Prediction of $Y^*\Xi K$ Couplings by Superconvergence

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We have constructed superconvergent sum rules for the process $\bar{K}N\rightarrow K\Xi$ and used them to predict the coupling constants for the vertices $Y_0^*(1405)\Xi K$, $Y_0^*(1520)\Xi K$, and $Y_1^*(1660)\Xi K$ by saturating the sum rules with the low-lying intermediate states.

I. INTRODUCTION

SUPERCONVERGENCE relations can be exploited[1] to predict those coupling constants which are either not amenable to experiments or difficult to evaluate by other methods. The purpose of this paper is to consider the process

$$\bar{K}+N\rightarrow K+\Xi \tag{1}$$

and construct possible superconvergent sum rules in order to obtain plausible values for the various $Y^*\Xi K$ couplings.

II. SUM RULES

We define the invariant matrix for the process (1) by

$$M=-A+\tfrac{1}{2}i\gamma\cdot(q_i+q_f)B\,, \tag{2}$$

where q_i and q_f are the initial and final four-momenta of the mesons and A and B are invariant amplitudes. The asymptotic behavior ($S\rightarrow\infty$) of the amplitudes A and B are

$$A\sim s^{\alpha_I(u)-1/2},\quad B\sim s^{\alpha_I(u)-1/2}. \tag{3}$$

When α_I is the leading Regge trajectory for the u channel, $\bar{K}+N\rightarrow K+\Xi$, then the allowed values of strangeness and isospin are $S=-1$ and $I=0$, 1. We can safely assume[2] that α_I $(u=0)<-\frac{1}{2}$, since the allowed trajectories are Λ for $I=0$ and Σ for $I=1$.

In view of the above discussion we can write the following superconvergent relations:

$$\int_{-\infty}^{\infty}\mathrm{Im}A_{I=0}{}^{(u)}ds=0\,, \tag{4}$$

$$\int_{-\infty}^{\infty}\mathrm{Im}A_{I=1}{}^{(u)}ds=0\,, \tag{5}$$

$$\int_{-\infty}^{\infty}\mathrm{Im}B_{I=0}{}^{(u)}ds=0\,, \tag{6}$$

$$\int_{-\infty}^{\infty}\mathrm{Im}B_{I=1}{}^{(u)}ds=0\,. \tag{7}$$

We get no contribution from the t-channel intermediate states because this would require an $S=2$ particle. Therefore, at $u=0$ all the four amplitudes will receive contributions only from the s-channel amplitudes. If $A_I{}^s$ and $B_I{}^s$ denote the eigenamplitudes of isospin in the s channel, then one can write sum rules (4)–(7) as

$$\int_0^{\infty}C_{00}{}^{us}\,\mathrm{Im}A_0{}^sds+\int_0^{\infty}C_{01}{}^{us}\,\mathrm{Im}A_1{}^sds=0\,, \tag{8}$$

$$\int_0^{\infty}C_{10}{}^{us}\,\mathrm{Im}A_0{}^sds+\int_0^{\infty}C_{11}{}^{us}\,\mathrm{Im}A_1{}^sds=0\,, \tag{9}$$

$$\int_0^{\infty}C_{00}{}^{us}\,\mathrm{Im}B_0{}^sds+\int_0^{\infty}C_{01}{}^{us}\,\mathrm{Im}B_1{}^sds=0\,, \tag{10}$$

$$\int_0^{\infty}C_{10}{}^{us}\,\mathrm{Im}B_0{}^sds+\int_0^{\infty}C_{11}{}^{us}\,\mathrm{Im}B_1{}^sds=0\,, \tag{11}$$

where $C_{ij}{}^{us}$ are elements of the isospin crossing matrix. These can be recombined to yield the following sum rules:

$$\int_0^{\infty}\mathrm{Im}A_0{}^sds=0\,, \tag{12}$$

$$\int_0^{\infty}\mathrm{Im}A_1{}^sds=0\,, \tag{13}$$

$$\int_0^{\infty}\mathrm{Im}B_0{}^sds=0\,, \tag{14}$$

$$\int_0^{\infty}\mathrm{Im}B_1{}^sds=0\,. \tag{15}$$

III. RESULTS

We follow the usual method[1,3,4] of saturating the relations (12)–(15) by the low-lying intermediate states Λ, Σ, $Y_1^*(1385)$, $Y_0^*(1405)$, $Y_0^*(1520)$, and $Y_1^*(1660)$. The resulting sum rules are

$$\begin{aligned}[\tfrac{1}{2}(m_\Xi+m_N)-m_\Lambda]&g_{\Lambda NK}g_{\Lambda\Xi K}\\&+[m_{Y_0^*(1405)}+\tfrac{1}{2}(m_\Xi+m_N)]g_{Y_0^*(1405)NK}g_{Y_0^*(1405)\Xi K}\\&\quad+a(m_{Y_0^*(1520)})g_{Y_0^*(1520)NK}g_{Y_0^*(1520)\Xi K}=0\,,\end{aligned} \tag{16}$$

$$\begin{aligned}[\tfrac{1}{2}(m_\Xi+m_N)-m_\Sigma]&g_{\Sigma NK}g_{\Sigma\Xi K}\\&+A\,(m_{Y_1^*(1385)})g_{Y_1^*(1385)NK}g_{Y_1^*(1385)\Xi K}\\&\quad+a(m_{Y_1^*(1660)})g_{Y_1^*(1660)NK}g_{Y_1^*(1660)\Xi K}=0\,,\end{aligned} \tag{17}$$

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[2] L. Bertocchi, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (Wiley-Interscience Inc., New York, 1968), p. 197.

[3] R. H. Graham and M. Huq, Phys. Rev. **160**, 1421 (1967).

[4] M. S. K. Razmi and Y. Ueda (unpublished).

$$g_{\Lambda NK}g_{\Lambda\Xi K}+g_{Y_0^*(1405)NK}g_{Y_0^*(1405)\Xi K}+b(m_{Y_0^*(1520)})g_{Y_0^*(1520)NK}g_{Y_0^*(1520)\Xi K}=0\,, \quad (18)$$

$$g_{\Sigma NK}g_{\Sigma\Xi K}+B(m_{Y_1^*(1385)})g_{Y_1^*(1385)NK}g_{Y_1^*(1385)\Xi K}+b(m_{Y_1^*(1660)})g_{Y_1^*(1660)NK}g_{Y_1^*(1660)\Xi K}=0\,, \quad (19)$$

where

$$a(x)=\{(x-\tfrac{1}{2}m_\Xi)[m_K^2-\tfrac{1}{2}x^2+\tfrac{1}{3}m_Nm_\Xi+\tfrac{1}{2}(E_2m_N-E_1m_\Xi)+\tfrac{2}{3}E_1E_2]-\tfrac{1}{2}m_N(m_K^2-\tfrac{1}{2}x^2+\tfrac{2}{3}E_1E_2)-\tfrac{1}{3}[(E_1-m_N)(m_K^2-x^2+\tfrac{1}{2}m_Nm_\Xi)+\tfrac{1}{2}E_2m_N^2]\}\,,$$

$$A(x)=-\{(x+\tfrac{1}{2}m_\Xi)[m_K^2-\tfrac{1}{2}x^2+\tfrac{1}{3}m_Nm_\Xi+\tfrac{1}{3}(E_1m_\Xi-E_2m_N)+\tfrac{2}{3}E_1E_2]+\tfrac{1}{2}m_N(m_K^2-\tfrac{1}{2}x^2+\tfrac{2}{3}E_1E_2)-\tfrac{1}{3}[(E_1+m_N)(m_K^2-x^2+\tfrac{1}{2}m_Nm_\Xi)+\tfrac{1}{2}E_2m_N^2]\}\,,$$

$$b(x)=-\tfrac{1}{3}(3m_K^2-\tfrac{3}{2}x^2+2E_1E_2+E_1m_\Xi+E_2m_N-m_Nm_\Xi)\,,$$

$$B(x)=\tfrac{1}{3}(-3m_K^2+\tfrac{3}{2}x^2-2E_1E_2+E_1m_\Xi+E_2m_N+m_Nm_\Xi)\,,$$

with

$$E_1=(x^2+m_N^2-m_K^2)/2x\,,$$

$$E_2=(x^2+m_\Xi^2-m_K^2)/2x\,.$$

Here the m's denote the masses of the particles.

We take the following values for the coupling constants[5]:

$$g_{\Lambda NK}=-(1/\sqrt{3})g(1+2\alpha)\,,$$

$$g_{\Lambda\Xi K}=(1/\sqrt{3})g(4\alpha-1)\,,$$

$$g_{\Sigma NK}=g(1-2\alpha)\,,$$

$$g_{\Sigma\Xi K}=-g\,,$$

and

$$g^2=g_{NN\pi}^2=14.7\times4\pi\,, \quad (20)$$

with[6]

$$\alpha=0.29\,. \quad (21)$$

Substituting these values in sum rules (16)–(19), we obtain

$$g_{Y_0^*(1405)NK}g_{Y_0^*(1405)\Xi K}/4\pi=0.17\,, \quad (22)$$

$$g_{Y_0^*(1520)NK}g_{Y_0^*(1520)\Xi K}/4\pi=-3.54\,, \quad (23)$$

$$g_{Y_1^*(1385)NK}g_{Y_1^*(1385)\Xi K}/4\pi=-3.20\,, \quad (24)$$

$$g_{Y_1^*(1660)NK}g_{Y_1^*(1660)\Xi K}/4\pi=-56.91\,. \quad (25)$$

We have information on $g_{Y_0^*(1405)NK}/\sqrt{(4\pi)}$ ($=0.74$, Ref. 7), $g_{Y_0^*(1520)NK}/\sqrt{(4\pi)}$ ($=6.23$, Ref. 8), $g_{Y_1^*(1385)NK}$ (Refs. 9–12), and $g_{Y_1^*(1385)\Xi K}$ (Refs. 9–12) from model-dependent calculations. From decay[13] we can estimate $g_{Y_1^*(1660)NK}/\sqrt{(4\pi)}=-2.64$. If we use these values, we find that relation (24) is reasonably well satisfied because the right-hand side comes out to be -3.11 (Ref. 9), -4.33 (Ref. 10), -1.99 (Ref. 11), or -1.68 (Ref. 12). We further find

$$g_{Y_0^*(1405)\Xi K}/\sqrt{(4\pi)}=0.23\,, \quad (26)$$

$$g_{Y_0^*(1520)\Xi K}/\sqrt{(4\pi)}=-0.57\,, \quad (27)$$

$$g_{Y_1^*(1660)\Xi K}/\sqrt{(4\pi)}=21.55\,. \quad (28)$$

We would now like to have some idea how reasonable are our predictions (26)–(28). For example, we can calculate $Y_0^*{}_{(1405)}\rightarrow\Sigma\pi$ decay width. Using

$$\frac{g_{Y_0^*(1405)\Sigma\pi}}{4\pi}=\frac{m_{Y_0^*(1405)}}{k_\Sigma(m_\Sigma+E_\Sigma)}\times\tfrac{1}{3}\Gamma\,, \quad (29)$$

where k_Σ (E_Σ) is the momentum (energy) in the rest frame of the $Y_0{}^*{}_{(1405)}$ and $\Gamma=50$ MeV is the decay width of $Y_0{}^*{}_{(1405)}\rightarrow\Sigma\pi$, we get $g_{Y_0^*(1405)\Xi K}/\sqrt{(4\pi)}=0.25$. This is of the same order as predicted by us. For $Y_0^*{}_{(1520)\Xi K}$ the exact SU_3 value will require information on the mixing parameter which is not known. However, Graham *et al.*[14] have given a model-dependent estimate which is 0.46 or -0.042. As the magnitude is very small, the difference in sign is not very significant. Thus, our predicted value is near the current estimates. Graham *et al.*[14] find that $g_{Y_1^*(1660)\Xi K}/\sqrt{(4\pi)}\sim4$. Our predicted value is five times larger than this.

IV. DISCUSSION

The plausible results of our superconvergent model at $u=0$ lend support to the following assumptions which we have made:

(a) The asymptotic behavior of the scattering amplitude for the process $\bar{K}N\rightarrow K\Xi$ at $u=0$ is determined by the Reggeized u-channel trajectories of Λ and Σ.

(b) The superconvergent relations are well approximated by taking contributions from low-lying intermediate states.

We would like to make a remark about the saturation of the sum rules. If we do not include $Y_0^*(1520)$ and $Y_1^*(1660)$, we get contradictions in Eqs. (16) and (18) because of the exclusion of the former and in Eqs. (17) and (19) because of the exclusion of the latter. Similar

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[13] We have used $\Gamma(Y_1^*(1660)\rightarrow\bar{K}N)\approx7.5$ MeV; see Ref. 14.

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contradictions were found by Griffiths and Palmer[15] in their sum rules for $u=0$ pion-nucleon scattering. Our results, although based on $SU(3)$-breaking model-dependent calculations, do not show any obvious inconsistency. Therefore, we feel that calculations based on the superconvergent model with these assumptions deserve further study.

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Asymptotic Behavior of Form Factors and Possible Free-Field Behavior for the Electromagnetic Current*†

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It is shown that the assumption that the electromagnetic current behaves at infinity like a free vector-meson field implies that elastic form factors should vanish at infinity *faster* than would have been expected from vector-meson dominance (VMD). In fact, it is shown that VMD for form factors at infinite momentum transfer is violated with increasing strength as the spin of the particle concerned increases. Under some further speculative assumptions, correlations between the mass spectrum of very heavy particles and the asymptotic behavior of their form factors are found.

I. INTRODUCTION

It is widely known that the electromagnetic form factors accessible to present day experiments (e.g., pion and nucleon form factors) cannot be explained by vector-meson dominance (VMD) in the region of medium and large momentum transfers. Specifically, VMD can explain the nucleon form factors[1] only for squared momentum transfers smaller than about 1.2 $(\text{BeV}/c)^2$, and only under the assumption that strange, totally not understood, cancellations are taking place among the ρ, ω, and φ contributions. For squared momentum transfers larger than about 1.2 $(\text{BeV}/c)^2$ in the spacelike region, VMD cannot explain the nucleon form factor. With respect to the pion form factor, VMD can very well explain it for small timelike momentum transfers[2] but it seems to fail more or less quickly in the spacelike region.

On the other hand, there have recently been some speculations that the electromagnetic current behaves like a free vector-meson field at infinity.[3] To explain this more specifically, let us define the "current propagator" by[4]

$$\Delta_{\mu\nu}(q)\equiv i\int d^4x\, e^{-iqx}\langle 0|T^*(j_\mu{}^{\text{e.m.}}(x)j_\nu{}^{\text{e.m.}}(0))|0\rangle$$

$$=\int\frac{\rho(m^2)}{q^2+m^2-i\epsilon}\left(\delta_{\mu\nu}+\frac{q_\mu q_\nu}{m^2}\right)dm^2,\quad(1.1)$$

where

$$\rho(-P^2)\equiv\tfrac{1}{3}(2\pi)^3\sum_n\delta(P_n-P)\times\langle 0|j_\mu{}^{\text{e.m.}}(0)|n\rangle\langle n|j_\mu{}^{\text{e.m.}}(0)|0\rangle.\quad(1.2)$$

The assumption that the electromagnetic current behaves like a free field at infinity now means that

$$\Delta_{\mu\nu}(q)\xrightarrow[q_2\to\infty]{}\frac{\delta_{\mu\nu}}{q^2}\int\rho(m^2)dm^2+\frac{q_\mu q_\nu}{q^2}\int\frac{\rho(m^2)}{m^2}dm^2,\quad(1.3)$$

where

$$\int\frac{\rho(m^2)}{m^2}dm^2=\text{finite}\quad(1.4)$$

and

$$\int\rho(m^2)dm^2=\text{finite}.\quad(1.5)$$

Evidently, since $\rho(m^2)$ is positive definite, Eq. (1.4) should be satisfied if (1.5) is satisfied. Notice that relations (1.3)–(1.5) are expected, if it is possible to approximate the spectral function by a finite sum of δ-function terms (e.g., at ρ, ω, and φ).

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