With this prescription, the covariant spin-projection operator for particles with spin 1, $\frac{3}{2}$, and 2 are given by the following expressions, respectively:

$$\sum_i \epsilon_\mu{}^i(k)\epsilon_\nu{}^i(k)=\delta_{\mu\nu}-k_\mu k_\nu/k^2, \quad \text{(A2)}$$

$$\begin{aligned}\sum_r u_\mu{}^{(r)}(p)\bar{u}_\nu{}^{(r)}(p) &=[\tfrac{2}{3}(\delta_{\mu\nu}-p_\mu p_\nu/p^2)+(\not{p}/3p^2)\epsilon_{\mu\nu\lambda\sigma}\gamma_5\gamma_\lambda p_\sigma] \\ &\quad\times(\not{p}+iM)/2iM \\ &=[(\not{p}+iM)/2iM][\tfrac{2}{3}(\delta_{\mu\nu}-p_\mu p_\nu/p^2) \\ &\quad+(\not{p}/3p^2)\epsilon_{\mu\nu\lambda\sigma}\gamma_5\gamma_\lambda p_\sigma], \quad \text{(A3)}\end{aligned}$$

$$\sum_i \epsilon_{\mu\nu}{}^i(k)\epsilon_{\mu'\nu'}{}^i(k)=\tfrac{1}{2}(P_{\mu\mu'}P_{\nu\nu'}+P_{\mu\nu'}P_{\nu\mu'})-\tfrac{1}{3}P_{\mu\nu}P_{\mu'\nu'}, \quad \text{(A4)}$$

where

$$P_{\mu\nu}\equiv\delta_{\mu\nu}-k_\mu k_\nu/k^2.$$

It may be easily verified that the above spin-$\frac{3}{2}$ projection operator is the same as that given in Ref. 32:

$$\begin{aligned}\sum_r u_\mu{}^{(r)}(p)\bar{u}_\nu{}^{(r)}(p) &=[\delta_{\mu\nu}-\tfrac{1}{3}\gamma_\mu\gamma_\nu-(1/3p^2)(\not{p}\gamma_\mu p_\nu+p_\mu\gamma_\nu\not{p})] \\ &\quad\times(\not{p}+iM)/2iM \\ &=[(\not{p}+iM)/2iM][\delta_{\mu\nu}-\tfrac{1}{3}\gamma_\mu\gamma_\nu \\ &\quad-(1/3p^2)(\not{p}\gamma_\mu p_\nu+p_\mu\gamma_\nu\not{p})]. \quad \text{(A5)}\end{aligned}$$

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Failure of Soft-Pion Techniques for the Reaction $pp\to d\pi^+$ at Threshold*

M. E. Schillaci and R. R. Silbar
University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544
(Received 19 May 1969)

The reaction $pp\to d\pi^+$ at threshold is studied using the low-energy theorem of Adler and Dothan. Predictions are found to be in serious disagreement with experimental results, both in the soft-pion limit and for physical pion mass. The inapplicability of the low-energy theorem for this process is traced to the rapid decrease of the deuteron vertex as a function of pion mass. Internal emission diagrams of the one-pion-exchange type, which vary much less rapidly with pion mass, are likely to dominate in the limit of physical pion mass. A similar analysis applies to the processes $K^-d\to\Lambda n,\Sigma^0 n$ for kaons at rest, where the soft-kaon result disagrees with experiment even more seriously than above.

I. INTRODUCTION

It has only been recently that one could calculate the emission of soft pions in nucleon-nucleon collisions in a model-independent way. In particular, the "unbound" process $NN\to NN\pi$ has been extensively investigated, both at threshold,[1–3] where the agreement with available data is good, and at intermediate energies,[4] where data will soon become available. The basis of these calculations is the use of the Adler-Dothan low-energy theorem,[5,6] which gives a prescription for finding the leading behavior of the production amplitude when expanded in terms of pion four-momentum q_μ.

One would naturally expect[2] that these techniques could be applied to the bound process $pp\to d\pi^+$, at least insofar as the s-wave contribution near threshold is concerned. Furthermore, since the dnp vertex is to be evaluated at a fairly large momentum transfer (≈ 2 fm^{-1}), one might expect to obtain additional information about the deuteron wave function (and thus about nucleon-nucleon potentials). Unfortunately this is not the case. We find that the extrapolation from the unphysical, zero-pion-mass amplitude, for which the Adler-Dothan theorem holds, to the amplitude for physical, massive pions is far from "smooth." As a result, the soft-pion prediction is quite a bit smaller than experiment, indicating that contributions to the amplitude which the Adler-Dothan theorem says are small at zero pion mass are in fact dominant at the physical mass. The source of this rapid variation as the pion mass becomes physical can be pinpointed and the success of earlier, more complicated calculations,[7] which invoke dynamical mechanisms not prescribed by the low-energy theorem, can be better understood.

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] M. E. Schillaci, R. R. Silbar, and J. E. Young, Phys. Rev. Letters **21**, 711 (1968); **21**, 1030(E) (1968); Phys. Rev. **179**, 1539 (1969).

[2] D. S. Beder, Nuovo Cimento **56A**, 625 (1968).

[3] R. Baier and H. Kühnelt, Nuovo Cimento **63A**, 135 (1969).

[4] C. T. Grant, M. E. Schillaci, and R. R. Silbar, Phys. Rev. **184**, 1737 (1969).

[5] S. Weinberg, Phys. Rev. Letters **16**, 879 (1966).

[6] S. L. Adler and Y. Dothan, Phys. Rev. **151**, 1267 (1966).

[7] See D. S. Koltun and A. Reitan, Phys. Rev. **141**, 1413 (1966), and references therein. Reitan [Arkiv for Det Fysiske Seminar i Trondheim Report No. 2, 1969 (unpublished); Nucl. Phys. **B11**, 170 (1969)] has recently recalculated the s-wave contribution and comments on a similar calculation by Il-T. Cheon and A. Tohsaki, *ibid.* **B6**, 585 (1968).

II. SOFT-PION CALCULATION

In accordance with the Adler-Dothan theorem,[5,6] we consider the nucleon exchange graph Fig. 1(a), with pseudovector pion-nucleon coupling, as the dominant contribution to the $pp\to d\pi^+$ amplitude at threshold. By the theorem graphs such as Figs. (1b) and 1(c), "internal-emission" graphs, contribute only in higher order in the pion mass and can be neglected to $O(\mu/m)$. Evaluating the nucleon exchange graph, we have[8]

$$T_{12}=\bar{v}_2\Gamma_d[\gamma\cdot(p_1-q)-m]^{-1}[(\sqrt{2}g/2m)\gamma\cdot q\gamma_5]u_1. \quad (1)$$

The deuteron vertex Γ_d is given by[9]

$$\begin{aligned}\Gamma_d=\xi_\nu^*(d,\lambda)\{&\gamma^\nu F_1(t)-\kappa^\nu F_2(t)\\&+[\gamma^\nu F_3(t)-\kappa^\nu F_4(t)][\gamma\cdot(p_1-q)-m]/m\}\\ \equiv\xi_\nu^*(d,\lambda)\{&F^\nu(t)+G^\nu(t)[\gamma\cdot(p_1-q)-m]/m\},\end{aligned} \quad (2)$$

where $\kappa_\nu=(p_1-q-p_2)_\nu/2m$ and $\xi_\nu(d,\lambda)$ is the deuteron polarization four-vector with helicity λ. The invariant deuteron vertex functions $F_i(t)$, $i=1,\cdots,4$, depend on the momentum transfer $t=(p_1-q)^2=(p_2-d)^2$ carried across by the virtual neutron.

Specializing to threshold kinematics in the c.m. system,

$$\begin{aligned}p_{1,2}=(p_0,\pm\mathbf{p}),\quad q=(\mu,\mathbf{0}),\quad d=(M,\mathbf{0}),\\ p_0=m+\tfrac{1}{2}\mu,\quad |\mathbf{p}|^2=\mu m,\end{aligned} \quad (3)$$

and carrying out some Dirac algebra, we obtain

$$\begin{aligned}T_{12}=-(\sqrt{2}g/2m)\xi_\nu^*\bar{v}_2[F^\nu(t)(1+\gamma_0)\\ -(\mu/m)G^\nu(t)\gamma_0]\gamma_5u_1.\end{aligned} \quad (4)$$

We see that the term with $G^\nu(t)$ is of higher order in μ/m, and thus, to lowest order, only the first two invariant functions, $F_1(t)$ and $F_2(t)$, contribute to the deuteron vertex. The full amplitude is obtained by antisymmetrizing in the spin and momentum indices of the initial protons. Using the fact that $\xi_0=0$ (since $\mathbf{d}=0$), it is easily shown that the second invariant function $F_2(t)$ does not survive the antisymmetrization procedure. Thus the full amplitude is given by

$$T=-(\sqrt{2}g/m)F_1(t)\bar{v}_2\gamma\cdot\xi\gamma_5u_1. \quad (5)$$

Squaring and averaging over spins, the production cross section is found to be

$$\sigma=4\pi\frac{q}{p}\frac{\langle|T|^2\rangle}{(8\pi W)^2}=\frac{1}{2}\left(\frac{g^2}{4\pi}\right)\left(\frac{\mu}{m}\right)^{3/2}\frac{F_1^2(t)}{m^2}\eta, \quad (6)$$

where W is the total c.m. energy, p and q are the initial and final c.m. three-momenta, $\eta=q/\mu$, and $g^2/4\pi\cong14.6$. As stated above, only the lowest order terms in μ/m have been retained in Eq. (6).

[8] We use the metric and gamma-matrix convention used by J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill Book Co., New York, 1964). μ, m, and M are the pion, nucleon, and deuteron masses, respectively.

[9] M. Gourdin, M. le Bellac, F. M. Renard, and J. Tran Thanh Van, Nuovo Cimento **37**, 524 (1965).

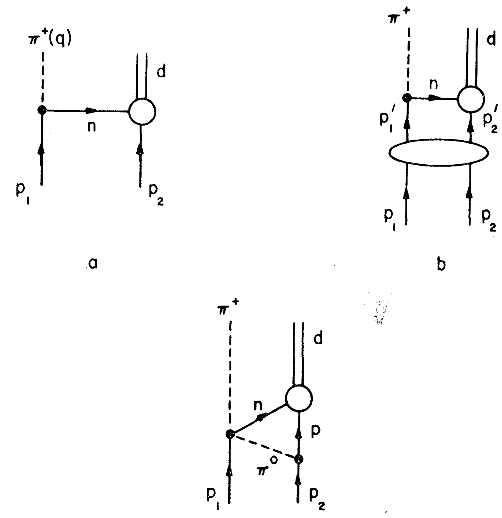

FIG. 1. (a) One-nucleon-exchange (ONE) graph; (b) DWBA graph; and (c) pion rescattering (or triangle) graph.

The invariant function $F_1(t)$ can be related to the momentum space wave functions of the deuteron.[9] We find

$$F_1(t)\approx(8\pi/m)^{1/2}(p^2+\alpha^2)[\tilde{u}(p)+\tilde{w}(p)/\sqrt{2}], \quad (7)$$

where the symbol $\approx$ indicates a nonrelativistic limit and, again, a neglect of higher orders in μ. Here p is related to t by $t=M^2+m^2-2M(p^2+m^2)^{1/2}$ and $\alpha^2=Bm=45.7\ \text{MeV}^2$, with B the deuteron binding energy. $\tilde{u}(p)$ and $\tilde{w}(p)$ are given by Bessel transforms of the s- and d-wave deuteron radial wave functions, namely,

$$\tilde{u}(p)=\int_0^\infty u(r)j_0(pr)rdr, \quad (8a)$$

$$\tilde{w}(p)=-\int_0^\infty w(r)j_2(pr)rdr. \quad (8b)$$

III. RESULTS AND DISCUSSION

Experimentally, the data for the reaction $pp\to d\pi^+$ (and its inverse, $\pi^+d\to pp$) in the vicinity of threshold can be well fitted by an expansion[10]

$$\sigma=a\eta+b\eta^3, \quad (9)$$

where the two terms correspond to s- and p-wave pions. The most recent data, closest to threshold, including Coulomb corrections, yield[11]

$$a_{\text{expt}}=240\pm20\ \mu\text{b},\quad b_{\text{expt}}=520\pm200\ \mu\text{b}. \quad (10)$$

Our soft-pion result, Eq. (6), predicts only the number a. [This restriction to s-wave production resulted in part from our setting $\mathbf{q}=0$ in Eq. (4).] The value of a

[10] M. Gell-Mann and K. M. Watson, Ann. Rev. Nucl. Sci. **4**, 219 (1954); A. H. Rosenfeld, Phys. Rev. **96**, 139 (1954).

[11] C. M. Rose, Phys. Rev. **154**, 1305 (1967).

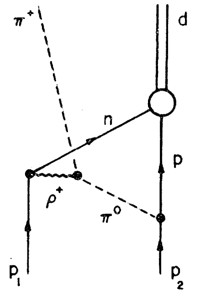

FIG. 2. Pion rescattering graph with ρ exchange.

predicted by Eq. (6) depends critically upon t through $F_1(t)$.

A first attempt[2] might be to set the virtual neutron on its mass shell, i.e., $t=m^2$. This would be the case if the pion were truly massless. In this way, using the normalizations of Ref. 9, we would have

$$a(t=m^2)=800\,\mu\text{b}\,, \qquad (11)$$

which is about 3.3 times larger than the experimental result.[12]

It is perhaps more in the spirit of what we have done in deriving Eq. (6) (i.e., keeping μ finite at every step) to evaluate a at $t=t_0$ appropriate to threshold:

$$t_0=(p_1-q)^2=m^2+\mu^2-2\mu p_0=m^2(1-2\mu/m)\,. \qquad (12)$$

To do this involves knowing the deuteron wave functions $\tilde{u}(p)$ and $\tilde{w}(p)$. We have evaluated $a(t_0)$ using a number of published and unpublished wave functions.[13–17] The results are summarized in Table I.

In general, the values of a predicted in Table I are too low by a factor of 3 to 10, but the variations among the various wave functions are quite large. This is because the relative momentum of the nucleons here probes the deuteron wave function to a distance of $\approx$0.5 fm, a region where there are considerable differences among the various wave functions used. Note also that at this relative momentum the contribution of the s wave is quite small and subtracts from that of the d wave. This is because $\tilde{u}(p)$ passes rapidly through zero at a value of p slightly larger than that corresponding to threshold.

In any case, the predictions of the cross section are much too small. What is worse, in going from the "zero-mass-pion" result given in Eq. (11) to those in Table I, the cross section changes by a factor of $\sim$20. This clearly indicates that the unphysical zero-mass production amplitude is not a good approximation for the physical amplitude; the soft-pion result cannot be extrapolated safely in this reaction. In particular, one cannot expect to learn much about the deuteron wave function in this way.

It is informative to compare this soft-pion calculation with the more ambitious work of Koltun and Reitan.[7] These authors included the effect of the nuclear potential on the initial two-proton state [distorted-wave Born approximation (DWBA)], which means internal emission graphs such as Fig. 1(b) were also considered. This DWBA is given by the sum of integrals I_1+I_2, where $I_{1,2}$ are defined by Eq. (13) of Ref. 7. It is easily seen that, by replacing the distorted $^3P_1$ radial wave function by the (free wave) spherical Bessel function $j_1(pr)$, and using recurrence relations for $j_l(pr)$, $I_1\to\tilde{u}(p)$, $I_2\to\tilde{w}(p)/\sqrt{2}$, thus recovering the combination $\tilde{u}(p)+\tilde{w}(p)/\sqrt{2}$ occurring in Eq. (7).

For Koltun and Reitan, I_1 and I_2 are nearly equal and opposite. They find that when the pion is emitted from one nucleon (and does not interact with the other), the contributions of the s- and d-wave parts of the deuteron strongly cancel. This is in contrast to the behavior exhibited in Table I, where the d-wave contribution was dominant. The effect of distortion (which, by the Adler-Dothan theorem, is ignored in Table I) evidently shifts the zero of the $\tilde{u}(p)$ (i.e., I_1) to a somewhat larger value of p than for the free-wave cases.

Most of the production cross section, according to Koltun and Reitan, actually comes from graphs like Fig. 1(c), where the pion is emitted from one nucleon and scatters off the other.[18,19] The πN scattering here must be s-wave and Koltun and Reitan assumed a phenomenological Hamiltonian in which the parameters were fitted to the experimental πN s-wave scattering lengths. In the light of recent current algebra results for s-wave πN scattering,[20] most of the contribution

TABLE I. Summary of results.

Wave function	$\tilde{u}(p_0)$ (fm$^{3/2}$)	$\tilde{w}(p_0)$ (fm$^{3/2}$)	$\lvert F_1(t_0)\rvert$	a (μb)
Moravcsik-Gartenhaus[a]	+0.030	−0.111	0.383	27
Hamada-Johnston[b]	+0.012	−0.133	0.655	80
Bressel-Kerman-Rouben[c]	+0.022	−0.108	0.431	34
Reid (soft core)[d]	+0.017	−0.109	0.479	43
Reid (hard core)[d]	+0.010	−0.108	0.527	52
Lomon-Feshbach[e]:				
(4.60% d state)	+0.009	−0.090	0.439	36
(5.20% d state)	+0.007	−0.097	0.487	44
(6.55% d state)	+0.002	−0.107	0.591	65

[a] Reference 13. [b] Reference 14. [c] Reference 15. [d] Reference 16. [e] Reference 17.

[12] The factor of 2 difference between this result and that of Beder (Ref. 2) is apparently due to a difference of $\sqrt{2}$ in the definitions of the deuteron vertex by C. Goebel and B. Sakita [Phys. Rev. **127**, 1787 (1962)] (used by Beder) and by R. Blankenbecler, M. L. Goldberger, and F. R. Halpern [Nucl. Phys. **12**, 629 (1959)] (used here).

[13] S. Gartenhaus, Phys. Rev. **100**, 900 (1956); M. J. Moravcsik, Nucl. Phys. **7**, 113 (1958).

[14] T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962).

[15] C. N. Bressel, A. K. Kerman, and B. Rouben, Nucl. Phys. **A124**, 624 (1969).

[16] R. V. Reid, Ann. Phys. (N. Y.) **50**, 411 (1968).

[17] E. L. Lomon and H. Feshbach, Ann. Phys. (N. Y.) 48, 94 (1968); and private communication.

[18] The same triangle graph was considered for this process at higher energies by T. Yao, Phys. Rev. **134**, B454 (1964).

[19] This graph also contributes an anomalous threshold; see, e.g., Huan Lee, Phys. Rev. **174**, 2130 (1968).

[20] See, e.g., J. J. Sakurai, *Currents and Mesons* (University of Chicago Press, Chicago, 1969).

of Koltun and Reitan's rescattering graph [Fig. 1(c)] can be interpreted as the ρ-exchange process depicted in Fig. 2. One can see the qualitative difference between this emission process and that of Fig. 1(b), where the initial proton-proton interaction might be thought of as proceeding through, say, pion exchange.

What Koltun and Reitan have done, essentially, is to propose the graph of Fig. 2 as a model for internal-emission contributions. Moreover, they find that this internal emission is dominant, in spite of what one might have expected from the Adler-Dothan theorem. The resolution of this lies in the nonsmooth extrapolation to physical pion mass, particularly through the strong dependence of the deuteron vertex function $F_1(t)$ as t varies from m^2 to $t_0 \cong 0.7m^2$. For the nucleon exchange graph, Fig. 1(a), all of this momentum transfer must be absorbed in the dnp vertex. On the other hand, the triangle graph of Fig. 1(c) can transfer the necessary momentum and energy through the exchanged pion, allowing the nucleon legs of the dnp vertex to be far less off-shell.[18] Thus, while the triangle graph is, by the Adler-Dothan theorem, small compared to the nucleon-exchange graph at nonphysical zero pion mass, in the extrapolation to finite pion mass it does not damp out nearly so rapidly as the nucleon exchange graph. At some point, it does indeed become the more important contribution.[21]

Another example of the same phenomenon can be seen in the reactions

$$\begin{aligned} K^-d &\to \Lambda n\,, \\ K^-d &\to \Sigma^0 n\,, \\ K^-d &\to \Sigma^- p\,, \end{aligned} \tag{13}$$

where the kaon is at rest. A recent experiment[22] gives branching ratios for these reactions in the ratio $\Lambda n:\Sigma^0 n:\Sigma^- p = 0.37:0.38:0.51\%$ with errors of 0.04%. The last ratio is inconsistent with the isospin prediction by ~ 4 standard deviations. Apart from that, however, if one were to naively apply the Adler-Dothan theorem for kaons[23] [assuming the hypothesis of partially conserved axial-vector current (PCAC) for the strangeness-changing axial-vector current], then the $\Lambda:\Sigma$ ratio would be very difficult to understand. For, these reactions would also presumably proceed by a nucleon exchange graph, as in Figs. 3(a) and 3(b). The $\Lambda:\Sigma$ ratio would then largely be determined by the ratio of

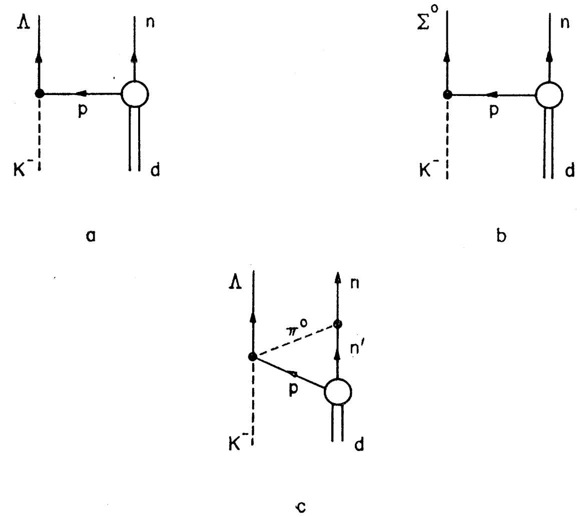

FIG. 3. (a) and (b) ONE graphs; (c) triangle graph.

the squares of the coupling constants at the KNY vertex, i.e.,

$$R(\Lambda)/R(\Sigma) \approx g_{KN\Lambda}{}^2/g_{KN\Sigma}{}^2\,. \tag{14}$$

But this ratio is known[24] to be something like 20 to 1, very different from the experimental ratio.

The momentum transfer dependence again comes in, even more strongly in this example, and indicates that triangle graphs such as in Fig. 3(c) might well be dominant. In fact, some time ago Burhop *et al.*[25] discussed these reactions by means of such triangle graphs, and the predicted ratios in that paper are not far from the experimental ratios.

Finally, we address ourselves to the question of why the soft-pion calculations of the unbound process $pp \to np\pi^+$, as in Ref. 1, were successful while those discussed here are not. Let us simply point out that the two calculations are qualitatively quite different. In the unbound case, the off-shell t matrix for the np scattering vertex was approximated by an appropriate on-shell value. There is some ambiguity in what on-shell point is chosen; the choices in Ref. 1 are only somewhat more obvious than others. The predicted cross sections are fairly sensitive to this choice, at least near threshold, where the elastic nucleon-nucleon t matrix varies rapidly with energy. We can conclude only that some on-shell points have been found which yield a favorable comparison between experimental cross sections and predictions of the Adler-Dothan theorem.

On the other hand, the calculation for the bound process is really an off-shell calculation throughout, since the dnp vertex was obtained from a deuteron wave function (or nucleon-nucleon potential). These two approaches, on-shell approximation and off-shell cal-

[21] V. Franco (private communication) has pointed out to us that a similar effect occurs in elastic pd scattering, for example. Here, the single-scattering contribution is much larger than the double-scattering contribution at $t=0$, but falls off much more rapidly with $|t|$. At large $|t|$, the double scattering dominates. The reason for this behavior is, as in the present case, due to the strong momentum-transfer dependence of the deuteron form factors.

[22] V. R. Veirs, R. A. Burnstein, and D. P. Novak, Bull. Am. Phys. Soc. **13**, 1440 (1968); and private communication. We would like to thank these authors for permission to use their results prior to publication.

[23] The unbound process $pp \to p\Lambda K^+$ at threshold is discussed by M. E. Schillaci, R. R. Silbar, and J. E. Young, Phys. Rev. **179**, 1546 (1969).

[24] J. H. Kim, Phys. Rev. Letters **19**, 1079 (1967); C. H. Chan and F. T. Meiere, *ibid.* **20**, 568 (1968).

[25] E. H. S. Burhop, A. K. Common, and K. Higgins, Nucl. Phys. **39**, 644 (1962). A recent, more critical, discussion is given by A. Reitan, Arkiv for Det Fysiske Seminar i Trondheim Report No. 1, 1969 (unpublished); Nucl. Phys. **B11**, 170 (1969).

culation, are really only equivalent in the zero-pion-mass limit. In view of the results in this paper, it would be interesting to reexamine the unbound case using a nucleon-nucleon potential, thus doing an off-shell calculation similar to that done here for the bound case.[26] However, at this point, it is not clear whether a potential calculation at such a far off-shell point would be sufficiently accurate to provide a test of the Adler-Dothan low-energy theorem.

[26] Koltun and Reitan (Ref. 7) have also considered the unbound reaction $pp \to pp\pi^0$ in a manner completely analogous to their $pp \to d\pi^+$ calculation. Direct comparisons cannot be made, however, since the isospin channels are quite different.

ACKNOWLEDGMENTS

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Asymptotic Virtual Photoabsorption Cross Sections and Sugawara's Theory of Currents*

EUGENE Y. C. LU†
Department of Physics, The Ohio State University, Columbus, Ohio 43210
(Received 11 December 1968)

Asymptotic bounds on transverse and longitudinal total virtual photoabsorption cross sections at high momentum transfer are obtained in Sugawara's theory of currents.

RECENTLY, Sugawara[1] has proposed a field theory in which the energy-momentum tensor $\theta_{\mu\nu}$ is expressed in terms of the weak and electromagnetic currents of the hadrons. The attractiveness of such a theory has been elegantly expressed and need not be repeated here. More crucial, however, is the task of searching for some simple experimental tests whose failure would tell us that a theory of this kind does not describe the real world. One such test has been proposed by Callan and Gross.[2] It is the purpose of the present note to propose another test. We shall show that Sugawara's theory implies that

$$\lim_{q^2\to-\infty} q^2\sigma_T(\omega,q^2)=0\,, \tag{1}$$

$$0<\lim_{q^2\to-\infty}\int_0^2 d\omega\, q^2\sigma_L(\omega,q^2)(1-\tfrac{1}{2}\omega)<4\pi^2\alpha\,, \tag{2}$$

where σ_T and σ_L are the total cross sections for the absorption of transverse and longitudinal virtual photons on protons; $\omega\equiv -q^2/\nu$, $\nu\equiv P\cdot q$, P, q are the proton and the photon four-momentum, respectively, and C is Sugawara's constant.

We begin with the covariant amplitude for virtual Compton scattering on protons

$$T_{\mu\nu}(p,q)=i\int d^4x\, e^{iqx}\langle p|T^*[J_\mu{}^{\rm em}(x),J_\nu{}^{\rm em}(0)]|p\rangle \tag{3}$$

$$=W_1(q^2,\nu)\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)+W_2(q^2,\nu)\left(p_\mu-\frac{\nu}{q^2}q_\mu\right)\left(p_\nu-\frac{\nu}{q^2}q_\nu\right), \tag{4}$$

where $J_\mu{}^{\rm em}$ is the hadronic electromagnetic current.

We further define

$$E_{0ij}\equiv\int d^4x\, e^{iqx}\delta(x_0)\langle p|[\partial_0 J_i{}^{\rm em}(x),J_j{}^{\rm em}(0)]|p\rangle\,, \tag{5}$$

where i and j are space indices.

In Sugawara's theory, we have

$$E_{0ij}=(-i/2C)\langle p|[V_i'(0),V_j'(0)]_+ +[V_i^2(0),V_j^2(0)]_+ +(V\leftrightarrow A)|p\rangle \tag{6}$$

$$=(-1/C)P_iP_jG_2+\delta_{ij}G_1\,, \tag{7}$$

where G_i are constants and G_2 may be expressed in terms of a Gallan-Gross integral over the differential cross section of electroproduction at large momentum transfer and is positive definite.

Now Bjorken[3] has recently shown that in the limit $q_0\to\infty$, $P_z\to\infty$ such that $q_0/P_0\to -\omega$ (fixed), we have

$$\int_0^2 d\omega[\delta_{ij}\omega F_1(\omega)-\delta_{i3}\delta_{j3}F_2(\omega)]=\lim_{q_0\to i\infty,\,-q_0/p_0\to\omega}\frac{E_{0ij}}{P_0{}^2}\,, \tag{8}$$

* Research supported by the United States Atomic Energy Commission.

† Present address: A. H. Compton, Laboratory of Physics, Washington University, St. Louis, Mo. 63130.

[1] H. Sugawara, Phys. Rev. 170, 1659 (1968).

[2] C. G. Callan, Jr., and D. J. Gross, Phys. Rev. Letters 21, 311 (1968).

[3] J. D. Bjorken, Phys. Rev. 179, 1547 (1969).