with even smaller values for the η-nucleon coupling constant, but we have not been able to find a solution which violates the inequality given by Eq. (11). In Fig. 4, we show the behavior of the total cross section in the neighborhood of threshold. The data are from Refs. 11 and 14 and were not used in minimizing χ^2.

It is interesting that Botke[10] predicts a wide bump in the total cross section centered at $T_\pi = 1.75$ GeV. Our model does not predict this bump.[28] We predict $\sigma_{\text{total}}(\pi^- p \rightarrow \eta n) = 0.56$ mb at $T_\pi = 1.75$ GeV, and Botke[10] obtains a value approximately twice as large. The data at $T_\pi = 1300$ MeV were hard to fit in Botke's model,[10] and we have no trouble in that energy region [see Figs. 2(g) and 3].

VI. CONCLUSIONS

We have been able to fit the data for the process $\pi^- p \rightarrow \eta n$ below 2 GeV c.m. energy by using a model which consists of direct-channel resonances and nucleon and A_2 pole terms for the nonresonant background. In order to develop this model, we had to calculate the rather complicated spin-2 contribution to the spin-flip and non-spin-flip amplitudes. It has, therefore, been possible to give a quantitative assessment of the relative importance of A_2 exchange in this process. The minimum χ^2 dropped from 98 to 82 (with 83 data) when the A_2 was added.

It was found that a realistic upper limit of 0.5 could be placed on the η-nucleon coupling constant, with a value in the neighborhood of 0.0025 favored. In unbroken $SU(3)$ symmetry, this corresponds to a D/F ratio between 2/1 and 3/1, with the favored value close to 3/1.

[28] Note that Botke includes resonances above 2 GeV c.m. energy, and we do not attempt to fit data at the higher energies.

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Implications of Direct- and Crossed-Channel Regge Self-Consistency

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By comparing the t-channel Regge-pole amplitude with the s-channel Regge amplitude, we apply self-consistency for near-forward scattering. Under certain plausible assumptions, this enables us to evaluate the high-energy behavior of the Regge-trajectory function as $\alpha(s) \rightarrow (s \ln s)^{1/2}$, and that of the residue function as $\beta(s) \rightarrow s^{\alpha_c(0)-1/2}(\ln s)^{-1/2}$. We also determine that two trajectories will have the same shape if their derivatives at $s=0$ are the same, since this derivative alone determines the trajectory shape completely. The resonance content of these trajectories at high s is also examined and found to be empty in the usual sense.

I. INTRODUCTION

RECENTLY, there has been a good deal of interest among Regge enthusiasts in examining the partial-wave projections of the leading crossed-channel Regge pole. Beginning with Schmid,[1] a number of theorists have demonstrated that such a projection produces partial-wave amplitudes which trace out arcs of circles in the Argand plane as the energy increases. Schmid originally conjectured this to be evidence for the existence of resonances in the direct channel. He worked in the region of 1–3 GeV for l between 2 and 6 and found a reasonable correspondence between generated resonances and experimentally known ones.

Combined with the work of Dolen, Horn, and Schmid[2] on finite-energy sum rules, this information was interpreted to give evidence of severe double counting in the intermediate-energy interference model of Barger and Cline.[3] Shortly thereafter, doubt began to arise about the resonance interpretation of the Argand circles.

Kugler[4] demonstrated that Argand circles also occur for high mass and high spin when $l \approx \sqrt{s}$. If these are really resonances, then he conjectured that for large s Regge trajectories must behave like $\sqrt{s}$ (omitting logarithmic factors). Collins, Johnson, and Squires[5] also demonstrated the existence of high-l Argand circles but doubted their interpretation as resonances partly because all such circles would have to be so interpreted.

[1] Christoph Schmid, Phys. Rev. Letters **20**, 689 (1968).
[2] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1768 (1968); Phys. Rev. Letters **19**, 402 (1967).
[3] V. Barger and D. Cline, Phys. Rev. Letters **16**, 913 (1966); Phys. Rev. **155**, 1792 (1967).
[4] M. Kugler, Phys. Rev. Letters **21**, 570 (1968).
[5] P. D. B. Collins, R. C. Johnson, and E. J. Squires, Phys. Letters **27B**, 23 (1968).

Further doubts were expressed by Alessandrini and Squires[6] because of certain problems with unitarity. Alessandrini, Freund, Oehme, and Squires[7] cast more doubt by revealing some unpleasant consequences of the resonance interpretation. In particular, they found the existence of many high-spin low-mass resonances, which they found unattractive. Kreps and Logan[8] have done a more careful study of Schmid's original N^* resonance identification and disagree with his conclusions. Chiu and Kotański[9] have recently demonstrated that the Argand circles are not just a consequence of the specific Regge form but are a more general property of asymptotic amplitudes.

In all the papers just mentioned, certain assumptions were necessary about the behavior of the residue function $\beta(t)$. The detailed nature of the circles, in other words, the positions and widths of the "resonances," depends sensitively on the choice of $\beta(t)$. This, of course, is also a weak part of the theory if one tries phenomenologically to investigate whether the generated resonances actually match experimentally known ones.

Besides the uncertainty in $\beta(t)$, the question as to the meaning of the Argand circles indicates that projecting the t-channel pole in the s-channel partial waves is unsatisfactory in a number of respects. We propose, instead, to investigate the correspondence between the t-channel Regge pole and the s-channel Regge poles. We shall do this for asymptotically high energy in the s channel and make the assumption that a single s pole dominates the reaction as well as a single t pole. We will discuss this assumption in some detail and indicate why we feel that it is plausible. Among other things, we are then able to derive the asymptotic form of the Regge trajectories assuming only that they are infinitely rising. We also are able to derive the value of the scale factor s_0 which determines where Regge asymptotics begins. We also find the asymptotic s behavior of the residue function. Our results enable us to see a possible reason why Regge trajectories in the resonance region are all similar in shape.

Recently, Khuri[10] has also related the t-channel pole to a sum of s-channel poles in order to apply crossing self-consistency and investigate a number of questions with which we are concerned. His approach and ours are complementary, in some sense, in that his is a many-s-pole program and ours is a single-s-pole approach. However, he starts with the assumption of linearly rising trajectories, and we do not.

Both approaches share the advantage that the discussion of whether certain "resonances" exist is bypassed since by using the s-channel Regge poles, and not the s-channel partial waves, the question does not arise. (See, however, Ref. 11.) However, we will discuss the relationship of these Regge s poles to physical resonances. For high s there appears to be no relation since the widths become very broad and the usual Breit-Wigner form loses its resonating structure.

II. ASSUMPTIONS AND DERIVATION

In the following, we shall treat the spinless case. In Sec. III, we shall discuss what additional considerations would be necessary for the inclusion of spin. In the s channel, the scattering amplitude for spinless particles may be written in its well-known form

$$A(s,t)=\tfrac{1}{4}i\int_{-\frac{1}{2}-\infty i}^{-\frac{1}{2}+\infty i} dl\frac{2l+1}{\sin\pi l}[P_l(-z)\pm P_l(z)]a^\pm(l,s) -\frac{\pi}{2}\sum_{i=1}^{N}\beta_i^\pm(s)\frac{2\alpha_i^\pm(s)+1}{\sin\pi\alpha_i^\pm(s)} \times[P_{\alpha_i^\pm(s)}(-z)\pm P_{\alpha_i^\pm(s)}(z)]. \quad (1)$$

For high energy in the s channel, with the usual assumption that a single t-channel pole dominates, we have (suppressing the signature indices)

$$A(s,t)\xrightarrow[s\to\infty]{} -\pi^{1/2}\frac{2\alpha(t)+1}{2\alpha(t)}\frac{\beta(t)\Gamma[\alpha(t)+\frac{3}{2}]}{\Gamma[\alpha(t)]\sin\pi\alpha(t)} \times\frac{1\pm e^{-i\pi\alpha(t)}}{(-q^2)^{\alpha(t)}}s^{\alpha(t)}, \quad (2)$$

where $\beta(t)$ in Eq. (2) is the same function that appears in Eq. (1). Normally the factor $(-q^2)^{-\alpha(t)}$ is absorbed into $\beta(t)$ to define a reduced residue, but we wish to display it explicitly. The q appearing in Eq. (2) is the t-channel momentum analytically continued to the s channel. For equal masses, for example, we have

$$q(t)=\tfrac{1}{4}(t-4m^2), \quad t\leq 0.$$

It is important to note that, while β is the same function in Eqs. (1) and (2), it refers to different particles, in general. In Eq. (1), we have the residues of s-channel poles while in Eq. (2) β refers to the residue of a t-channel pole. The same situation is obviously true for the trajectory function α.

Since for large s Eqs. (1) and (2) must be the same, a self-consistency restriction arises from crossing. If it turned out that a *single* s-channel pole in Eq. (1) dominated at high s, then important consequences would result in a more or less tractable way. However, the situation cannot be quite that simple. It is well known that although $A(s,t)$ has a t-plane cut running from $t_0>0$ to $+\infty$, the background integral in Eq. (1) and the

[6] V. A. Alessandrini and E. J. Squires, Phys. Letters **27B**, 300 (1968).

[7] V. A. Alessandrini, P. G. O. Freund, R. Oehme, and E. J. Squires, Phys. Letters **27B**, 456 (1968).

[8] R. E. Kreps and R. K. Logan, Phys. Rev. **177**, 2328 (1969).

[9] C. B. Chiu and A. Kotański, Nucl. Phys. **B7**, 615 (1968).

[10] N. N. Khuri, Phys. Rev. **176**, 2026 (1968).

[11] We have no doubt that the assumption about the relative smallness of the background integral is in some way connected to the assumption that the Argand circles are resonances. By trading one assumption for the other, however, new features of Regge theory are accessible which were not so before.

poles both have cuts beginning at $t=0$. These are easily shown[12] to be mutually cancelling for $0\leq t<t_0$.

For small t, an expansion may be made of the Legendre functions in Eq. (1) and the singular parts (near $z=-1$) may be explicitly cancelled. We shall do this below. What of the finite remainder which now has the correct analytic structure? We shall show that, providing $\mathrm{Re}\alpha(s)\rightarrow\infty$ as $s\rightarrow\infty$ (infinitely rising trajectory) and $\mathrm{Im}\alpha(s)$ does not approach too near zero as $s\rightarrow\infty$, *a single pole will dominate all the others in the s channel*. Such single-pole dominance depends on the behavior of $\beta(s)$ for large s, too, but the behavior determined by crossing for $\beta(s)$ does give self-consistent single-pole dominance.

Finally, there is the question of the contribution of the finite part of the background integral. In the s channel for high energy, little is known about this integral. However, there are a number of arguments that would indicate that it may well be small relative to the poles. If the arguments of Dolen *et al.*[2] are correct, this integral contributes the local fluctuations to the amplitude in the intermediate energy region as they claim. The bulk of the amplitude comes from the direct-channel resonances which are represented (once) by the t-channel pole. As the energy increases these "remaining wiggles" die away and the amplitude is smooth. Presumably it is the background integral that dies away for large s.

Another way of looking at the situation is this: At intermediate energies the resonances are nearly everything. Now s-channel resonances are s-channel Regge poles when $\mathrm{Im}\alpha(s)$ is small. Hence it is poles which lie near the real l axis in the s channel that make up nearly all the amplitude. As s increases these poles may move away from the real axis and lose their identification as physical resonances, but they will still dominate the amplitude and the background-integral contribution stays small.

In light of these considerations, we shall make the necessary assumption that the background integral in Eq. (1) may be neglected at high s. It turns out to be a most fruitful assumption in that much information about $\alpha(s)$ and $\beta(s)$ can now be obtained.[11]

To proceed, we must make use of a standard representation of the Legendre function near its singular point which is where its argument is -1. We have[13]

$$P_\alpha(z)=F[-\alpha,\alpha+1;1,\tfrac{1}{2}(1-z)]$$
$$=\frac{1}{\Gamma^2(-\alpha)\Gamma^2(\alpha+1)}\sum_{n=0}^{\infty}\frac{\Gamma(-\alpha+n)\Gamma(\alpha+1+n)}{(n!)^2}\times\left(\frac{1+z}{2}\right)^n[2\psi(n+1)-\psi(n-\alpha)-\psi(\alpha+1+n)-\ln\tfrac{1}{2}(1+z)]. \quad (3)$$

In this expression, F is the hypergeometric function and ψ is the logarithmic derivative of the γ function. For near-forward scattering, $t\approx 0$ or $z\approx 1$, we have

$$P_\alpha(-z)\approx\pi^{-1}\sin\pi\alpha[\ln\tfrac{1}{2}(1-z)+2\gamma+2\psi(\alpha+1)+\pi\cot\pi\alpha], \quad (4)$$

where γ is the Euler constant, $\psi(1)=-\gamma$, and we have used $\psi(-\alpha)=\psi(\alpha+1)+\pi\cot\pi\alpha$ as well as $\Gamma(\alpha)\Gamma(1-\alpha)=\pi/\sin\pi\alpha$.

We now make the observation that if the $P_l(-z)$ in Eq. (1) is expanded according to Eq. (4), then the first three terms in the brackets in Eq. (4) exactly cancel the corresponding Regge-pole contributions in Eq. (1). This is easily seen by closing the background integral by a semicircle in the right half l plane and evaluating the residues, which come *only* from the poles of $a(l,s)$. However, it is the $\pi\cot\pi\alpha$ term (which has poles for $\mathrm{Re}\alpha>-\frac{1}{2}$) which constitutes that part of the background integral which does not identically cancel the pole contributions. It is this integral that we assume to be small, as discussed above.

Assuming[14] only that $\mathrm{Im}\alpha(s)$ remains large enough so that $\cot\pi\alpha\rightarrow -i$ as $\mathrm{Re}\alpha(s)\rightarrow\infty$, we may equate Eq. (4) and the constant (in t) term in Eq. (2). We obtain

$$i\pi\beta(s)\alpha(s)=-\pi^{1/2}\beta_0\frac{2\alpha(0)+1}{2\alpha(0)}\times\frac{\Gamma[\alpha(0)+\frac{3}{2}][1\pm e^{-i\pi\alpha(0)}]}{\Gamma[\alpha(0)]\sin\pi\alpha(0)m^{2\alpha(0)}}s^{\alpha(0)}, \quad (5)$$

where we have put $\beta(0)=\beta_0$. This yields

$$\beta(s)\underset{s\rightarrow\infty}{\longrightarrow}\Omega_c s^{\alpha_c(0)}/\alpha(s), \quad (6)$$

with

$$\Omega_c=\frac{i}{\sqrt{\pi}}\beta_0\left(\frac{2\alpha_c(0)+1}{2\alpha_c(0)}\right)\frac{\Gamma[\alpha_c(0)+\frac{3}{2}]}{\Gamma[\alpha_c(0)]}\frac{1\pm e^{-i\pi\alpha_c(0)}}{\sin\pi\alpha_c(0)m^{2\alpha_c(0)}}, \quad (7)$$

where the subscript c indicates the crossed-channel quantity.

We have neglected the $P_\alpha(+z)$ term for the reason that in the approximation that $\cot\pi\alpha\approx -i$, which requires $e^{-2\pi\,\mathrm{Im}\alpha(0)}\ll 1$, the constant term in $P_\alpha(+z)$ is down by just this exponential factor from the constant term in $P_\alpha(-z)$. Similar considerations hold when we consider higher terms in t below.

Going now to the *largest* linear term in t, we may again equate Eq. (1) and Eq. (2). By the largest linear term, we mean that part of the coefficient of t that dominates

[12] R. G. Newton, *The Complex j-Plane* (W. A. Benjamin, Inc., New York, 1964), p. 7.

[13] *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (National Bureau of Standards, Washington, D. C., 1966), 5th ed., p. 559.

[14] The requirement on $\mathrm{Im}\alpha(s)$ is very minimal. As stated below Eq. (7) in the text it is that $e^{-2\pi\,\mathrm{Im}\alpha(s)}\ll 1$. Hence for $\mathrm{Im}\alpha(s)\geq\frac{1}{4}$, our approximation should be good. Evidence of the value of $\mathrm{Im}\alpha(s)$ can be found in Ref. 17, where the indications are that such a condition is satisfied even at low energies.

for large s. In Eq. (2), this will come from the expansion of $s^{\alpha(t)}$. We cannot consider anything but the leading s part of each t term, because lower-s parts could come from parts of $\alpha(s)$ and $\beta(s)$ in Eq. (1) which are not the leading parts of these functions at high s. We have, using the $n=1$ term in Eq. (3) for large s,

$$[i\pi\beta(s)\alpha^3(s)/s]t=i\pi\Omega_c s^{\alpha_c(0)}\alpha_c'(0)(\ln s)t\,, \qquad (8)$$
$$\beta(s)\alpha^3(s)/s=\Omega_c s^{\alpha_c(0)}\alpha_c'(0)\ln s\,,$$

where $z=1+t/2q^2$ so that $\frac{1}{2}(1-z)=t/s$ since $4q^2\to s$ for large s. Using Eq. (6), we may write Eq. (8) as

$$\alpha^2(s)/s=\alpha_c'(0)\ln s\,,$$
$$\alpha(s)\underset{s\to\infty}{\longrightarrow}\sqrt{[\alpha_c'(0)]}\sqrt{(s\ln s)} \qquad (9)$$

and Eq. (6) becomes

$$\beta(s)\underset{s\to\infty}{\longrightarrow}\frac{2\Omega_c}{\sqrt{[\alpha_c'(0)]}}\frac{s^{\alpha_c(0)-1/2}}{\ln^{1/2}s}\,. \qquad (10)$$

We need not stop at the linear t term, but may continue to equate the largest part of each t term from Eqs. (1) and (2). In Eq. (2), it will always come from the expansion of $s^{\alpha_c(t)}$ which introduces $\ln s$ factors along with higher and higher derivatives of $\alpha(s)$ at the origin. For example, a little calculation reveals that the t^2 term gives an equation corresponding to Eq. (9) which is, using $n=2$ in Eq. (3),

$$\alpha^4(s)/4s^2=\tfrac{1}{2}\{\alpha_c''(0)+[\alpha_c'(0)]^2\}\ln^2 s\,. \qquad (11)$$

Using Eq. (9), we have

$$\alpha_c''(0)=-\tfrac{1}{2}[\alpha_c'(0)]^2\,. \qquad (12)$$

The t^3 term has three terms on the right-hand side and gives $\alpha_c'''(0)=-(7/3)[\alpha_c'(0)]^3$, and so on. We note the interesting fact that the shape of the trajectory is completely determined by its slope at $s=0$. Hence, any trajectories that have the same slope there (as they all seem to have) will be the same except possibly for a vertical displacement relative to each other. This is precisely what is observed physically. In Eq. (9), we see even at high s that this same shape stipulation holds under the same requirement: that the slope be the same at $s=0$.

In a paper by Mandula and Slansky,[15] it was shown that Regge asymptotics (finite-energy sum rules) are inconsistent with the assumption of a finite number of s-channel trajectories if all t values are considered. This might imply that the series in powers of t that we have generated [discussed below Eq. (10)] may not hold for all t. It may be asymptotic, not convergent, in which case only a certain number of the derivatives of $\alpha(t)$ at $t=0$ may be accurate. On the other hand, since we do *not* require the narrow-resonance approximation—in fact, Ref. 14 seems to exclude it—and this *is* required by Mandula and Slansky, the two papers may be fully consistent.

Since for nearly all trajectories, except possibly the Pomeranchon,[16] $\alpha_c'(0)\approx 1$, it is clear from Eq. (12) that the trajectories cannot be exactly straight lines in the physical resonances region of $1\lesssim s\lesssim 10$. Since the Taylor series for $\alpha(s)$ arising from its derivatives [as determined by Eq. (12) and like equations] converges extremely slowly, it is not possible to determine its shape without calculating a prohibitive number of derivatives (the equations for the higher derivatives develop excessive numbers of terms). We note[17] that the physical curve may not be a straight line exactly, but can be matched by a nonlinear function which is nearly linear in the physical region only. For a trajectory for which $\alpha_c'(0)\approx 0$ (the Pomeranchon) then the shape will be very close to linear in the physical region.

It appears from Eq. (9) that the scale parameter for Regge asymptotics is given by

$$s_0=[\alpha_c'(0)]^{-1}.$$

This is consistent with Eq. (12) and would indicate a value of $s_0\approx 1$ GeV, a very reasonable value. We note that the scale parameter is determined by the derivative at $s=0$ of the crossed-channel pole, not the derivative of the pole itself. For example, if the Pomeranchon has $\alpha_0'(0)\approx 0$, it implies a very large s_0 for some other direct-channel pole.

III. ANALYSIS

Knowing the asymptotic $\alpha(s)$ and $\beta(s)$ forms, the first question to be answered is whether a single Regge pole does indeed dominate. For large s, the ratio of the contribution of two poles is

$$\frac{\beta_1(s)\alpha_1(s)\ln\alpha_1(s)}{\beta_2(s)\alpha_2(s)\ln\alpha_2(s)}\propto s^{\alpha_1(0)-\alpha_2(0)}. \qquad (13)$$

If $\alpha_1(0)$ is the Regge pole with the largest real part, then clearly one s-channel pole dominates all the others. The next order in t is also dominated by the single pole that dominates in the leading order. Our assumption is self-consistent, but it should be borne in mind that in a real two-s-trajectory problem, it would not be true that Eq. (13) *must* hold.

Since $\alpha_c(0)\leq 1$, $\beta(s)$ can never grow as fast as $\sqrt{s}$ and will decrease if $\alpha_c(0)\leq\frac{1}{2}$. It is important to determine the structure of the partial waves in the s channel to see if reasonable behavior is obtained and also to see if any high-energy resonances are distinguishable. We begin with the standard expression[12]

$$a_l(s)=\frac{-\beta(s)[2\alpha(s)+1]}{[l-\alpha(s)][l+\alpha(s)+1]}[1\pm(-1)^l]. \qquad (14)$$

[15] J. E. Mandula and R. C. Slansky, Phys. Rev. Letters **20**, 1402 (1968).

[16] For a study of whether the Pomeranchon is really like the other particles see H. Harari, Stanford Linear Accelerator Report No. SLAC-PUB 463 (unpublished).

[17] R. M. Spector, Phys. Rev. **173**, 1761 (1968).

For $l \ll \alpha(s)$, we have

$$a_l(s) \approx \frac{\beta(s)}{\alpha(s)} = \frac{s^{\alpha_c(0)-1}}{\ln s}, \tag{15}$$

which goes to zero as $s \to \infty$ since $\alpha_c(0) \leq 1$. In the region $l \gg \alpha$, Eq. (14) yields

$$\begin{aligned} a_l(s) &\approx \frac{\beta(s)\alpha(s)}{l^2} = \frac{\beta(s)}{l}\left(\frac{\alpha(s)}{l}\right) \\ &= \frac{s^{\alpha_c(0)-1/2}}{l \ln^{1/2} s}\left(\frac{\alpha(s)}{l}\right) \\ &= \frac{s^{\alpha_c(0)-1}}{\ln s}\left(\frac{\alpha(s)}{l}\right)^2. \end{aligned} \tag{16}$$

This too goes to zero for large s. Hence, the main contribution to the partial waves comes in the region $l \approx \alpha(s) \approx \sqrt{(s \ln s)}$ as we might expect (see Kugler[8]).

In the region $l \approx \alpha(s)$, it is convenient to use the resonance form of Eq. (14), which is[11,17]

$$a_l(s) \propto \frac{\beta(s)}{\alpha'(s)} \frac{1}{[s - s_0 + \Delta s + \frac{1}{2} i\Gamma]}, \tag{17}$$

with

$$\Gamma = 2 \,\mathrm{Im}\alpha \,\mathrm{Re}\alpha' / |\alpha'|^2$$

and

$$\Delta s = \mathrm{Im}\alpha \,\mathrm{Im}\alpha' / |\alpha'|^2$$

and where $\alpha' = d\alpha/ds|_{s=s_0}$ and $\mathrm{Re}\alpha(s_0) = l$. Under our usual assumption that $\mathrm{Re}\alpha'(s) \gg \mathrm{Im}\alpha'(s)$ and, additionally, $\mathrm{Im}\alpha'(s) \to$ const, at worst, we have

$$\begin{aligned} \Gamma &\propto s^{1/2}/\ln^{1/2} s, \\ \Delta s &\propto s \,\mathrm{Im}\alpha'/\ln s. \end{aligned} \tag{18}$$

Equation (18) indicates that while possibly $\Delta s \to 0$, we always have Γ increasingly large. This indicates that though we may have large partial waves for $l \approx \alpha(s)$ (which grow like $\beta(s)$ since $\Gamma \propto [\alpha'(s)]^{-1}$ and $s \approx s_0$), these do *not* appear as resonances. They are too broad and become increasingly so for larger and larger s. Even if $a_l(s)$ decreases due to decreasing $\beta(s)$, it will always dominate those waves for which $l \ll \alpha(s)$ and $l \gg \alpha(s)$. This is apparent from inspection of Eqs. (15), (16), and (18).

In the analysis leading to Eq. (18) and the conclusions following it, we have really examined the single-trajectory problem because we used our earlier results which come from the assumption of a single dominant s pole. In an infinite-trajectory model with daughters (such as the Veneziano model) our conclusions could be altered. In this case, lower trajectories could dominate due to small residues of the leading trajectories.

Until now we have considered only a two-pole problem (one t and one s pole) and neglected spin. In the t channel, no problems arise from spin since one need consider only the single leading pole which contributes to the helicity amplitude in question. Hence, the identification of $\alpha_c(0)$ and $\beta_c(0)$ is clear. The correct asymptotic form equivalent to Eq. (2) will be modified in both the exponent of s and the exact t factors multiplying it. However, in principle the t channel presents no difficulty.

In the s channel, the problem is more difficult since the dominant pole in the j plane will in general contribute to a number of amplitudes (for example, to both a spin-flip and a non-spin-flip). Indeed, the dominant s pole for one amplitude may not be the same as that for another. Clearly, though, the *leading* term of the most dominant s pole will contribute *only* to the largest amplitude in the reaction. The next largest amplitude could come from two sources. It could come from (a) the *next* leading term in that most dominant pole, or (b) it could come from the leading term in a second (less dominant) pole.

Generally the situation is complicated, but in case (b) above, Eq. (10) would still hold for the residue of the second pole; but the $\alpha_c(0) - \frac{1}{2}$ exponent of s would be replaced by a lesser exponent. For example, in a spin-flip amplitude it might be $\alpha_c(0) - \frac{3}{2}$ with a different $\alpha_c(0)$. In case (a) if we have $\beta(s) = \beta_0(s) + \beta_1(s)$ with $\lim_{s\to\infty}[\beta_1(s)/\beta_0(s)] \to 0$, then Eq. (10) would hold for $\beta_1(s)$ with the same remarks applying to the exponent of s. This is true for case (a), however, only if this next leading term in the most dominant pole comes from the leading term in $\alpha(s)$ and the $\beta_1(s)$ term in $\beta(s)$. It could happen that a lower term in $\alpha(s)$ along with $\beta_0(s)$ would contribute the next leading term; then we can say nothing.

Finally, we observe that while the behavior for $\beta(s)$ in Eq. (10) is different from other suggested behaviors,[10,18] it is perfectly acceptable. The high-s behavior of $\alpha(s)$ as given in Eq. (9) it is precisely that as determined by Childers[19] from dispersion theory, by Chu and Tan[20] from a bootstrap model, and Brower and Harte[21] from certain dynamical assumptions.

After this paper was submitted for publication the author became aware of a paper by Mohapatra.[22] With similar single-channel assumptions, but with extremely different techniques, he reaches the identical results that we do. Compare his Eq. (9) and our Eq. (9).

[18] V. Teplitz and C. E. Jones, Phys. Rev. Letters **19**, 135 (1967).

[19] R. W. Childers, Phys. Rev. Letters **21**, 868 (1968); **21**, 1669(E) (1968). The fact that we do not have $\lim_{s\to\infty} \arg\alpha(s) = \frac{1}{2}\pi$ as suggested by Childers presumably is due to a violation of his proposed condition (3).

[20] S. Chu and C. Tan, University of California Radiation Laboratory Report No. UCRL-17511, 1967 (unpublished).

[21] R. C. Brower and J. Harte, Phys. Rev. **164**, 1841 (1967).

[22] R. N. Mohapatra, Phys. Rev. Letters **22**, 735 (1969).