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Operational Formulation of the Principle of Equivalence

J. L. ANDERSON
Physics Department, Stevens Institute of Technology, Hoboken, New Jersey

AND

R. GAUTREAU
Physics Department, Newark College of Engineering, Newark, New Jersey 07102

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A formulation of the principle of equivalence is given that avoids some of the difficulties in the more usual statements of the principle. In particular, we give an operational definition of local systems for which the principle is applicable. The relation between our statement and other statements of the principle is discussed. To investigate the relation of our statement to general relativity, we construct model reference frames from physical systems whose dynamics is sufficiently simple that their behavior can be described within the framework of the general theory without the need to introduce extraneous assumptions. With the help of these reference frames, we then show that it is possible to formulate gedankenexperiments yielding results that satisfy our statement of the principle.

1. INTRODUCTION

It is generally accepted that the principle of equivalence (PE) constitutes one of the cornerstones of general relativity. What is less generally accepted is just what is the content of the principle, there being almost as many formulations of the principle as there are authors writing about it. All of these formulations more or less take as their starting point the similarity between inertial and gravitational forces, at least in sufficiently small regions of space-time. It is only when one tries to make this similarity more precise that differences arise. Dicke[1] has, for example, distinguished two principles of equivalence, a weak principle and a strong principle. Fock,[2] on the other hand, considers the PE as a heuristic principle and makes no attempt to give it a precise form. Finally, we have the eloquent statement of Synge[3]: "I have never been able to understand this principle. . . . The principle of equivalence performed the essential office of midwife at the birth of general relativity. . . . I suggest that the midwife be now buried with appropriate honours."

If indeed the PE is a cornerstone of general relativity, then it is necessary to give it a precise statement in order to assess its role in the theory. It is also necessary to make its content and range of validity precise if one wishes to test its observational consequences. In this paper we shall give a formulation of the PE that, at least in principle, is capable of experimental verification on the one hand, and on the other hand exhibits its essential role in general relativity. We shall then discuss a class of space-time measurements that can be carried out on a simple system in and out of a gravitational field that can serve as a basis for the implementation of the principle as we have formulated it.

[1] R. H. Dicke, in *Gravitation and Relativity*, edited by H.-Y. Chiu and W. F. Hoffman (W. A. Benjamin, Inc., New York, 1964), p. 1.

[2] V. Fock, *The Theory of Space Time and Gravitation* (Pergamon Press., Inc., New York, 1959).

[3] J. L. Synge, *Relativity: The General Theory* (North-Holland Publishing Co., Amsterdam, 1960).

2. FORMULATION OF A PRINCIPLE OF EQUIVALENCE

The form of the PE that is usually accepted has been stated most succinctly by Bergamnn.[4] According to him the principle asserts that "we cannot distinguish between gravitational and inertial forces that employ the kinetic effects of these forces on test particles at a fixed location in space and time." In discussing this formulation, Bergmann is careful to emphasize its essential character, hence the use of test bodies. Dicke has extended the range of the statement somewhat and called it the "strong" principle of equivalence. The strong principle, according to Dicke, says that "in a free-fall laboratory, if one experiments locally, one observes the same laws of physics— including all the numerical content—that one observes any place else, including any gravity-free place." Again there is included in the Dicke statement a restriction to local experiments.

Both Bergmann and Dicke have taken the strong PE to be fundamental to general relativity. It is our contention, however, that this is not the case, that the strong principle is a principle separate from those that underly general relativity, and whose validity does not affect the validity of that theory. For us, the essential feature of general relativity is its assertion concerning the symmetry of all physical systems embodied in what we have called the principle of general invariance.[5] According to this principle the group of arbitrary mappings of the space-time manifold onto itself constitutes the minimum invariance group of all physical systems. While this principle requires that the laws of physics admit the manifold mapping group as a covariance group as does the older principle of general covariance it requires more, namely, that no absolute

[4] P. G. Bergmann, in *Encyclopedia of Physics*, edited by S. Flugge (Springer-Verlag, Berlin, 1962), Vol. 4, p. 205.

[5] J. L. Anderson, *Principles of Relativity Physics* (Academic Press Inc., New York, 1967).

objects appear in these laws in an essential way.[6] An immediate consequence of this principle is that one must introduce at least one additional dynamical object into the description of physical systems over and above those used for their description in special relativity. What the principle does not tell us is how many such additional objects are needed or how they couple to a given physical system. As we shall see, it is these latter questions that are decided by the PE.

In general relativity, one introduces one additional dynamical object, the gravitational field, to comply with the requirements of the principle of general invariance. The strong PE is then a statement of how this additional field couples to other physical systems. It asserts, in effect, that only the gravitational field, represented by a symmetric, second-rank tensor $g_{\mu\nu}$, and its first derivatives appear in the dynamical laws describing these systems. For this reason the strong PE can rightly be called the principle of minimal coupling for the gravitational field. However, it is possible to construct quite reasonable dynamical laws for which the requirement of minimal coupling is violated. In the theory of spinning bodies developed by Mathisson,[7] Papapetrou,[8] Tulczyjew,[9] and others, just such a violation occurs. In this theory, the Reimann tensor appears explicitly in the equations governing the spin of the body. At present there is no compelling evidence for ruling out the possibility of the existence of such systems, nor is there any compelling needed to do so since their existence is quite compatible with general relativity as we know it today.

There is, however, another version of the PE which Dicke calls the "weak" principle of equivalence and is supported by the experimental evidence of the Eötvös-type experiments. The weak principle asserts, again quoting Dicke, " . . . up to the great accuracy of the Eötvös experiment, all bodies move along the same geodesic paths (under the conditions for which the experiment was performed)." Trautman[10] has emphasized the essential role played by this weak principle, and has generalized it in a form that appears to be consistent with Einstein's original views on the subject. This generalization asserts that measurements made on any physical system will always serve to determine the same affine connection in a given space-time region to the extent that one can neglect the effect of the system on the sources of the affinity. This weak principle makes no assertion of how the system is coupled to the affinity and hence is not restricted in its applicability to "local" systems. However, it does allow one to conclude two things. One, there is a *single* universal affinity that couples to all physical systems, and two, whatever the coupling of this affinity to any given system, it is impossible to separate the interaction into a dynamical and an absolute part. In order to effect such a separation, it would be necessary that at least one system couples only to the absolute part of the affinity. But then we would not always measure the same affinity. It is just this second consequence of the weak principle that forces us to the conclusion that there are no absolute objects in nature. This latter conclusion, coupled with the requirement of general covariance, constitutes the principle of general invariance. Thus, the weak PE together with the principle of general invariance, in our view, constitutes the basis of the general theory of relativity.

Nevertheless, and in spite of its possible nonuniversality, the strong PE is a useful principle. Among other things, it allows us to predict the behavior of a local physical system in a given gravitational field from a knowledge of its reaction to inertial force. We know, for instance, that the rates of nuclear "clocks" are unaffected by strong accelerations,[11] and hence will be unaffected by strong gravitational fields to the extent that these clocks can be considered local. The problem in applying the strong principle then lies in ascertaining whether or not a given physical system can be considered to be local. We shall give an operational definition of a local system and a reformulation of the strong PE that is equivalent to the usual formulation of the strong principle when that formulation applies.

Let S_A designate a particular physical system, e.g., an electromagnetic field, a spinning body, etc. Further, let R_{Gn} and R_{Fn} designate reference frames in the presence of a real gravitational field and in a flat space-time (absence of a real gravitational field), respectively. Finally, let $R_{Gn}S_A$ and $R_{Fn}S_A$ denote the results of space-time measurements made on S_A in these two frames. We say that a system is a local system if, given an R_{Gn}, there exists an R_{Fn}, and vice versa, such that

$$R_{Gn}S_A = R_{Fn}S_A. \tag{2.1}$$

A local system thus defined allows us to establish an equivalence relation between the class $\{R_{Gn}\}$ of gravitational reference frames and the class $\{R_{Fn}\}$ of flat-space reference frames. However, it might be that this equivalence relation depends upon which local system is used to establish the relation. We take the strong PE to be an assertion that the equivalence relation between $\{R_{Gn}\}$ and $\{R_{Fn}\}$ is independent of which local system is used to establish the relation.

Clearly, this statement of the PE contains within it the essential feature of the strong principle, namely, the similarity of gravitational and intertial effects on local systems. Furthermore, its fulfillment is obviously necessary for the weak principle to be valid. What is not

[6] Roughly speaking, an absolute object affects the behavior of other objects but is not affected by these objects in turn. For a more rigorous definition see Ref. 5.

[7] M. Mathisson, Acta Phys. Polon. **6**, 167 (1937).

[8] A. Papapetrou, Proc. Roy. Soc. (London) **A209**, 248 (1951).

[9] W. Tulczyjew, Acta Phys. Polon. **18**, 37 (1959).

[10] A. Trautman, Usp. Fiz. Nauk **89**, 3 (1966) [English transl.: Soviet Phys.—Usp. **9**, 319 (1966)].

[11] R. F. Marzke and J. A. Wheeler, in Ref. 1, p. 40.

so obvious is that even one local system, in our sense, exists within the framework of general relativity. The remainder of this paper is devoted to a discussion of measurements of space-time quantities made on a simple system in and out of a gravitational field. With its help we will show how to establish the equivalence relation between references frames discussed above. Once this relation is established for one local system, it is then a matter of experimental verification that it is the same for any other local system.

3. REFERENCE FRAMES

Given a complete physical theory, one can predict the outcome of physical measurements made on any system described by the theory. In addition, such a theory should also be capable of describing the behavior of the measuring devices used to observe a system without recourse to additional assumptions such as "proper time is measured by ideal clocks." Consequently, given such a description of a set of space-time measuring devices within the framework of general relativity, it should be possible to check whether or not the PE, as we have stated it, is consistent with this theory. The dynamical laws governing most measuring devices are, however, so complicated that their behavior cannot be easily described. What we shall show now is that it is possible to construct models of space-time reference systems whose elements have dynamics sufficiently simple that their behavior can be simply described within the framework of general relativity.

The reference frames that we shall employ consist of a Fock[2] "radar station," together with a simple model a proper-time clock proposed by Marzke and Wheeler.[11] With such a reference system it is possible to calculate the effects of accelerations and gravitational fields on the elements comprising it. By then analyzing various gedankenexperiments performed with these reference systems, we shall show that the results of these experiments are indeed consistent with our statement of the PE.

The photon clock of Marzke and Wheeler is formed by reflecting a light signal back and forth between two slightly separated particles. The number of clicks, i.e., the number of reflections of the light pulse, is taken as a measure of the time recorded by the clock. Since the dynamics of particles and light signals are describable within the framework of general relativity, it is possible to determine how the time recorded by the photon clock will be related to the mathematical proper time.

In order to find this relationship, we consider a light signal reflected between two particles that have an infinitesimal coordinate separation, with one of the particles moving along an arbitrary timelike world with coordinates $x^\mu(\tau)$, where τ is the mathematical proper time along the trajectory. The separation between the two particles is allowed to vary arbitrarily, except for the restrictions that the separation should remain infinitesimal and any variation in the separation should be smooth. If one analyzes such a photon clock,[12] it is found that the number of clicks on the clock are related to the proper time along the clock's trajectory by

$$n=\frac{1}{2}\int\frac{d\tau}{(\eta_\mu\eta^\mu)^{1/2}}. \tag{3.1}$$

Here η^μ is a vector given by the (infinitesimal) difference between the coordinates $x^\mu{}_1$ and $x_2{}^\mu$ of the particles forming the photon clock in a direction orthogonal to the world line of the clock, i.e.,

$$\eta^\mu=x_2{}^\mu-x_1{}^\mu,\quad \eta_\mu dx^\mu/d\tau=0. \tag{3.2}$$

Thus, if the separation between the two particles is specified, one can determine the time recorded by the photon clock.

It is seen from (3.1) that for the number of clicks on the photon clock to be proportional to the integrated proper time along the clock's world line the separation of the particles forming the clock must vary such that $\eta_\mu\eta^\mu=\text{const}$. This condition is equivalent to Born's criterion of rigidity.[12] Thus the clicks on a "Born rigid" photon clock are proportional to the integrated proper time along the world line of the clock.

The problem that now remains is now operationally to construct a Born rigid photon clock. This problem has been solved by Marzke and Wheeler,[11] who developed a method for constructing a parallel to a given geodesic world line. Their construction employs only light signals and intersecting world lines of free particles and is valid even if space-time is curved. In this analysis, Marzke and Wheeler considered only the problem of constructing parallels to geodesic world lines, but it is easy to show[12] that their analysis can be extended to construct operationally a parallel to *any* given world line. In addition, it can be shown[12] that two infinitesimally separated world lines rendered parallel by the Marzke-Wheeler method satisfy Born's criterion of rigidity.

Having established that it is possible to construct a proper time clock, we assume that all observers are equipped with such a clock. We now discuss the procedure by which a particular observer makes space-time measurements, that is, how an observer goes about constructing a reference frame.

We consider the case where observers employ the "radar" method of Fock[2] for the description of space and time measurements. With the radar method an observer reflects a light signal from the event that he is interested in measuring. By recording the (proper) time on his clock when the light signal is emitted (t_1) and received (t_2), the observer can assign a distance s and a time t to the distant event by defining

$$s=\tfrac{1}{2}(t_2-t_1),\quad t=\tfrac{1}{2}(t_2+t_1). \tag{3.3}$$

[12] R. Gautreau, Ph.D. thesis, Stevens Institute of Technology (unpublished).

In this manner the observer can measure distances, velocities, accelerations, etc., of particles. When used in conjunction with a photon clock, the radar method constitutes a space-time measuring device based on only two primitive elements, a light signal and a particle, and the dynamics of both of these elements is simply describable within the framework of general relativity.

The above measurement prescription provides a method by which an observer can, in principle, operationally assign coordinates to events that occur in the space-time manifold. Whether or not these coordinates conform to any particular criteria, for example, Born's criterion of rigidity, is of course immaterial. The main function of coordinates is to catalog events, and radar coordinates accomplish this task. Moreover, as we show in Sec. 4, it is possible because of the simple dynamics of the measuring devices to determine completely the results of particular types of gedanken-experiments involving radar measurements.

4. LOCAL VELOCITY AND ACCELERATION MEASUREMENTS

Because the dynamics of particles and light signals are known, one should, in principle, be able to determine the results of any experiment involving radar measurements. In paractice, however, it is found that, except for a few special cases, when one actually tries to analyze general radar measurements made by an arbitrary observer, the mathematics become much too difficult to handle. Fortunately, however, the special case where an observer makes only local measurements can be treated in all generality.

We here use the word "local" space-time measurements in the following sense. Consider an observer moving along an arbitrary time-like world line whose coordinates are $x^\mu(\tau)$, where τ is the proper time along the trajectory. Consider also a particle moving along a second completely arbitrary world line with coordinates $x_p{}^\mu(\lambda_p)$, where λ_p is some as yet unspecified monotonically increasing parameter, and let the two world lines intersect at $x_p{}^\mu(\lambda_{p_i})=x^\mu(\tau_i)$. Local space-time measurements are defined to be measurements made on the particle in an infinitesimal neighborhood of the coincidence event $x^\mu(\tau_i)=x_p{}^\mu(\lambda_{p_i})$.

Without any loss in generality, we now specify the parameter λ_p as follows:

(a) If V^μ is timelike, choose

$$\lambda_p=\tau_p \quad \text{(proper time)};$$

(b) if V^μ is spacelike, choose

$$\lambda_p=s_p \quad \text{(proper distance)}; \tag{4.1}$$

(c) if V^μ is null, choose

$$\lambda_p=\lambda_p \quad \text{(affine parameter)},$$

where we have set $V^\mu=dx_p{}^\mu/d\lambda_p$. Thus we may write $V_\mu V^\mu=-\epsilon$,

$$\begin{aligned}\epsilon&=+1 \quad \text{for } V^\mu \text{ timelike}\\ &=\ \ 0 \quad \text{for } V^\mu \text{ null}\\ &=-1 \quad \text{for } V^\mu \text{ spacelike}.\end{aligned} \tag{4.2}$$

What we shall do now is analyze local velocity and acceleration measurements that are made on the particle. The results of these measurements can be calculated by assuming that light signals travel along null geodesics and by using a power-series expansion to determine the results to any desired accuracy. The values of the local velocity and acceleration measurements can then be obtained by taking the limit as the two world lines coincide. To express these results we make use of a vector U^μ that is proportional to the infinitesimal coordinate separation between the observer and the particle in a direction orthogonal to the world line, i.e.,

$$U^\mu=\Delta x^\mu/\Delta l,\quad \Delta x^\mu=x_p{}^\mu(\lambda_p)-x^\mu(\tau), \tag{4.3}$$
$$(\Delta l)^2=g_{\mu\nu}\Delta x^\mu\Delta x^\nu,$$

with the direction of Δx^μ defined by

$$U_\mu A^\mu=0,\quad A^\mu=dx^\mu/d\tau. \tag{4.4}$$

From the definition of U^μ it is seen that $U_\mu U^\mu\equiv 1$, so that U^μ is a unit vector.

The measured velocity of the particle at the coincidence event, defined as ds/dt, is found to be[12]

$$ds/dt=-U_\mu V^\mu/A_\alpha V^\alpha. \tag{4.5}$$

From this expression, one can show that the unit vector U^μ can be expressed in terms of V^μ and A^μ as

$$-(ds/dt)U^\mu=V^\mu/A_\alpha V^\alpha+A^\mu. \tag{4.6}$$

The measured acceleration of the particle, d^2s/dt^2, at the coincidence event is found to be[12]

$$\left(\frac{ds}{dt}\right)\frac{d^2s}{dt^2}=-\epsilon A_\mu\frac{\delta V^\mu}{\delta\lambda_p}\left[1-\left(\frac{ds}{dt}\right)^2\right]^2 +V_\mu\frac{\delta A^\mu}{\delta\tau}\left\{\epsilon\left[1-\left(\frac{ds}{dt}\right)^2\right]\right\}^{3/2}. \tag{4.7a}$$

Using (4.6), we can rewrite this expression in terms of U^μ as

$$\frac{d^2s}{dt^2}=\epsilon U_\mu\frac{\delta V^\mu}{\delta\lambda_p}\left[1-\left(\frac{ds}{dt}\right)^2\right]^2-U_\mu\frac{\delta A^\mu}{\delta\tau}\left[1-\left(\frac{ds}{dt}\right)^2\right]. \tag{4.7b}$$

As is seen from inspection of (4.5) and (4.7), the expressions for the measured local velocity and acceleration are manifestly covariant. This, of course, is to be expected, since all measurements are defined operationally and are independent of any coordinates. It is also seen from (4.7) that if both $x^\mu(\tau)$ and $x_p{}^\mu(\lambda_p)$ are geodesic world lines, i.e., if $\delta A^\mu/\delta\tau=\delta V^\mu/\delta\tau=0$, the

TABLE I. Comparison of various equivalent local gedankenexperiments performed in flat space and in an arbitrary gravitational field.

Type of experiment	Flat-space result	Gravitational-field result
A_1: Nongeodesic observer measures the velocity and acceleration of a geodesic particle	$\dfrac{d^2s_a}{dt_a^2}=-a\left[1-\left(\dfrac{ds_a}{dt_a}\right)^2\right]$	$\dfrac{d^2s_F}{dt_F^2}=-U_\mu\dfrac{\delta F^\mu}{\delta\tau_F}\left[1-\left(\dfrac{ds_F}{dt_F}\right)^2\right]$
A_2: Nongeodesic observer measures the variation of the proper time on a geodesic clock	$\dfrac{d^2\tau_f}{dt_a^2}=2a\left(\dfrac{ds_a}{dt_a}\right)\left[1-\left(\dfrac{ds_a}{dt_a}\right)^2\right]^{1/2}$	$\dfrac{d^2\tau_G}{dt_F^2}=2U_\mu\dfrac{\delta F^\mu}{\delta\tau_F}\left(\dfrac{ds_F}{dt_F}\right)\left[1-\left(\dfrac{ds_F}{dt_F}\right)^2\right]^{1/2}$
B_1: Geodesic observer measures the velocity and acceleration of a nongeodesic particle	$\dfrac{d^2s_f}{dt_f^2}=a\left[1-\left(\dfrac{ds_f}{dt_f}\right)^2\right]^{3/2}$	$\dfrac{d^2s_G}{dt_G^2}=U_\mu\dfrac{\delta F^\mu}{\delta\tau_F}\left[1-\left(\dfrac{ds_G}{dt_G}\right)^2\right]^{3/2}$
B_2: Geodesic observer measures the variation of the proper time on a nongeodesic clock	$\dfrac{d^2\tau_a}{dt_f^2}=-a\left(\dfrac{ds_f}{dt_f}\right)\left[1-\left(\dfrac{ds_f}{dt_f}\right)^2\right]$	$\dfrac{d^2\tau_F}{dt_G^2}=-U_\mu\dfrac{\delta F^\mu}{\delta\tau_F}\left(\dfrac{ds_G}{dt_G}\right)\left[1-\left(\dfrac{ds_G}{dt_G}\right)^2\right]$

measured acceleration will vanish for all values of the measured velocity.

It is to be stressed that in calculating these expressions for the measured velocity and acceleration, no assumptions have been made concerning the "strength" of the gravitational field or the relative magnitude of the measured velocity. The only restriction that has been imposed is that the measurements should be local in the sense described above.

It should be pointed out that the acceleration expression (4.7) cannot be obtained by directly differentiating the velocity expression (4.5) The measured velocity (4.5) was obtained from a power-series expansion, which yielded a result of the form

$$ds/dt=-U_\mu V^\mu/A_\alpha V^\alpha+f_1(\Delta l)+f_2(\Delta l)^2 \\ f_3(\Delta l)^3+\cdots, \quad (4.8)$$

where the f_i are known functions of A^μ and V^μ. The measured acceleration is obtained by differentiating this expression with respect to t. It is seen, however, that differentiation of the first-order term $f_1(\Delta l)$ yields a zeroth-order term that does not vanish in the limit $\Delta l\rightarrow 0$; this zeroth-order term, of course, cannot be neglected. Similarly, if the third derivative were desired it would be necessary to consider the second-order term $f_2(\Delta l)^2$; this would have the effect of bringing the Reimann-Christoffel tensor into the expression.

5. ANALYSIS OF LOCAL GEDANKEN-EXPERIMENTS

In the calculation of the results of an observer's local velocity and acceleration measurements, no use was made of any statement of the PE. We are free, therefore, to use these results to illustrate that there exist local physical systems and reference frames which are in accord with our statement of the PE. What we shall now do is to employ the results of Sec. 4 to describe various gedankenexperiments that can be performed on local systems. These experiments will define the equivalence of a particular set of reference frames R_{Gi} and R_{Fi} that correspond to radar-station reference frames in and out of gravitational fields. By examining a series of gedankenexperiments performed on diffeent local systems, we shall then demonstrate that this equivalence relation is independent of these local systems, as is required by our statement of the PE.

To illustrate the types of gedankenexperiments that can be analyzed, consider an observer in an arbitrary gravitational field moving along an arbitrary timelike world line with tangent vector F^μ. Let this observer measure the local velocity and acceleration of a particle moving along a timelike geodesic world line with tangent vector G^μ. The relationship between the measured velocity and acceleration can be found by setting $A^\mu=F^\mu$ and $V^\mu=G^\mu$ in (4.7b), yielding

$$\frac{d^2s}{dt^2}=-U_\mu\frac{\delta F^\mu}{\delta\tau_p}\left[1-\left(\frac{ds}{dt}\right)^2\right]. \quad (5.1)$$

Let us now analyze a similar set of measurements carried out in a flat space. To this end, we consider a hyperbolically accelerated trajectory described by

$$(x^1-x_i^1+1/a)^2-(x^0-x_i^0)^2=1/a^2. \quad (5.2)$$

A particle following such a trajectory has the property that its acceleration is found to be a constant a, when measured by any free observer who is instantaneously at rest with respect to the particle. Hence in this sense the particle can be considered to move with "constant" acceleration. Consider now an observer in flat space following such a hyperbolic trajectory who measures the velocity and acceleration of a free particle at the instant when it is in coincidence with him. An analysis of this experiment,[13] which makes use of only special relativity, shows that the measured velocity and acceleration of the free particle are related by

$$\frac{d^2s}{dt^2}=-a\left[1-\left(\frac{ds}{dt}\right)^2\right]. \quad (5.3)$$

[13] J. L. Anderson and R. Gautreau, Am. J. Phys. 37, 108 (1969).

If the quantity $U_\mu \delta F^\mu / \delta\tau_p$ is regarded as the gravitational acceleration, then it is seen that the results of the two gedankenexperiments described here are identical. By our definition, the equivalence of the gravitational field and flat-space radar-station reference frames is thereby established.

Another type of experiment that can be analyzed is the case where the roles of observer and particle are reversed, so that a geodescially moving observer measures the velocity and acceleration of an arbitrarily moving particle. One can also consider a situation where a clock is moving past an observer, and one can determine how this clock's time is related to its velocity as measured by the observer. A description of four such experiments and their results is given in Table I.[14]

Table I shows the results of measurements made with two different sets of radar-station reference frames. In each set, one frame is in a gravitational field while the other is in flat space. In one set, the frames move alone geodesics (experiments B_1 and B_2), while in the other they move along nongeodesic trajectories (experiments A_1 and A_2). The local systems on which the measurements are made are a particle and a proper time clock. From Table I it is seen that when an equivalence relation is established between radar-station reference frames, e.g., experiment A_1 or B_1, this equivalence relation is maintained when the local system being measured is changed, e.g., experiment A_2 or B_2. Therefore, we see that one can demonstrate the existence of reference frames that satisfy our statement of the PE.

In closing this section, we point out the following. The unit vector U^μ is defined such that it is always orthogonal to the world line F^μ, i.e., $U_\mu U^\mu = 1$ and $U_\mu F^\mu = 0$. Therefore, the term $U_\mu \delta F^\mu / \delta\tau_p$ is a quantity that depends only on the trajectory F^μ and is independent of the geodesic world line G^μ. Hence the quantity $U_\mu \delta F^\mu / \delta\tau_p$ can be regarded as characterizing the acceleration of the trajectory F^μ in the same manner as the constant a characterizes the acceleration of the hyperbolically accelerated trajectory in flat space. Since the only restriction on the world line F^μ is that it should be timelike, the trajectory F^μ can describe the motion of a particle that is stationary with respect to a source of the field or a particle that is moving about with arbitrary motion. Hence the quantity $U_\mu \delta F^\mu / \delta\tau_p$ can represent the gravitational acceleration produced only by the sources of the field or produced by some combination of field strength and accelerative motion.

[14] See Ref. 12 for a complete analysis.

6. SUMMARY

In this paper we have attempted to formulate a statement of the principle of equivalence that is both precise and nonempty. Our statement is equivalent to the more usual statements of the principle in those cases when these latter statements are applicable. In formulating this statement we have had to introduce the notion of a local system and equivalent reference frames. By considering especially simple systems, we have shown that it is possible, within the framework of general relativity, to construct reference frames employing these systems as measuring devices that satisfy our statement of the principle. We emphasize that, in order for this demonstration to be meaningful, the behavior of these systems as measuring devices must be determined from their dynamics and not postulated *ab initio*. Thus we do not employ any such hypothesis as Synge's chronometric hypothesis that ideal clocks measure proper time.

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