Asymptotic Symmetry, Lagrangian Gauge Model, and the *PVV* Vertex*

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We study a model in which $SU(3)$ breaking in the PVV vertex is determined by the requirement of asymptotic nonet symmetry. A vector-meson gauge model is constructed which satisfies this asymptotic symmetry condition. The model also incorporates the usual asymptotic symmetry result for the VPP vertex, the field-current identities, and the algebra of fields. Nevertheless, we obtain a second Weinberg sum rule of the Das-Mathur-Okubo form, and consequently, a quadratic mass formula as in a mass-mixing model. The predictions of the model are compared with available experimental data on meson decays involving the PVV vertex. The predicted rates for radiative decays of vector mesons are also given.

It has been intimated[1,2] that any gauge-field theory that incorporates the field-current identities and the algebra of currents must lead to an inverse square mass formula. Furthermore, in such a model, the required renormalization leads to a symmetry breaking in the PVV vertex proportional to the vector-meson masses.[3] In the present article we give a simple argument based on asymptotic symmetry leading to a breaking in the PVV couplings proportional to the inverse masses. By construction we demonstrate the existence of a Lagrangian model that contains the usual field-current identities and the algebra of currents as well as this latter form of splitting in the PVV vertex. The model leads to the two Weinberg sum rules,[4,5] providing, therefore, a general agreement with asymptotic symmetry considerations.

In Sec. I, we introduce our notation by recasting the Weinberg sum rules in a convenient form. An asymptotic symmetry condition for the PVV vertex is formulated in Sec. II, and in Sec. III we present our Lagrangian gauge-field model. The important distinction of this model relative to Refs. 1–3 is that $G_{\mu\nu}$ is treated as an independent field under the variation of the Lagrangian and no derivatives occur except in the kinetic term. In Secs. IV and V, we compare with experiment and discuss our results.

I. INTRODUCTION

We begin by defining the vector-current spectral function $\rho_{ab}(m^2)$ according to the equation

$$-i\int d^4x\, e^{-iq\cdot x}\langle 0|T(J_\mu{}^a(x)\tilde{J}_\nu{}^b(0))|0\rangle = \int dm^2\rho_{ab}(m^2)\left(\frac{g_{\mu\nu}+q_\mu q_\nu/m^2}{q^2+m^2-i\epsilon}+\frac{g_{\mu 0}g_{\nu 0}}{m^2}\right), \quad (1)$$

where we ignore scalar contributions to the propagator of the currents and the usual $SU(3)$ commutation relations are assumed to hold. We may write the above equation in matrix notation as

$$-i\int d^4x\, e^{-iq\cdot x}\langle 0|T(J_\mu(x)\tilde{J}_\nu(0))|0\rangle = \int dm^2\rho(m^2)\left(\frac{g_{\mu\nu}+q_\mu q_\nu/m^2}{q^2+m^2-i\epsilon}+\frac{g_{\mu 0}g_{\nu 0}}{m^2}\right), \quad (2)$$

where $\rho(m^2)$ is now a 9×9 matrix.

The Weinberg sum rules can be written in the form

$$\int \rho(m^2)\frac{dm^2}{m^2}=(m_0{}^2/g_0{}^2)1\,, \quad (3)$$

$$\int \rho(m^2)dm^2=(m_0{}^2/g_0{}^2)B\,, \quad (4)$$

where m_0 and g_0 are arbitrary constants and B is a symmetry-breaking matrix whose most general form consistent with isospin invariance is

$$B_{ab}=b_0\delta_{ab}+\sqrt{3}b_1d_{8ab}+b_2\delta_{a8}\delta_{b8}+b_3\delta_{a0}\delta_{b0}+b_4(\delta_{a0}\delta_{b8}+\delta_{a8}\delta_{b0})\,. \quad (5)$$

The indices a, b run from 0 to 8 with $d_{808}=\sqrt{\tfrac{2}{3}}$. These sum rules may be regarded as a consequence of an asymptotic symmetry in Eq. (1). In the presence of symmetry breaking, ρ can have off-diagonal elements. However, it can be put into diagonal form:

$$\rho=S\rho'\tilde{S}\,, \quad (6)$$

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where ρ' is of the form[6] $\rho_\alpha'\delta_{\alpha\beta}$ and in the single-particle saturation picture

$$\rho_\alpha' = (m_0^4/g_0^2)\delta(m^2 - m_\alpha^2). \quad (7)$$

In general, S is not unitary.

We must also define the (diagonal) physical vector-meson mass matrix

$$M_{\alpha\beta} \equiv m_\alpha \delta_{\alpha\beta}. \quad (8)$$

In terms of the matrices M and S, the spectral-function sum rules can be written

$$m_0^2 S M^{-2} \tilde{S} = 1, \quad (9)$$

$$m_0^2 S \tilde{S} = B. \quad (10)$$

Finally, we define the matrix U:

$$U \equiv S M^{-1} m_0. \quad (11)$$

From Eq. (9), U is unitary:

$$U\tilde{U} = \tilde{U}U = 1. \quad (12)$$

From Eq. (10) we see that U is the unitary matrix that diagonalizes B:

$$M^2 = \tilde{U} B U. \quad (13)$$

U is, therefore, the unit matrix in the subspace of the ρ and the K^* and is the 2×2 orthogonal matrix in the space of ω and ϕ:

$$U = \begin{array}{c} \\ \\ \\ 8 \\ 0 \end{array} \left(\begin{array}{cc|cc} \rho & K^* & \phi & \omega \\ 1 & & & \\ & 1 & & \\ \hline & & \cos\theta & \sin\theta \\ & & -\sin\theta & \cos\theta \end{array} \right). \quad (14)$$

Through Eq. (13) all the b's of Eq. (5) can be solved in terms of the physical masses of ρ, ω, K^*, and ϕ and the angle θ:

$$b_0 + b_1 = m_\rho^2, \quad (15)$$

$$b_0 - \tfrac{1}{2} b_1 = m_{K^*}^2, \quad (16)$$

$$b_2 - b_1 - b_3 = (m_\phi^2 - m_\omega^2)\cos 2\theta, \quad (17)$$

$$2b_0 - b_1 + b_2 + b_3 = m_\phi^2 + m_\omega^2, \quad (18)$$

$$\sqrt{2}b_1 + b_4 = -\tfrac{1}{2}(m_\phi^2 - m_\omega^2)\sin 2\theta. \quad (19)$$

Any *a priori* relations assumed among the b's lead to relations between the physical masses and θ. For example, the nonet symmetry assumption that $b_2 = b_3 = b_4 = 0$ leads to the relations

$$\tan\theta = 1/\sqrt{2}, \quad (20)$$

$$m_\phi^2 + m_\omega^2 = 2m_{K^*}^2, \quad (21)$$

$$m_\rho^2 = m_\omega^2. \quad (22)$$

TABLE I. Mass and mixing-angle predictions for various simple symmetry-breaking models. The errors on the last three entries correspond to the assumption $m_\rho = 770\pm10$ MeV. $B_{ab} = b_0\delta_{ab} + \sqrt{3}b_1 d_{8ab} + b_2\delta_{a8}\delta_{b8} + b_3\delta_{a0}\delta_{b0} + b_4(\delta_{a8}\delta_{b0} + \delta_{a0}\delta_{b8})$.

Okubo[a]	$b_2 = b_3 = b_4 = 0$	$\theta = \theta_0 = 35.26°$	$m_\rho^2 = m_\omega^2$, $2m_{K^*}^2 = m_\omega^2 + m_\phi^2$
Schwinger[b]	$b_2 = b_4 = 0$	$\theta = 39.3$	$m_\rho = 760$ MeV
27 model[c]	$b_3 = \frac{1}{8}b_2$, $b_4 = -\frac{1}{5}\sqrt{2}b_2$	$\theta = 32.8°$	$m_\rho = 778$ MeV
	$b_2 = b_3 = 0$	$\theta = 35.8$	$2m_{K^*}^2 = m_\omega^2 + m_\phi^2$
	$b_3 = b_4 = 0$	$\theta = 31.5°$	$m_\rho = 771$ MeV
Rosenfeld[d]	$b_2 = 0$	$\theta = 39.9° \pm 1.1°$	
	$b_3 = 0$	$\theta = 31.3° \pm 0.6°$	
	$b_4 = 0$	$\theta = 32.5° \pm 4.0°$	

[a] See Ref. 7.
[b] See Ref. 8.
[c] See Ref. 9.
[d] See the mixing angle quoted in Ref. 10.

Other symmetry-breaking models[7–10] can also be formulated in terms of assumptions about the b's in Eq. (5). The predictions of some of these models are summarized in Table I. We note that the mixing angle is to be regarded as an experimental parameter on a par with, and in general independent of, the empirical masses.

We may now define the field-current coupling matrix:

$$\langle 0 | J_\mu{}^a | \phi^\alpha \rangle \equiv (g^{-1}M^2)_{a\alpha}\epsilon_\mu. \quad (23)$$

In terms of pole dominance, the spectral function of Eq. (1) can be written

$$\rho_{ab}(m^2) = \sum_\alpha (g^{-1}M^2)_{a\alpha}\delta(m^2 - m_\alpha^2)(M^2\tilde{g}^{-1})_{\alpha b}. \quad (24)$$

Comparing this with Eqs. (6) and (7), we have

$$g^{-1}M^2 = S(m_0^2/g_0), \quad (25)$$

$$g^{-1}M = (m_0/g_0)U. \quad (26)$$

Using Eqs. (14) and (26), we see that

$$\frac{m_0}{g_0} = \frac{m_\rho}{g_\rho} = \frac{m_{K^*}}{g_{K^*}} = (g^{-1})_{0\omega}m_\omega(\cos\theta)^{-1} = (g^{-1})_{8\omega}m_\omega(\sin\theta)^{-1}$$
$$= (g^{-1})_{8\phi}m_\phi(\cos\theta)^{-1} = -(g^{-1})_{0\phi}m_\phi(\sin\theta)^{-1}. \quad (27)$$

Equation (27) is the usual asymptotic symmetry or current-mixing result for the VPP vertex assuming vector dominance. Equations (23)–(25) are equivalent

[6] Here and in the following the early italic letters (a, b, c, etc.) are used for $SU(3)$ indices running from 0 to 8; early Greek letters (α, β, γ, etc.) denote the physical vector mesons ρ, ω, K^*, ϕ, and the late Greek letters (μ, ν, etc.) are reserved for space-time indices.

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[9] L. Clavelli and R. Torgerson, Nuovo Cimento (to be published).
[10] Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).

to writing a matrix field-current identity[11]

$$J^\mu=(m_0^2/g_0)S\Phi^\mu, \tag{28}$$

where Φ is the 9-vector containing the renormalized ρ, K^*, ϕ, and ω fields.

II. ASYMPTOTIC SYMMETRY

We may now require asymptotic symmetry in the matrix elements of the current-current product between vacuum and the one-pseudoscalar state. We assume a unitary transformation between the physical pseudoscalar mesons and the $SU(3)$ eigenstates. This is equivalent to a mass-mixing model for the pseudoscalars:

$$P^a=(U_p)^{a\alpha}P^\alpha,$$

with

$$U_p=\begin{array}{r} \\ \\ \\ 8 \\ 0 \end{array}\left[\begin{array}{c:c:cc} \pi & K & \eta & \eta' \\ 1 & & & \\ \hdashline & 1 & & \\ \hdashline & & \cos\gamma & \sin\gamma \\ & & -\sin\gamma & \cos\gamma \end{array}\right]. \tag{29}$$

We now consider

$$U_p{}^{c\gamma}\int d^4x\,d^4y\,e^{-iq\cdot x+ik\cdot y}\langle P^\gamma|T(J_\mu{}^a(x)\tilde{J}_\nu{}^b(y))|0\rangle \equiv (H^c)_{ab}\epsilon_{\mu\nu\sigma\tau}q_\sigma k_\tau, \tag{30}$$

where H is a form factor depending on k^2, q^2, and $\mathbf{k}\cdot\mathbf{q}$. Since H has poles in both k^2 and q^2, we will take asymptotic symmetry in the present context to mean

$$\lim_{k^2,q^2\to\infty}k^2q^2H^c=h_0(m_0^4/g_0^2)D^c, \tag{31}$$

where D^c is a matrix whose elements are the Gell-Mann d's:

$$(D^c)_{ab}=d_{acb}, \tag{32}$$

and h_0 is a new constant. From Eqs. (28) and (30), we have

$$U_p{}^{c\gamma}\int d^4x\,d^4y\,e^{-iq\cdot x+ik\cdot y}\langle P^\gamma|T(\Phi_\mu{}^\alpha(x)\tilde{\Phi}_\nu{}^\beta(y))|0\rangle =(g_0^2/m_0^4)(S^{-1}H^c\tilde{S}^{-1})_{\alpha\beta}\epsilon_{\mu\nu\sigma\tau}q_\sigma k_\tau, \tag{33}$$

but

$$\int d^4x\,d^4y\,e^{-iq\cdot x+ik\cdot y}\langle P^\gamma|T(\Phi_\mu{}^\alpha(x)\tilde{\Phi}_\nu{}^\beta(y))|0\rangle =(h^\gamma)_{\alpha\beta}\frac{\epsilon_{\mu\nu\sigma\tau}q_\sigma k_\tau}{(q^2+m_\alpha{}^2)(k^2+m_\beta{}^2)}, \tag{34}$$

where we have defined the PVV coupling matrix

$$(h^\gamma)_{\alpha\beta}\equiv h_{P^\gamma V^\alpha V^\beta}.$$

Assuming that h^γ is a momentum-independent coupling constant, we have from Eqs. (31), (33), and (34) the asymptotic symmetry condition

$$(h^\gamma)_{\alpha\beta}=h_0\sum_c(\tilde{U}_p)^{\gamma c}(S^{-1}D^c\tilde{S}^{-1})_{\alpha\beta}. \tag{35}$$

Using Eq. (11), this may be written

$$(Mh^\gamma M)_{\alpha\beta}=m_0^2h_0\sum_c(\tilde{U}_p)^{\gamma c}(U^{-1}D^cU)_{\alpha\beta}. \tag{36}$$

Equation (36) is the PVV analog of the asymptotic symmetry condition, Eq. (26) for the VPP vertex. We have more explicitly

$$h_{P^\gamma V^\alpha V^\beta}=\frac{h_0m_0^2}{m_\alpha m_\beta}\sum_{abc}(\tilde{U}_p)^{\gamma c}(\tilde{U})^{\alpha a}d_{acb}(U)^{b\beta}. \tag{37}$$

The factor $m_0^2/m_\alpha m_\beta$ gives the $SU(3)$ symmetry breaking. In general, Eq. (37) implies

$$h_{P^\gamma V^\alpha V^\beta}=h_{P^\gamma V^\alpha V^\beta}{}^{(\mathrm{sym})}(m_0^2/m_\alpha m_\beta), \tag{38}$$

where $h^{(\mathrm{sym})}$ is the nonet symmetric value. The usual current-mixing model[3] leads to a breaking, directly (instead of inversely) proportional to the vector-meson masses.

III. LAGRANGIAN MODEL

We now wish to point out the existence of a Lagrangian model that incorporates all the previous results of asymptotic symmetry[12]:

$$\begin{aligned}\mathfrak{L}=&-\tfrac{1}{2}\tilde{G}_0{}^{\mu\nu}[\partial_\mu\phi_\nu{}^0-\partial_\nu\phi_\mu{}^0+ig_0(\tilde{\phi}_\mu{}^0F\phi_\nu{}^0)]\\&+\tfrac{1}{4}\tilde{G}_0{}^{\mu\nu}(B/m_0^2)G_{\mu\nu}{}^0-\tfrac{1}{2}m_0^2\tilde{\phi}_0{}^\mu\phi_\mu{}^0\\&+g_0\tilde{\phi}_0{}^\mu K_\mu+\mathfrak{L}_1(\chi^0)+\mathfrak{L}_2(\chi^0,G_{\mu\nu}{}^0).\end{aligned} \tag{39}$$

Wherever possible, the notation is that of Schwinger.[13] The unrenormalized quantities are denoted by a subscript or superscript zero. When necessary to avoid confusion, the space-time index 0 will be put in parentheses. B is the symmetry-breaking matrix which in the most general case is given by Eq. (5). In the limit of exact symmetry, B/m_0^2 becomes 1. $G_{\mu\nu}{}^0$ and $\phi_\mu{}^0$ are canonical nine-vectors describing the vector mesons. For purposes of varying the Lagrangian, $G_{\mu\nu}{}^0$ is treated as an independent variable. χ stands for any matter field, i.e., any field other than that of the vector mesons. K_μ is the matter current. It is sufficient for our present

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[13] J. Schwinger, Phys. Rev. **125**, 1043 (1962).

purposes to restrict χ to the pseudoscalar nonet P. Then

$$\mathfrak{L}_1(P)=-\tilde{P}_0{}^{\mu}\partial_{\mu}P_0+\tfrac{1}{2}\tilde{P}_0{}^{\mu}P_{\mu}{}^0-\tfrac{1}{2}\tilde{P}_0AP_0, \quad (40)$$

$$\mathfrak{L}_2(P,G_{\mu\nu})=\tfrac{1}{4}h_0\epsilon_{\mu\nu\sigma\tau}\tilde{P}_0(\tilde{G}_0{}^{\mu\nu}DG_0{}^{\sigma\tau}). \quad (41)$$

A is the mass-squared matrix of the pseudoscalar mesons and h_0 is the unrenormalized PVV coupling constant. The matrix D is defined in Eq. (32).

We wish to consider the behavior of $\mathfrak{L}$ under the infinitesimal local $SU(3)$ transformation,

$$\delta\phi_{\mu}{}^0=i\,`F\cdot\delta\lambda'\phi_{\mu}{}^0+(1/g_0)\partial_{\mu}\delta\lambda, \quad (42)$$

$$\delta G_{\mu\nu}{}^0=i\,`F\cdot\delta\lambda'G_{\mu\nu}{}^0, \quad (43)$$

$$\delta P_0=i\,`F\cdot\delta\lambda'P_0, \quad (44)$$

$$\delta P_{\mu}{}^0=i\,`F\cdot\delta\lambda'P_{\mu}{}^0. \quad (45)$$

F is the antisymmetric octet representation of $SU(3)$:

$$(F^a)_{bc}=if_{bac}. \quad (46)$$

$\delta\lambda$ is a function of space-time behaving as a nine-vector under the internal symmetry.

We also define

$$`F\cdot\delta\lambda'\equiv\textstyle\sum_a F^a\delta\lambda^a. \quad (47)$$

Under the transformation of Eqs. (42)–(45),

$$\delta\mathfrak{L}_2=0, \quad (48)$$

$$\delta\mathfrak{L}_1=-i\tilde{P}_0{}^{\mu}\,`F\cdot\partial_{\mu}\delta\lambda'P_0-\tfrac{1}{2}i\tilde{P}_0[A,`F\cdot\delta\lambda']P_0. \quad (49)$$

According to the action principle,

$$\delta\mathfrak{L}_1=-\partial_{\mu}(\tilde{K}^{\mu}\delta\lambda) \quad (50)$$

and, therefore,

$$K_{\mu}{}^a=i\tilde{P}_{\mu}{}^0F^aP_0, \quad (51)$$

$$\partial_{\mu}K_a{}^{\mu}=\tfrac{1}{2}i\tilde{P}_0[A,F^a]P_0. \quad (52)$$

From Eq. (51) it can be seen that

$$\delta K_{\mu}=i\,`F\cdot\delta\lambda'K_{\mu}. \quad (53)$$

The space integral of the fourth component of K_{μ} is the generator of $SU(3)$ transformations on $\mathfrak{L}_1$ and therefore satisfies the usual commutation relations.

The variation of the total Lagrangian is

$$\delta\mathfrak{L}=-(m_0{}^2/g_0)\tilde{\phi}_0{}^{\mu}\partial_{\mu}\delta\lambda+\tfrac{1}{2}\tilde{G}_0{}^{\mu\nu}[B/m_0{}^2,`F\cdot\delta\lambda']G_{\mu\nu}{}^0 -\tfrac{1}{2}i\tilde{P}_0[A,`F\cdot\delta\lambda']P_0. \quad (54)$$

But from the action principle,

$$\delta\mathfrak{L}=-\partial_{\mu}(\tilde{J}^{\mu}\delta\lambda), \quad (55)$$

where J_{μ} is the current for the total system. Comparing Eqs. (54) and (55), we have the field-current identity

$$J^{\mu}=(m_0{}^2/g_0)\phi_0{}^{\mu}. \quad (56)$$

The essential distinction of the present model is that explicit derivatives occur neither in symmetry-breaking terms nor in the interaction terms. This guarantees the form of Eq. (56) and the canonical commutation relations.

$$(1/i)[P_{0a}{}^{(0)}(\mathbf{x}),P_{0b}(\mathbf{y})]=\delta_{ab}\delta^3(\mathbf{x}-\mathbf{y}), \quad (57)$$

$$(1/i)[G_{0a}{}^{(0)k}(\mathbf{x}),\phi_{0b}{}^i(\mathbf{y})]=\delta_{ab}\delta_{ik}\delta^3(\mathbf{x}-\mathbf{y}). \quad (58)$$

It can be shown that the charge

$$Q_{\delta\lambda}=\int d^3x\,\tilde{J}_{(0)}(x)\delta\lambda(x) \quad (59)$$

indeed generates the correct transformation of the fields. In particular,

$$\delta\phi_k{}^0=(1/i)[Q_{\delta\lambda},\phi_k{}^0]=i\,`F\cdot\delta\lambda'\phi_k{}^0+(1/g_0)\partial_k\delta\lambda, \quad (60)$$

and therefore

$$[J_{(0)}{}^a,J_k{}^b]=if_{abc}J_k{}^c\delta^3(\mathbf{x}-\mathbf{y}) +i(m_0{}^2/g_0{}^2)\delta_{ab}\partial_k\delta^3(\mathbf{x}-\mathbf{y}) \quad (61)$$

together with the other algebra-of-fields commutators

$$[J_{(0)}{}^a,J_{(0)}{}^b]=if_{abc}J_{(0)}{}^c\delta^3(\mathbf{x}-\mathbf{y}), \quad (62)$$

$$[J_i{}^a,J_j{}^b]=0. \quad (63)$$

In spite of the symmetry breaking in Eq. (39), the Schwinger term in Eq. (61) is $SU(3)$-symmetric, which guarantees the first Weinberg sum rule [Eq. (3)] for the spectral function of the J's. The second sum rule also follows in the form of Eq. (4). We now wish to diagonalize Eq. (39) in terms of the physical fields by means of the following canonical transformations. The matrix S will later be identified with the S of the previous sections:

$$\phi_{\mu}{}^0=S\Phi_{\mu}, \quad (64)$$

$$G_{\mu\nu}{}^0=\tilde{S}^{-1}G_{\mu\nu}, \quad (65)$$

$$P_0=U_PP, \quad (66)$$

$$P_{\mu}{}^0=\tilde{U}_P{}^{-1}P_{\mu}. \quad (67)$$

The Lagrangian of Eq. (39) takes the form

$$\mathfrak{L}=-\tfrac{1}{2}\tilde{G}^{\mu\nu}[\partial_{\mu}\Phi_{\nu}-\partial_{\nu}\Phi_{\mu}+ig_0S^{-1}(\tilde{\Phi}_{\mu}\tilde{S}FS\Phi_{\nu})] +\tfrac{1}{4}\tilde{G}^{\mu\nu}S^{-1}(B/m_0{}^2)\tilde{S}^{-1}G_{\mu\nu}-\tfrac{1}{2}m_0{}^2\tilde{\Phi}^{\mu}\tilde{S}S\Phi_{\mu} +ig_0\tilde{\Phi}^{\mu}\tilde{S}(\tilde{P}_{\mu}U_p{}^{-1}FU_pP)+\mathfrak{L}_1(U_pP,\tilde{U}_p{}^{-1}P_{\mu}) +\mathfrak{L}_2(U_pP,\tilde{S}^{-1}G_{\mu\nu}), \quad (68)$$

where

$$\mathfrak{L}_1=-\tilde{P}^{\mu}\partial_{\mu}P+\tfrac{1}{2}\tilde{P}^{\mu}U_p{}^{-1}\tilde{U}_p{}^{-1}P_{\mu}-\tfrac{1}{2}\tilde{P}\tilde{U}_pAU_pP, \quad (69)$$

$$\mathfrak{L}_2=\tfrac{1}{4}h_0\epsilon_{\mu\nu\sigma\tau}\tilde{P}\tilde{U}_p(\tilde{G}^{\mu\nu}S^{-1}D\tilde{S}^{-1}G^{\sigma\tau}). \quad (70)$$

For the correct normalization and physical masses, we must take

$$S^{-1}(B/m_0{}^2)\tilde{S}^{-1}=1, \quad (71)$$

$$m_0{}^2\tilde{S}S=M^2, \quad (72)$$

where $M_{\alpha\beta}{}^2=m_{\alpha}{}^2\delta_{\alpha\beta}$, the physical mass matrix. If we define a matrix U such that

$$S=UMm_0^{-1},\tag{73}$$

we see from Eq. (72) that U is unitary:

$$\tilde{U}U=U\tilde{U}=1,\tag{74}$$

and from Eq. (71) that U diagonalizes the symmetry-breaking matrix B:

$$M^2=\tilde{U}BU.\tag{75}$$

Any simple form for B leads to a quadratic mass formula.

The field-current identity from Eq. (56) becomes

$$J^\mu=(m_0^2/g_0)S\Phi^\mu=(m_0/g_0)UM\Phi^\mu.\tag{76}$$

If we define the field-current matrix through the equation

$$J^\mu=g^{-1}M^2\Phi^\mu,\tag{77}$$

the comparison of Eqs. (76) and (77) yields

$$g=g_0(M/m_0)\tilde{U}=g_0\tilde{S}.\tag{78}$$

U_p must be chosen to diagonalize the pseudoscalar masses

$$\mu^2=\tilde{U}_pAU_p,\tag{79}$$

and the correct normalization of the kinetic term in Eq. (69) requires that U_p be unitary. We can now write the Lagrangian in terms of the physical particles.

$$\begin{aligned}\mathfrak{L}=-\tfrac{1}{2}\tilde{G}^{\mu\nu}[\partial_\mu\Phi_\nu-\partial_\nu\Phi_\mu+iM^{-1}\tilde{U}(\tilde{\Phi}_\mu gFUM\Phi_\nu)]\\+\tfrac{1}{4}\tilde{G}^{\mu\nu}G_{\mu\nu}-\tfrac{1}{2}\tilde{\Phi}^\mu M^2\Phi_\mu\\+i\tilde{\Phi}^\mu g(\tilde{P}^\mu\tilde{U}_pFU_pP)+\mathfrak{L}_1+\mathfrak{L}_2,\end{aligned}\tag{80}$$

with

$$\mathfrak{L}_1=-\tilde{P}^\mu\partial_\mu P+\tfrac{1}{2}\tilde{P}^\mu P_\mu-\tfrac{1}{2}\tilde{P}\mu^2P,\tag{81}$$

$$\mathfrak{L}_2=\tfrac{1}{4}m_0^2h_0\epsilon_{\mu\nu\sigma\tau}\tilde{P}\tilde{U}_p(\tilde{G}^{\mu\nu}M^{-1}\tilde{U}DUM^{-1}G^{\sigma\tau}).\tag{82}$$

The VPP coupling constant can be immediately extracted from Eq. (80) and, using Eq. (78), one sees that

$$g_{\rho\pi\pi}/m_\rho=g_{K^*K\pi}/m_{K^*}=g_0/m_0,\tag{83}$$

in agreement with the asymptotic symmetry condition [Eq. (27)] and vector dominance. The VVP coupling matrix h is defined through

$$\begin{aligned}\mathfrak{L}_2&=\tfrac{1}{4}\epsilon_{\mu\nu\sigma\tau}\tilde{P}(\tilde{G}^{\mu\nu}hG^{\sigma\tau})\\&=\sum_{\alpha\beta\gamma}\tfrac{1}{4}h_{P^\alpha V^\beta V^\gamma}P_\alpha G_\beta{}^{\mu\nu}G_\gamma{}^{\sigma\tau}\epsilon_{\mu\nu\sigma\tau}.\end{aligned}\tag{84}$$

By comparison with Eq. (82), we see that

$$(h^\alpha)_{\beta\gamma}=h_{P^\alpha V^\beta V^\gamma}=m_0^2h_0(\tilde{U}_p)^{\alpha a}(M^{-1}\tilde{U}D^aUM^{-1})_{\beta\gamma}.\tag{85}$$

Equation (85) is identical to the asymptotic-symmetry prediction, Eq. (35).

In short, one sees by the comparison of the matrix definitions here with those of Secs. I and II that the proposed Lagrangian contains all the properties of those sections including the two spectral-function sum rules (3) and (4).

IV. EXPERIMENTAL CONSEQUENCES

To begin the comparison of the present model with experiment, we list for convenience the physical coupling constants. In the case of dissimilar vector mesons, the physical g's of the following equations are twice the corresponding h's of Eq. (85), as can be seen by grouping the terms in Eq. (84) that differ only by interchange of β and γ:

$$g_{\pi\rho\omega}=(h_0m_0^2/m_\rho m_\omega)2\cos(\theta_c-\theta),\tag{86a}$$

$$g_{\pi\rho\phi}=(h_0m_0^2/m_\rho m_\phi)2\sin(\theta_c-\theta),\tag{86b}$$

$$g_{\eta\rho^0\rho^0}=(h_0m_0^2/m_\rho^2)\sin(\theta_c-\gamma),\tag{86c}$$

$$g_{\eta'\rho^0\rho^0}=(h_0m_0^2/m_\rho^2)\cos(\theta_c-\gamma),\tag{86d}$$

$$\begin{aligned}g_{\eta\omega\omega}=(h_0m_0^2/m_\omega^2)[\sin(\theta_c-\gamma)\\-\sqrt{3}\cos\gamma\sin^2(\theta_c-\theta)],\end{aligned}\tag{86e}$$

$$\begin{aligned}g_{\eta'\omega\omega}=(h_0m_0^2/m_\omega^2)[\cos(\theta_c-\gamma)\\-\sqrt{3}\sin\gamma\sin^2(\theta_c-\theta)],\end{aligned}\tag{86f}$$

$$\begin{aligned}g_{\eta\phi\phi}=(h_0m_0^2/m_\phi^2)[-\sqrt{2}\cos(\theta_c-\gamma)\\+\sqrt{3}\cos\gamma\sin^2(\theta_c-\theta)],\end{aligned}\tag{86g}$$

$$\begin{aligned}g_{\eta'\phi\phi}=(h_0m_0^2/m_\phi^2)[\sqrt{2}\sin(\theta_c-\gamma)\\+\sqrt{3}\sin\gamma\sin^2(\theta_c-\theta)],\end{aligned}\tag{86h}$$

$$\begin{aligned}g_{\eta\omega\phi}=(h_0m_0^2/m_\omega m_\phi)[2\sqrt{3}\cos\gamma\sin(\theta_c-\theta)\\\times\cos(\theta_c-\theta)],\end{aligned}\tag{86i}$$

$$\begin{aligned}g_{\eta'\omega\phi}=(h_0m_0^2/m_\omega m_\phi)[2\sqrt{3}\sin\gamma\sin(\theta_c-\theta)\\\times\cos(\theta_c-\theta)],\end{aligned}\tag{86j}$$

$$g_{K^+\rho K^{*+}}=-g_{K^0\rho K^{*0}}=h_0m_0^2/m_\rho m_{K^*},\tag{86k}$$

$$\begin{aligned}g_{K\omega K^*}=(h_0m_0^2/m_\omega m_{K^*})\\\times[2(\sqrt{\tfrac{2}{3}})\cos\theta-(\sqrt{\tfrac{1}{3}})\sin\theta],\end{aligned}\tag{86l}$$

$$\begin{aligned}g_{K\phi K^*}=(h_0m_0^2/m_\phi m_{K^*})\\\times[-2(\sqrt{\tfrac{2}{3}})\sin\theta-(\sqrt{\tfrac{1}{3}})\cos\theta],\end{aligned}\tag{86m}$$

where θ_c is the canonical angle 35.26°. θ and γ are the vector and pseudoscalar mixing angles, respectively.

From the field-current identity [Eq. (76)], one has the electromagnetic current

$$\begin{aligned}J_{3+8/\sqrt{3}}{}^\mu=e(m_0/g_0)(m_\rho\Phi_\rho{}^\mu+m_\omega\Phi_\omega{}^\mu\sin\theta/\sqrt{3}\\+m_\phi\Phi_\phi{}^\mu\cos\theta/\sqrt{3}),\end{aligned}\tag{87}$$

where $m_0/g_0=m_\rho/g_\rho$.

From this equation and Eq. (86), we can write the $VP\gamma$ and $P\gamma\gamma$ coupling constants defined by the rates

$$\Gamma(V\to P\gamma)=\frac{g_{VP\gamma}{}^2}{4\pi}\frac{(m_V{}^2-m_P{}^2)^3}{24m_V{}^3},\tag{88a}$$

$$\Gamma(P\to V\gamma)=\frac{g_{VP\gamma}{}^2}{4\pi}\frac{(m_P{}^2-m_V{}^2)^3}{8m_P{}^3},\tag{88b}$$

$$\Gamma(P\to 2\gamma)=\frac{g_{P\gamma\gamma}{}^2m_P{}^3}{64\pi}.\tag{88c}$$

They are

$$g_{\pi\gamma\gamma}=\frac{2e^2}{\sqrt{3}g_\rho^2}\left(g_{\pi\rho\omega}\frac{m_\rho}{m_\omega}\sin\theta+g_{\pi\rho\phi}\frac{m_\rho}{m_\phi}\cos\theta\right), \tag{89a}$$

$$g_{\eta\gamma\gamma}=\frac{2e^2}{g_\rho^2}\left(g_{\eta\rho\rho}+g_{\eta\omega\omega}\frac{m_\rho^2}{m_\omega^2}\frac{\sin^2\theta}{3}+g_{\eta\phi\phi}\frac{m_\rho^2}{m_\phi^2}\frac{\cos^2\theta}{3}+g_{\eta\omega\phi}\frac{m_\rho^2}{m_\omega m_\phi}\frac{\cos\theta\sin\theta}{3}\right), \tag{89b}$$

$$g_{\eta'\gamma\gamma}=\frac{2e^2}{g_\rho^2}\left(g_{\eta'\rho\rho}+g_{\eta'\omega\omega}\frac{m_\rho^2}{m_\omega^2}\frac{\sin^2\theta}{3}+g_{\eta'\phi\phi}\frac{m_\rho^2}{m_\phi^2}\frac{\cos^2\theta}{3}+g_{\eta'\omega\phi}\frac{m_\rho^2}{m_\omega m_\phi}\frac{\cos\theta\sin\theta}{3}\right), \tag{89c}$$

$$g_{\omega\pi\gamma}=(e/g_\rho)g_{\pi\rho\omega}, \tag{90a}$$

$$g_{\phi\pi\gamma}=(e/g_\rho)g_{\pi\rho\phi}, \tag{90b}$$

$$g_{\rho\pi\gamma}=(em_\rho/\sqrt{3}g_\rho)\times[g_{\pi\rho\omega}(\sin\theta/m_\omega)+g_{\pi\rho\phi}(\cos\theta/m_\phi)], \tag{90c}$$

$$g_{\omega\eta\gamma}=(em_\rho/\sqrt{3}g_\rho)\times[2g_{\eta\omega\omega}(\sin\theta/m_\omega)+g_{\eta\omega\phi}(\cos\theta/m_\phi)], \tag{90d}$$

$$g_{\phi\eta\gamma}=(em_\rho/\sqrt{3}g_\rho)\times[2g_{\eta\phi\phi}(\cos\theta/m_\phi)+g_{\eta\omega\phi}(\sin\theta/m_\omega)], \tag{90e}$$

$$g_{\rho\eta\gamma}=(2e/g_\rho)g_{\eta\rho\rho}, \tag{90f}$$

$$g_{\phi\eta'\gamma}=(em_\rho/\sqrt{3}g_\rho)\times[2g_{\eta'\phi\phi}(\cos\theta/m_\phi)+g_{\eta'\omega\phi}(\sin\theta/m_\omega)], \tag{90g}$$

$$g_{K^*K\gamma}=(em_\rho/g_\rho)[g_{K\rho K^*}/m_\rho+g_{K\omega K^*}(\sin\theta/\sqrt{3}m_\omega)+g_{K\phi K^*}(\cos\theta/\sqrt{3}m_\phi)]. \tag{90h}$$

All the above equations depend on four independent parameters which we may choose to be the two mixing angles θ and γ and the two coupling constants $g_{\pi\rho\omega}$ and g_ρ $(=g_{\rho\pi\pi})$. For the pseudoscalar mixing angle γ, we adopt the conventional $-10.4°$, which can be derived by putting $b_2=0$ in the pseudoscalar equivalent of Eqs. (5) and (13).[14]

From Eq. (87), one can see that

$$\tan^2\theta=\frac{m_\omega\Gamma(\omega\to e^+e^-)}{m_\phi\Gamma(\phi\to e^+e^-)}. \tag{91}$$

The recent Orsay[15] measurement of this ratio yields $\theta=33.5°\pm3.2°$.

From Eqs. (86) and (90), one finds the alternative determinations in our model

$$\tan^2(\theta_c-\theta)=\frac{m_\omega\Gamma(\phi\to\pi\gamma)}{m_\phi\Gamma(\omega\to\pi\gamma)} \tag{92}$$

and

$$\tan^2(\theta_c-\theta)=\frac{m_\phi^2}{m_\omega^2}\frac{\Gamma(\phi\to3\pi)}{\Gamma(\omega\to3\pi)}\frac{1}{(36.6\pm5.0)}. \tag{93}$$

From Eq. (92) and the Bonn-Pisa[16] upper limit on $\phi\to\pi\gamma$, we have $\theta=35.3°\pm5.1°$. From Eq. (93) and the latest data[10] on $\phi\to3\pi$, we calculate either

$$\theta=32.2°\pm0.6° \tag{94a}$$

or

$$\theta=38.4°\pm0.6°. \tag{94b}$$

In the following computations we adopt the angle of Eq. (94a) because it provides agreement among processes (91)–(93). We note, however, that our remaining calculations are very insensitive to fluctuations in the angle within the larger limits of the Orsay experiment. The angle of Eq. (94a) is also in reasonable agreement with several of the theoretical models listed in Table I.

Although we could take g_ρ from outside information, we prefer to treat it as a free parameter. We have of course assumed throughout that $g_\rho=g_{\rho\pi\pi}$. Therefore, using the above mixing angles, we can write the five known decay rates in terms of g_ρ and $g_{\pi\rho\omega}$ and a numerical factor. Defining for convenience

$$Y=g_{\pi\rho\omega}^2(m_\omega^2/4\pi), \tag{95}$$

$$X=g_\rho^2/4\pi, \tag{96}$$

we have

$$\Gamma(\omega\to3\pi)=XY(0.247\pm0.039)\text{ MeV}, \tag{97}$$

$$\Gamma(\omega\to\pi\gamma)=(Y/X)(0.218)\text{ MeV}, \tag{98}$$

$$\Gamma(\pi\to2\gamma)=(Y/X^2)(5.40\pm0.22)\text{ eV}, \tag{99}$$

$$\Gamma(\eta\to2\gamma)=(Y/X^2)(0.486\pm0.02)\text{ keV}, \tag{100}$$

$$\Gamma(\eta\to\pi\pi\gamma)=Y(11.5\pm1.1)\text{ eV}. \tag{101}$$

The errors in the numerical factors come from the uncertainty in the mixing angle [Eq. (94a)] and in the intermediate ρ-meson mass. The phase-space calculations in processes (97) and (101) are somewhat sensitive to the ρ-meson mass, which we have taken to be 770 ± 10 MeV. The best fit to the above processes and their branching ratios[17] yields

$$Y=\tfrac{1}{4}g_{\pi\rho\omega}^2m_\omega^2=13.3, \tag{102}$$

$$X=g_\rho^2/4\pi=2.74. \tag{103}$$

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[14] Actually, the sign of γ is undetermined by the mass formula. The minus sign is preferred in the best fit to the data.

[15] J. E. Augustin *et al.*, Phys. Letters **28B**, 503 (1969); **28B**, 508 (1969); **28B**, 513 (1969); **28B**, 517 (1969).

[16] C. Bemporad *et al.* Phys. Letters **29B**, 383 (1969).

[17] We have used the data averages of Ref. 10. Since the branching ratios of η and ω are known more accurately than the individual rates, they provide additional constraints on X and Y.

TABLE II. Decay rates depending on the PVV vertex.

Decay mode	Present model (keV)	Experiment (keV)
$\eta \to 2\gamma$	0.86 ± 0.04	1.00 ± 0.22
$\eta \to \pi\pi\gamma$	0.153 ± 0.015	0.145 ± 0.050
$\pi \to 2\gamma$	$(9.6 \pm 0.4) \times 10^{-3}$	$(7.3 \pm 1.7) \times 10^{-3}$
$\phi \to 3\pi$	570 ± 30	730 ± 210
$\omega \to 3\pi$	$(9.0 \pm 1.3) \times 10^3$	$(11.3 \pm 1.0) \times 10^3$
$\omega \to \pi\gamma$	1.05×10^3	$(1.17 \pm 0.19) \times 10^3$
$\omega \to \eta\gamma$	8.6	<190
$\phi \to \pi\gamma$	4.2	<13
$\phi \to \eta\gamma$	33.5	<290
$\phi \to \eta'\gamma$	0.19	
$\rho \to \pi\gamma$	101	<500
$\rho \to \eta\gamma$	69.8	
$K^{*+} \to K^+\gamma$	96.6	
$K^{*0} \to K^0\gamma$	123	
$\eta' \to \omega\gamma$	9.5	
$\eta' \to \rho\gamma$	142	
$\eta' \to 2\gamma$	6.1	

The value of X is seen to be in very close agreement with the Kawarabayashi-Suzuki relation,[18] which predicts a value of 2.66 corresponding to a ρ-meson width of 144 MeV. We note, however, that the leptonic rates of the vector mesons as given, for instance, by Ref. 15 seem to require a lower value for X. The value of Y is in rough agreement with $SU(6)$ predictions. In Table II, we list all the relevant meson-decay rates predicted in the present model using Eqs. (102) and (103).

V. CONCLUSION

It can be seen from Table II that the present model based on asymptotic symmetry allows a reasonable fit to all the known meson-decay rates involving the PVV vertex. This is in contrast to the often-voiced view that these decay rates can be brought into agreement only through extra $SU(3)$ and nonet symmetry-breaking parameters. It is well known, for example, that the usual current-mixing model leads to widespread disagreement with experiment unless one introduces a number of extra symmetry-breaking parameters.[3] In particular, we note that either exact U-spin symmetry or the usual current-mixing model without $\eta\eta'$ mixing leads to discrepancies with experiment for the $(\eta \to 2\gamma)/(\pi \to 2\gamma)$ and $(\eta \to \pi\pi\gamma)/(\eta \to 2\gamma)$ ratios by factors of 6 and 3.5, respectively.[3] Discrepancies by a factor of 3 remain even if one introduces the conventional $\eta\eta'$ mixing angle of $-10.4°$. Because of the uncertain status of this angle, we should mention that varying γ by $\pm 10°$ in the present model can affect certain of the predicted rates by up to 50%, although the over-all quality of the fit and especially the best-fit values for g_ρ and $g_{\pi\rho\omega}$ do not change significantly. With regard to the rare decay modes included in Table II, we note that, because of the inverse-mass factors in Eqs. (86), processes involving the ϕ meson are especially sensitive to our model. Most interesting perhaps is the large suppression of the $\phi \to \eta\gamma$ rate. Only a slight improvement on the present experimental upper limit should discriminate between our model and the exact-symmetry prediction.[19] The present upper limit on $\phi \to \eta\gamma$ is already in disagreement with the current-mixing model of Ref. 3 in the absence of extra symmetry-breaking parameters. Also of interest is the ratio of the radiative decay rates of charged to neutral K^*'s, where there is a factor of 3 difference between exact symmetry and the present model. Finally, we remark that the model proposed here can be extended to predict symmetry breaking in all the nonminimal (i.e., through $G_{\mu\nu}$) couplings of the vector mesons to other particles such as baryons and tensor mesons.

[18] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

[19] See, for example, Anisovich *et al.*, Phys. Letters **16**, 194 (1965), who give $\Gamma(\phi \to \eta\gamma) = 250$ keV.