Gauge Invariance and Regge-Pole Theory in Compton Scattering*

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The kinematics of Compton scattering by a boson target are studied. It is shown that a special interpretation of gauge invariance follows as a requirement of Lorentz invariance of S-matrix elements in the limit of zero photon mass. As a consequence, it is demonstrated that the Pomeranchuk trajectory can contribute to the forward-Compton-scattering amplitude without introducing a singular Regge residue function, in contrast with results obtained via the conventional formulation. The close correspondence with perturbation theory (not required *a priori*) is also discussed. It is found, however, that a similar treatment of gauge invariance for the Yang-Mills field cannot be given consistently.

I. INTRODUCTION

THE formulation and enforcement of gauge-invariance requirements are interesting problems in the use of helicity amplitudes (HA's). Invariance of S-matrix elements under Lorentz transformations restricts the massless photon to two transverse polarization states and introduces trouble in interpretation and understanding even for perturbation-theory results in terms of the HA formalism. For example, in one of the simpler cases, Compton scattering by the K meson,[1] which is discussed in detail in this paper, the s-channel (dynamical) K-meson pole in perturbation theory must be considered as a kinematical reflection of the u-channel *dynamical* pole in the HA formalism, because s-channel transversely polarized photons cannot couple to the K pole in the s channel. Furthermore, as is well known,[2] gauge invariance (if not handled properly) forces poles in Regge amplitudes involving photons to be kinematically interpreted.

In perturbation theory, gauge invariance is satisfied through certain combinations of (dynamical) poles in different channels. It is essential that each of these poles represent a state of definite spin and that these states have a universal charge. Such a "dynamical" fulfillment of the gauge-invariance requirement is rather difficult to imagine for the case of Regge theory, since the Regge poles in crossed channels are assumed to determine the asymptotic behavior.

Another more or less practical question is to what extent the widely accepted "phenomenological" Lagrangian $\rho_\mu{}^0 A_\mu$, where ρ^0 and A represent a neutral vector meson and a photon, respectively, is consistent with Regge theory.

These three mutually related questions were studied previously by the present authors[3] in the case of photopion production. In this paper we study whether the formalism used earlier[3] can be applied to cases where two photons are involved.

As is well known, there exists a problem in Compton scattering[4] if the "classical" Pomeranchuk trajectory is invoked to explain constant asymptotic cross sections in hadron-hadron scattering. Specifically, it has been argued that the Pomeranchuk trajectory with $\alpha(0)=1$ cannot contribute to Compton scattering at $t=0$. This leads to the paradoxical relation $\sigma^{\text{inel}}>\sigma^{\text{el}}>\sigma^{\text{tot}}$, where σ^{inel} is the inelastic cross section, etc. Furthermore, this result seriously contradicts experimentally supported notions concerning the Lagrangian $\rho_\mu{}^0 A_\mu$, since the asymptotic total ρ-hadron scattering cross section is expected to be constant due to Pomeranchuk trajectory exchange.

We can state our view of gauge invariance briefly as follows: Suppose there exists an amplitude for producing or absorbing a $J=1$ state V $(C=-1)$ with mass m; then the requirement that this amplitude be continuable to the point $m=0$ for the state with fixed $J=1$ is equivalent to gauge invariance.[5] This requires that the HA's involving the longitudinally polarized V's must vanish smoothly as $m\to 0$, so that relations exist among certain of the *kinematically* independent invariant amplitudes as well as among those HA's contained in these relations after the mass factors are considered appropriately. Boson Compton scattering in the u channel is treated with this approach in Sec. II, and

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[1] The intent in taking the K meson as an example is merely to preserve some of the features of the nucleon. The content of this paper can be generalized to the case of nucleon Compton scattering without difficulty.

[2] For example, S. Frautschi and L. Jones, Phys. Rev. **163**, 1820 (1967).

[3] T. Ebata and K. E. Lassila, Phys. Rev. Letters **21**, 250 (1968); Phys. Rev. **183**, 1425 (1969).

[4] See, e.g., V. D. Murr, Zh. Eksperim. i Teor. Fiz. **44**, 2173 (1963) [English transl.: Soviet Phys.—JETP **17**, 1458 (1963)]; H. D. I. Abarbanel and S. Nussinov, Phys. Rev. **158**, 1462 (1967); H. K. Shepard, *ibid.* **159**, 1331 (1967).

[5] This interpretation of gauge invariance allows us to understand the coupling $\rho_\mu{}^0 A_\mu$ in the framework of Regge theory even if the photon A_μ is not a Regge particle. The ρ^0 appearing in the Lagrangian should be regarded as a state with $J=1$, not necessarily the ρ pole on a Chew-Frautschi plot, but one of the vector mesons.

the resulting gauge-invariance relation for helicity amplitudes is specifically applied, in Sec. III, to the problem of Reggeization of the K meson. Here, difficulties similar to those in Reggeization of the pion in photoproduction amplitudes are found when one compares with perturbation-theory expressions.

In Sec. IV it is shown that the generalized gauge principles formulated in Sec. II can be applied to the t-channel HA's. The result is a conspiracy condition not expected in the usual formalism, where only transverse HA's are considered. This conspiracy condition implies that the Pomeranchuk trajectory can contribute at $t=0$ just as in, e.g., ρ-K scattering. The paradox mentioned above can thus be resolved without introducing singular Regge residue functions, as seems necessary in the conventional approach. Further elaboration of this point is reserved for Sec. V.

From the work discussed in Secs. II–V, it appears that the photon can and should be regarded as the zero-mass limit of a pure $J=1$ state to understand the similarity between a photon and the vector mesons (ρ^0, ω, and φ). In Sec. VI, however, we see that a similar situation is not expected for the Yang-Mills field. It is shown that the massless charged Yang-Mills fields $A_\mu^\pm$ cannot couple to $\rho_\mu^\pm$ by $\rho_\mu^\pm A_\mu^\pm$, as contrasted with the neutral case, with $\rho_\mu^0 A_\mu$ coupling.

The last section contains some closing remarks.

II. COMPTON SCATTERING (u CHANNEL)

In this section we consider the kinematics of Compton scattering,

$$V_\mu(k)+K(p_1)\rightarrow V_\nu(k')+K(p_2)\,, \tag{1}$$

where $V_\mu(k)$ is the neutral vector meson with mass m and K is the isodoublet K meson with mass M. The momentum of each particle is written in parentheses. We define reaction (1) as occurring in the u channel and study the asymptotic behavior in the s channel. We assume V_μ represents a field of pure spin 1; i.e., it satisfies the condition

$$k_\mu V_\mu(k)=0\,. \tag{2}$$

The amplitude for process (1) can be written as a sum of kinematical-singularity-free invariant amplitudes $A_i(s,t,u;m^2)$:

$$\epsilon_\nu' T_{\nu\mu}\epsilon_\mu=(\epsilon'\cdot\epsilon)A_1+(P\cdot\epsilon')(P\cdot\epsilon)A_2+[(P\cdot\epsilon')(k'\cdot\epsilon)+(P\cdot\epsilon)(k\cdot\epsilon')]A_3+(k'\cdot\epsilon)(k\cdot\epsilon')A_4\,, \tag{3}$$

where ϵ and ϵ' represent the polarization vector of the initial and final vector particle, respectively, and $P=p_1+p_2$. It is clear that the information of Eq. (2) should be contained in the four independent HA's. We choose these four to be f_{++}, f_{0+}, f_{-+}, and f_{00}. The explicit relations between the f's and A_i's are given in

TABLE I. u-channel HA's expressed in terms of invariant amplitudes.

$f_{++}=\frac{1}{2}(1+\cos\theta)[-A_1-k^2(1-\cos\theta)(A_2-2A_3+A_4)]$
$f_{-+}=-\frac{1}{2}(1-\cos\theta)[A_1+k^2(1+\cos\theta)(-A_2+2A_3-A_4)]$
$f_{0+}=-(1/\sqrt{2})m^{-1}\sin\theta[\omega A_1-k^2(2E+\omega+\omega\cos\theta)(A_2-A_3)-k^2\omega(1-\cos\theta)(A_3-A_4)]$
$f_{00}=m^{-2}[(k^2-\omega^2\cos\theta)A_1+k^2(2E+\omega+\omega\cos\theta)^2A_2+2k^2\omega(1-\cos\theta)(2E+\omega+\omega\cos\theta)A_3+k^2\omega^2(1-\cos\theta)^2A_4]$

Table I. To get the results of Table I, we have chosen the following coordinate system:

$$\begin{aligned}&k=(\omega,0,0,k)\,, && k'=(\omega,k\sin\theta,0,k\cos\theta)\,,\\ &p_1=(E,0,0,-k)\,, && p_2=(E,-k\sin\theta,0,-k\cos\theta)\,,\end{aligned} \tag{4}$$

where $\omega^2=m^2+k^2$ and $E^2=M^2+k^2$. The polarization vector for a longitudinally polarized vector particle with momentum k is given by

$$\epsilon_0=(k/m,0,0,\omega/m)\,. \tag{5}$$

Also, $u=(k+p_1)^2$, $s=(k-p_2)^2$, and $t=(k-k')^2$.

We define the parity-conserving HA's (PCHA's) X_1, X_2, Y, and Z as

$$\begin{aligned}X_{1,2}&\equiv[f_{++}/(1+\cos\theta)]\pm[f_{-+}/(1-\cos\theta)]\,,\\ Y&\equiv(\sqrt{2}/\sin\theta)f_{+0}\,,\\ Z&\equiv f_{00}\,.\end{aligned} \tag{6}$$

It is easy to see that the $K^*(1^-)$ and $K_V(2^+)$ Regge poles contribute only to X_2, whereas the $K(0^-)$ and $K_A(1^+)$ Regge poles can contribute to X_1, Y, and Z.

As one may see from Table I, the amplitudes Y and Z will go to infinity as m^{-1} and m^{-2}, respectively, as m goes to zero, if the A_i's are mutually dynamically independent. As shown by various authors,[3,6] gauge invariance, which correlates those *kinematically independent* A_i's, is a requirement that Y and Z vanish as $m\rightarrow 0$ despite the aforementioned mass "singularity." In the following, it will be shown implicitly that such a gauge-invariance requirement means, for $m\neq 0$, that the helicity amplitudes X_1, Y, and Z are not entirely independent of each other and that Y and Z are explicitly proportional to m and m^2, respectively.

In order to obtain a generalized gauge-invariance relation, we notice that X_2 cannot be correlated with X_1, Y, and Z, in general. From a study of the relations among X_1, Y, and Z, we obtain a general relation for $m=0$ (interpretable as "continued" from the $m\neq 0$ gauge condition):

$$\begin{aligned}&A_1-(2E\omega+\omega^2+k^2\cos\theta)(A_2-A_3)-(\omega^2-k^2\cos\theta)(A_3-A_4)=m^2\varphi_1\,,\\ &(2E\omega+\omega^2+k^2\cos\theta)A_2+(\omega^2-k^2\cos\theta)A_3=m^2\{-[\omega(1-\cos\theta)/(2E+2\omega)]\varphi_1+m^2\varphi_2\}\,.\end{aligned} \tag{7}$$

[6] S. Weinberg, Phys. Rev. 134, B882 (1964).

TABLE II. Invariant amplitudes expressed in terms of u-channel KSF HA's.

$$A_1=-u^{-1}[\xi^2\tilde{X}_1+(\lambda^2+2ut)\tilde{X}_2]$$
$$A_2=-(m^2-\tfrac{1}{2}t)u^{-1}(\tilde{X}_1+\tilde{X}_2)+\tfrac{1}{2}m^2[(\xi^2+\lambda^2)t-2m^2\lambda^2]\bar{\varphi}_1+m^4\bar{\varphi}_2$$
$$A_3=(u-M^2+\tfrac{1}{2}t)u^{-1}(\tilde{X}_1+\tilde{X}_2)+\tfrac{1}{2}m^2[2(u-M^2)\lambda^2+(\xi^2+\lambda^2)t]\bar{\varphi}_1+m^4\bar{\varphi}_2$$
$$A_4=(2\xi-m^2+\tfrac{1}{2}t)\tilde{X}_1u^{-1}-\{2[u+(M^2-m^2)]+m^2-\tfrac{1}{2}t\}\tilde{X}_2u^{-1}+m^2[2\xi\lambda^2+\lambda^2(t-m^2)+2m^2ut]\bar{\varphi}_1+m^4\bar{\varphi}_2$$

In terms of the invariants, Eqs. (7) can be rewritten as

$$A_1-(u-M^2+\tfrac{1}{2}t)(A_2-A_3)+(\tfrac{1}{2}t-m^2)(A_3-A_4)=m^2\varphi_1\,,$$
$$(u-M^2+\tfrac{1}{2}t)A_2-(\tfrac{1}{2}t-m^2)A_3=m^2[(\xi t/2\lambda^2)\varphi_1+m^2\varphi_2]\,, \quad (8)$$

where

$$\xi\equiv u-(M^2-m^2)\,,\ \lambda^2=[u-(M+m)^2][u-(M-m)^2]\,,$$

and where the φ_i are defined in terms of analytic functions $\bar{\varphi}_i$ free of kinematic singularities as $\bar{\varphi}_1\equiv\varphi_1\xi^{-2}\lambda^{-2}$ and $\bar{\varphi}_2\equiv\varphi_2/\xi$. Equations (7) and (8) reduce to the usual gauge condition at $m=0$. Equations (7) imply the following relations among HA's X_1, $Y'\equiv Y/m$, and $Z'\equiv Z/m^2$:

$$X_1=-\omega Y'+k^2\varphi_1\,,$$
$$\omega Z'=-\cos\theta\ Y'+2k^2(E+\omega)\varphi_2\,. \quad (9)$$

With the help of Eqs. (9), we can construct kinematical-singularity-free PCHA's (KSFPC HA's) $\tilde{X}_1$, $\tilde{X}_2$, $\tilde{Y}$, and $\tilde{Z}$ as

$$X_1=(\xi^2/u)\tilde{X}_1\,,\quad X_2=(\lambda^2/u)\tilde{X}_2\,,$$
$$Y'=(\xi/\sqrt{u})\tilde{Y}\,,\quad Z'=\lambda^{-2}\tilde{Z}\,, \quad (10)$$

and we must have a conspiracy relation at $u=0$:

$$\tilde{X}_1(u=0)=-\tilde{X}_2(u=0)\,. \quad (11)$$

This conspiracy relation must hold independent of Regge theory, since it is merely an identity in terms of the invariant amplitudes. With Eqs. (10), we can rewrite Eqs. (9) as

$$\tilde{Y}=-(2\tilde{X}_1-\tfrac{1}{2}\lambda^4\bar{\varphi}_1)\,,\ \tilde{Z}=-[2(\lambda^2+2ut)\tilde{Y}-\lambda^4\bar{\varphi}_2]\,. \quad (12)$$

The invariant amplitudes A_i can be expressed by the KSFPC HA's $\tilde{X}_1$ and $\tilde{X}_2$ as given in Table II. The kinematical factors for X_1 and X_2 given in Eqs. (10) coincide, in the limit $m\rightarrow 0$, with those one would obtain if one had started with $m=0$.[7] It can also be shown easily that expressions given in Table II are consistent with the generalized gauge conditions (8) at the special points $u=M^2-\frac{1}{2}t$ and $t=2m^2$.

III. REGGEIZATION (u CHANNEL)

We are particularly interested in Reggeizing the u-channel K-meson pole, since in the perturbation calculation this u pole along with an s-channel pole and a contact interaction term form a set of graphs which satisfy gauge invariance. Explicitly, the results for the perturbation graphs are (for $m=0$)

$$A_1=-2e^2\,,$$
$$A_2=A_4=e^2[(u-M^2)^{-1}+(s-M^2)^{-1}]=-e^2t/(s-M^2)(u-M^2)\,, \quad (13)$$
$$A_3=e^2[(u-M^2)^{-1}-(s-M^2)^{-1}]=e^2(s-u)/(s-M^2)(u-M^2)\,,$$

where e is the VKK coupling constant.

We assume $\tilde{Z}$ has a dynamical Regge K pole of the form

$$\tilde{Z}^K=\beta_K\frac{1+e^{-i\pi\alpha_K(u)}}{\sin\pi\alpha_K(u)}P_{\alpha_K}(z)\,, \quad (14)$$

where

$$\alpha_K(M^2)=0\,, \quad (15)$$

with $z=\cos\theta$. Then we obtain the following form for the K Regge-pole contribution to $\tilde{X}_1$, $\tilde{Y}$, $\bar{\varphi}_1$, and $\bar{\varphi}_2$:

$$\tilde{X}_1^K=\frac{\beta_K}{4\lambda^2}\frac{1}{\alpha_K^2}\frac{1+e^{-i\pi\alpha_K}}{\sin\pi\alpha_K}\times[P_{\alpha_K}'(z)+zP_{\alpha_K}''(z)+a_KP_{\alpha_K}''(z)]\,,$$
$$\tilde{Y}^K=-\frac{\beta_K}{2\lambda^2}\frac{1}{\alpha_K}\frac{1+e^{-i\pi\alpha_K}}{\sin\pi\alpha_K}P_{\alpha_K}'(z)\,,$$
$$\bar{\varphi}_1^K=\frac{\beta_K}{\lambda^6}\frac{1}{\alpha_K^2}\frac{1+e^{-i\pi\alpha_K}}{\sin\pi\alpha_K}a_KP_{\alpha_K}''(z)\,, \quad (16)$$
$$\bar{\varphi}_2^K=-\frac{\beta_K}{\lambda^4}\frac{1}{\alpha_K}\frac{1+e^{-i\pi\alpha_K}}{\sin\pi\alpha_K}P_{\alpha_K-1}'(z)\,.$$

In order that $\bar{\varphi}_1$ be KSF, we must demand $a_K\propto\lambda^2$ for $\lambda^2\sim 0$. With Eqs. (16) we can satisfy the relations (12) by using the identities

$$zP_\alpha'(z)-P_{\alpha-1}'(z)=\alpha P_\alpha(z)\,,$$
$$P_\alpha'(z)+zP_\alpha''(z)=\alpha P_\alpha'(z)+P_{\alpha-1}''(z)\,. \quad (17)$$

If we abbreviate the expression for $\tilde{X}_1^K$ given by Eq. (16) as

$$\tilde{X}_1^K\approx\bar{\beta}_K\frac{1+e^{-i\pi\alpha_K}}{\sin\pi\alpha_K}\left(\frac{s}{s_0}\right)^{\alpha_K-1}\,,\quad s\rightarrow\infty \quad (18)$$

then at $u=M^2$ (with $m=0$), we obtain from Table II the following expressions for the A_i's:

$$A_1^K=0\,,$$
$$A_j^K=\frac{M^2-s}{2M^2}\bar{\beta}_K\frac{2}{\pi\alpha_K'(M^2)}\left(\frac{s}{s_0}\right)^{-1}\frac{1}{u-M^2}\,,\quad j=2,3,4. \quad (19)$$

[7] D. Horn, California Institute of Technology Report/CTSL Internal Report No. 34, 1967 (unpublished).

TABLE III. t-channel HA's expressed in terms of invariant amplitudes.

$$f_{++}\equiv\bar{f}_{++}=A_1-2p^2\sin^2\theta\,A_2$$
$$f_{+-}\equiv\sin^2\theta\,\bar{f}_{+-}=2p^2\sin^2\theta\,A_2$$
$$f_{+0}=2\sqrt{2}m^{-1}\sin\theta(p^2\omega\cos\theta\,A_2-\omega kpA_3)$$
$$f_{00}=m^{-2}[(k^2+\omega^2)A_1-4p^2\omega^2\cos^2\theta\,A_2+8p\omega^2k\cos\theta\,A_3-4\omega^2k^2A_4]$$

Therefore, with the choice of the parameter

$$\frac{1}{M^2}\beta_K\frac{1}{\pi\alpha'(M^2)}=-e^2, \qquad (20)$$

we recover the perturbation results of Eqs. (13), with A_1 as the only exception. In perturbation theory, the contact interaction term associated with the kaon poles through gauge invariance remains independent of the energy, in contrast with our Reggeized result showing

$$A_1{}^K\approx s^{\alpha_K-1}\quad\text{for large } s. \qquad (21)$$

Thus, from our present point of view, the existence of the contact interaction in perturbation theory does not restrict the high-energy behavior of A_1 at all (see, however, Sec. VII).

IV. COMPTON SCATTERING (t CHANNEL)

In this section, we discuss the Reggeization of the t-channel amplitudes. The t-channel amplitude describing the process

$$\bar{V}_\mu(k)+V_\nu(k')\rightarrow K(p_1)+\bar{K}(p_2) \qquad (22)$$

can be written as

$$\epsilon_\mu T_{\mu\nu}\epsilon_\nu'=(\epsilon\cdot\epsilon')A_1+(P\cdot\epsilon)(P\cdot\epsilon')A_2+[(P\cdot\epsilon')(k'\cdot\epsilon)-(P\cdot\epsilon)(k\cdot\epsilon')]A_3-(k'\cdot\epsilon)(k\cdot\epsilon')A_4, \qquad (23)$$

with $P=p_1-p_2$. Using the coordinate system defined by the four-vectors

$$\begin{aligned}k&=(\omega,0,0,k),\quad k'=(\omega,0,0,-k),\\ p_1&=(E,p\sin\theta,0,p\cos\theta),\\ p_2&=(E,-p\sin\theta,0,-p\cos\theta),\end{aligned} \qquad (24)$$

we find the t-channel HA's as listed in Table III. With the generalized gauge-invariance conditions in Eq. (8), the HA's containing the longitudinal, polarized vector mesons can be rewritten as

$$\begin{aligned}f_{+0}&\equiv 2\sqrt{2}m\sin\theta\;\bar{f}_{+0}\\ &=2\sqrt{2}m\sin\theta\,\{-A_2+A_3-\omega^2k(k-p\cos\theta)\\ &\qquad\times[(\omega^2-kp\cos\theta)^2-M^2m^2]^{-1}\varphi_1+m^2\varphi_2\},\\ f_{00}&\equiv m^2\bar{f}_{00}\\ &=m^2\{k^{-2}[A_1-\omega^2(A_2-2A_3+A_4)]\\ &\qquad+\varphi_1\omega^2[-k^{-2}+2p^2\sin^2\theta/((\omega^2-kp\cos\theta)^2-M^2m^2)]\\ &\qquad+\varphi_2 2\omega^2k^{-2}(k^2-kp\cos\theta)\}.\end{aligned} \qquad (25)$$

TABLE IV. Invariant amplitudes expressed in terms of the t-channel KSF HA's.

$$A_1=\tilde{f}_{++}+\Phi^2\tilde{f}_{+-}$$
$$A_2=2(t-4m^2)\tilde{f}_{+-}$$
$$A_3=(t-4m^2)(\tilde{f}_{+0}+2\tilde{f}_{+-})-\tfrac{1}{2}t\xi^3\bar{\varphi}_1-m^2\xi\bar{\varphi}_2$$
$$\begin{aligned}A_4=&-[(t-4m^2)/t]\tilde{f}_{00}+2(t-4m^2)\tilde{f}_{+0}\\ &+[2(t-4m^2)+4\Phi^2/t]\tilde{f}_{+-}+(4/t)\tilde{f}_{++}\\ &+\xi^2(-\lambda^2-t\xi+\tfrac{1}{2}\Phi^2)\bar{\varphi}_1-\xi(\xi+2m^2)\bar{\varphi}_2\end{aligned}$$

Equations (25), together with the expression for f_{++} and f_{+-} in Table III, can be used to express the A_i's in terms of f_{++}, f_{+-}, f_{+0}, f_{00}, φ_1, and φ_2, where $\bar{f}_{++}=f_{++}$ and $\bar{f}_{+-}=f_{+-}/\sin^2\theta$. These relations can be used to construct the KSFPC HA's. After straightforward calculation, we find the KSF HA's can be defined as

$$\begin{aligned}\tilde{f}_{++}&=\bar{f}_{++},\\ \tilde{f}_{+-}&=\bar{f}_{+-}[(t-4m^2)(t-4M^2)]^{-1},\\ \tilde{f}_{+0}&=\bar{f}_{+0}/(t-4m^2),\\ \tilde{f}_{00}&=\bar{f}_{00}.\end{aligned} \qquad (26)$$

The invariant amplitudes A_i can be written in terms of KSFPC HA's and are given in Table IV, and the KSF HA's can be reexpressed in terms of the A_i's as listed in Table V. In these tables Φ^2 is defined by

$$\begin{aligned}\sin^2\theta&=[(t-4m^2)(t-4M^2)]^{-1}[(t-4m^2)\\ &\qquad\times(t-4M^2)-(u-s)^2]\\ &\equiv[(t-4m^2)(t-4M^2)]^{-1}\Phi^2.\end{aligned} \qquad (27)$$

The generalized gauge conditions of Eq. (8) can be written in terms of these t-channel KSF HA's as

$$\begin{aligned}&4\xi\tilde{f}_{+-}-(t-2m^2)\tilde{f}_{+0}=\xi[-\tfrac{1}{2}t\xi^2\bar{\varphi}_1-m^2\bar{\varphi}_2],\\ &t^{-1}[-\tilde{f}_{++}-\Phi^2\tilde{f}_{+-}+\xi t\tilde{f}_{+0}+\tfrac{1}{2}(t-2m^2)\tilde{f}_{00}]\\ &\qquad=\tfrac{1}{4}\xi^2\Phi^2\bar{\varphi}_1-\tfrac{1}{2}\xi^2\bar{\varphi}_2.\end{aligned} \qquad (28)$$

The expression for A_4 in Table IV deserves further comment; it appears that A_4 might have a kinematical pole at $t=0$. However, more than one KSF HA is involved and a conspiracy relation exists at $t=0$:

$$\tilde{f}_{++}+m^2\tilde{f}_{00}+\Phi^2\tilde{f}_{+-}=0, \qquad (29)$$

which, as one can check using the amplitudes of Table V, is essentially such as to guarantee the identity $A_4=A_4$. Equation (29) is derived for $m\neq0$. If we put

TABLE V. t-channel KSF HA's in terms of the invariant amplitudes.

$$\tilde{f}_{++}=A_1-\tfrac{1}{2}[\Phi^2/(t-4m^2)]A_2$$
$$\tilde{f}_{+-}=\tfrac{1}{2}(t-4m^2)^{-1}A_2$$
$$\tilde{f}_{+0}=(t-4m^2)^{-1}(-A_2+A_3+\tfrac{1}{2}t\xi^3\bar{\varphi}_1+m^2\xi\bar{\varphi}_2)$$
$$\begin{aligned}\tilde{f}_{00}=(t-4m^2)^{-1}[&4A_1-t(A_2-2A_3+A_4)\\ &+(-\xi^2\lambda^2+\tfrac{1}{2}\xi^2\Phi^2)t\bar{\varphi}_1-\xi^2t\bar{\varphi}_2]\end{aligned}$$

$m=0$ in Tables IV and V, the conspiracy relation is slightly modified to the form

$$4\tilde{f}_{++}-t\tilde{f}_{00}+4\Phi^2\tilde{f}_{+-}=0 \tag{30}$$

at $t=0$. Note that Tables IV and V are consistent when m is zero; A_2 has a kinematical zero at $t=0$ and $t\tilde{f}_{00}$ is KSF. The first equation is Eq. (28) may appear too stringent, since it requires that $\tilde{f}_{+0}$ vanish at $\xi=0$. This is, however, a very desirable situation, as seen from the earlier analysis of u-channel HA's. From Table V we see that this requirement on $\tilde{f}_{+0}$ at $\xi=0$ is equivalent to a requirement on A_2-A_3, which, from Table II, is

$$A_2-A_3=\xi[-u^{-1}(\tilde{X}_1+\tilde{X}_2)-m^2\lambda^2\bar{\varphi}_1]. \tag{31}$$

V. ASYMPTOTIC CROSS SECTION

One of the unsatisfactory results of the conventional treatment of Compton scattering in Regge theory[4] is that the asymptotic total cross section cannot remain finite if the Pomeranchuk trajectory ($\alpha_p=1$ at $t=0$) is assumed. On the other hand, we know that the vector-meson–K-meson scattering total cross section can remain finite even with the assumption of the Pomeranchuk trajectory exchange. Thus, it would appear that the well-accepted (phenomenological) Lagrangian $\rho_\mu{}^0A_\mu$, where $\rho_\mu{}^0$ is any one of the neutral vector mesons, is incompatible with a moving Pomeranchuk trajectory.

The reason why such a conclusion is derived is easily seen. If one starts, as *conventionally*, with $m\equiv0$, there exist only *two* HA's namely, f_{++} and f_{+-}, and gauge invariance is introduced *a priori*, with

$$\begin{aligned} 2A_1+(u-s)A_3-tA_4&=0,\\ (u-s)A_2-tA_3&=0. \end{aligned} \tag{32}$$

These A_i's are all expressed by two HA's, $\bar{f}_{++}$ and $\bar{f}_{+-}$, defined in Table III:

$$\begin{aligned} A_1&=\bar{f}_{++}+[t(t-4M^2)]^{-1}[t(t-4M^2)-(u-s)^2]\bar{f}_{+-},\\ A_2&=2(t-4M^2)^{-1}\bar{f}_{+-},\\ A_3&=2(u-s)[t(t-4M^2)]^{-1}\bar{f}_{+-},\\ A_4&=2[\bar{f}_{++}+\bar{f}_{+-}]/t. \end{aligned} \tag{33}$$

The only possible way to avoid kinematical singularities in the A_i's is to define KSF HA's $\tilde{f}_{++}$ and $\tilde{f}_{+-}$ as

$$\begin{aligned} \tilde{f}_{++}&=\bar{f}_{++}/t,\\ \tilde{f}_{+-}&=\bar{f}_{+-}[t(t-4M^2)]^{-1}. \end{aligned} \tag{34}$$

Then the invariant amplitudes A_i, Eq. (33), can be readily written in terms of the KSF HA's of Eq. (34), so that, for example, A_1 is

$$A_1=t\tilde{f}_{++}+[t(t-4M^2)-(u-s)^2]\tilde{f}_{+-}. \tag{35}$$

For a Regge-pole contribution of the usual form,

$$\begin{aligned} \tilde{f}_{++}{}^P&\approx\beta_{++}\frac{1+e^{-i\pi\alpha_P}}{\sin\pi\alpha_P}\left(\frac{s}{s_0}\right)^{\alpha_P},\\ \tilde{f}_{+-}{}^P&\approx\beta_{+-}\frac{1+e^{-i\pi\alpha_P}}{\sin\pi\alpha_P}\alpha_P(\alpha_P-1)\left(\frac{s}{s_0}\right)^{\alpha_P-2}, \end{aligned} \tag{36}$$

one sees from Eq. (35) that the Pomeranchuk trajectory cannot contribute to A_1 (or, e.g., A_2) at $t=0$, and consequently cannot contribute to the total cross section. On the other hand,[4] it is known that the Pomeranchuk trajectory can contribute to photo-ρ^0-meson production, as well as ρ-K scattering, at $t=0$. These results lead one, in the conventional approach, to conjecture that the (possibly) constant asymptotic total cross section in Compton scattering is obtained by means of a different mechanism than in usual hadron-hadron scattering,[8] e.g., singular residue functions must be introduced.

In the present work, however, we have no such difficulties. The expression for A_1 in Table IV shows that $\tilde{f}_{++}$ can contribute to A_1 at $t=0$. Thus, in our approach asymptotic cross sections for Compton scattering *can* be obtained by the same mechanism as in hadron-hadron scattering, and the (phenomenological) Lagrangian $\rho_\mu A_\mu$ is compatible with Regge theory.

VI. YANG-MILLS FIELDS

In comparing the relations of Tables II, IV, and V with the perturbation-theory results of Eq. (13), we note that, remarkably, the crossing relations between u and s are very important in establishing the KSF HA's. For example, from Tables II and IV, A_2 must have a kinematical zero at $t=0$ when $m=0$, and we see that the Born terms in Eq. (13) are consistent with this statement because of the crossing relation.

It is interesting, therefore, to study a similar case with a different type of crossing relation, namely, the $I=1$ exchange part of the "Compton scattering" for Yang-Mills fields. The process we consider is, in the u channel,

$$V_\mu{}^\alpha(k)+K(p_1)\to V_\nu{}^\beta(k')+K(p_2)\quad(\alpha\neq\beta), \tag{37}$$

where α and β are the isospin indices, and the mass associated with the Yang-Mills field V is m.

To see whether there exists a physical limiting procedure $m\to0$ for reaction (37), as was the case for reaction (1), we study the u-channel HA's. The contents of Table I are independent of crossing relations, and straightforward inversion leads to

$$\begin{aligned} A_2=\lambda^{-2}[(-m^2+\tfrac{1}{2}t\xi^2\lambda^{-2})X_1-\tfrac{1}{2}(2m^2-t)X_2\\ +2m^2(\sqrt{u})t\xi\lambda^{-2}Y'+m^4Z']. \end{aligned} \tag{38}$$

[8] H. D. I. Abarbanel, F. E. Low, I. J. Muzinich, S. Nussinov, and J. H. Schwarz, Phys. Rev. **160**, 1329 (1967).

This equation shows that if Y' and Z' are finite (i.e., Y and Z vanish) in the limit $m \to 0$, then the only way to avoid the appearance of a $t=0$ kinematical zero in A_2 is to assume one of the X_i's has a kinematical pole at $t=0$ when $m \to 0$.

On the other hand, the Born terms in this case take the form

$$\begin{aligned} A_1 &= \frac{2(s-u)}{t-m^2}, \\ A_2 &= \frac{s-u}{(s-M^2)(u-M^2)}, \\ A_3 &= \frac{-(t-2m^2)}{(s-M^2)(u-M^2)} + \frac{4}{t-m^2}, \\ A_4 &= \frac{s-u}{(s-M^2)(u-M^2)}, \end{aligned} \tag{39}$$

where the coupling constant as well as the isospin matrices are suppressed. The contribution of the Born diagram to A_2 is clearly incompatible with the existence of a kinematical zero at $t=0$ when $m \to 0$.

This shows that if the existence of the smooth limit $m \to 0$ is assumed, we cannot construct the u-channel KSFPC HA's by factoring out a kinematical function of u only. We believe that this is (kinematical) evidence against the existence of a *massless charged* "photon" $A_\mu^\pm$ (Yang-Mills field) coupled to the $\rho_\mu^\pm$ meson through $\rho_\mu^\pm A_\mu^\pm$ in analogy with the corresponding neutral case $\rho_\mu^0 A_\mu$.

Actually, the perturbation-theory result (39) is not compatible with our generalized gauge condition (8) in the sense that the function φ_1, defined by the two equations in (8), cannot be the same[9] in both. Stated otherwise, the Born terms (39) do not allow the existence of the smooth limit for Z' as $m \to 0$.

Therefore, we conclude that, for the case of the Yang-Mills field, $m=0$ is an isolated physically allowed situation.

[9] This is in contrast with the case of usual Compton scattering.

VII. REMARKS

In Secs. IV and V, it is shown that even the "classical" Pomeranchuk trajectory can be incorporated into the Regge theory for Compton scattering, and from the t-channel Reggeization we obtain, for example, $A_1 \sim s^{\alpha_P(t)}$. On the other hand, the same amplitudes A_i can be represented by the sum of u-channel Regge poles (which carries hypercharge in the example of K-meson Compton scattering). Consistency of the t- and u-channel Reggeization can be achieved, however, by assuming appropriate subtractions in writing the dispersion relations for HA's.[10] The consistency of t- and u-channel analyticity, however, gives rather stringent kinematical restrictions at some specific points. For example, from Table IV we have $A_2(t=4m^2)=0$, which means, from Table II,

$$m^2\{u^{-1}(\tilde{X}_1+\tilde{X}_2)+m^2[(2\xi^2+\lambda^2)\bar{\varphi}_1+\bar{\varphi}_2]\}_{t=4m^2}=0. \tag{40}$$

The generalized gauge conditions of Eq. (8) are not crossing-symmetric. The odd parts, however, are always of the order of m^2. Therefore, reformulation of Eq. (8) in a crossing-symmetric way is merely a redefinition of φ_1 and φ_2.

From the study of Compton and Yang-Mills field scattering, we find that the photon and the Yang-Mills field behave differently when they are in virtual states. For a Yang-Mills field, the virtual state cannot be realized as a pure spin-1 state, in contrast with the photon case. For such a field (Yang-Mills), we believe that the approach taken by Ball and Jacob[11] is more appropriate; the virtual Yang-Mills field could be regarded as a mixture of spin-1 and spin-0 states.

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[10] This situation is not peculiar to Compton scattering.

[11] J. S. Ball and M. Jacob, Nuovo Cimento **54**, 620 (1968).