where

$$A=\frac{\sin\theta_N}{\sin(\theta_N-\theta_\gamma)},\quad B=\frac{\sin(\theta_\gamma+\theta_N)}{\sin(\theta_\gamma-\theta_N)}.$$

Only one of the solutions is physical, depending on the value of θ_γ. Once p_2 is found, Eq. (A3) can be solved for P_2 and k. The new value of P_2 can be plugged into Eq. (A4) to obtain a better value of p_2; then Eq. (A3) can be solved for better values of P_2 and k. This process can be repeated until any desired accuracy is obtained. It was found that a very small number of iterations provides excellent accuracy.

In the extraction of the πN amplitude from the elastic scattering data, the center-of-mass angle for the elastic reaction was obtained from the formula

$$\cos\theta_{\text{c.m.}}=\frac{-\gamma(\beta_N\cos\theta_N-\beta)}{[\gamma^2(\beta_N\cos\theta_N-\beta)^2+\beta_N{}^2\sin^2\theta_N]^{1/2}},\quad\text{(A5)}$$

where $\beta=p_1/(\omega_1+m)$, $\gamma=1/(1-\beta^2)^{1/2}$, and $\beta_N=P_2/E_2$.

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Use of Spin-Density Matrix Elements to Test Feynman-Diagram Models of Reggeization

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In the Van Hove–Durand model of Reggeization, as extended by Blankenbecler and Sugar to include particles with spin, it is shown that the spin-density matrix elements of ρ mesons produced by scattering pions off nucleons have the property that $(\rho_{11}-\rho_{1,-1})/\rho_{00}$, in the high-energy limit, is dependent only on abnormal-parity Reggeons, of which the pion is the dominant contributor. Combining this result with the relation $\rho_{00}(\rho_{11}-\rho_{1,-1})\simeq2(\text{Re}\rho_{10})^2$, which is shown to hold in the same limit, it is concluded that the quantity $(\text{Re}\rho_{10})/\rho_{00}$ should depend mainly on pion exchange. Using this latter ratio, it is shown that neither minimal nor maximal derivative coupling alone in the model is consistent with data at 8-GeV/c pion momentum.

I. INTRODUCTION

ONE of the outstanding problems in Regge-pole phenomenology is how Reggeons couple to other Reggeons and to particles with spin. A rather interesting model of Reggeization which contains information on such couplings is that based on Feynman diagrams originally proposed by Van Hove[1] and Durand[2] for spinless particles and extended to particles with spin by Blankenbecler and Sugar.[3] The purpose of the present work is to investigate this model by applying it to the calculation of the ρ-meson spin-density matrix elements in $\pi^\pm p\to\rho^\pm p$ reactions at high energies. Actually, because of nonuniqueness of the couplings, only certain functions of the spin-density matrix elements are compared to experiment.

Specifically, it is shown that the quantity $(\rho_{11}-\rho_{1,-1})/\rho_{00}$ is independent of the known normal-parity Reggeons at high energies and thereby measures pion exchange. Combining this with the relation $\rho_{00}(\rho_{11}-\rho_{1,-1})\simeq2(\text{Re}\rho_{10})^2$, which is shown also to hold at high energies, the quantity $(\text{Re}\rho_{10})/\rho_{00}$ is then seen to measure pion exchange. Independently of these remarks, this latter ratio for separate minimal and maximal derivative coupling is compared to experimental data[4,5] at 8-GeV/c pion laboratory momentum. Both couplings give disagreement with the data, indicating that a mixture of the two is needed.

The analysis presented here is applicable to the spin-density matrix elements of any vector meson produced by scattering high-energy pseudoscalar mesons off nucleons. Charged ρ mesons are chosen for investigation simply because a reasonable amount of data[4,5] at a high energy is available.

In Sec. II the model is defined and the high-energy limits of the spin-density matrix elements are found. Section III then contains a comparison with the data. Presumably some of the results found in Sec. II are model-independent and could therefore be used to test other models of Regge couplings.

II. SPIN-DENSITY MATRIX ELEMENTS IN THE MODEL

In the Van Hove–Durand model,[1,2] Regge behavior is viewed as being generated by an infinite sum of Born diagrams each containing the exchange of an elementary

[1] L. Van Hove, Phys. Letters **24B**, 183 (1967).
[2] L. Durand, III, Phys. Rev. **154**, 1537 (1967); **161**, 1610 (1967).
[3] R. Blankenbecler and R. L. Sugar, Phys. Rev. **168**, 1597 (1968).
[4] Aachen-Berlin-CERN Collaboration, Phys. Letters **22**, 533 (1966).
[5] I. Derado, J. A. Poirier, N. N. Biswas, N. M. Cason, V. P. Kenney, and W. D. Shepard, Phys. Letters **24B**, 112 (1967).

particle or resonance of different fixed spin. The prototype diagram is shown in Fig. 1, and the model is fully defined by specification of the vertex couplings. Let $\pi(x)$, $\rho_\mu(x)$, $\psi(x)$, $A_{\mu_1\mu_2\cdots\mu_J}(x)$, and $N_{\mu_1\mu_2\cdots\mu_J}(x)$ denote the fields of the pion, the ρ meson, the proton, the abnormal-parity $[P=-(-1)^J]$ exchanged particle of spin J, and the normal-parity $[P=(-1)^J]$ exchanged particle of spin J, respectively. Then the effective interaction (Hermitian) Hamiltonian to be used for the reaction $\pi^\pm p\to\rho^\pm p$ is[6]

$$
\begin{aligned}
H_I=g(A,\rho,\min)\rho_{\mu_1}\overleftrightarrow{\partial}_{\mu_2}\overleftrightarrow{\partial}_{\mu_3}\cdots\overleftrightarrow{\partial}_{\mu_J}\pi A_{\mu_1\mu_2\cdots\mu_J}&\\
+g(A,\rho,\max)\rho_\nu\overleftrightarrow{\partial}_{\mu_1}\overleftrightarrow{\partial}_{\mu_2}\cdots\overleftrightarrow{\partial}_{\mu_J}\pi\overleftrightarrow{\partial}_\nu A_{\mu_1\mu_2\cdots\mu_J}+g(N,\rho)\epsilon_{\alpha\beta\gamma\mu_1}\rho_\alpha\overleftrightarrow{\partial}_\beta\overleftrightarrow{\partial}_{\mu_2}\overleftrightarrow{\partial}_{\mu_3}\cdots\overleftrightarrow{\partial}_{\mu_J}\pi\overleftrightarrow{\partial}_\gamma N_{\mu_1\mu_2\cdots\mu_J}&\\
+i^{J-1}g(A,p,\min)A_{\mu_1\mu_2\cdots\mu_J}\bar\psi\gamma_5\gamma_{\mu_1}\overleftrightarrow{\partial}_{\mu_2}\overleftrightarrow{\partial}_{\mu_3}\cdots\overleftrightarrow{\partial}_{\mu_J}\psi+i^{J+1}g(A,p,\max)A_{\mu_1\mu_2\cdots\mu_J}\bar\psi\gamma_5\overleftrightarrow{\partial}_{\mu_1}\overleftrightarrow{\partial}_{\mu_2}\cdots\overleftrightarrow{\partial}_{\mu_J}\psi&\\
+i^{J-1}g(N,p,\min)N_{\mu_1\mu_2\cdots\mu_J}\bar\psi\gamma_{\mu_1}\overleftrightarrow{\partial}_{\mu_2}\overleftrightarrow{\partial}_{\mu_2}\cdots\overleftrightarrow{\partial}_{\mu_J}\psi+i^Jg(N,p,\max)N_{\mu_1\mu_2\cdots\mu_J}\bar\psi\overleftrightarrow{\partial}_{\mu_1}\overleftrightarrow{\partial}_{\mu_2}\cdots\overleftrightarrow{\partial}_{\mu_J}\psi,&\quad(1)
\end{aligned}
$$

where

$$\overleftrightarrow{\partial}_\mu\equiv\tfrac{1}{2}(\overrightarrow{\partial}_\mu-\overleftarrow{\partial}_\mu)$$

and

$$
\begin{aligned}
\epsilon_{\alpha\beta\gamma\delta}&=+1\ (-1)\text{ for }\alpha,\beta,\gamma,\delta\text{ an even (odd) permutation of }0,1,2,3\\
&=0\text{ for two or more indices equal.}
\end{aligned}
$$

The labeling on the coupling parameters should be clear except perhaps for the min-max notation, which signifies minimal-maximal derivative coupling, respectively. Taking the meson fields to be Hermitian, these parameters are real; they are also viewed as depending implicitly on J (necessary for ghost-eliminating purposes[3]).

With the kinematics of Fig. 1 and with the following definitions:

$$
\begin{gathered}
Q=\tfrac{1}{2}(p_1+p_2),\quad Q'=\tfrac{1}{2}(q_1+q_2),\\
P=p_1-p_2=q_2-q_1,\quad s=(p_1+q_1)^2,\quad t=P^2,\\
u=(p_2-q_1)^2,\quad p_i^2=m^2,\quad q_i^2=\mu_i^2,
\end{gathered}\qquad(2)
$$

the required amplitude may be constructed. Letting S_1 and S_2 denote the spin projections of the initial and final proton, respectively, and M the spin projection of the ρ meson, the Born amplitude for diagrams like Fig. 1 coming from (1) is

$$
\begin{aligned}
f(J,M,S_1,S_2)=f_A(J,M,S_1,S_2)/[t-M_A^2(J)]&\\
+f_N(J,M,S_1,S_2)/[t-M_N^2(J)],&\quad(3)
\end{aligned}
$$

where

$$
\begin{aligned}
f_A(J,M,S_1,S_2)=&\tfrac{1}{2}\bar u(p_2,S_2)[g(A,\rho,\min)\epsilon_{\mu_1}{}^{M*}\\
&+g(A,\rho,\max)\epsilon_\alpha{}^{M*}q_{1\alpha}Q_{\mu_1}{}']Q_{\mu_2}{}'Q_{\mu_3}{}'\cdots Q_{\mu_J}{}'\Gamma_{\mu;\nu}{}^J\\
&\times i^{J-1}[-g(A,p,\min)\gamma_{\nu_1}+ig(A,p,\max)Q_{\nu_1}]\\
&\times\gamma_5Q_{\nu_2}Q_{\nu_3}\cdots Q_{\nu_J}u(p_1,S_1)
\end{aligned}
$$

and

$$
\begin{aligned}
f_N&(J,M,S_1,S_2)\\
&=-\tfrac{1}{2}\bar u(p_2,S_2)[g(N,\rho)\times\tfrac{3}{4}\epsilon_{\alpha\beta\gamma\mu_1}\epsilon_\alpha{}^{M*}Q_\beta{}'P_\gamma]\\
&\quad\times Q_{\mu_2}{}'Q_{\mu_3}{}'\cdots Q_{\mu_J}{}'\Gamma_{\mu;\nu}{}^Ji^{J-1}[g(N,p,\min)\gamma_{\nu_1}\\
&\quad+g(N,p,\max)Q_{\nu_1}]Q_{\nu_2}Q_{\nu_3}\cdots Q_{\nu_J}u(p_1,S_1),
\end{aligned}
$$

where $(-1)^J\Gamma_{\mu;\nu}{}^J\equiv(-1)^J\Gamma_{\mu_1\mu_2\cdots\mu_J;\nu_1\nu_2\cdots\nu_J}{}^J$ is the propagator of a spin-J particle.[2,3,6] The convention for Dirac spinors and matrices is that of Bjorken and Drell.[7]

To Reggeize the amplitude a sum over J is taken, a Sommerfeld-Watson transformation is made, and Regge poles located at the zeros of $t-M^2(J)$ are picked up.[1,3] Retaining only the leading trajectories of each type (normal and abnormal parity), the result is

$$
\begin{aligned}
F(M,S_1,S_2)=f_A(\alpha_\pi,M,S_1,S_2)\frac{d\alpha_\pi}{dt}\frac{\pi}{\sin(\pi\alpha_\pi)}e^{i\pi\alpha_\pi}&\\
+f_N(\alpha_\omega,M,S_1,S_2)\frac{d\alpha_\omega}{dt}\frac{\pi}{\sin(\pi\alpha_\omega)}e^{i\pi\alpha_\omega},&\quad(4)
\end{aligned}
$$

where the π and ω mesons have been selected as the dominant abnormal- and normal-parity contributors, respectively.[8] Signature which has so far been ignored will added later in an *ad hoc* fashion.

In defining and calculating the spin-density matrix elements, it is convenient to use the Gottfried-Jackson frame[9] shown in Fig. 2. In this frame, the ρ meson is at rest, the z axis is chosen in the direction of the incoming pion, and the y axis is chosen in the direction of the

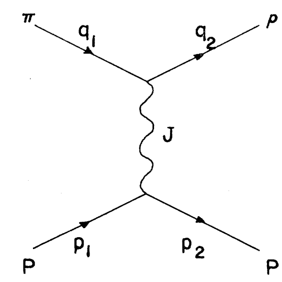

FIG. 1. Prototype diagram for the generation of Regge behavior.

[6] M. D. Scadron, Phys. Rev. **165**, 1640 (1968). Only on-shell couplings are used in the present work.

[7] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill Book Co., New York, 1965).

[8] For a discussion see J. D. Jackson, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (Universtity of California Press, Berkeley, 1967), p. 149.

[9] K. Gottfried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964).

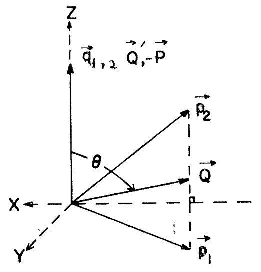

FIG. 2. The Gottfried-Jackson frame for the analysis of the spin-density matrix elements of the ρ meson.

normal ($\mathbf{p}_1\times\mathbf{p}_2$) to the production plane. The spin-density matrix elements are then

$$\rho_{MM'}=\frac{1}{N}\sum_{S_1,S_2}F(M,S_1,S_2)F(M',S_1,S_2)^\dagger\,, \qquad (5)$$

where N is chosen to make $\sum_M\rho_{MM}=1$.

To compute the asymptotic behavior of these $\rho_{MM'}$, the corresponding forms of some kinematic quantities in this frame are needed. In terms of the variables (2), it is simple to show the following:

$$\begin{aligned}
|\mathbf{q}_1|&=\lambda(t,u_1^2,\mu_2^2)^{1/2}/2\mu_2\,,\\
|\mathbf{Q}|&=[2\lambda(u,m^2,\mu_2^2)+2\lambda(s,m^2,\mu_2^2)\\
&\qquad-\lambda(t,\mu_1^2,\mu_2^2)]^{1/2}/4\mu_2\to s/2\mu_2\,,\\
\cos\theta&=[\lambda(s,m^2,\mu_2^2)-\lambda(u,m^2,\mu_2^2)]/8\mu_2^2|\mathbf{q}_1|\,|\mathbf{Q}|\\
&\qquad\to-(t+\mu_2^2-\mu_1^2)/\lambda(t,\mu_1^2,\mu_2^2)^{1/2}\,,\\
Q'\cdot Q&=\tfrac{1}{4}(s-u)\to\tfrac{1}{2}s\,,\\
\epsilon^M\cdot Q&=-|\mathbf{Q}|\cos\theta\to s(t+\mu_2^2-\mu_1^2)/\qquad\qquad (6)\\
&\qquad[2\mu_2\lambda(t,\mu_1^2,\mu_2^2)^{1/2}]\quad\text{for } M=0\\
&=\mp|\mathbf{Q}|\sin\theta/\sqrt{2}\to\mp s(-2\mu_2^2t)^{1/2}/\\
&\qquad[2\mu_2\lambda(t,\mu_1^2,\mu_2^2)^{1/2}]\quad\text{for } M=\pm1\,,\\
\epsilon^M\cdot q_1&=-|\mathbf{q}_1|\delta_{M0}\,,\\
\epsilon_{\alpha\beta\gamma\delta}\epsilon_\alpha{}^MQ_\beta'P_\gamma Q_\delta&=\mu_2|\mathbf{q}_1|\,|\mathbf{Q}|\sin\theta\epsilon_2{}^M\to\tfrac{1}{2}s(-t)^{1/2}\epsilon_2{}^M\\
&=-i2^{-3/2}s(-t)^{1/2}\delta_{|M|1}\,,
\end{aligned}$$

where

$$\lambda(x,y,z)=x^2+y^2+x^2-2xy-2yz-2zx\,,$$

and where the polarization basis vectors of the ρ meson are

$$\epsilon_\mu{}^1=-(\sqrt{\tfrac{1}{2}})(0,1,i,0)\,,$$
$$\epsilon_\mu{}^{-1}=(\sqrt{\tfrac{1}{2}})(0,\,1,\,-i,\,0),\quad \epsilon_\mu{}^0=(0,0,0,1)\,.$$

Next, recognizing that the dominant part of (4) comes from those terms in $\Gamma_{\mu;\nu}{}^J$ of the form $g_{\mu_i\nu_j}g_{\mu_k\nu_l}\cdots g_{\mu_m\nu_m}$ (J factors), the asymptotic form of the spin-density matrix elements (5) may be determined straightforwardly. After carrying this calculation out it is found that $\sum_{S_1,S_2}F(M,S_1,S_2)F(M',S_1,S_2)^\dagger$ splits into two terms, one dependent only on pion exchange and the other only on ω exchange. This allows a corresponding decomposition of the spin-density matrix elements:

$$\rho_{MM'}\to\rho_{MM'}(\pi)+\rho_{MM'}(\omega)\quad\text{as } s\to\infty\,,$$

with

$$\begin{aligned}
&\rho_{MM'}(\pi)/N\\
&=\{[g(A,p,\min)]^2-\tfrac{1}{4}[g(A,p,\max)]^2t\}\\
&\quad\times[g(A,\rho,\min)(\epsilon^{M*}\cdot Q)_a+g(A,\rho,\max)(\epsilon^{M*}\cdot q_1)_a(\tfrac{1}{2}s)]\\
&\quad\times[g(A,\rho,\min)(\epsilon^{M'}\cdot Q)_a+g(A,\rho,\max)\\
&\qquad\times(\epsilon^{M'}\cdot q_1)_a(\tfrac{1}{2}s)](\tfrac{1}{2}s)^{2\alpha_\pi-2}|\sigma_\pi|^2,\qquad (7)\\
&\rho_{MM'}(\omega)/N\\
&=\{[g(N,p,\min)]^2+2mg(N,p,\min)g(N,p,\max)\\
&\quad+(m^2-\tfrac{1}{4}t)[g(N,p,\max)]^2\}[\epsilon_{\alpha\beta\gamma\delta}\epsilon_\alpha{}^{M*}Q_\beta'P_\gamma Q_\delta]_a\\
&\quad\times[\epsilon_{\alpha\beta\gamma\delta}\epsilon_\alpha{}^{M'}Q_\beta'P_\gamma Q_\delta]_a(\tfrac{1}{2}s)^{2\alpha_\omega-2}\\
&\qquad\times(\tfrac{3}{4})^2g(N,\rho)^2|\sigma_\omega|^2,
\end{aligned}$$

where

$$\sigma_i=\left(\frac{d\alpha_i}{dt}\right)\left(\frac{\pi}{\sin\pi\alpha_i}\right)e^{i\pi\alpha_i}\frac{(1+\tau_ie^{i\pi\alpha_i})}{2^{3/2}m}$$

for $i=\pi$, ω with signature added[10]: $\tau_\pi=1$, $\tau_\omega=-1$. The subscript a on a quantity indicates that its asymptotic form as given in (6) is to be used.

In deriving the above expressions, terms of order $s^{\alpha_\pi+\alpha_\omega-1}$, coming from the interference between π and ω exchange, have been dropped from $\rho_{1,\pm1}/N$ and ρ_{10}/N, while terms of order $s^{2\alpha_\omega-2}$, coming from ω exchange, have been dropped from $\rho_{1,\pm1}/N$, both in comparison to terms of order $s^{2\alpha_\pi}$. This is justified by the fact that

$$\alpha_\omega-1<\alpha_\pi\,. \qquad (8)$$

The $\rho_{MM'}/N$ in (7) are therfore correct to order $s^{2\alpha_\pi}$.

From (7) it is easy to see that the following relations among the spin-density matrix elements hold, where arrows denote the high-energy limit:

$$\begin{aligned}
\rho_{00}/N\to\rho_{00}(\pi)/N,\quad \rho_{11}/N&\to\rho_{11}(\pi)/N+\rho_{11}(\omega)/N\,,\\
\rho_{10}/N&\to\rho_{10}(\pi)/N\,,\qquad (9)
\end{aligned}$$

$$\rho_{1,-1}(\pi)/N=-\rho_{11}(\pi)/N,\ \text{and}\ \rho_{1,-1}(\omega)/N=\rho_{11}(\omega)/N\,.$$

Thus, at high energy,

$$(\rho_{11}-\rho_{1,-1})/\rho_{00}\simeq[\rho_{11}(\pi)-\rho_{1,-1}(\pi)]/\rho_{00}(\pi)\qquad (10)$$

and is independent of ω exchange. Consequently, although in the model $\rho_{00},\rho_{10}\to0$, $\rho_{11},\rho_{1,-1}\to0.5$ at infinite energy because of the dominance of ω exchange, they do so in a manner such that (10) holds. The ratio (10) therefore offers a means of checking on pion exchange independently of the presence of ω exchange.

A further interesting relation follows in a straight-

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[10] Signature is incorporated in the asymptotic limit simply by a multiplicative factor $\frac{1}{2}(1+\tau_ie^{i\pi\alpha_i})$. See, for example, R. Blankenbecler, R. L. Sugar, and J. D. Sullivan, Phys. Rev. **172**, 1451 (1968).

forward manner from (7) and (9), namely,[11]

$$\rho_{00}(\rho_{11}-\rho_{1,-1})\simeq 2|\rho_{10}|^2\simeq 2(\mathrm{Re}\rho_{10})^2. \qquad (11)$$

Putting (10) and (11) together, it is seen that

$$\frac{\rho_{11}(\pi)-\rho_{1,-1}(\pi)}{\rho_{00}(\pi)}\simeq\frac{2(\mathrm{Re}\rho_{10})_2}{\rho_{00}{}^2},$$

and, consequently, that $(\mathrm{Re}\rho_{10})/\rho_{00}$ at high energy also measures the pion contribution.

Some of the relations just derived hold under more general circumstances. In particular, (9) and (10) are valid when any number of normal- and abnormal-parity trajectories, denoted collectively by ω and π, respectively, are present provided that there is not much dispersion in each set, since generalizations of conditions like (8) must hold. On the other hand, the validity of (11) requires in addition that only one abnormal-parity trajectory be present.

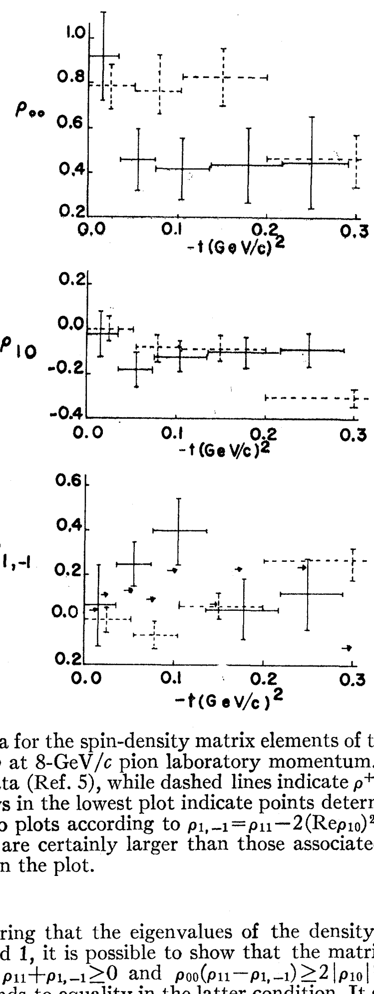

FIG. 3. Data for the spin-density matrix elements of the ρ meson in $\pi^\pm p\to\rho^\pm p$ at 8-GeV/c pion laboratory momentum. Full lines indicate ρ^- data (Ref. 5), while dashed lines indicate ρ^+ data (Ref. 4). The arrows in the lowest plot indicate points determined from the upper two plots according to $\rho_{1,-1}=\rho_{11}-2(\mathrm{Re}\rho_{10})^2/\rho_{00}$. Their uncertainties are certainly larger than those associated with the other points in the plot.

[11] By requiring that the eigenvalues of the density matrix lie between 0 and 1, it is possible to show that the matrix elements must satisfy $\rho_{11}+\rho_{1,-1}\geq 0$ and $\rho_{00}(\rho_{11}-\rho_{1,-1})\geq 2|\rho_{10}|^2$. Relation (11) corresponds to equality in the latter condition. It should also be noted that $(\rho_{11}-\rho_{1,-1})/\rho_{00}\geq 2(\mathrm{Re}\rho_{10})^2/\rho_{00}{}^2$, rigorously.

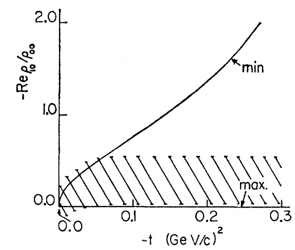

FIG. 4. The cross-hatched area indicates the location of the data from Fig. 3. The theoretical predictions for minimal (min) and maximal (max) derivative coupling in the model are shown.

III. APPLICATION

Relations derived above are now to be applied to the reaction $\pi^\pm p\to\rho^\pm p$ at 8-GeV/c pion laboratory momenta. Data for both ρ^+ and ρ^- are shown together in Fig. 3. This joint plot is made because the density matrix elements for the two processes should differ only with respect to the interference term between π and ω exchange,[12] and this term was shown in the last section not to contribute in the high-energy limit. The arrows in the $\rho_{1,-1}$ plot are the points in the ρ_{00} and ρ_{11} plots reflected by use of (11). The reasonable location of these arrows shows that within the large experimental uncertainties (11) holds.

From the general nature of these plots, two interesting conclusions can be drawn. First, since $\rho_{1,-1}\geq 0$, it follows from (9) that ω exchange must be present in the model. Second, since $\rho_{00},\rho_{10}\neq 0$, it follows from (3) that π exchange must be present (because $f_N=0$ if $M=0$). This second point is well known.[13]

Turning next to a check of basic couplings, a plot of $(\mathrm{Re}\rho_{10})/\rho_{00}$ is made in Fig. 4, with the cross-hatched area indicating the possible spread of the experimental points. This is to be compared with the model for both minimal and maximal derivative coupling. These theoretical predictions are apparent from (7):

$$\frac{(\mathrm{Re}\rho_{10})}{\rho_{00}}\to\frac{(\epsilon^{1*}\cdot Q)_a}{(\epsilon^0\cdot Q)_a}=\frac{-(-2\mu_2{}^2t)^{1/2}}{(t+\mu_2{}^2-\mu_1{}^2)}\quad\text{for minimal derivative coupling}$$

$$\to 0 \text{ for maximal derivative coupling}$$

and are shown in Fig. 4. Clearly, the minimal derivative coupling case is ruled out. The maximal derivative coupling curve might be questioned in view of the fact that asymptotic formulas have been used. However, an exact calculation with maximal derivative coupling at all vertices shows that $\rho_{00}\neq 0$, $\rho_{11}=\rho_{1,-1}$, and $\rho_{10}=0$. Thus $(\mathrm{Re}\rho_{10})/\rho_{00}\equiv 0$ and the data rule out this case. It then appears that the correct field coupling in the model must be a combination of minimal and maximal derivative coupling.

[12] See Ref. 8, p. 159.

[13] J. D. Jackson, Nuovo Cimento 34, 1644 (1964).