Spin-2 Exchange in the Process $\pi^- + p \rightarrow \eta + n$ and the η-Nucleon Coupling Constant

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A model which consists of direct-channel resonances and a nonresonant background of nucleon and A_2 pole terms is used to fit the data below 2-GeV c.m. energy for the process $\pi^- + p \rightarrow \eta + n$. A good fit is achieved with $\chi^2 = 82$ for 83 data. The branching fractions are calculated for the decay of the isospin-$\frac{1}{2}$ nucleon resonances into the ηN channel, and a realistic upper limit of 0.5 is placed upon the η-nucleon coupling constant.

I. INTRODUCTION

SEVERAL phenomenological models have been developed[1–10] in an attempt to explain the behavior of the total and differential cross sections[11–14] for the reaction $\pi^- + p \rightarrow \eta + n$ at low energy (below 2 GeV c.m. energy). The lowest mass meson which can be exchanged in this process is the A_2 meson, and all of these models[1–10] have neglected this contribution. There are several reasons for leaving out the A_2. (i) The A_2 is a spin-2 particle, and the spin-2 contribution to the production amplitudes is rather complicated. (ii) Several additional unknown parameters are introduced, and until the experimental situation is significantly improved, there is no hope for a unique determination of these parameters. (iii) A fairly good fit to the data below 2 GeV c.m. energy has been obtained[7,9,10] without the A_2. (iv) There may be the risk of a small amount of double counting[15]; however, this is questionable at low energy with particle exchange rather than Regge-pole exchange. Although these may be good reasons for neglecting the A_2, there are also several reasons for doing a calculation which includes the A_2. (i) We can find the rather complicated spin-2 contribution to the spin-flip and non-spin-flip amplitudes. (ii) The success of A_2 Regge pole exchange at higher energy[16–18] suggests that A_2 exchange may be important at lower energy. (iii) Although there is no hope for a unique determination of the parameters until the experimentalists provide us with a complete set of measured observables,[19] it is possible to set a realistic upper limit (within the framework of the model) on the η-nucleon coupling constant and to find a solution for the resonance contributions with a background composed of both nucleon poles and A_2 exchange rather than just nucleon pole terms. (iv) The differential cross-section data indicate a slight trend toward peaking in the forward direction which may be an A_2-exchange effect. This could mean that the resonance parameters obtained with the A_2 included in the nonresonant background may be a better set than those obtained without the A_2. We believe these are sufficient reasons for considering a pole-resonance model for the process $\pi^- + p \rightarrow \eta + n$ which includes A_2 exchange.

Our procedure will be first to calculate the A_2-exchange contribution to the spin-flip and non-spin-flip amplitudes. Second, we will develop a pole-resonance model for the process under consideration. Finally, we will discuss the values of the pole and resonance parameters obtained by fitting the data and their implications for unitary symmetry.

II. A_2-EXCHANGE CONTRIBUTION

We shall calculate the A_2-exchange contribution to the A and B amplitudes, defined in Ref. 20, for the process $\pi^- + p \rightarrow \eta + n$. The Feynman graph for A_2 exchange is shown in Fig. 1, and the four-momenta of the particles are given by k, q, p_1, and p_2 for π^-, η, p, and n, respectively. We use the convention of Feynman

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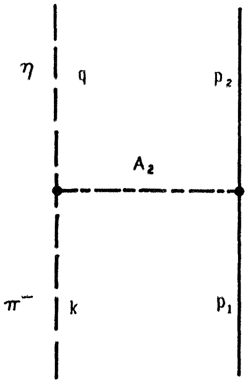

FIG. 1. A_2-exchange diagram.

rules developed by Sakurai[21] to calculate the Feynman amplitude

$$-iM_{fi}=\bar{U}(p_2)V_{\mu\nu}P(A_2)_{\mu\nu\alpha\beta}v_{\alpha\beta}U(p_1)\,, \qquad (1)$$

where $V_{\mu\nu}$ is the vertex factor at the NNA_2 vertex, $v_{\alpha\beta}$ is the vertex factor at the $\pi\eta A_2$ vertex, and $P(A_2)_{\mu\nu\alpha\beta}$ is the A_2 propagator. (There is an understood sum over the repeated indices.) $U(p_1)$ and $\bar{U}(p_2)$ are the Dirac spinors associated with the incoming and outgoing nucleons.

The vertex factors are given by[22,23]

$$V_{\mu\nu}=[(1/M)(\gamma_\mu P_\nu+\gamma_\nu P_\mu)G_1+P_\mu P_\nu G_2/4iM^2 + (1/iM^2)(Q^2\delta_{\mu\nu}-Q_\mu Q_\nu)G_3] \qquad (2)$$

and

$$v_{\alpha\beta}=(1/im^*)[K_\alpha K_\beta F_1+(L^2\delta_{\alpha\beta}-L_\alpha L_\beta)F_2]\,, \qquad (3)$$

where

$$P=p_1+p_2\,,\quad Q=p_1-p_2\,,\quad K=q+k\,,\quad L=k-q\,,\quad m^*=m_\pi+m_\eta\,, \qquad (4)$$

M is the nucleon mass, m is the mass of the A_2 meson, and the γ_μ are the Dirac γ matrices. We have chosen the vertex factors so that the quantities F_1, F_2, G_1, G_2, and G_3 are dimensionless. We shall assume that these quantities are constants which can be adjusted to fit the experimental data.

The spin-2 propagator is developed by Pilkuhn[24] and is given by

$$P(A_2)_{\mu\nu\alpha\beta}=[1/i(m^2-t)][\tfrac{1}{2}(P_{\mu\alpha}P_{\nu\beta}+P_{\mu\beta}P_{\nu\alpha}) - \tfrac{1}{3}P_{\mu\nu}P_{\alpha\beta}]\,, \qquad (5)$$

where

$$P_{\mu\nu}=\delta_{\mu\nu}+Q_\mu Q_\nu/m^2 \qquad (6)$$

and

$$t=-(k-q)^2\,.$$

We now insert Eqs. (2), (3), and (5) into Eq. (1), and after a rather long calculation, we obtain

$$M_{fi}=\bar{U}(p_2)(A+\tfrac{1}{2}i\gamma_\mu K_\mu B)U(p_1)\,, \qquad (7)$$

where

$$\begin{aligned}A=&[12m^2m^*M^2(m^2-t)]^{-1}\\ &\times\{16M^2F_1G_1[m^2(2m_\pi^2+2m_\eta^2-t)-(m_\pi^2-m_\eta^2)^2]\\ &+m^2F_1G_2[(t-4M^2)(t-2m_\eta^2-2m_\pi^2)\\ &+(t-4M^2)(m_\pi^2-m_\eta^2)^2m^{-2}-3(u-s)^2]\\ &+12m^2F_1G_3[t(t-2m_\eta^2-2m_\pi^2)+(m_\pi^2-m_\eta^2)^2]\\ &-12m^2M^2(4t)F_2G_1+3m^2F_2G_2[t(t-4M^2)]\}\end{aligned} \qquad (8)$$

and

$$B=4F_1G_1(s-u)/Mm^*(m^2-t)\,. \qquad (9)$$

The variables s and u are given by $s=-(p_1+k)^2$ and $u=-(p_1-q)^2$. The expressions for A and B have been obtained under the assumption that the initial and final nucleons have the same mass. If this assumption is not made, then these equations have several additional terms.

It is now a simple matter to calculate the A_2-exchange contribution to the spin-flip and non-spin-flip amplitudes F_S and F_N from the A and B amplitudes. In the

TABLE I. Parameters for model A in the notation of Refs. 8 and 9.[a]

Resonance parameters				
Resonance Γ (MeV)	$(\gamma_{\pi^-p}\gamma_{\eta n})^{1/2}$ (MeV)	$\Gamma_{\eta n}/\Gamma$	$\alpha\,(\eta n)$	$\alpha\,(\Lambda K)$[b]
$S_{11}(1550)$, 130	11.0 ± 0.7	$0.4\ \pm0.1$	$\genfrac{}{}{0pt}{}{2.0}{-0.5}\pm0.2$	$\genfrac{}{}{0pt}{}{1.5}{1.5}\pm0.5$
$S_{11}(1710)$, 300	6.9 ± 2.0	$0.03\ \pm0.02$	$\genfrac{}{}{0pt}{}{0.9}{0.6}\pm0.1$	$\genfrac{}{}{0pt}{}{1.9}{1.1}\pm0.2$
$P_{11}(1470)$, 210	6.8 ± 2.2	$\cdots$	$\genfrac{}{}{0pt}{}{0.9}{0.6}\pm0.1$	$\genfrac{}{}{0pt}{}{3.2}{-0.2}\pm0.3$[c]
$P_{11}(1751)$, 327	9.8 ± 2.4	$0.09\ \pm0.05$	$\genfrac{}{}{0pt}{}{1.2}{0.3}\pm0.2$	$\genfrac{}{}{0pt}{}{1.8}{1.2}\pm0.2$
$P_{13}(1863)$, 296	-1.0 ± 1.2	0.003 ± 0.003	$\genfrac{}{}{0pt}{}{0.8}{0.7}\pm0.2$	$\genfrac{}{}{0pt}{}{1.7}{1.3}\pm0.3$
$D_{13}(1525)$, 115	8.9 ± 1.3	0.003 ± 0.001	$\genfrac{}{}{0pt}{}{1.1}{0.4}\pm0.1$	$\genfrac{}{}{0pt}{}{2.2}{0.8}\pm0.3$
$D_{13}(1700)$, 150	2.0 ± 2.0	$0.02\ \pm0.02$	$\genfrac{}{}{0pt}{}{1.2}{0.3}\pm0.5$	$\genfrac{}{}{0pt}{}{1.6}{1.4}\pm1.0$
$D_{15}(1680)$, 170	-2.7 ± 1.2	0.006 ± 0.004	$\genfrac{}{}{0pt}{}{0.9}{0.6}\pm0.1$	$\genfrac{}{}{0pt}{}{1.8}{1.2}\pm0.2$
$F_{15}(1690)$, 130	-4.5 ± 1.8	0.003 ± 0.002	$\genfrac{}{}{0pt}{}{0.9}{0.6}\pm0.2$	$\genfrac{}{}{0pt}{}{1.7}{1.3}\pm0.2$
$F_{17}(1983)$, 225	2.6 ± 1.7	$0.02\ \pm0.02$	$\genfrac{}{}{0pt}{}{1.1}{0.4}\pm0.3$	$\genfrac{}{}{0pt}{}{1.8}{1.2}\pm0.3$

Pole parameters	
Parameter (dimensionless)	Best value
$g_{\eta NN}^2/4\pi$	0.0025
F_1G_1	0.47
F_1G_2	−0.68
F_1G_3	−4.13
F_2G_1	5.10
F_2G_2	−7.37
χ^2	82

[a] 83 data pt were used when adjusting the parameters to minimize χ^2.

[b] The values of α in this column are from S. R. Deans, W. G. Holladay, and J. E. Rush, Vanderbilt University, 1968 (unpublished), where further work was done on the process $\pi^-p\to K^\circ\Lambda$. The mixing parameter α is given in terms of the partial reduced widths by $\alpha(\Lambda K)=\frac{3}{2}\pm\frac{1}{2}\sqrt{6}(\gamma_{\Lambda K}/\gamma_{\pi^-p})^{1/2}$ and $\alpha(\eta n)=\frac{3}{4}\pm\frac{1}{4}\sqrt{6}(\gamma_{\eta n}/\gamma_{\pi^-p})^{1/2}$.

[c] Note that this is the only case where there is no common possible value for α.

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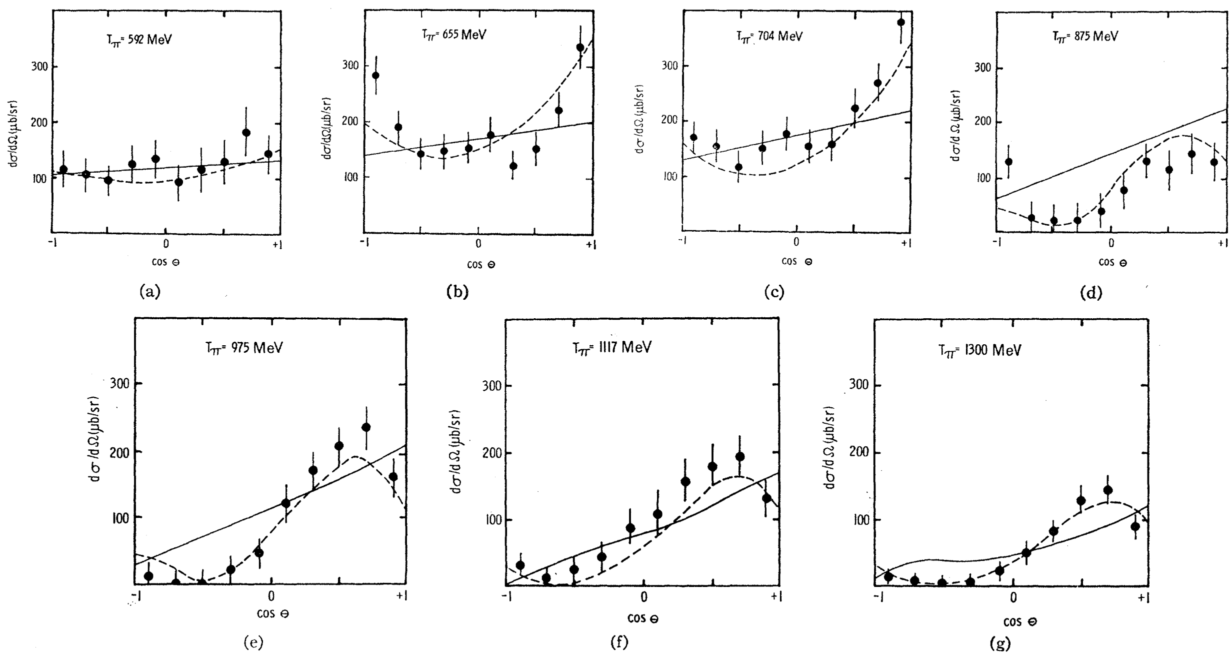

FIG. 2. Differential cross section for model A (dashed line) and model B (solid line). The data are from Ref. 12.

notation of Ref. 8, the amplitudes A and B can be used to calculate amplitudes a and b which are related to F_S and F_N by $F_N=a$ and $F_S=b\sin\theta$.

III. POLE-RESONANCE MODEL

The model consists of ten isospin-$\frac{1}{2}$ nucleon resonances in the direct channel (see Table I), together with a nonresonant background composed of nucleon-pole terms in the s and u channels, and A_2 exchange in the t channel. This is exactly the same model considered by Deans[9] plus the addition of the A_2 exchange. The notation for resonance parameters, the details of calculation, and the treatment of data have been presented in Refs. 8 and 9 and need not be repeated here.

We wish to emphasize that this model can be applied only at low energy—below approximately 2 GeV c.m. energy. At high energy, the contribution from the spin-2 particle would dominate and lead to divergent cross sections.

IV. COMPARISON WITH EXPERIMENT

In making a comparison with the data from Refs. 12 and 13, the parameters were allowed to vary until χ^2 was minimized, where

$$\chi^2=\sum_{\text{data}}\left[\frac{d\sigma/d\Omega(\text{theor})-d\sigma/d\Omega(\text{expt})}{\text{expt error}}\right]^2. \qquad (10)$$

The best values obtained for the parameters (in the notation of Refs. 8 and 9) are given in Table I. Figures 2(a)–2(g) show the differential cross sections at seven energies for (i) the model discussed in Sec. III, which we shall refer to as model A, and (ii) a model which represents a best fit to the data using A_2 and nucleon-pole terms only—no resonance terms included (model B). The parameters obtained for this model are given in Table II. Figure 3 shows the total cross sections for these two models.

V. DISCUSSION OF RESULTS

The minimum χ^2 obtained for the model consisting of resonance terms and pole terms (model A) was 82 with 83 data. (In view of disagreement among certain points of the two sets of experimental data,[12,13] we have increased the error bars on three of the 83 data. This was also done in Ref. 9.) This is to be compared with $\chi^2=98$ when the A_2 is not included[9] in the calculation. Thus, we find some improvement in the minimum χ^2. The major improvement comes from (i) a more rapid rise close to threshold, which yields better fits to the

TABLE II. Parameters for model B. All parameters are dimensionless.[a]

Parameters	Best values
$g_{\eta NN}^2/4\pi$	0.14
F_1G_1	−0.46
F_1G_2	0.11
F_1G_3	12.08
F_2G_1	2.81
F_2G_2	−0.69
χ^2	260

[a] 83 data pt were used when adjusting the parameters to minimize χ^2.

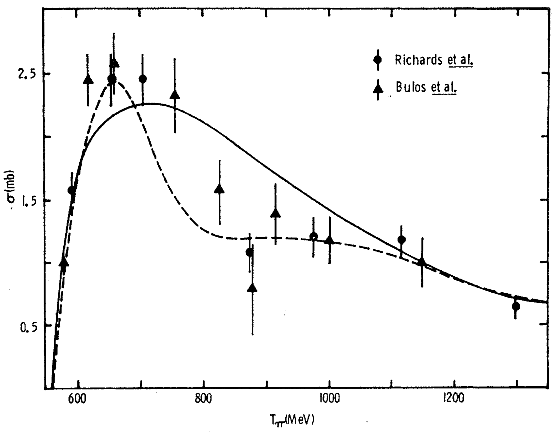

FIG. 3. Total cross section for model A (dashed line) and model B (solid line).

very low-energy data, and (ii) more background provided at the higher energies ($T_\pi=1117$, 1300 MeV) which improves the fits in that region.

Model B, which consists of nonresonant background alone, was included for comparison ($\chi^2=260$). This model yields a good fit near threshold; however, it is not satisfactory in the energy region where the differential cross section ceases to be isotropic. Thus, we see the importance of the resonance contributions in fitting all of the data. It is interesting to note that a model consisting of only nucleon-pole terms yields a minimum χ^2 of 1288, and a model consisting of only A_2 exchange gives 855. The combined effect ($\chi^2=260$) is significantly better.

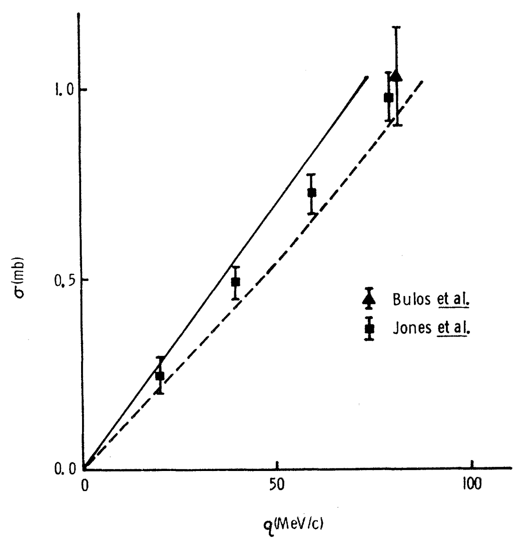

FIG. 4. Total cross section close to threshold as a function of the c.m. momentum of the η. The dashed line ($g_{\eta NN}{}^2/4\pi=0.0025$) and the solid line ($g_{\eta NN}{}^2/4\pi=0.14$) are for model A and model B, respectively. The data of Jones *et al.* (Ref. 14) have been normalized to the data of Bulos *et al.* (Ref. 11) with $R_{\gamma\gamma}=0.38$, where $R_{\gamma\gamma}=(\eta\rightarrow\gamma\gamma)/(\eta\rightarrow \text{all modes})$.

The resonance parameters obtained here, although different, have not undergone significant change from the earlier results.[9] We calculate the branching fraction $\Gamma_{\eta n}/\Gamma$ for each resonance which lies above threshold and give these values in Table I.

If we make the assumptions that each of the resonant states is a member of an $SU(3)$ octet, and the partial reduced widths in the Breit-Wigner formula measure the $SU(3)$ invariant coupling strengths; then in the limit of exact $SU(3)$ symmetry, we can calculate the mixing parameter α for PBB' coupling, as was done by Rush[25] for low-energy ΛK° production. The results of these calculations are given in Table I. The values obtained for α are to be compared with those given by Rush.[25]

We find a value for the η-nucleon coupling constant of $g_{\eta NN}{}^2/4\pi=0.0025\pm0.0020$. It was felt that since the error came from the error matrix,[26] it might be somewhat optimistic. We therefore attempted to fit the data with larger fixed values for the η-nucleon coupling constant. The other parameters were allowed to vary and χ^2 was minimized. We eliminate all solutions with $\chi^2>2\langle\chi^2\rangle_{\text{av}}$, and we find that realistic limits on the η-nucleon coupling constant are

$$0\leq g_{\eta NN}{}^2/4\pi < 0.5. \tag{11}$$

We find that the above inequality still holds even if the $P_{11}(1470)$ resonance, which is below threshold, is inserted as a pole with the same quantum numbers as the nucleon. In order to obtain a cancellation effect by this approach, it would be necessary to assume an energy dependence for $g_{\pi N^*N}g_{\eta N^*N}$ such that $g_{\pi N^*N}g_{\eta N^*N}$ ($T_\pi=1300$ MeV)≈ 10 $g_{\pi N^*N}g_{\eta N^*N}$ (threshold), and we know of no justification for such an assumption.

Our limits on the η-nucleon coupling constant are in agreement with the value found by Botke,[10] but in disagreement with the value obtained by Sasaki.[27] We believe that the disagreement may be traced either to the use of Eq. (2) in the paper by Sasaki[27] or to the problem of multiple minima. Several approximations must be made in order to obtain the equation, and it is not clear that all of them are beyond question. (If t-channel effects were important at threshold, then the additional terms would render the equation useless; however, we find that these effects are probably negligible compared to u-channel effects.) The problem of multiple minima is always present in this type of calculation. Is the Sasaki solution[7,27] for the P_{13} partial-wave amplitude unique? Perhaps the solution which Sasaki abandons ($g_{\eta NN}/4\pi=0.4$) should be studied more carefully. This solution was abandoned because it did not properly reproduce the enhancement of the η-production cross section near threshold. We have no difficulty reproducing the enhancement (see Fig. 4)

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with even smaller values for the η-nucleon coupling constant, but we have not been able to find a solution which violates the inequality given by Eq. (11). In Fig. 4, we show the behavior of the total cross section in the neighborhood of threshold. The data are from Refs. 11 and 14 and were not used in minimizing χ^2.

It is interesting that Botke[10] predicts a wide bump in the total cross section centered at $T_\pi = 1.75$ GeV. Our model does not predict this bump.[28] We predict $\sigma_{\text{total}}(\pi^- p \to \eta n) = 0.56$ mb at $T_\pi = 1.75$ GeV, and Botke[10] obtains a value approximately twice as large. The data at $T_\pi = 1300$ MeV were hard to fit in Botke's model,[10] and we have no trouble in that energy region [see Figs. 2(g) and 3].

VI. CONCLUSIONS

We have been able to fit the data for the process $\pi^- p \to \eta n$ below 2 GeV c.m. energy by using a model which consists of direct-channel resonances and nucleon and A_2 pole terms for the nonresonant background. In order to develop this model, we had to calculate the rather complicated spin-2 contribution to the spin-flip and non-spin-flip amplitudes. It has, therefore, been possible to give a quantitative assessment of the relative importance of A_2 exchange in this process. The minimum χ^2 dropped from 98 to 82 (with 83 data) when the A_2 was added.

It was found that a realistic upper limit of 0.5 could be placed on the η-nucleon coupling constant, with a value in the neighborhood of 0.0025 favored. In unbroken $SU(3)$ symmetry, this corresponds to a D/F ratio between 2/1 and 3/1, with the favored value close to 3/1.

[28] Note that Botke includes resonances above 2 GeV c.m. energy, and we do not attempt to fit data at the higher energies.

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Implications of Direct- and Crossed-Channel Regge Self-Consistency

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By comparing the t-channel Regge-pole amplitude with the s-channel Regge amplitude, we apply self-consistency for near-forward scattering. Under certain plausible assumptions, this enables us to evaluate the high-energy behavior of the Regge-trajectory function as $\alpha(s) \to (s \ln s)^{1/2}$, and that of the residue function as $\beta(s) \to s^{\alpha_c(0)-1/2}(\ln s)^{-1/2}$. We also determine that two trajectories will have the same shape if their derivatives at $s=0$ are the same, since this derivative alone determines the trajectory shape completely. The resonance content of these trajectories at high s is also examined and found to be empty in the usual sense.

I. INTRODUCTION

RECENTLY, there has been a good deal of interest among Regge enthusiasts in examining the partial-wave projections of the leading crossed-channel Regge pole. Beginning with Schmid,[1] a number of theorists have demonstrated that such a projection produces partial-wave amplitudes which trace out arcs of circles in the Argand plane as the energy increases. Schmid originally conjectured this to be evidence for the existence of resonances in the direct channel. He worked in the region of 1–3 GeV for l between 2 and 6 and found a reasonable correspondence between generated resonances and experimentally known ones.

Combined with the work of Dolen, Horn, and Schmid[2] on finite-energy sum rules, this information was interpreted to give evidence of severe double counting in the intermediate-energy interference model of Barger and Cline.[3] Shortly thereafter, doubt began to arise about the resonance interpretation of the Argand circles.

Kugler[4] demonstrated that Argand circles also occur for high mass and high spin when $l \approx \sqrt{s}$. If these are really resonances, then he conjectured that for large s Regge trajectories must behave like $\sqrt{s}$ (omitting logarithmic factors). Collins, Johnson, and Squires[5] also demonstrated the existence of high-l Argand circles but doubted their interpretation as resonances partly because all such circles would have to be so interpreted.

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