the commutator $[V_0^{\pi^0}(x),A_{\pi^+}]=A_0^{\pi^+}(x)$ together with the constraint given by $[V_0^{\pi^0}(x),\dot{A}_{\pi^+}]=\partial_\mu A_\mu^{\pi^+}(x)$, we have previously shown[14] that the Gell-Mann–Zachariasen relation or Sakurai's ρ dominance follows in our approach,

$$\tfrac{1}{2}\sqrt{2}G_\rho G_{\rho\pi^+\pi^-}=m_\rho^2,\quad(22)$$

which is not modified in the present case. However, the K^* analog of Eq. (22) gets modified. Combining Eqs. (17), (20), and (22), we can write

$$\begin{aligned}\sqrt{2}G_{K^*}G_{K^*K^-\pi^0}=\sqrt{2}\tan\theta_{KK'}G_{K^*}G_{K^*K'^-\pi^0}&\\+m_{K^*}^2\cos^2\theta_{K^*K^{*\prime}}/\cos\theta_{KK'}&,\end{aligned}$$

and from Eqs. (18), (21), and (22),

$$\begin{aligned}\sqrt{2}G_{K^{*\prime}}G_{K^{*\prime}K^-\pi^0}=\sqrt{2}\tan\theta_{KK'}G_{K^{*\prime}}G_{K^{*\prime}K^-\pi^0}&\\+m_{K^{*\prime}}^2\sin^2\theta_{K^*K^{*\prime}}/\cos\theta_{KK'}&.\end{aligned}$$

[25] The factor m_{K^*}/m_ρ expresses the symmetry-breaking effect and predicts the ρ width around 128 MeV, if we take $\Gamma(K^*\to K\pi)\simeq 49$ MeV. This factor, in fact, significantly improves the discrepancy between the experimental ρ width and the one predicted by exact $SU(3)$ symmetry. The remaining discrepancy might be blamed for the mixing described by Eq. (17).

[26] These vertices involve two independent form factors, and the simplest way to satisfy the constraint is to assume that both of these vertices satisfy superconvergence relations.

Eliminating $\theta_{K^*K^{*\prime}}$ from the above two equations,

$$\sqrt{2}\left(\frac{G_{K^*}G_{K^*K^-\pi^0}}{m_{K^*}^2}+\frac{G_{K^{*\prime}}G_{K^{*\prime}K^-\pi^0}}{m_{K^{*\prime}}^2}\right)=\frac{1}{\cos\theta_{KK'}}+\sqrt{2}\tan\theta_{KK'} \times\left(\frac{G_{K^*}G_{K^*K'^-\pi^0}}{m_{K^*}^2}+\frac{G_{K^{*\prime}}G_{K^{*\prime}K'^-\pi^0}}{m_{K^{*\prime}}^2}\right). \quad (23)$$

One can simplify further. Consider now

$$\langle K^{*0}(\mathbf{p})|[V_{K^0},A_{\pi^-}]|\pi^{+\prime}(\mathbf{p})\rangle=0$$

and

$$\langle K^{*\prime0}(\mathbf{p})|[V_{K^0},A_{\pi^-}]|\pi^+(\mathbf{p})\rangle=0$$

with $|\mathbf{p}|=\infty$; we obtain, corresponding to Eqs. (17) and (18),

$$G_{K^*K'^+\pi^-}\cos\theta_{KK'}+G_{K^*K^+\pi^-}\sin\theta_{KK'}=\tfrac{1}{2}\sqrt{2}\sin\theta_{K^*K^{*\prime}} \times(m_{K^*}/m_{\rho'})G_{\rho'\pi'^+\pi'^-} \quad (24)$$

and

$$G_{K^{*\prime}K'^+\pi^-}\cos\theta_{KK'}+G_{K^{*\prime}K^+\pi^-}\sin\theta_{KK'}=\tfrac{1}{2}\sqrt{2}\cos\theta_{K^*K^{*\prime}} \times(m_{K^{*\prime}}/m_{\rho'})G_{\rho'\pi'^+\pi^-}. \quad (25)$$

If the mixings are small, Eq. (25) implies[25] that $G_{K^{*\prime}K'^+\pi^-}=\frac{1}{2}\sqrt{2}(m_{K'^*}/m_{\rho}')G_{\rho'\pi'^+\pi^-}$ and Eq. (24) gives $G_{K^*K'\pi}\simeq0$, i.e., $G_{K^*K'\pi}$ is zero in the $SU(3)$ limit and becomes nonvanishing only through mixings in the $SU(3)$ approximation. Multiplying Eqs. (24) and (25) by $G_{K^*}/m_{K^*}^2$ and $G_{K^{*\prime}}/m_{K^{*\prime}}^2$, respectively, and using Eqs. (20) and (21), one obtains

$$\frac{G_{K^*}G_{K^*K'^+\pi^-}}{m_{K^*}^2}+\frac{G_{K^{*\prime}}G_{K'^*K'^+\pi^-}}{m_{K^{*\prime}}^2}=-\tan\theta_{KK'} \times\left(\frac{G_{K^*}G_{K^*K^+\pi^-}}{m_{K^*}^2}+\frac{G_{K^{*\prime}}G_{K^{*\prime}K^+\pi^-}}{m_{K^{*\prime}}^2}\right). \quad (26)$$

By combining Eqs. (23) and (26), we obtain

$$\frac{G_{K^*}G_{K^*K^+\pi^-}}{m_{K^*}^2}+\frac{G_{K^{*\prime}}G_{K^{*\prime}K^+\pi^-}}{m_{K^{*\prime}2}}=\cos\theta_{KK'}. \quad (27)$$

This is the modification of our previous K^* analog[14] of ρ dominance, Eq. (22), i.e., $G_{K^*}G_{K^*K^+\pi^-}=m_{K^*}^2$.

Combining now Eqs. (16) and (27), we finally arrive at the desired result:

$$\frac{F_K}{F_\pi}=\cos\theta_{KK'}+\left(\frac{F_\kappa G_{\kappa K^-\pi^+}}{m_\kappa^2-m_K^2}+\frac{F_{\kappa'}G_{\kappa'K^+\pi^-}}{m_{\kappa'}^2-m_{K2}}\right). \quad (28)$$

Equation (28) reduces to previously obtained Eq. (10) when there is no K-K' mixing and no κ' meson. The magnitude of F_κ and $F_{\kappa'}$ would be of comparable magnitude (or $|F_{\kappa'}|$ might be smaller[27] than $|F_\kappa|$) since both of them are of first order in the symmetry breaking. However, we notice that the $S'PP$ coupling vanishes in the $SU(3)$ limit whereas the $SU(3)$-symmetric SPP coupling can exist. Thus, we expect that[28] $G_{\kappa K\pi}\gg G_{\kappa'K\pi}$. Therefore, unless the value of $m_{\kappa'}$ is very close to m_K, we may neglect the κ' term in Eq. (28), i.e.,

$$F_K/F_\pi=\cos\theta_{KK'}+F_\kappa G_{\kappa K^-\pi^+}/(m_\kappa^2-m_K^2). \quad (29)$$

On the other hand, Eq. (13) will also be modified to

$$F_\kappa G_{\kappa K^+\pi^-}+F_{\kappa'}G_{\kappa'K^+\pi^-}=(m_K^2-m_\pi^2)\cos\theta_{KK'}. \quad (30)$$

However, by the same reasoning which led to Eq. (29), we may also neglect the κ' term in Eq. (30). Therefore, from Eqs. (29) and (30) we obtain

$$F_K/F_\pi=1+[(m_K^2-m_\pi^2)/(m_\kappa^2-m_K^2)]\cos\theta_{KK'}. \quad (31)$$

The mixing angle $\theta_{KK'}$ must be small.[29] If it is large it will affect the Gell-Mann–Okubo mass formula for the pseudoscalar mesons. In fact the $F_+(0)$ of the K_{e3} decay is now given by $\cos\theta_{KK'}$. A reasonable guess may be that $\cos\theta_{KK'}$ is 1 within an error of about 5%. Thus in Eq. (31) we may take $\cos\theta_{KK'}\simeq1$. Therefore, except for the case when $m_{\kappa'}\simeq m_K$, the prediction of Eqs. (14) or (15) will be approximately preserved and at the same time the problem of $F_K=F_\pi$ has been taken care of by the introduction of the κ' meson. It may be noted that our result is free from the knowledge of the F_κ and the magnitude of the $\kappa K\pi$ coupling and depends essentially only on the mass of the κ meson. The above argument indicates that our previous discussion on the K_{l3}-decay form factors[30] will not be modified significantly by the inclusion of the κ' meson. We have $F_+(0)=\cos\theta_{KK'}\simeq1$, so we expect that the parameter ξ will remain small and negative.

IV. κ MASS AROUND 1.1 BeV

By using commutators such as $[V_{K^0},A_{\pi^-}]=0$ and $[\dot{V}_{K^0},A_{\pi^-}]=0$ and the $SU(3)$ approximation, the following intermultiplet mass formulas of the hybrid type can be derived[6,31]:

$$\begin{aligned}K^2-\pi^2&=K^{*2}-\rho^2=K^{**2}(1420)-A_{2H}^2\\&=K_A^2-A_1^2=K^{*\prime}-\rho'^2\\&=K'^2-\pi'^2=\kappa^2-\delta^2=\kappa'^2-\delta'^2=\cdots. \quad (32)\end{aligned}$$

Here K^2, for example, denotes the square of the K-meson mass. In the derivation we have ngelected the mixing effect in the $I=\frac{1}{2}$ states. The formulas include the $SU(6)$ result, $K^2-\pi^2=K^{*2}-\rho^2$, and agree with experiment rather well for the cases where identifications of resonances are established. If we identify[16] either the $\delta(962)$ or the $\pi_N(1016)$ with the $I=1$ 0^{++} meson, δ, then Eq.

[27] For the F_κ one can consider the diagram $\kappa\to K+\pi\to$ vacuum. Since $G_{kK\pi}\gg G_{\kappa'K\pi}$, a similar mechanism for the $F_{\kappa'}$ leads to $|F_{\kappa'}|<|F_\kappa|$.

[28] As in Eq. (18), the $\kappa'K\pi$ coupling takes a nonvanishing value under the $SU(3)$ approximation only through the κ-κ' mixing. We expect this mixing to be small since charge-conjugation invariance forbids the 0^{++}-0^{+-} mixing except for the κ-κ' mixing.

[29] The same argument as for κ-κ' mixing also holds here. See Ref. 28.

[30] S. Matsuda and S. Oneda, Phys. Rev. **169**, 1172 (1968).

[31] S. Matsuda and S. Oneda, Phys. Rev. **179**, 1301 (1969).

(32) predicts that m_κ is around 1.10–1.13 BeV. Then from Eq. (14) [or from Eq. (31) assuming $\cos\theta_{KK'}\simeq 1$], we obtain[32] $F_K/F_\pi \simeq 1.22$. The fact that we arrive at the same value of the mass of the κ meson from two different sources of information seems to strengthen our argument since both the derivations are consistently based on the same $SU(3)$ approximation and the chiral $SU(3)\otimes SU(3)$ algebra.

There is some preliminary evidence[33] for the existence of a κ meson with mass around 1.1 BeV.

V. VECTOR-MESON $\to l+\bar{l}$ DECAY SUM RULES

Previously we have obtained in our approach sum rules for the vector-meson $\to l+\bar{l}$ decays.[14] In particular, we have obtained

$$G_\phi = \tfrac{1}{2}\sqrt{2}G_\rho \cos\theta(m_\phi/m_\rho) \tag{33}$$

and

$$G_\omega = \tfrac{1}{2}\sqrt{2}G_\rho \sin\theta(m_\omega/m_\rho)\,. \tag{34}$$

Here θ is the usual ω–ϕ mixing angle defined by $\omega = \cos\theta\, \omega_1 + \sin\theta\, \omega_8$ and $\phi = \cos\theta\, \omega_8 - \sin\theta\, \omega_1$. [$\omega \to \omega_1$ and $\phi \to \omega_8$ in the $SU(3)$ limit.] Eliminating θ from these equations, we obtain the so-called first spectral-function sum rule,[9,10,14]

$$G_\rho^2/m_\rho^2 = G_\omega^2/m_\omega^2 + G_\phi^2/m_\phi^2\,. \tag{35}$$

We remark here that even if we consider the existence of the exotic vector meson V', these results do not change. In the computation, the contribution of the V' states appears through mixing as in Eqs. (20) and (21). However, in the final result it drops out and Eqs. (33)–(35) are not affected.

On the other hand, the existence of the V' meson will affect the Gell-Mann–Okubo mass formula. In our approach, consideration of $\langle K^{*0}(\mathbf{p})|[V_{K^0},\dot{V}_{\bar{K}^0}]|\bar{K}^{*0}(\mathbf{p})\rangle = 0$ with $|\mathbf{p}|=\infty$ and the $SU(3)$ approximation leads, in the absence of V' mesons, to an ω–ϕ mixing angle θ_m given by[14]

$$\sin^2\theta_m = \frac{3m_\varphi^2 - 4m_{K^*}^2 + m_\rho^2}{3(m_\varphi^2 - m_\omega^2)}\,. \tag{36}$$

In the presence of V' mesons, this will be modified. Therefore, the discrepancy between the value of θ determined from the $V\to l+\bar{l}$ decays and that of θ_m, if it is found, may be attributed to the existence of V' mesons. Experiment, at present, is still not accurate enough to make a definite statement. Of course, we must also keep in mind the possibility that there may exist a recurrence of the 1^{--} meson in the higher-mass range. In that case, Eqs. (33)–(35) will also be modified, though the effect may not be very large if the masses of the new vector mesons are large.

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APPENDIX

We consider the commutator $[A_{K^0}, V_0^{\pi^0}(x)] = \frac{1}{2}A_0^{K^0}(x)$ and sandwich it between the states $\langle 0|$ and $|\bar{K}^0(\mathbf{p})\rangle$, with $|\mathbf{p}|=\infty$. Then we obtain

$$\lim_{|\mathbf{p}|\to\infty} [\langle 0|V_0^{\pi^0}(x)|\rho\rangle\langle\rho|A_{K^0}|\bar{K}^0(\mathbf{p})\rangle - \langle 0|A_{K^0}|\bar{K}^0\rangle\langle\bar{K}^0|V_0^{\pi^0}(x)|\bar{K}^0(\mathbf{p})\rangle] = \lim_{|\mathbf{p}|\to\infty} -\tfrac{1}{2}\langle 0|A_0^K(x)|\bar{K}^0(\mathbf{p})\rangle\,. \tag{A1}$$

Again, if one uses the commutator $\partial_\mu[A_{K^0}, V_\mu^{\pi^0}(x)] = \frac{1}{2}\partial_\mu A_\mu^{K^0}(x)$ sandwiched between the same states as in Eq. (A1) and assumes the gentleness condition for the matrix elements of the operators $\partial_\mu A_\mu^{K^0}(x)$, one obtains

$$\lim_{|\mathbf{p}|\to\infty} \langle 0|A_{K^0}|\bar{K}^0\rangle\langle\bar{K}^0|V_0^{\pi^0}(x)|\bar{K}^0(\mathbf{p})\rangle = 0\,.$$

Thus by replacing A_{K^0} by $2[V_{K^0}, A_{\pi^0}]$, Eq. (A1) can be written in the form

$$\lim_{|\mathbf{p}|\to\infty} \langle 0|V_0^{\pi^0}(x)|\rho^0\rangle\langle\rho^0|[V_{K^0}, A_{\pi^0}]|\bar{K}^0(\mathbf{p})\rangle = \lim_{|\mathbf{p}|\to\infty} \tfrac{1}{4}\langle 0|A_0^{K^0}(x)|\bar{K}^0(\mathbf{p})\rangle\,.$$

Using the $SU(3)$ approximation for V_{K^0} and PCAC for A_π, this equation gives

$$\frac{1}{2}\frac{2G_{K^*K^0\pi^0}}{G_{\rho\pi^+\pi^-}}\frac{G_\rho}{\sqrt{2}}G_{\rho^0\pi^+\pi^-}\frac{1}{2m_\rho m_{K^*}}F_\pi = \tfrac{1}{4}F_K\,.$$

On the other hand, we have previously[14] obtained $G_\rho/\sqrt{2}G_{\rho\pi^+\pi^-} = m_\rho^2$ [see Eq. (22)], and $2G_{K^*K^0\pi^0}/G_{\rho\pi^+\pi^-} = m_{K^*}/m_\rho$.[25] Therefore, we are again led to the equation $F_K = F_\pi$. However, similar to the case discussed in Sec. II, the above conclusion will not be obtained if there are two $I=\frac{1}{2}$ 0^{-+} mesons K and K'.

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[32] This value is, of course, subject to error due to the approximation $F_+(0) = \cos\theta_{KK'} = 1$ and the neglect of the κ' contribution. The determination of m_κ will certainly be helpful in obtaining a feeling for these approximations.

[33] T. Trippe, C. Y. Chien, E. Malamud, J. Mellema, P. E. Schlein, W. E. Slater, D. H. Storkland, and H. K. Ticho, Phys. Letters **24B**, 203 (1968).