tude for the trajectory $\alpha=-0.7+0.95u$ as determined by Λ_α particle mass spectra.[20]

(iii) The A and B FESR results for Λ_γ-Λ_δ exchanges fail to change sign at $\alpha=-\frac{3}{2}$ ($u\simeq-0.8$). A zero in the amplitudes at this point is expected in conventional Regge theory. We are inclined not to take this disagreement at this large negative u value too seriously at the moment, since the $n=0$ FESR integrals may involve appreciable background contributions that are not accounted for (particularly in the Z^* region) in our resonance-saturation approximation. In fact, the A amplitude does change sign at a larger u value ($u\simeq-1.3$).

(iv) The signs of the Λ_γ and Λ_α contributions to A and B amplitudes are consistent with exchange degeneracy for the Regge residues[20] (such that the imaginary Regge exchange amplitude due to Λ_α and Λ_γ vanishes).

The K^+p backward elastic scattering data in the momentum range 2–7 GeV/c have been successfully described by Λ_α and Λ_γ Regge poles with exchange-degenerate trajectories and residues.[20] We have evaluated the corresponding Regge contribution to the FESR for comparison with the $I=0$, $n=0$, 1 low-energy integrals. The results from the Regge fit are shown by the dashed curves in Fig. 5. Except for the Λ_α-Λ_β FESR results on the B amplitude, the low-energy integrals are not in good agreement with the Regge integrals obtained from fits to high-energy data, which indicates that the resonance evaluations of the low-energy integrals are at present too crude to serve as a quantitative tool.

In conclusion, we should emphasize that even at the present stage the u-channel FESR have provided useful information on the properties of the N, Λ, and Σ exchanges. As further experimental information becomes available and as more sophisticated models for unphysical regions are developed, the FESR will become a quantitative probe of u-channel baryon exchanges. Alternatively, these FESR relations can supplement studies of unphysical regions.

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Form Factors and Photoproduction in the $\rho+\rho'$ Model*

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The nucleon isovector form factor $F_1{}^V(t)$ is described by two isovector resonances, the ρ and the ρ'. The effect of the ρ' resonance on current-algebra sum rules and on the normalization of the pion form factor $F_\pi(t)$ is investigated. The widths $\Gamma_{(\rho\to 2\pi)}$ and $\Gamma_{(\rho\to 2l)}$ are determined within fairly narrow limits to be in excellent agreement with a recent analysis of the colliding-beam data. The pion form factor in the spacelike region is found to behave like t^{-1} at large t. Using the γ-ρ' coupling determined by this analysis, a possible explanation is given for the apparent discrepancies reported recently in photoproduction experiments. A discussion of the $u=0$, πN superconvergence relations is given. A detailed account of the nucleon form factors $G_E{}^p(t)$ and $G_E{}^n(t)$ is presented.

I. INTRODUCTION AND BASIC EQUATIONS

ONE of the major theoretical problems in electromagnetic interactions in recent years has been the explanation of the so-called dipole fit for the nucleon form factors. Recent experiments[1] have confirmed that the form factor $G_M{}^p(t)/\mu_p$ behaves according to the empirical relation

$$G_M{}^p(t)/\mu_p=(1-t/0.71)^{-2}, \tag{1.1}$$

for $0\leq -t\lesssim 25$ (GeV/c)^2. While, experimentally, it is difficult to obtain $G_E{}^p(t)$ for large t, it has been observed that for $0\leq -t\lesssim 3$ (GeV/c)^2 the scaling laws

$$G_E{}^p(t)=G_M{}^p(t)/\mu_p, \tag{1.2}$$

$$G_M{}^p(t)/\mu_p=G_M{}^n(t)/\mu_n, \tag{1.3}$$

and

$$G_E{}^n(t)=0 \tag{1.4}$$

are consistent with the data, except for a small region of t near zero, where it is known that $G_E{}^n(t)$ has a small but nonzero slope.[2]

The strong t dependence of the form factor is difficult to understand on the basis of the usual pole dominance by vector particles of the dispersion relations for $G_E(t)$ and $G_M(t)$. It is clear, for example, that the ρ meson is not sufficient to explain the t^{-2} behavior of $G_E{}^V(t)$, since in dispersion theory a single resonance leads to a t^{-1} behavior far from the resonance, unless some *ad hoc* structure is assumed for the ρNN vertex function.[3] An obvious explanation for this t dependence would be the

* Supported in part by the National Research Council of Canada.

[1] D. H. Coward *et al.*, Phys. Rev. Letters **20**, 292 (1968).

[2] R. Wilson, in *Lectures at 1966 Scottish Universities Summer School in Physics*, edited by T. W. Priest and L. L. J. Vick (Oliver and Boyd, London, 1967).

[3] See, e.g., the fit by T. Massam and A. Zichichi, Nuovo Cimento **44**, 309 (1966).

existence of another isovector resonance, the ρ', with the same quantum numbers as the ρ, especially since such a resonance could also explain the nonzero polarization in high-energy π-N charge-exchange scattering.[4] Assuming that the Sach's isovector form factors $G_E{}^V$ and $G_M{}^V$ are dominated by these two resonances leads[2] to $m_{\rho'}<1$ GeV. This is unsatisfactory, since no such particle has been observed in this region. However, it is well known that there are alternatives to the Sachs choice of form factors, including the original Dirac and Pauli form factors, F_1 and F_2. Indeed, the hypotheses of ρ dominance of the isovector current and universality were originally applied to the isovector Dirac form factor $F_1{}^V(t)$ and seem to be valid for small t. It would seem natural, therefore, to attempt to extend this approach to higher $|t|$ by including the second isovector resonance in a dispersion relation for $F_1{}^V(t)$. Since

$$G_E=F_1+\tau F_2\,, \tag{1.5}$$

$$G_M=F_1+F_2\,, \tag{1.6}$$

where $\tau=t/4M^2$ for both the isovector and isoscalar form factors, it follows that $F_2{}^{V,S}\sim t^{-3}$ from Eq. (1.1), provided the scaling laws (1.2)–(1.4) hold. If the scaling laws are assumed to be valid, however, $G_M{}^p/\mu_p$ can be expressed entirely[5,6] in terms of $F_1{}^V$:

$$\frac{G_M{}^p(t)}{\mu_p}=\frac{(1-\tau)F_1{}^V(t)}{1-(\mu_p-\mu_n)\tau}. \tag{1.7}$$

Assuming that $F_1{}^V$ satisfies an unsubtracted dispersion relation, and dominating the absorptive part by the ρ and ρ', leads to, in the zero-width approximation,

$$F_1{}^V(t)=\frac{\gamma_{\rho NN}F_\rho}{m_\rho{}^2-t}+\frac{\gamma_{\rho' NN}F_{\rho'}}{m_{\rho'}{}^2-t}, \tag{1.8}$$

where F_ρ is defined by

$$\langle 0|V_\mu{}^{(3)}(0)|\rho^0(q)\rangle=F_\rho\epsilon_\mu(q) \tag{1.9}$$

and $\gamma_{\rho NN}$ is the coupling constant for the vector part of the ρNN interaction;

$$\langle N(p')|J_\mu{}^\rho(0)|N(p)\rangle=\bar{u}(p')(\gamma_{\rho NN}\gamma_\mu + i\gamma_{\rho NN}{}^{(T)}\sigma_{\mu\nu}q_\nu)u(p)\,, \tag{1.10}$$

where $q=p'-p$. $F_{\rho'}$ and $\gamma_{\rho' NN}$ are defined similarly. The normalization condition $F_1{}^V(0)=1$, and the requirement that $F_1{}^V(t)\sim t^{-2}$ for large t, gives two relations between the coupling constants appearing in Eq. (1.8), which results in

$$F_1{}^V(t)=m_\rho{}^2m_{\rho'}{}^2/(m_\rho{}^2-t)(m_{\rho'}{}^2-t)\,. \tag{1.11}$$

[4] T. J. Gajdicar, R. K. Logan, and J. W. Moffat, Phys. Rev. **170**, 1599 (1968).

[5] We emphasize that the form (1.7) is an empirical relationship valid in the spacelike region $t<0$ to the extent that the scaling laws (1.2)–(1.4) hold.

[6] J. G. Cordes and P. J. O'Donnell, Phys. Rev. Letters **20**, 1462 (1968).

This leads to a good representation of the data for $0\leq -t\lesssim 25$ (GeV/c)^2, using Eq. (1.7) when $m_{\rho'}\simeq 2.0$ GeV $\pm 5\%$. Although no new form-factor data have appeared since our previous analysis,[6] the results of a series of colliding-beam experiments and of photoproduction of ρ mesons from complex nuclei have been reported for which our $\rho+\rho'$ model makes some interesting statements.

The requirement that the ρ and ρ' completely describe the nucleon form factor $F_1{}^V(t)$ implies that the couplings of ρ and ρ' to the photon and to the $N\bar{N}$ system should be comparable in magnitude. This might lead one to expect some difficulty in maintaining the first Weinberg sum rules for[7] $SU(2)\times SU(2)$ and[8] $SU(3)$ and even the vector dominance of the π-N scattering lengths.[9] As we shall show, apart from the radiative decay of the pion, which depends on the $SU(2)\times SU(2)$ first Weinberg sum rule, the ρ' couplings actually serve to bring the vector-dominance model into agreement with the colliding-beam experiments[10,11] on the decays of the ρ, ω, and ϕ. Thus, if we consider the conventional first Weinberg sum rule for $SU(3)$

$$\frac{F_\rho{}^2}{m_\rho{}^2}=\frac{3}{4}\left(\frac{F_\omega{}^2}{m_\omega{}^2}+\frac{F_\phi{}^2}{m_\phi{}^2}\right), \tag{1.12}$$

the Orsay[10] result for the right-hand side of (1.12) is 0.0324 ± 0.0038 GeV^2, whereas the left-hand side is found both from the Orsay experiment alone and from the fit by Roos and Pisut[12] to the combined Orsay-Novosibirsk data to be 0.0254 ± 0.0024 GeV^2, leaving a discrepancy of 0.0070 ± 0.0045 GeV^2. This discrepancy is increased to about 2 standard deviations by corrections for the finite width of the ρ. It is evident that a ρ' contribution to the left-hand side of (1.12) is welcome, and in our analysis we shall, in fact, require $F_\rho{}^2/m_\rho{}^2+F_{\rho'}{}^2/m_{\rho'}{}^2\geq 0.0286$ GeV^2 (to be within 1 standard deviation of the right-hand side).

A similar situation exists with regard to the relation proposed by Sakurai,[9]

$$\frac{\gamma_{\rho\pi\pi}\gamma_{\rho NN}}{m_\rho{}^2}=\frac{1}{2F_\pi{}^2}, \tag{1.13}$$

obtained by equating the values obtained from ρ exchange and from current algebra for the π-N s-wave scattering-length difference. Since the normalization

[7] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967).

[8] R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters **19**, 1266 (1967); T. Das, V. S. Mathur, and S. Okubo, *ibid.* **19**, 470 (1967); J. G. Cordes and P. J. O'Donnell, Nuovo Cimento Letters **1**, 107 (1969).

[9] J. J. Sakurai, Phys. Rev. Letters **17**, 552 (1966).

[10] J. E. Augustin *et al.*, Phys. Letters **28B**, 503 (1969). We define F_ω by $\langle 0|V_\mu{}^{(8)}(0)|\omega(q)\rangle=(\sqrt{\tfrac{3}{2}})F_\omega\epsilon_\mu(q)$, and F_ϕ by $\langle 0|V_\mu{}^{(8)}(0)|\phi(q)\rangle=(\sqrt{\tfrac{3}{2}})F_\phi\epsilon_\mu(q)$. The width formulas are $\Gamma(\omega\to e^+e^-)=\pi\alpha^2F_\omega{}^2/3m_\omega{}^3$, $\Gamma(\phi\to e^+e^-)=\pi\alpha^2F_\phi{}^2/3m_\phi{}^3$, and $\Gamma(\rho\to e^+e^-)=4\pi\alpha^2F_\rho{}^2/3m_\rho{}^3$, in the approximation $m_e\simeq 0$.

[11] V. L. Auslander *et al.*, Phys. Letters **25B**, 433 (1967).

[12] M. Roos and J. Pisut, CERN Report, 1969 (unpublished).

conditions for $F_1{}^V(t)$ and for the pion electromagnetic form factor $F_\pi(t)$ in a ρ-dominant model lead one to expect $\gamma_{\rho NN}=\gamma_{\rho\pi\pi}$, the sum rule (1.13) may be tested by using the colliding-beam values for the ρ width. Using the value for $\Gamma_{\rho\pi\pi}$ found in the Orsay experiment,[10] 111±6 MeV, leaves a discrepancy which is (20.4±4.2)% of the right-hand side of (1.13), while the result of the fit of the combined data,[12] $\Gamma_{\rho\pi\pi}=122\pm6$ MeV, leaves a discrepancy of (12.5±4.3)%. In the subsequent analysis, we shall require that the generalization of (1.13),

$$\frac{\gamma_{\rho\pi\pi}\gamma_{\rho NN}}{m_\rho{}^2}+\frac{\gamma_{\rho'\pi\pi}\gamma_{\rho'NN}}{m_{\rho'}{}^2}=\frac{1}{2F_\pi{}^2},\qquad(1.14)$$

be satisfied. It should be noted, however, that with the inclusion of the ρ', the form-factor normalization conditions no longer imply universality of the vector-meson couplings. Separate universality for the ρ and ρ' couplings would follow if, in addition, it was required that the pion form factor superconverge, that is, $F_\pi(t)\sim t^{-2}$ for large $|t|$; however, we shall see in Sec. II that this is probably inconsistent with various experimental limits.

II. THE $A_1\rho\pi$ PROBLEM AND RADIATIVE PION DECAY

In the radiative decay of the pion, $\pi^+\to e^+\nu\gamma$, there exists one measurement of the number N of structure-dependent events,[13] which may be expressed in terms of a quantity[14] χ by

$$N=68.4\left(\frac{0.89\times10^{-16}\text{ sec}}{\tau_{\pi^0}}\right)\pm144.3\left(\frac{0.89\times10^{-16}\text{ sec}}{\tau_{\pi^0}}\right)^{1/2}\chi+102.3\chi^2.\qquad(2.1)$$

In the pole-dominance approximation, χ may be related[14] to the matrix elements $\langle\pi|V_\mu|A_1\rangle$, $\langle\pi|A_\mu|\rho\rangle$, and $\langle\pi|A_\mu|\rho'\rangle$. Application of the hypothesis of partially conserved axial-vector current, together with the conservation condition on the vector current, leads to the following three equations:

$$f_{A_1\rho\pi}-g_{A_1\rho\pi}=\frac{m_A{}^2F_\rho}{m_\rho{}^2F_AF_\pi}\left(1-\frac{2F_\pi{}^2\gamma_{\rho\pi\pi}}{F_\rho}\right),\qquad(2.2\text{a})$$

$$f_{\rho'A_1\pi}-g_{\rho'A_1\pi}=\frac{m_A{}^2F_{\rho'}}{m_{\rho'}{}^2F_AF_\pi}\left(1-\frac{2F_\pi{}^2\gamma_{\rho'\pi\pi}}{F_{\rho'}}\right),\qquad(2.2\text{b})$$

and

$$f_{A_1\rho\pi}+g_{A_1\rho\pi}+\frac{m_\rho{}^2F_{\rho'}}{m_{\rho'}{}^2F_\rho}(f_{\rho'A_1\pi}+g_{\rho'A_1\pi})=\frac{m_\rho{}^2F_A}{m_A{}^2F_\rho F_\pi},\qquad(2.2\text{c})$$

where we define $f_{A_1\rho\pi}$ and $g_{A_1\rho\pi}$ by

$$[(m_\pi{}^2-q^2)/m_\pi{}^2F_\pi]\langle\rho^a(k)|\partial^\mu A_\mu{}^b|A_1{}^c(p)\rangle=-i\epsilon_{abc}[(m_A{}^2-m_\rho{}^2)f_{A_1\rho\pi}\epsilon^\rho\cdot\epsilon^{A_1}+2g_{A_1\rho\pi}(\epsilon^\rho\cdot p)(\epsilon^{A_1}\cdot k)],$$

and $f_{\rho'A_1\pi}$ and $g_{\rho'A_1\pi}$ are similarly defined. F_A is defined by

$$\langle0|A_\mu{}^a|A_1{}^b(P)\rangle=-i\delta_{ab}F_A\epsilon_\mu{}^{A_1(P)}.$$

One then finds

$$\chi=\frac{F_A}{F_\pi}\left(\frac{F_\rho}{m_\rho{}^2}g_{A_1\rho\pi}+\frac{F_{\rho'}}{m_{\rho'}{}^2}g_{\rho'A_1\pi}\right)\qquad(2.3\text{a})$$

$$=\left(\frac{2\gamma_{\rho\pi\pi}}{F_\rho}-\frac{1}{2F_\pi{}^2}\right)\left(\frac{F_\rho{}^2}{m_\rho{}^2}+\frac{F_{\rho'}{}^2}{m_{\rho'}{}^2}\right)-\frac{1}{2},\qquad(2.3\text{b})$$

where the notation of Geffen[15] has been used in Eqs. (2.2) and (2.3) for the $A_1\rho\pi$ and $A_1\rho'\pi$ coupling constants. In obtaining (2.3b), use has also been made of the superconvergence condition for $F_1{}^V(t)$

$$F_\rho\gamma_{\rho NN}+F_{\rho'}\gamma_{\rho'NN}=0,\qquad(2.4)$$

the normalization conditions for $F_1{}^V(t)$ and $F_\pi(t)$

$$F_\rho\gamma_{\rho NN}/m_\rho{}^2+F_{\rho'}\gamma_{\rho'NN}/m_{\rho'}{}^2=1,\qquad(2.5)$$

$$F_\rho\gamma_{\rho\pi\pi}/m_\rho{}^2+F_{\rho'}\gamma_{\rho'\pi\pi}/m_{\rho'}{}^2=1,\qquad(2.6)$$

and Eq. (1.14). The first Weinberg sum rule for $SU(2)\times SU(2)$, in the form

$$F_A{}^2/m_A{}^2+F_\pi{}^2=F_\rho{}^2/m_\rho{}^2+F_{\rho'}{}^2/m_{\rho'}{}^2,$$

has also been used to eliminate F_A from Eq. (2.3a). Including the uncertainty in the π^0 lifetime, one finds from (2.1) that the observed number of events (110±15) implies $\chi\simeq0.25_{-0.19}{}^{+0.15}$.

III. RESULTS

The restrictions on the ρ-coupling constants imposed by the requirements of Eqs. (1.14) and (2.4)–(2.6), and by the experimental limits on $V\equiv F_\rho{}^2/m_\rho{}^2+F_{\rho'}{}^2/m_{\rho'}{}^2$ and χ, are best described graphically. In Fig. 1, we have plotted in the F_ρ-$\Gamma_{\rho\pi\pi}$ plane contours of fixed V, χ, $\Gamma_{\rho'\pi\pi}$, and $|F_\pi(m_\rho{}^2)|^2$ for several values of these quantities. Also shown is the line corresponding to the additional requirement that the pion form factor $F_\pi(t)$ be superconvergent, i.e., the solution of the equation $F_\rho\gamma_{\rho\pi\pi}+F_{\rho'}\gamma_{\rho'\pi\pi}=0$. As an illustration of the way in which this graph may be used, the region in which the three conditions $V\geq0.0286$ GeV^2, $\chi\leq0.40$, and $\Gamma_{\rho'\pi\pi}\leqslant200$ MeV are simultaneously satisfied has been shaded. It will be seen that this allowed region of the ρ coupling constants is extremely small and can be approximately described by $\Gamma_{\rho\pi\pi}\simeq118\pm2$ MeV, $\Gamma_{\rho\to e^+e^-}\simeq7.34\pm0.12$ keV, and branching ratio$\equiv\Gamma_{\rho\to e^+e^-}/\Gamma_{\rho\pi\pi}\simeq(6.22\pm0.15)$

[13] P. De Pommier *et al.*, Phys. Letters **7**, 285 (1963).

[14] S. G. Brown and G. B. West, Phys. Rev. **168**, 1605 (1967). We use $F_\pi=0.094$ GeV, $m_\pi=0.138$ GeV, $m_\rho=0.770$ GeV, and $m_A=\sqrt{2}m_\rho$. Our χ differs from that of Brown and West by a factor $(1.058\text{ GeV}/m_A)^{-2}=1.059$.

[15] D. A. Geffen, Phys. Rev. Letters **19**, 770 (1967).

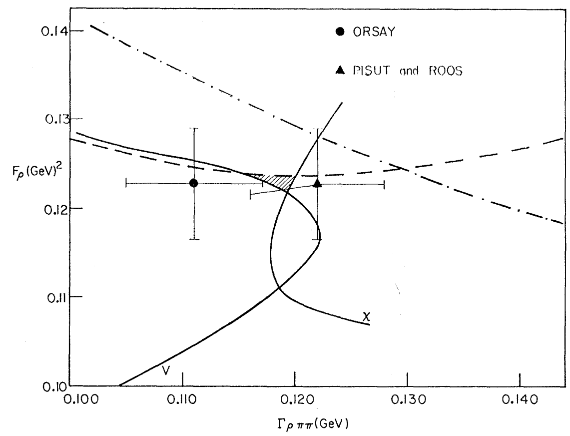

FIG. 1. Solid lines show curves of constant $\chi=0.4$ and $V=0.0286$ GeV^2 obtained as outlined in the text, for $m_{\rho'}=2.0$ GeV. The dashed line represents $\Gamma_{\rho'\pi\pi}=200$ MeV, and the dot-dash curve is the locus of points for which $F_\pi(t)\sim t^{-2}$. The allowed region is shaded and represents $\chi\lesssim 0.4$, $V\gtrsim 0.0286$ GeV^2, and $\Gamma_{\rho'\pi\pi}\lesssim 200$ MeV. We have also plotted the points $F_\rho=0.1227$ GeV^2, $\Gamma_{\rho\pi\pi}=112$ MeV (Ref. 10) and $F_\rho=0.1225$ GeV^2, $\Gamma_{\rho\pi\pi}=122$ MeV (Ref. 12).

$\times 10^{-5}$. We also find that in this region we have[11] $\Gamma_{\rho'\pi\pi}\simeq 170\pm 30$ MeV, $\chi\simeq 0.37\pm 0.03$ (and hence $N\simeq 136_{-23}^{+31}$), $F_{\rho'}^2/m_{\rho'}^2\simeq 0.0038\pm 0.0007$ GeV^2, $\gamma_{\rho NN}^2/4\pi\simeq 2.60\pm 0.10$, $\gamma_{\rho' NN}^2/4\pi\simeq 2.57\pm 0.51$, and[16]

$$V\simeq 0.0291\pm 0.0005 \text{ GeV}^2.$$

In Fig. 2 we have plotted similar contours, but with $m_{\rho'}=1.75$ GeV as input. This illustrates the dependence of the allowed region on the ρ' mass. It will be seen that the allowed region is much larger than in the $m_{\rho}'=2.0$ GeV case, which allows a lower partial width $\Gamma_{\rho'\pi\pi}$ than was previously required. This opens the tempting possibility of identifying the ρ' with one of the narrow-width meson resonances seen in the so-called R region, e.g., $R_2(1720)$ or $R_3(1740)$. A ρ' with a mass in this region will not give such a good fit to the nucleon form factors using Eq. (1.7). We shall discuss this problem below.

As an indication of a possible solution in the $m_{\rho'}=1.75$ GeV case, we list the following qualities obtained at a value of $F_\rho=0.1227$ and $\Gamma_{\rho\pi\pi}=0.111$ GeV (corresponding to the Orsay point[10]):

$$\Gamma_{\rho'\pi\pi}=43.2 \text{ MeV},\quad \chi=0.31 \text{ (and } N=123),$$
$$F_{\rho'}^2/m_{\rho'}^2=0.00385 \text{ GeV}^2,$$
$$\gamma_{\rho NN}^2/4\pi=2.86, \text{ and } \gamma_{\rho' NN}^2/4\pi=3.65.$$

[16] It has been pointed out in Ref. 10 that the vector-dominance prediction for the combination $\Gamma(\omega\to 2l)\Gamma(\omega\to\pi^0\gamma)/\Gamma(\pi^0\to 2\gamma)$ is in disagreement with the measured values by a factor of almost 2.5. However, a least-squares fit of the most recent data on the five decays $\omega\to\pi^0\gamma$, $\omega\to 2l$, $\pi^0\to 2\gamma$, $\phi\to K\bar{K}$, and $\phi\to 2l$ using the width formulas in terms of $g_{\omega\pi\gamma}$, V, and θ as given in Cordes and O'Donnell (Ref. 8) gives $\Gamma(\omega\to\pi^0\gamma)\simeq 1.05$ MeV, $V\simeq 0.03$ GeV^2, and $\theta\simeq 28°$, with a χ^2 of $\simeq 4.5$ from two degrees of freedom (corresponding to a confidence level of 11%). These values then imply the following widths: $\Gamma(\omega\to 2l)\simeq 0.63$ keV (0.94 ± 0.18 keV), $\Gamma(\pi^0\to 2\gamma)\simeq 8.78$ eV (7.46 ± 1.51 eV), $\Gamma(\phi\to K\bar{K})\simeq 3.37$ MeV (3.33 ± 0.53 MeV), and $\Gamma(\phi\to 2l)\simeq 1.71$ keV (1.64 ± 0.26 keV), where the experimental widths (Refs. 10 and 18) used are shown in brackets.

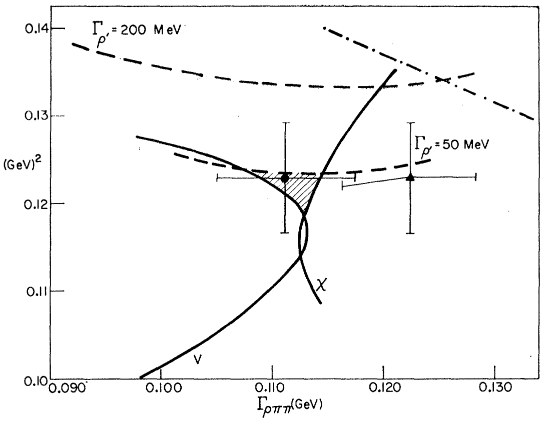

FIG. 2. Shows the same curves as in Fig. 1, calculated for $m_{\rho'}=1.75$ GeV. Two contours corresponding to $\Gamma_{\rho'\pi\pi}=200$ MeV and $\Gamma_{\rho'\pi\pi}=50$ MeV are shown as dashed lines.

We now consider Eqs. (2.2) for the $A_1\rho\pi$ and $\rho' A_1\pi$ coupling constants. Since there are four coupling constants to determine only three equations involving them, it will not be possible to determine them all solely from the values previously found for F_ρ, $\gamma_{\rho\pi\pi}$, etc. It turns out, however, because of the large amount of phase space available, that the partial width $\Gamma_{\rho'\to\pi A_1}$ is very sensitive to the ratio $f_{\rho' A_1\pi}/g_{\rho' A_1\pi}$, and hence any reasonable choice of upper limit for this partial width, say, 100 or 150 MeV, is sufficient to determine all the coupling constants accurately. This procedure leads to the following range of partial widths over the region of interest:

(i) For $m_{\rho'}=2.0$ GeV, $\Gamma_{A_1\rho\pi}\simeq 120\pm 2$ MeV, $\Gamma_{\rho'\pi A_1}\gtrsim 75\pm 15$ MeV, and the ratio of transverse to longitudinal couplings in the $A_1\to\rho\pi$ decay is determined[17] to be $g_T/g_L\simeq -1.22\pm 0.02$.

(ii) For $m_{\rho'}=1.75$ GeV, $\Gamma_{A_1\rho\pi}=137$ MeV, $\Gamma_{\rho' A_1\pi}=22$ MeV, and the ratio of transverse to longitudinal couplings $g_T/g_L=-1.23$ (for the solution corresponding to the Orsay measurement).

The superconvergence relation for π-ρ scattering proposed by de Alfaro *et al.* becomes, on saturation with the π, ω, A_1, and A_2 mesons,

$$\frac{g_{\omega\rho\pi}^2}{m_\omega^2}=\frac{4\gamma_{\rho\pi\pi}^2}{m_\rho^2}+\frac{g_L^2}{8}-\frac{g_T^2}{16}-\frac{1}{8}\frac{(m_{A_2}^2-m_\rho^2)^2}{m_{A_2}^2}g_{A_2\rho\pi}^2. \quad (3.1)$$

Using the values for $\gamma_{\rho\pi\pi}$, g_T, and g_L found here, and the experimental $A_2\rho\pi$ coupling, gives, for $m_{\rho'}=2.0$ (1.75) GeV, $g_{\omega\rho\pi}=10.95$ (10.58). Neglecting the $\rho'\pi\omega$ coupling, which we expect to be small due to the large amount of phase space available for the decay, then

[17] J. Ballam *et al.* [Phys. Rev. Letters **21**, 934 (1968)] give $|g_T/g_L|^2=0.16\pm 0.08$. S. G. Brown and G. B. West [Phys. Rev. **180**, 1613 (1969)] quote a private communication from this group revising this value to 0.64 ± 0.25.

gives $g_{\omega\pi\gamma}\simeq F_\rho g_{\omega\rho\pi}/m_\rho^2=2.26$ (2.19) and $\Gamma_{\omega\to\pi^0\gamma}\simeq1.10$ $(1.04)\pm0.06$ MeV over the bounded region, in excellent agreement with the experimental value[18] 1.17 ± 0.14 MeV. A similar calculation, assuming π-ρ dominance of the $\omega\to3\pi$ decay, gives the prediction $\Gamma_{\omega\to3\pi}\simeq6.2$ $(5.4)\pm0.3$ MeV, in poor agreement[19–21] with the experimental value[18] $\sim11.0\pm1.1$ MeV.

IV. APPLICATION TO ρ^0 PHOTOPRODUCTION

The value of F_ρ found by the graphical method described above using Fig. 1, namely, $F_\rho\simeq0.1225$ ±0.001 GeV^2, which is in excellent agreement with the Orsay determination[10] [see Eq. (4.4)], corresponds to a value of $\gamma_\rho^2/4\pi\simeq0.466\pm0.008$, where we have introduced γ_ρ by $F_\rho\equiv m_\rho^2/2\gamma_\rho$. Two recent experiments[22,23] on ρ^0 photoproduction on complex nuclei have obtained values for $\gamma_\rho^2/4\pi$ around 1.1 ± 0.2. We shall now indicate how inclusion of the ρ' as well as the ρ in the expression for the forward vector-meson production amplitude affects this determination. Extending the usual expression[24] in an obvious way gives

$$\frac{d\sigma}{dt}\bigg|_{t=0}(\gamma A\to\rho^0A)=\frac{\alpha}{64\pi}\left(\frac{\gamma_\rho^2}{4\pi}\right)^{-1}\sigma_T^2(\rho^0A)\left|1+\frac{\gamma_\rho}{\gamma_{\rho'}}\frac{F_{\rho'\rho}}{F_{\rho\rho}}\right|^2, \quad (4.1)$$

where $F_{\rho(\rho')\rho}$ is the forward ($t=0$) amplitude for the process $\rho(\rho')A\to\rho A$, with A representing a nucleus of mass number A, and $\gamma_{\rho'}\equiv m_{\rho'}^2/2F_{\rho'}$. Note that the process $\rho'A\to\rho A$ is quasielastic in the sense that it can proceed diffractively with no quantum number exchange, just as for $\rho A\to\rho A$. The usual assumption that the real part of the amplitude $F_{\rho\rho}$ is negligible has already been assumed in writing Eq. (4.1) and, if we make the same assumption for $F_{\rho'\rho}$, we see that, rather than measuring $\gamma_\rho^2/4\pi$, the photoproduction experiments are actually determining the combination

$$\left(1+\frac{\gamma_\rho}{\gamma_{\rho'}}\frac{F_{\rho'\rho}}{F_{\rho\rho}}\right)^{-2}\left(\frac{\gamma_\rho^2}{4\pi}\right). \quad (4.2)$$

In our previous analysis we found $\gamma_\rho/\gamma_{\rho'}\simeq-0.15\pm0.01$; to form an initial crude estimate of the effect of the first factor in (4.2), it seems reasonable to assume $F_{\rho'\rho}\simeq F_{\rho\rho}$. If we do this, the Cornell[22] and SLAC[23] results then imply

$$\gamma_\rho^2/4\pi\simeq(0.85)^2(1.1\pm0.2)\simeq0.80\pm0.15, \quad (4.3)$$

to be compared with the colliding-beam value[10,12]

$$(\gamma_\rho^2/4\pi)_{\text{colliding beam}}\simeq0.46\pm0.05. \quad (4.4)$$

The discrepancy is thus reduced from 0.64 ± 0.21 to about 0.34 ± 0.16, a considerable improvement.[25] For $m_{\rho'}=1.75$, we find $\gamma_\rho/\gamma_{\rho'}=-0.17$, and the Cornell and SLAC results imply $\gamma_\rho^2/4\pi\simeq0.75\pm0.14$, reducing the discrepancy still further to 0.29 ± 0.15. Another way of expressing this is to evaluate (4.1) using the colliding-beam value for $\gamma_\rho^2/4\pi$, $\gamma_\rho/\gamma_{\rho'}=-0.15$ (-0.17), and $\sigma_T(\rho p)=30$ mb. Then (4.1) gives[26]

$$d\sigma/dt|_{t=0}(\gamma p\to\rho p)\simeq132\ \mu\text{b GeV}^{-2}\ [125\ \mu\text{b GeV}^{-2}]; \quad (4.5)$$

without the ρ' contribution, one finds this cross section to be $\simeq182$ μb GeV^{-2}. Another application of vector dominance to the photoproduction process is the calculation of the total γ-p cross section[27]:

$$\sigma_T(\gamma p)=\sum_V\left[4\pi\alpha\left(\frac{\gamma_V^2}{4\pi}\right)^{-1}\frac{d\sigma}{dt}\bigg|_{t=0}(\gamma p\to Vp)\right]^{1/2}, \quad (4.6)$$

where we have assumed that the real parts of all relevant scattering amplitudes are negligible. If we extend the assumption made previously on the equality of $F_{\rho\rho}$ and $F_{\rho'\rho}$ to include $F_{\rho\rho'}$ and $F_{\rho'\rho'}$, the $\rho+\rho'$ contribution to (4.6) can be written in the form

$$\left(1-\frac{\gamma_\rho^2}{\gamma_{\rho'}^2}\right)\frac{\alpha}{4}\left(\frac{\gamma_\rho^2}{4\pi}\right)^{-1}\sigma_T(\rho p), \quad (4.7)$$

which only differs from the usual ρ contribution by the factor $1-\gamma_\rho^2/\gamma_{\rho'}^2$. To illustrate the effect of including the ρ' contribution in this expression, using the values γ_ρ, $\gamma_{\rho'}$, and $\sigma_T(\rho p)$ used previously to evaluate (4.1), we find the $\rho+\rho'$ contribution to $\sigma_T(\gamma p)\simeq116$ μb. The ρ contribution alone is 119 μb.[28] Under the same assumptions, the forward differential cross section for $\gamma p\to\rho' p$ is predicted to be the same as (4.5).

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[18] Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).

[19] The difficulty here may lie in the momentum dependence of the vertices which has an effect (Ref. 20) on the calculation of the phase-space factor in the Gell-Mann–Sharp–Wagner model (Ref. 21).

[20] S. G. Brown and G. B. West, Phys. Rev. **174**, 1777 (1968).

[21] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).

[22] G. McClellan *et al.*, Phys. Rev. Letters **22**, 374 (1969).

[23] F. Bulos *et al.*, Phys. Rev. Letters **22**, 490 (1969).

[24] See the review by S. C. C. Ting, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 43.

[25] For reactions induced by photons of lower incident energy, the assumption $F_{\rho'\rho}\simeq F_{\rho\rho}$ may be less reasonable, and the part played by the ρ' in the production process would be expected to decrease. This would be consistent with the result of J. G. Asbury *et al.*, Phys. Rev. Letters **19**, 865 (1967); **19**, 869 (1967).

[26] Experimentally $d\sigma/dt|_{t=0}(\gamma p\to\rho p)$ seems to be (Ref. 22) approximately constant and equal to 130 μb GeV^{-2} for photon energies between 4 and 9 GeV. Some of the measured values of $\sigma_T(\rho p)$ are 30_{-4}^{+6} mb (Ref. 23), 38 ± 3 mb (Ref. 22), and 31.3 ± 2.3 mb (Ref. 25).

[27] L. Stodolsky, Phys. Rev. Letters **18**, 135 (1967).

[28] J. Ballam *et al.* [Phys. Rev. Letters **21**, 1544 (1968)] find $\sigma_T(\gamma p)=126\pm17$ μb for a photon energy of 7.5 GeV.

Brodsky and Pumplin[29] have recently taken the point of view that the value $\gamma_\rho^2/4\pi \simeq 1.1$ may be the actual value of the ρ-photon coupling on the photon mass shell. Since this then implies a ρ contribution to $\sigma_T(\gamma p)$ of only $\simeq 50\ \mu$b, they are led to consider the contribution of additional states in the photon channel. However, the possible contribution of these additional states on the original analysis leading to $\gamma_\rho^2/4\pi \simeq 1.1$ does not seem to have been taken into account.

V. FORM FACTORS

One of the more interesting conclusions that we can draw from this analysis is that the pion form factor $F_\pi(t)$ almost certainly does not fall off as fast as $F_1{}^V(t)$, although their respective charge radii are not very different. For, if we define r_π and r_{1V} by

$$F_\pi'(0) = \tfrac{1}{6} r_\pi{}^2$$

and

$$F_1{}^{V\prime}(0) = \tfrac{1}{6} r_{1V}{}^2 ,$$

then, for $m_{\rho'} = 2.0$ (1.75) GeV, we obtain $r_\pi = 0.66$ (0.65) F and $r_{1V} = 0.67$ (0.69) F. The difference in t behavior in the case of $m_{\rho'} = 1.75$ should begin to show up at $t \approx -1$ (GeV/c)^2, where we would expect $F_\pi(t)$ to be $\approx 20\%$ greater than $F_1{}^V(t)$. For $m_{\rho'} = 2.0$ GeV, however, the deviation of $F_\pi(t)$ from $F_1(t)$ would not show up until higher t values are obtained. Figure 3 shows $F_\pi(t)$ for $m_{\rho'} = 2.0$ GeV.

The proton charge radius r_{Ep} can be obtained immediately from Eq. (1.5) using $F_2{}^V(0) = \mu_p - \mu_n - 1$, the experimental[30] value of $G_E{}^{n\prime}(0) = -0.0193$ F^2 and our value of $F_1{}^{V\prime}(0) = m_\rho{}^{-2} + m_{\rho'}{}^{-2}$. For $m_{\rho'} = 2.0$ (1.75) GeV, this leads to $r_{Ep} = 0.76$ (0.77) F, to be compared with the experimental value[31] of 0.81 ± 0.01 F. The alternative method of calculating r_{Ep} using Eq. (1.7) will introduce errors arising from the neglect of $G_E{}^n(t)$ and leads to $r_{Ep} = 0.84$ (0.85) F.

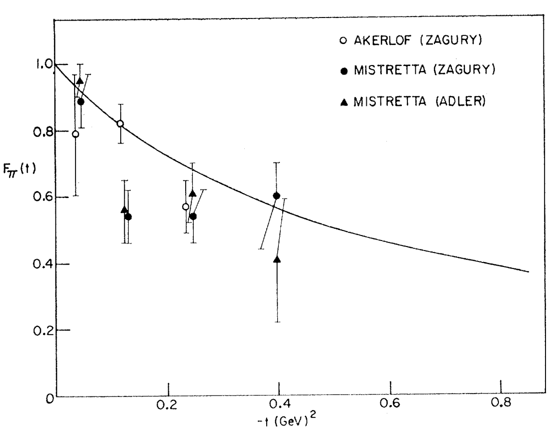

FIG. 3. $F_\pi(t)$ calculated for $m_{\rho'} = 2.0$ GeV. The coupling constants used were $\gamma_{\rho\pi\pi} = 5.354$, $\gamma_{\rho'\pi\pi} = 3.598$, $F_\rho = 1.0225$ GeV^2, and $F_{\rho'} = -0.1184$ GeV^2. The curve corresponding to $m_{\rho'} = 1.75$ is barely distinguishable in this region from the curve shown. The data are taken from C. W. Akerlof *et al.*, Phys. Rev. **163**, 1482 (1967), and from C. Mistretta, Phys. Rev. Letters **20**, 1523 (1968). The latter data have been analyzed with both the Zagury and the Adler theories of electroproduction, giving the two sets of points shown.

The determination of $\gamma_{\rho NN}$ has usually been obtained in one of two different ways, either by assuming "universality," $\gamma_{\rho NN} = \gamma_{\rho\pi\pi}$, or by fitting $F_1{}^V$ to the nucleon form-factor data for small values of t, in which case $\gamma_{\rho NN}/\gamma_{\rho\pi\pi} \approx 1.6$.[32] The value taken for $\gamma_{\rho NN}$ has a large effect on some finite-energy sum rules and, in particular, the $u=0$ $\pi - N$ superconvergence relations proposed by some authors.[33] For the latter relations, it is generally found necessary[34] to use $\gamma_{\rho NN}/\gamma_{\rho\pi\pi} = 1.6$ in order to obtain agreement. In our model, which fits $F_1{}^V(t)$ well, we find $\gamma_{\rho NN}/\gamma_{\rho\pi\pi} = 1.07$ (1.16) for $m_{\rho'} = 2.0$ (1.75). The ρ' contribution must be included as well, however, and almost exactly compensates for the reduced ρNN coupling. We illustrate this in the case of the superconvergence relation for the $B^{(1/2)}$ amplitude in π-N scattering at fixed $u=0$. The superconvergences relation in this case reads, at $u=0$,

$$\int_0^\infty ds' \,\mathrm{Im} B^{(1/2)}(s',0) - \int_0^\infty dt' \,\mathrm{Im} B^{(1/2)}(t',0) = 0 .$$

With ρ and ρ' assumed to dominate the t-channel contributions to the sum rule and the N and Δ taken[35] to saturate the s-channel integral, we find, in the narrow-width approximation, that

$$-2\mu_V\left(\frac{\gamma_{\rho\pi\pi}\gamma_{\rho NN}}{4\pi} + \frac{\gamma_{\rho'\pi\pi}\gamma_{\rho' NN}}{4\pi}\right) + \frac{g_{\pi NN}{}^2}{4\pi} - \frac{4}{3}\frac{g_{\Delta N\pi}{}^2}{4\pi}\frac{B^*}{m_\pi{}^2} \simeq 0 ,$$

where $\mu_V = \mu_p - \mu_n = 4.7$, and we have assumed $\gamma_{\rho NN}{}^{(T)} = (\mu_V - 1)\gamma_{\rho NN}/2M$. This follows with the assumption of $\rho + \rho'$ dominance of $F_2{}^V$. Using $g_{\pi NN}{}^2/4\pi \simeq 14.6$, $g_{\Delta N\pi}{}^2/4\pi = 0.36$, and $B^* = -0.667 M_\Delta{}^2$, we find that the s channel contributes 40.4, whereas the t-channel contribution is 38.2 (made up with 22.8 from the ρ part and 15.4 from the ρ'). The above numbers refer to the ρ' having a mass of 2.0 GeV. For $m_{\rho'} = 1.75$ GeV, the ρ

[29] S. J. Brodsky and J. Pumplin, Phys. Rev. **182**, 1794 (1969). We received their paper after a preliminary version of the present paper was circulated.

[30] V. E. Krohn and G. R. Ringo, Phys. Rev. **148**, 1303 (1966).

[31] L. H. Chan *et al.*, Phys. Rev. **141**, 1298 (1966).

[32] T. D. Spearman, Phys. Rev. **129**, 1847 (1963).

[33] D. Beder and J. Finkelstein, Phys. Rev. **160**, 1363 (1967); D. Griffiths and W. Palmer, *ibid.* **161**, 1606 (1967); R. Ramachandran, *ibid.* **166**, 1528 (1968); R. F. Amann, Nuovo Cimento **56A**, 1125 (1968); see also C. B. Chiu and M. Der Sarkissian, *ibid.* **55A**, 396 (1968).

[34] The work of Ramachandran (Ref. 33) appears to imply that universality is sufficient to satisfy the sum rules. However, there appears to be an inconsistency of a factor of 2 in the normalization of his ρ contribution.

[35] Contributions from higher π-N resonances do not appear to alter the conclusions below. See Ref. 33 for details.

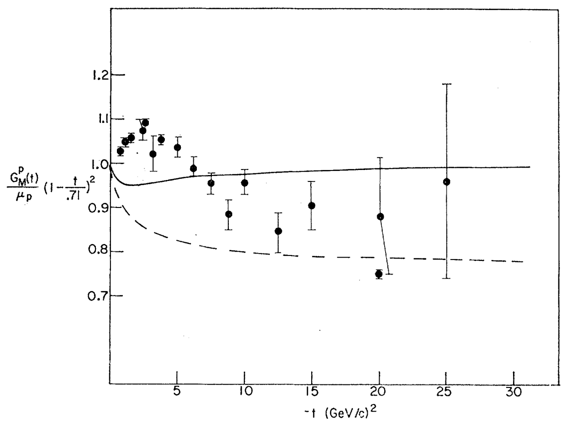

FIG. 4. Solid line is a plot of the predicted $G_M{}^p(t)/\mu_p$ using $m_{\rho'}=2.0$ GeV in Eq. (1.7) and is shown as a ratio to the empirical dipole $[1-t/(0.71\text{ GeV}^2)]^{-2}$. The dashed line is the corresponding curve in the $m_{\rho'}=1.75$ GeV case, and the experimental points are taken from Coward *et al.* (Ref. 1).

contribution is 23.2 and the ρ' contribution is 10. A similar situation holds for the $B^{(3/2)}$ sum rule.

In the expression we obtained for $\gamma_{\rho NN}{}^{(T)}$, we assumed $\rho+\rho'$ dominance for $F_2{}^V(t)$ as well as for $F_1{}^V(t)$. We note that this form for $F_2{}^V$ does not fall off as rapidly with increasing $|t|$ as the data require. This is evident in calculations of the slope of $F_2{}^V(0)$, where the predicted value of $r_{2V}=1.29$ (1.32) F is to be compared with the experimental value[31] $r_{2V}\simeq 1.60\pm 0.02$ F. This may indicate that both width effects and background structure in the $N\bar{N}\rightarrow\pi\pi$ amplitude are more important for $F_2{}^V$ than for $F_1{}^V$. However, we can avoid this problem in calculating $G_M{}^p(t)/\mu_p$ by making use of the scaling laws (1.2)–(1.4) to express $G_M{}^p/\mu_p$ solely in terms of $F_1{}^V$ as in Eq. (1.7). In Fig. 4, we plot $G_M{}^p(t)/\mu_p$ as a function of t for the two values of $m_{\rho'}$ that we considered above, relative to the empirical dipole fit $G_M{}^p(t)/\mu_p=(1-t/0.71)^{-2}$. This method of presentation clearly shows deviations of the data from theory that would go unnoticed in plots on a logarithmic scale. The discrepancy for $-t\lesssim 5$ $(\text{GeV}/c)^2$ may be due to using zero-width approximations and neglecting $G_E{}^n(t)$. For example, we can construct a functional form for $G_E{}^n(t)$ similar to Eq. (1.7), but with the factor $1-\tau$ replaced by $A\tau$, where A is chosen to reproduce the known slope $G_E{}^{p\prime}(0)$. This is consistent with the known measurements of $G_E{}^n(t)$ for $-t>0$. With a nonzero $G_E{}^n(t)$, Eq. (1.7) must be modified to read

$$G_M{}^p(t)/\mu_p=\frac{(1-\tau)F_1{}^V(t)}{(1-\mu_V\tau)}+\frac{G_E{}^n(t)}{(1-\mu_V\tau)}. \qquad (5.1)$$

It turns out that with a $G_E{}^n(t)$ of the form $A\tau F_1{}^V(t)/(1-\mu_V\tau)$, the best representation of the data over the whole region is given by having a ρ' of mass $m_{\rho'}\simeq 1.88$ GeV. Figure 5 shows the corresponding $G_E{}^n(t)$, and Fig. 6 shows $G_M{}^p(t)/\mu_p$ including the $G_E{}^n(t)$ term.

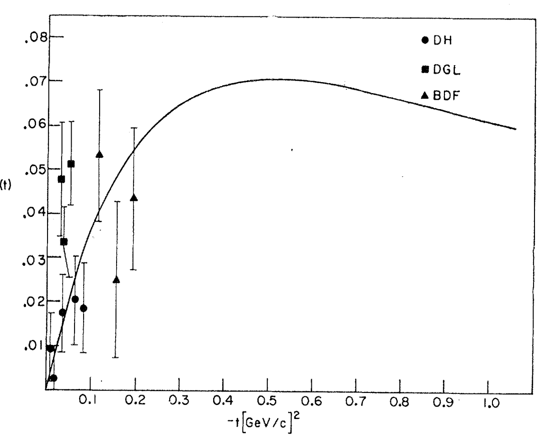

FIG. 5. Curve shown is $A\tau F_1{}^V(t)/(1-\mu_V\tau)$, where $\tau=t/4M^2$, $\mu_V=\mu_p-\mu_n\simeq 4.7$, and $A=0.495$ GeV^{-2} is chosen to reproduce the known slope of $G_E{}^n(t)$ at $t=0$. This form is suggested in the text as a possible parametrization of $G_E{}^n(t)$ and is plotted for the case $m_{\rho'}=1.88$ GeV. The data are basically from D. Drickey and L. Hand, Phys. Rev. Letters **9**, 521 (1962); D. J. Drickey, B. Grossetête, and P. Lehmann, in *Proceedings of the Sienna International Conference on Elementary Particles and High-Energy Physics, 1963*, edited by G. Bernardini and G. Puppi (Societa Italiana di Fisica, Bologna, 1963); D. Benaksas, D. Drickey, and D. Frèrejacque, Phys. Rev. **148**, 1327 (1966). However, relativistic corrections to elastic electron-deutron scattering [see B. M. Casper and F. Gross, Phys. Rev. **155**, 1607 (1967)] have been applied to obtain the actual points shown, which we have taken from Fig. 4 of the review by W. Panofsky, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 23.

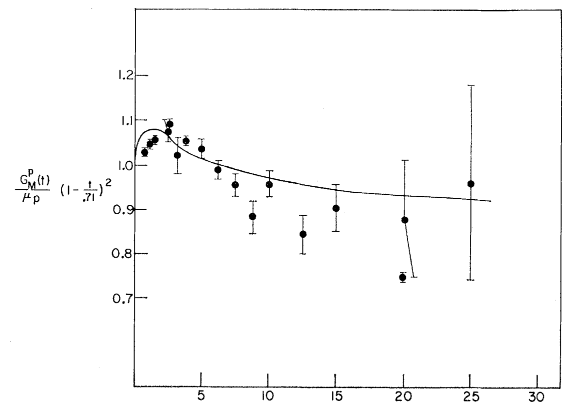

FIG. 6. Shows the effect of including a nonzero $G_E{}^n(t)$ in the calculation of $G_M{}^p(t)/\mu_p$. The details will, of course, depend on the particular parametrization of $G_E{}^n(t)$ chosen; here we have taken $m_{\rho'}=1.88$ GeV and used the $G_E{}^n(t)$ given in Fig. 5. It will be seen that the shape of the experimental curve is at least qualitatively reproduced by including $G_E{}^n(t)$ (compare with Fig. 4). The data points are the same as in Fig. 4.