contradictions were found by Griffiths and Palmer[15] in their sum rules for $u=0$ pion-nucleon scattering. Our results, although based on $SU(3)$-breaking model-dependent calculations, do not show any obvious inconsistency. Therefore, we feel that calculations based on the superconvergent model with these assumptions deserve further study.

[15] David Griffiths and William Palmer, Phys. Rev. 161, 1606 (1967).

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Asymptotic Behavior of Form Factors and Possible Free-Field Behavior for the Electromagnetic Current*†

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It is shown that the assumption that the electromagnetic current behaves at infinity like a free vector-meson field implies that elastic form factors should vanish at infinity *faster* than would have been expected from vector-meson dominance (VMD). In fact, it is shown that VMD for form factors at infinite momentum transfer is violated with increasing strength as the spin of the particle concerned increases. Under some further speculative assumptions, correlations between the mass spectrum of very heavy particles and the asymptotic behavior of their form factors are found.

I. INTRODUCTION

It is widely known that the electromagnetic form factors accessible to present day experiments (e.g., pion and nucleon form factors) cannot be explained by vector-meson dominance (VMD) in the region of medium and large momentum transfers. Specifically, VMD can explain the nucleon form factors[1] only for squared momentum transfers smaller than about 1.2 $(\text{BeV}/c)^2$, and only under the assumption that strange, totally not understood, cancellations are taking place among the ρ, ω, and φ contributions. For squared momentum transfers larger than about 1.2 $(\text{BeV}/c)^2$ in the spacelike region, VMD cannot explain the nucleon form factor. With respect to the pion form factor, VMD can very well explain it for small timelike momentum transfers[2] but it seems to fail more or less quickly in the spacelike region.

On the other hand, there have recently been some speculations that the electromagnetic current behaves like a free vector-meson field at infinity.[3] To explain this more specifically, let us define the "current propagator" by[4]

$$\Delta_{\mu\nu}(q)\equiv i\int d^4x\, e^{-iqx}\langle 0|T^*(j_\mu{}^{\text{e.m.}}(x)j_\nu{}^{\text{e.m.}}(0))|0\rangle$$

$$=\int\frac{\rho(m^2)}{q^2+m^2-i\epsilon}\left(\delta_{\mu\nu}+\frac{q_\mu q_\nu}{m^2}\right)dm^2,\quad(1.1)$$

where

$$\rho(-P^2)\equiv\tfrac{1}{3}(2\pi)^3\sum_n\delta(P_n-P)$$
$$\times\langle 0|j_\mu{}^{\text{e.m.}}(0)|n\rangle\langle n|j_\mu{}^{\text{e.m.}}(0)|0\rangle.\quad(1.2)$$

The assumption that the electromagnetic current behaves like a free field at infinity now means that

$$\Delta_{\mu\nu}(q)\xrightarrow[q_2\to\infty]{}\frac{\delta_{\mu\nu}}{q^2}\int\rho(m^2)dm^2+\frac{q_\mu q_\nu}{q^2}\int\frac{\rho(m^2)}{m^2}dm^2,\quad(1.3)$$

where

$$\int\frac{\rho(m^2)}{m^2}dm^2=\text{finite}\quad(1.4)$$

and

$$\int\rho(m^2)dm^2=\text{finite}.\quad(1.5)$$

Evidently, since $\rho(m^2)$ is positive definite, Eq. (1.4) should be satisfied if (1.5) is satisfied. Notice that relations (1.3)–(1.5) are expected, if it is possible to approximate the spectral function by a finite sum of δ-function terms (e.g., at ρ, ω, and φ).

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[1] L. H. Chan, K. W. Chen, J. R. Dunning, Jr., N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. 141, 1298 (1966).

[2] J. E. Augustin, J. C. Bizot, J. Buon, J. Haissinski, D. Lalanne, P. Marin, H. Nguyen Ngoc, J. Perez-Y-Jorba, F. Rumpf, E. Silva, and S. Tavernier, Phys. Letters 28B, 508 (1968).

[3] J. J. Sakurai (to be published). Speculations along this line seem to have been made first by S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

[4] By T^* we mean the covariant part of the time-ordered product of the two currents.

The main purpose of the present work is to investigate possible consequences on the asymptotic behavior of the electromagnetic form factors of the various particles which stem from the requirement (1.5). From (1.2) we see that any set of intermediate states $|n\rangle$ gives a semipositive definite contribution to $\rho(m^2)$, and therefore the corresponding contribution to the integral

$$\int \rho(m^2)dm^2 \qquad (1.6)$$

should be finite if Eq. (1.5) is to hold. In the following, we shall investigate the consequences of the assumption that any two-body (particle-antiparticle) contribution to the integral (1.6), as well as the total two-body contribution to the same integral, is finite. For convenience we treat the boson and the fermion intermediate states separately. Our most important result is that requirement (1.5) implies that VMD *cannot* be valid for form factors at infinity. To express it in more dramatic terms, we demonstrate that VMD for the electromagnetic current propagator at infinity *excludes* VMD for the electromagnetic form factors at infinity. We further show that the larger the spin of a particle, the stronger the violation of VMD for its form factor at infinity. We hasten to say that the arguments leading to our result have nothing to do with the usual plausibility arguments that VMD is a low-momentum-transfer theory and should not be expected to be valid at high energies. In order to derive our result, the only assumptions made is that the electromagnetic current behaves like a free field at infinity.

An outline of the paper is as follows. In Sec. II elastic form factors for particles of any spin are reviewed and, assuming Eq. (1.5), a condition on their asymptotic behavior is found. This condition excludes the possibility of VMD for the form factors at infinity. In Sec. III a stronger condition on the asymptotic behavior of the form factors is derived, using assumptions and techniques similar to those of Terazawa.[5] This latter condition imposes a certain correlation between the mass spectrum of very heavy particles and the asymptotic behavior of their form factors. Finally, in Sec. IV we summarize our results.

II. ASYMPTOTIC BEHAVIOR OF FORM FACTORS; ASSUMPTION THAT ELECTROMAGNETIC CURRENT BEHAVES LIKE A FREE FIELD AT INFINITY

We are interested in the transition matrix elements of the form[6]

$$_{\rm out}\langle\lambda_1p_1,\lambda_2p_2|j_\mu^{\rm e.m.}(0)|0\rangle, \qquad (2.1)$$

where λ_1p_1 and λ_2p_2 are the helicities and momenta of a particle and its antiparticle, respectively, of spin J. We define

$$P=p_1+p_2 \quad \text{and} \quad s=-P^2. \qquad (2.2)$$

Following Terazawa,[5] who has translated the treatments of Yennie *et al.*[7] and Durand *et al.*[8] from the spacelike region to the timelike region of the momentum transfer, we work in the c.m. frame of particles 1 and 2, and take

$$\mathbf{p}_1=\mathbf{p}=\text{along the } z \text{ axis}.$$

Of course, $\mathbf{p}_2=-\mathbf{p}$. Current conservation implies that[9]

$$\langle\lambda_1p_1,\lambda_2p_2|j_0^{\rm e.m.}|0\rangle=0. \qquad (2.3)$$

Now we define

$$j_\pm^{\rm e.m.}=\mp(1/\sqrt{2})(j_1^{\rm e.m.}\pm ij_2^{\rm e.m.}). \qquad (2.4)$$

Using the commutation relations between current and total angular momentum along the z axis, we define

$$\langle\lambda,\mathbf{p};\lambda',-\mathbf{p}|j_3^{\rm e.m.}|0\rangle\equiv A^{(J)}(\lambda,s)\delta_{\lambda',\lambda}, \qquad (2.5)$$

$$\langle\lambda\mathbf{p};\lambda',-\mathbf{p}|j_\pm^{\rm e.m.}|0\rangle\equiv B_\pm^{(J)}(\lambda,s)\delta_{\lambda',\lambda\mp1}, \qquad (2.6)$$

where J is the spin of the particle. The number of independent form factors can be further reduced by using the transformation properties of the electromagnetic current under parity and charge conjugation. In this way we obtain[5,10,11]

$$A^{(J)}(\lambda,s)=A^{(J)}(-\lambda,s), \qquad (2.7)$$

$$B_\pm^{(J)}(\lambda,s)=B_\mp^{(J)}(-\lambda,s), \qquad (2.8)$$

$$B_\pm^{(J)}(\lambda,s)=B_\mp^{(J)}(\lambda\mp1,s). \qquad (2.9)$$

In terms of these amplitudes, we can express the contribution to $\rho(s)$ due to a particle-antiparticle pair of spin J as

$$\rho_J(s)\equiv[\mathfrak{F}_J(s)/12\pi^2]m_J^2(1-4m_J^2/s)^{1/2}\times\theta(s-4m_J^2), \qquad (2.10)$$

where

$$\mathfrak{F}_J(s)\equiv\sum_\lambda|A^{(J)}(\lambda,s)|^2+2\sum_\lambda|B_+^{(J)}(\lambda,s)|^2. \qquad (2.11)$$

From (2.10), (2.11), and requirement (1.5), we see immediately that[12]

$$s\mathfrak{F}_J(s)\to0 \qquad (2.12)$$

as $s\to\infty$.

[5] H. Terazawa, Phys. Rev. **177**, 2159 (1969).

[6] We use covariant normalization common to both fermions and bosons. For single-particle states it is given by $\langle p'\lambda'|p\lambda\rangle=(2\pi)^3\delta(\mathbf{p}-\mathbf{p}')\delta_{\lambda\lambda'}(p_0/m)$.

[7] D. R. Yennie, M. M. Lévy, and D. G. Ravenhall, Rev. Mod. Phys. **29**, 144 (1957).

[8] L. Durand, III, P. C. DeCelles, and R. B. Marr, Phys. Rev. **126**, 1882 (1962).

[9] Actually, this result is independent of current conservation, as can be seen in the same manner as in Ref. 7.

[10] Our results look somewhat different from those of Terazawa (Ref. 5) owing to different definitions of $j_\pm$ and owing to the fact that we use the helicities in order to describe the various particle states.

[11] Notice that in deriving these equations we have followed the phase conventions associated with particle 2 of Jacob and Wick. We take as particle 2 the second particle stated in a two-particle state.

[12] Actually, requirement (1.5) demands that $s\mathfrak{F}_J(s)$ should go to zero faster than $1/\ln^2 s$ as $s\to\infty$.

In the remaining part of this section we shall study the structure of $A^{(J)}(\lambda,s)$, $B_\pm^{(J)}(\lambda,s)$, and $\mathfrak{F}_J(s)$. To start with we observe[5] that Eqs. (2.5)–(2.9) give us all possible constraints on the various helicity amplitudes $A^{(J)}(\lambda,s)$ and $B_\pm^{(J)}(\lambda,s)$. Therefore, these relations can be used to find the number of independent amplitudes and consequently the number of independent form factors, free of kinematical singularities, for any spin J. We also remark that the amplitudes $A^{(J)}(\lambda,s)$ and $B_\pm^{(J)}(\lambda,s)$ have actually been defined through (2.5) and (2.6) only for $s\geq 4m_J^2$. Of course, they can be analytically continued in the whole s plane, and crossing relations, like the ones invented by Trueman and Wick for four-point functions, can be formulated to determine (after being analytically continued) their relation to the various helicity amplitudes for $s<0$ in, say, the brick-wall frame with the outgoing particle moving along the z axis. Here we do not need to examine the intricacies of this crossing. It suffices to know that $A^{(J)}(\lambda,s)$ and $B_\pm^{(J)}(\lambda,s)$ can be analytically continued in the whole s plane, that they contain only singularities (both dynamical and kinematical) along the real axis, and that they are polynomially bounded as $|s|\to\infty$.

We now study in detail the kinematical structure of $A^{(J)}(\lambda,s)$, $B_\pm^{(J)}(\lambda,s)$, and $\mathfrak{F}_J(s)$ for the pion, proton, and ρ meson, and discuss briefly the general J case. This problem has been studied in complete generality by Yndurain and by Trueman.[13] Our treatment is far more specific and stresses only a few points of relevance to our present work.

Electromagnetic Form Factors of p and π^+

We define as usual[14]

$$\langle p_1,p_2|j_\mu^{\text{e.m.}}(0)|0\rangle=(1/2m_\pi)(p_1-p_2)_\mu F_\pi(s)\,, \quad (2.13)$$

$$\langle p_1\lambda_1,p_2\lambda_2|j_\mu^{\text{e.m.}}(0)|0\rangle=i\bar{u}^{\lambda}(p_1)[\gamma_\mu F_1(s)-(\kappa/2m_p)\times P_\nu\sigma_{\mu\nu}F_2(s)]v^{\lambda_2}(p_2)\,, \quad (2.14)$$

where κ is the anomalous magnetic moment of the proton.

From these expressions, working in the c.m. system and taking

$$\mathbf{p}_1=\mathbf{p}=\text{along the } z \text{ axis},$$

we find after a little algebra[11] that

$$A^{(\pi)}(0,s)=(|2\mathbf{p}|/2m_\pi)F_\pi(s)\,, \quad (2.15a)$$

$$B_\pm^{(\pi)}(0,s)=0\,, \quad (2.15b)$$

$$\mathfrak{F}_\pi(s)=|(s/4m_\pi^2)-1|\,|F_\pi(s)|^2 \quad (2.15c)$$

and

$$A^{(p)}(\tfrac{1}{2},s)=A^{(p)}(-\tfrac{1}{2},s)=F_1(s)+[\kappa s/(2m_p)^2]F_2(s)=G_E{}^p(s)\,, \quad (2.16a)$$

$$-\sqrt{2}B_+^{(p)}(\tfrac{1}{2},s)=-\sqrt{2}B_-^{(p)}(-\tfrac{1}{2},s)=(\sqrt{2}/m_p)[F_1(s)+\kappa F_2(s)]=(\sqrt{s}/m_p)G_M{}^p(s)\,, \quad (2.16b)$$

$$\mathfrak{F}_p(s)=2|F_1(s)+[\kappa s/(2m_p)^2]F_2(s)|^2+(s/m_p^2)|F_1(s)+\kappa F_2(s)|^2=2|G_E{}^p(s)|^2+(s/m_p^2)|G_M{}^p(s)|^2. \quad (2.16c)$$

From (2.16c) we have

$$\mathfrak{F}_p(s)\sim s(1/s^2)^2\sim 1/s^3\,,$$

as $s\to\infty$, if the usual dipole fit holds at asymptotic energies. This is consistent with the requirement (2.12). Notice also that if we use simple pole formulas for the π and p form factors at *high* momentum transfer (without strange concellations among the ρ, ω, and φ contributions in the proton case), then condition (2.12) is *not* satisfied.

Electromagnetic Form Factors of the ρ Meson

The most general ρ-ρ-γ interaction can be written as

$$\begin{aligned}\langle p_1\lambda_1,p_2\lambda_2|j_\lambda^{\text{e.m.}}(0)|0\rangle &=(\boldsymbol{\phi}_1^*\times\boldsymbol{\phi}_2^*)_3\epsilon_\mu^{\lambda_1*}(p_1)\epsilon_\nu^{\lambda_2*}(p_2)\{\delta_{\mu\nu}(p_2-p_1)_\lambda f_1(s)\\&+[\delta_{\nu\lambda}(-2p_2-p_1)_\mu+\delta_{\lambda\mu}(2p_1+p_2)_\nu]f_2(s)\\&+(p_1-p_2)_\lambda(2p_2+p_1)_\mu(-2p_1-p_2)_\nu f_3(s)\}\times 1/2m_\rho\,. \quad (2.17)\end{aligned}$$

Above, $\boldsymbol{\phi}_1$ and $\boldsymbol{\phi}_2$ are the isotopic wave functions of the ρ mesons 1 and 2. Expression (2.17) is the most general one which has the right properties under space, spin, and isospin exchange of the two ρ mesons.[15] Evidently, $f_1(s)$, $f_2(s)$, and $f_3(s)$ can only have dynamical singularities. We mention in particular that if it is assumed that the ρ meson dominates the form factors and if, according to the Yang-Mills prescription, we write the three ρ vertex as

$$\boldsymbol{\varrho}_\mu\cdot(\boldsymbol{\varrho}_\nu\times\partial_\mu\boldsymbol{\varrho}_\nu)\,,$$

then

$$f_1(s)=f_2(s)=m_\rho^2/m_\rho^2-s\,,\quad f_3(s)=0\,. \quad (2.18)$$

Taking as usual $\mathbf{p}_1=\mathbf{p}=$ along the z axis, we have

$$A^\rho(0,s)=-(s/4m_\rho^2-1)^{1/2}(\boldsymbol{\phi}_1^*\times\boldsymbol{\phi}_2^*)_3[f_1(s)\times(s/2m_\rho^2-1)-(s/m_\rho^2)f_2(s)-(s^2/m_\rho^2-4s)f_3(s)]\,, \quad (2.19a)$$

$$A^\rho(1,s)=A^\rho(-1,s)=(s/4m_\rho^2-1)^{1/2}\times(\boldsymbol{\phi}_1^*\times\boldsymbol{\phi}_2^*)_3 f_1(s)\,, \quad (2.19b)$$

$$B_+{}^\rho(1,s)=B_+{}^\rho(0,s)=B_-{}^\rho(-1,s)=B_-{}^\rho(0,s)=(s/4m_\rho^2-1)^{1/2}(\boldsymbol{\phi}_1^*\times\boldsymbol{\phi}_2^*)_3(\sqrt{s}/m_\rho)\times f_2(s)\,, \quad (2.19c)$$

[13] F. J. Yndurain, CERN Report (unpublished); T. L. Trueman, Phys. Rev. **182**, 1469 (1969).

[14] T. L. Trueman and G. C. Wick, Ann. Phys. (N. Y.) **26**, 322 (1964).

[15] Actually, the fact that for the ρ meson we should have three form factors could be deduced from Eqs. (2.6)–(2.10), according to what has been stated after Eq. (2.12).

and

$$\mathfrak{F}_\rho(s)=|s/4m_\rho^2-1|\,|(\boldsymbol{\phi}_1^*\times\boldsymbol{\phi}_2^*)_3|^2\{|f_1(s)(s/2m_\rho^2-1)-(s/m_\rho^2)f_2(s)-(s^2/m_\rho^2-4s)f_3(s)|^2+2|f_1(s)|^2+(4|s|/m_\rho^2)|f_2(s)|^2\}. \quad (2.19\text{d})$$

Notice the various kinematical factors present. First of all, there is a factor $|s/(4m_\rho^2)-1|\sim|\mathbf{p}|$ which is common to all amplitudes $A^\rho(\lambda,s)$ and $B_\pm^\rho(\lambda,s)$. This factor comes from the p-wave threshold behavior and therefore should always be present in boson form factors. Second, notice the extra kinematical factors in the expression in the brackets in Eq. (2.19a). For a pointlike ρ meson for which $f_1(s)$, $f_2(s)$, and $f_3(s)$ should be constants, not all zero, this expression is going to diverge at least linearly as s increases. Finally, notice from (2.18) and (2.19d) that if the ρ form factor is supposed to be dominated for large s by a simple pole, then $\mathfrak{F}_\rho(s)$ will behave *at least* like a constant for large s, strongly violating condition (2.12). Actually even a "dipole fit" of the type suggested by the experiment for the proton form factor, will not be adequate for the ρ form factor in order to satisfy condition (2.12). This can be easily seen from (2.19d) assuming that there are no strange cancellations among the form factors $f_1(s)$ and $f_2(s)$ and $f_3(s)$ in the first term in brackets.

General Spin Case

We here discuss briefly the transition amplitudes of a photon going to two bosons of spin J, $T=1$. As can be seen from (2.5)–(2.9), we now have $2J+1$ form factors to express these transition amplitudes. In the following we shall, for simplicity, be interested only in one of them. This consideration will be enough to enable us to emphasize the points we want to make. But in order to do this we need first an explicit representation of the wave function in momentum space of a particle of spin J and helicity λ. This problem has been solved for us by Brudnoy[16] using the Rarita-Schwinger formalism; we need only translate his formula for bosons. The result is

$$\varphi_{\mu_1\cdots\mu_J}{}^\lambda(\mathbf{p})=\frac{2^J(J+\lambda)!(J-\lambda)!}{(2J)!}\times\sum_{\lambda_1\cdots\lambda_J}^{1}\frac{\epsilon_{\mu_1}{}^{\lambda_1}(\mathbf{p})\cdots\epsilon_{\mu_J}{}^{\lambda_J}(\mathbf{p})}{[(1+\lambda_1)!(1-\lambda_1)!\cdots(1+\lambda_J)!(1-\lambda_J)!]^{1/2}}, \quad (2.20\text{a})$$

where

$$\lambda=\lambda_1+\lambda_2+\cdots+\lambda_J, \quad (2.20\text{b})$$

and $\varphi_{\mu_1\cdots\mu_J}{}^\lambda(\mathbf{p})$ is the wave function (in momentum space) of a particle of spin J, helicity λ, momentum $\mathbf{p}$, and mass m_J. Correspondingly, $\epsilon_{\mu_i}{}^{\lambda_i}(\mathbf{p})$ is the wave function of a vector meson of helicity λ_i, momentum $\mathbf{p}$, and mass m_J. After this preparation we write[6]

$$\langle Jp\lambda;Jp'\lambda'|j_\nu{}^{\text{e.m.}}(0)|0\rangle=(\boldsymbol{\phi}_1^*\times\boldsymbol{\phi}_2^*)_3(1/2m_J)\varphi_{\mu_1\cdots\mu_J}{}^{\lambda*}(\mathbf{p})\varphi_{\mu_1'\cdots\mu_J'}{}^{\lambda'*}(\mathbf{p}')\times[-\delta_{\mu_1\mu_1'}\cdots\delta_{\mu_J\mu_J'}(p'-p)_\nu f_1{}^J(s)(-1)^J+\cdots]. \quad (2.21)$$

The dots stand for other omitted form factors. Now let us specialize in the c.m. system (i.e., $\mathbf{p}'=-\mathbf{p}$). Observe that in this frame

$$\epsilon_\mu{}^\lambda(\mathbf{p})\epsilon_\mu{}^{\lambda'}(\mathbf{p}')=-\delta_{\lambda\lambda'}[1+2(1-\lambda^2)(|\mathbf{p}|^2/m_J{}^2)]. \quad (2.22)$$

Combining (2.20)–(2.22), we obtain

$$\langle Jp\lambda;Jp'\lambda'|j_\nu{}^{\text{e.m.}}|0\rangle=(\boldsymbol{\phi}_1^*\times\boldsymbol{\phi}_2^*)_3[2^J(J+\lambda)!(J-\lambda)!/(2J)!]\delta_{\lambda\lambda'}\times\left(\sum_{\lambda_1\cdots\lambda_J=-1}^{1}\frac{[1+2(1-\lambda_1^2)(|\mathbf{p}|^2/m_J{}^2)]\cdots[1+2(1-\lambda_J{}^2)(|\mathbf{p}|^2/m_J{}^2)]}{(1+\lambda_1)!(1-\lambda_1)!\cdots(1+\lambda_J)!(1-\lambda_J)!}\right)(p'-p)_\nu f_1{}^J(s)+\cdots, \quad (2.23)$$

where

$$\lambda_1+\lambda_2+\cdots+\lambda_J=\lambda=\lambda'. \quad (2.24)$$

Several things should be observed in (2.23):

(1) Notice the factor $(p'-p)_\nu$ which, as it has been explained previously, comes from the p-wave threshold behavior.

(2) Notice the expression in angular brackets in Eq. (2.23). For high energies (or high momentum transfer) this expression behaves as

$$\sim s^{J-|\lambda|}. \quad (2.25)$$

We can conclude, therefore, that such behavior should be expected for a spin J pointlike particle which has only charge, but no higher multipole moments. (This does not seem very reasonable and is perhaps an indication that pointlike massive charged particles of spin higher than or equal to 1 do not exist.)

(3) Let us now specialize to the case that the particle of spin J, whose electromagnetic interactions we are studying, is described by only one form factor, namely, $f_1(s)$. We see immediately from (2.23) that VMD for this form factor at infinity violates condition (2.12) and consequently violates the assumption that the electromagnetic current behaves like a free field at infinity. Actually, we should have $s^{2J+1}|f_1{}^J(s)|^2\to 0$ as $s\to\infty$, in order that condition (2.12) be satisfied. We see, therefore, that the larger the spin of the particle, the stronger the violation of VMD for its form factor at infinite energies. (The form factors must

[16] D. M. Brudnoy, Phys. Rev. **145**, 1229 (1966), and references therein.

vanish faster and faster at $s \to \infty$ as the spin of the particle increases.) We remark that these conclusions are independent of our assumption that the electromagnetic interactions of the particle of spin J we are studying are described by only one form factor, namely, $f_1{}^J(s)$.

(4) It is evident from our treatment and Brudnoy's[16] formalism that expressions having similar dependence on s, as in Eq. (2.23), will appear in the transition amplitudes of a photon (off its mass shell) to any boson-antiboson or fermion-antifermion system. Therefore, if we assume that these amplitudes are dominated, for *every* J and *high* s by a *finite* number of more or less the same vector mesons, then for large s, we will have

$$\mathfrak{F}_J(s) \sim s^{2J}\chi(s,J),$$

where $\chi(s,J)$ does not have terms of the form

$$\sim s^J.$$

Therefore, as J increases,[17] we shall eventually violate condition (2.12) unless the number of vector mesons used to describe the form factor at infinity also increase. In connection with this it may be reasonable to conclude that for high s we will need an *infinite number* of vector mesons in order to describe $\mathfrak{F}_J(s)$.

Thus the assumption that the electromagnetic current behaves like a free field at infinity implies that the electromagnetic form factors of any particle should vanish at infinity *faster* than would have been expected from VMD. Actually, the violation of VMD at infinity (for the form factors) becomes stronger as the spin of the particle increases. It is important to realize that our arguments are entirely in the framework of VMD for the two-point and three-point functions at infinity, and that they have nothing to do with the usual plausibility arguments that VMD is a low-momentum-transfer theory and should not be expected to be valid for high momentum transfer. Actually, one purpose of the present work is to investigate how far (with respect to the energy) we can in principle hope that VMD may be a good approximation simultaneously for the two-point and three-point functions.

Another interesting point is that our conclusions are not going to be upset greatly even if the integral

$$\int \rho(m)^2 dm^2$$

diverges, provided that it does not diverge like an exponential. For example, if this integral diverges *linearly*, we can conclude (in exactly the same way as above), that VMD for the electromagnetic form factors of the pion and the proton *may* be valid at infinity, but that it has to fail for higher-spin particles. In exactly the same way we can also see that the violation of VMD for particles of high spin becomes stronger as the spin increases. Consequently, if particles of spin up to infinity exist, we still need an infinite number of vector mesons to describe their form factors.

[17] We assume that resonances exist with masses going up to infinity. This is a fashionable assumption nowadays.

III. AN ASSUMPTION ON $\mathfrak{F}_J(s)$ FOR PARTICLES OF VERY HIGH MASS AND ITS CONSEQUENCES

We make the assumption[18] that for particles of *very high mass* $s \geq 4m_J{}^2$, and of spin J, we have

$$\mathfrak{F}_{J,n}(s) = (2J+1)|s/4m_{J,n}{}^2 - 1| \times [\mu^2(J,n)/s]^{2l(J,n)} \quad (3.1a)$$

for $J = 0, 1, \cdots$, and

$$\mathfrak{F}_{J,n}(s) = (2J+1)[\mu^2(J,n)/s]^{2l(J,n)} \quad (3.1b)$$

for $J = \frac{1}{2}, \frac{3}{2}, \cdots$.

Let us explain these expressions. The index n serves to distinguish among different particles with the same spin; it can be thought of as distinguishing among various Regge trajectories. The factor $2J+1$ can be justified from the definition of $\mathfrak{F}_{J,n}(s)$ given in Eq. (2.11). The difference between the expressions (3.1a) and (3.1b) comes from the fact that for bosons at threshold we have p waves, while for fermions we have s waves. To justify somewhat the last factor in (3.1a) and (3.1b), we invoke the Sugawara-Kanazawa[19] theorem which says that $\mathfrak{F}_{J,n}(s)$ should have the same limit when $|s| \to \infty$ along any direction in the complex s plane provided that $\mathfrak{F}_{J,n}(s)$ is polynomially bounded, and finite as $s \to \infty$ along the positive real axis.[20] These provisions of the theorem are seen from (2.12) to be true. We can therefore perhaps argue that, for $s \geq 4m_{J,n}{}^2$ and $m_{J,n}$ *very large*, we are already in the asymptotic region of the form factor, and that the form factor in this region is simply described by (3.1a) and (3.1b). From (2.12) we have, for bosons,

$$l(J,n) > 1,$$

and, for fermions,

$$l(J,n) > \tfrac{1}{2}.$$

But from assumptions (3.1a) and (3.1b) we can actually derive stronger conditions. We demand, as stated in the Introduction, that the total particle-antiparticle contribution to the integral

$$\int \rho(s) ds$$

should be finite both for bosons and for fermions. Let us do it first for bosons. The following series should converge [see Eq. (2.10)]:

$$\sum_{J,n} \left[\int_{4m^2{}_{J,n}}^{\infty} \mathfrak{F}_{J,n}(s) m_{J,n}{}^2 \left(1 - \frac{4m_{J,n}{}^2}{s}\right)^{1/2} ds \right]. \quad (3.2)$$

[18] This assumption is very similar to the one made by Terazawa (Ref. 5).

[19] M. Sugawara and A. Kanazawa, Phys. Rev. **123**, 1895 (1961).

[20] Notice that we can always arrange for all the dynamical and kinematical singularities of $\mathfrak{F}_{J,n}(s)$ to lie along the positive real axis.

All the terms in this series are positive. For this series to converge, it is sufficient and necessary that the following series converge:

$$\sum_{J,n;\ m_{J,n}>m}\left[\int_{4m_{J,n}^2}^{\infty}\mathfrak{F}_{J,n}(s)m_{J,n}^2\left(1-\frac{4m_{J,n}^2}{s}\right)^{1/2}ds\right], \tag{3.3}$$

where m may be chosen as large as we please.[17] Now, we can use (3.1a) in (3.3). We calculate first the expression in the square brackets. We have

$$\int_{4m_{J,n}^2}^{\infty}\mathfrak{F}_{J,n}(s)m_{J,n}^2\left(1-\frac{4m_{J,n}^2}{s}\right)^{1/2}ds$$
$$=\tfrac{1}{4}(2J+1)(4m_{J,n}^2)^2[\mu^2(J,n)/4m_{J,n}]^{2l(J,n)}$$
$$\times B(2l(J,n)-2,\tfrac{5}{2}). \tag{3.4}$$

To derive this result we used the fact that $l(J,n)>1$ for bosons. Thus we now have to require that the following series converge:

$$\sum_{J,n;\ m_{J,n}>m}\left[(2J+1)(4m_{J,n}^2)^2\left(\frac{\mu^2(J,n)}{4m_{J,n}^2}\right)^{2l(J,n)}\times B(2l(J,n)-2,\tfrac{5}{2})\right]. \tag{3.5}$$

This result is too general to be useful. Therefore, let us further assume that

$$l(J,n)=l=\text{const independent of } J \text{ and } n, \tag{3.6a}$$

$$\mu(J,n)=\mu=\text{const independent of } J \text{ and } n. \tag{3.6b}$$

The only justification we can give for this additional assumption is the analogy between the three-point and the four-point function case. As is well known, Regge theory tells us that the asymptotic behavior of a four-point function from which no kinematic singularities have been extracted is given by the leading Regge singularity and does not depend on the external spins and helicities. Assumptions (3.6a) and (3.6b) are analogous for the three-point function case. Making these assumptions, we are led to the convergence of the following series:

$$\sum_{J,n;\ m_{J,n}>m}\left[(2J+1)\frac{1}{(4m_{J,n}^2)^{2l-2}}\right]. \tag{3.7}$$

Evidently the convergence of this series will depend on the mass spectrum of very-heavy-mass bosons. For example, a Veneziano-type mass spectrum for very high masses will demand that

$$l>\tfrac{5}{2}. \tag{3.8}$$

If asymptotically the spectrum looks like a single Regge trajectory, then

$$l>2 \tag{3.9}$$

in (3.1a).

For the fermions the corresponding series to that in expression (3.5) is

$$\sum_{J,n;\ m_{J,n}>m}\left[(2J+1)(4m_{J,n}^2)^2\left(\frac{\mu^2(J,n)}{4m_{J,n}^2}\right)^{2l(J,n)}\times B(\tfrac{3}{2},2l(J,n)-1)\right]. \tag{3.10}$$

Under the same assumptions as before concerning the expressions $l(J,n)$ and $\mu(J,n)$ (i.e., l and μ=const), we derive identical results with the boson case for the connection between the spectrum of very-high-mass particles and their asymptotic form factors.

IV. SUMMARY AND CONCULSIONS

We have found that the assumption that the electromagnetic current behaves like a free field at infinity implies that the expression $\mathfrak{F}_J(s)$ defined in (2.11) should satisfy the condition

$$s\mathfrak{F}_J(s)\to 0 \tag{2.12}$$

as $s\to\infty$. We have studied in detail the consequences of this condition for the pion, proton, and ρ-meson form factor, and, rather briefly, for the general spin case. We have found the following:

(i) Condition (2.12) requires that form factors should vanish as $s\to\infty$ *faster* than would have been expected from VMD.

(ii) As the spin of the particle increases, a stronger violation of VMD for the form factors at infinity is required in order to satisfy (2.12). For example, for the ρ-meson form factors we found that even a "dipole fit" of the type suggested by the experiment for the proton will violate (2.12).

In Sec. III we have made some additional, largely speculative, assumptions about $\mathfrak{F}_J(s)$ for particles of very high mass and spin,[17] and consequently we have found some correlations between the spectrum of very-high-mass particles and the asymptotic behavior of their form factor.

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