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# X-Ray Production in $C^+$ - C Collisions in the Energy Range 20 keV to 1.5 MeV*

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Cross sections for carbon *K*-x-ray production for $C^+$ ions incident upon a carbon target have been measured in the energy range 20 keV–1.5 MeV. A theoretical model based on the Landau-Zener theory of level crossing is formulated and is found to fit the experimental data within experimental uncertainty.

## I. INTRODUCTION

In a previous paper,[1] cross sections for carbon *K*-x-ray production have been measured for heavy ions incident on a carbon target in the energy range from 20 to 80 keV. It was found that the cross sections are several orders of magnitude larger than those predicted by direct scattering theory. It was suggested that the mechanism for electron excitation was level crossing, as proposed by Fano and Lichten[2] to explain the results of Kessel and Everhart[3] for $Ar^+$-Ar collisions. An extensive discussion of the level-crossing mechanism is found in Ref. 4. As reported below, the data for $C^+$ ions incident on carbon are extended to 1. 5 MeV, and a theoretical model based on the level-crossing mechanism is formulated and compared with the experimental data.

## II. Experimental Measurements

The experimental techniques were essentially those described in earlier papers on protons.[5] A thick carbon target was used, and carbon *K*-x rays were detected by a gas-flow proportional counter with a $\frac{1}{4}$- or $\frac{1}{2}$-mil Mylar window. The directly measured quantity in these experiments was the number of detected x rays per unit charge incident upon the target. The thick-target yield $I$ (x rays per ion) was obtained by bombarding the carbon target alternately with protons and $C^+$ ions and normalizing to the known carbon x-ray yield for protons.[5] The measurements were made on two separate accelerators. The data from 20 to 120 keV were obtained by using a dc power supply (15 mA at voltages up to 120 kV) employing a duoplasmatron ion source. A Van de Graaff generator employing a conventional rf electrodeless discharge source provided ions in the 100 keV–1.5 MeV range. In both accelerators, $CO_2$ gas was used, and the $C^+$ ions were resolved by use of a bending magnet.

The x-ray production cross section $\sigma_X$ for a given projectile energy $E$ was calculated from the thick-target yield $I$ by the relation

$$\sigma_X = \frac{dI}{dE} S + \frac{1}{n}\frac{\mu}{\rho} I, \tag{1}$$

where $S$ is the target stopping cross section for carbon ions in carbon, $n$ is the number of target atoms per gram, and $\mu/\rho$ is the target mass absorption coefficient for the carbon x rays. The values for the stopping cross sections were a combination of calculated nuclear stopping cross section of Lindhard, Scharff, and Schiott[6] and measured electronic stopping cross sections of Ormrod and Duckworth,[7] Porat and Ramavataram,[8] and Fastrup, Hvelplund, and Sautter.[9] The $K$-shell excitation cross section $\sigma_I$ is related to $\sigma_X$ by

$$\sigma_X = \omega_K \sigma_I , \tag{2}$$

where $\omega_K$ is the $K$-shell fluorescence yield.

We have taken the value of the fluorescence yield to be 0.0009.[10] This value is 38% smaller than the theoretical value,[11] indicating that the uncertainty for $\sigma_I$ might be quite large. The experimental uncertainty is 15% for $I$ and 30% for $\sigma_X$. The shape of the cross-section curve, which will be compared with the theoretical model derived in Sec. III, is influenced only by the error in $\sigma_X$. The results are summarized in Table I.

### III. Theoretical Model

During an ion-atom collision the electronic states undergo a continuous change. At large ion-atom separations the electron states are the normal atomic states characteristic of either the incident ion or the target atom. At zero distance of separation, the electron states are assumed to be characteristic of an atom whose atomic number is the sum of the atomic numbers of the ion and the atom. At intermediate distances, the electron energy states are quasimolecular states which change as the ion-atom distance of separation varies.

At some distance of separation, two different quasimolecular states may have the same energy. This phenomenon is called "level crossing" and the distance of separation between the ion and the atom at level crossing is called the "level-crossing radius." A model for electron transitions between electron states at level crossing is provided by the Landau-Zener theory.[12] If level crossings occur, and electron transitions take place, electrons can remain in higher-energy states after the collision, creating an inner shell vacancy. Lichten[4] has shown that a $2p\sigma$-$2p\pi$ crossing at small internuclear distances gives rise to $K$-shell vacancies. In heavy-ion–atom collisions the excitation cross section due to level crossing appears to be much larger than the excitation cross section due to the direct scattering model.[13]

In this section, the cross section for creating inner shell vacancies (i.e., the excitation cross section) is calculated. This calculation is similar to that applied to charge exchange in atomic collisions. In this calculation, it is assumed that electron excitation occurs at the level-crossing radius and that the probability of excitation obeys the Landau-Zener theory.

The excitation cross section $\sigma_I$ is given by

$$\sigma_I = \int_0^{b_m} P(b) \times 2\pi b \, db, \tag{3}$$

where $b$ is the impact parameter, $b_m$ is the impact parameter for which the distance of closest approach is equal to the level-crossing radius, and $P(b)$ is the probability that the target atom has an inner shell vacancy after the collision. If $p$ is the probability that an electron transition can occur at level crossing, then simple statistical arguments[14] lead to

$$P(b) = 2p(1-p). \tag{4}$$

Equation (4) assumes that the probability of a vacancy existing in the level to which the electron is excited is unity. According to the Landau-Zener theory,

$$p = \exp(-y/v_x), \tag{5}$$

where $v_x$ is the radial velocity of relative motion at level crossing, and $y$ is a parameter which depends on the dynamics of the specific level crossing.

To evaluate Eq. (3), we must determine $v_x$ as a function of the impact parameter $b$ and we must also determine $b_m$. Using the classical equations of motion,

$$E = \tfrac{1}{2}\mu v^2 + l^2/2\mu r^2 + V(r) \tag{6}$$

$$= \tfrac{1}{2}\mu v^2 + E(b^2/r^2) + V(r), \tag{7}$$

where $E$ is the energy of the ion in the c.m. system, $v$ is the radial velocity, $\mu$ is the reduced

TABLE I. Experimental results. For column 3, the experimentally measured quantity was $N(E)$, x rays detected per $\mu$C. The thick-target yield I is determined by the expression $N(E)(\Lambda e \times 10^6)/(T_w A_c)$, where $e$ is the electron charge in Coulombs; $\Lambda$ (geometrical correction factor) $= 4\pi R^2$/area of counter window; $T_w$ is the counter window transmission; and $A_c$ is the counter absorption. Typical values in this experiment were $\Lambda = 3364$, $T_w = 0.05$, and $A_c = 1.0$. In column 7, we have used the values $n = 5.02 \times 10^{22}$ atoms/g and $\mu/\rho = 2.17 \times 10^3$ cm$^2$/g. In column 9, we have used the value $\omega_K = 0.0009$.

| $E$ (keV) | $v$ $\left[\frac{\text{keV}}{\text{amu}}\right]^{1/2}$ | $I$ $10^{-3}$ $\frac{\text{x rays}}{\text{ion}}$ | $\frac{dI}{dE}$ $10^{-5}$ $\frac{\text{x rays}}{\text{ion-keV}}$ | $S(E)$ $10^{-17}$ $\frac{\text{keV-cm}^2}{\text{atom}}$ | $S(E)\frac{dI}{dE}$ $10^{-22}$ cm$^2$ | $\frac{1}{n}\frac{\mu}{\rho}I$ $10^{-22}$ cm$^2$ | $\sigma_X$ $10^{-22}$ cm$^2$ | $\sigma_I$ $10^{-18}$ cm$^2$ |
|---|---|---|---|---|---|---|---|---|
| 20 | 1.29 | 0.035 | 0.67 | 3.67 | 2.45 | 0.02 | 2.47 | 0.28 |
| 30 | 1.58 | 0.15 | 1.20 | 3.82 | 4.59 | 0.07 | 4.66 | 0.52 |
| 40 | 1.82 | 0.31 | 1.74 | 4.00 | 6.96 | 0.13 | 7.09 | 0.79 |
| 50 | 2.04 | 0.52 | 2.30 | 4.15 | 9.55 | 0.22 | 9.77 | 1.08 |
| 60 | 2.24 | 0.78 | 2.81 | 4.30 | 12.1 | 0.34 | 12.4 | 1.37 |
| 70 | 2.42 | 1.20 | 3.30 | 4.45 | 14.7 | 0.46 | 15.2 | 1.68 |
| 80 | 2.57 | 1.53 | 3.80 | 4.60 | 17.2 | 0.65 | 17.9 | 1.99 |
| 90 | 2.74 | 1.85 | 4.30 | 4.75 | 20.4 | 0.80 | 21.2 | 2.36 |
| 100 | 2.89 | 2.34 | 4.75 | 4.90 | 23.3 | 1.01 | 24.3 | 2.69 |
| 110 | 3.03 | 2.85 | 5.14 | 5.05 | 25.9 | 1.23 | 27.1 | 3.01 |
| 120 | 3.16 | 3.31 | 5.37 | 5.16 | 27.7 | 1.43 | 29.1 | 3.23 |
| 150 | 3.54 | 5.20 | 6.05 | 5.50 | 33.3 | 2.25 | 35.6 | 3.95 |
| 200 | 4.07 | 8.64 | 6.75 | 6.00 | 40.5 | 3.73 | 44.2 | 4.92 |
| 300 | 5.00 | 15.8 | 7.68 | 6.90 | 53.0 | 6.80 | 59.8 | 6.64 |
| 400 | 5.76 | 23.4 | 7.92 | 7.60 | 60.2 | 10.1 | 70.3 | 7.82 |
| 500 | 6.46 | 31.2 | 8.10 | 8.20 | 66.5 | 13.5 | 80.0 | 8.87 |
| 600 | 7.08 | 39.2 | 8.17 | 8.70 | 71.0 | 16.9 | 87.9 | 9.8 |
| 700 | 7.65 | 48.3 | 8.30 | 9.20 | 76.4 | 20.9 | 97.3 | 10.8 |
| 800 | 8.17 | 56.3 | 8.00 | 9.60 | 77.0 | 24.4 | 101.4 | 11.3 |
| 900 | 8.67 | 63.8 | 7.65 | 10.0 | 76.5 | 27.6 | 104.1 | 11.5 |
| 1000 | 9.15 | 70.7 | 7.40 | 10.3 | 76.2 | 30.5 | 106.7 | 11.8 |
| 1100 | 9.56 | 78.0 | 7.16 | 10.7 | 76.6 | 33.6 | 110.2 | 12.2 |
| 1200 | 10.0 | 85.0 | 6.90 | 10.9 | 75.0 | 36.7 | 111.7 | 12.5 |
| 1300 | 10.4 | 92.0 | 6.62 | 11.3 | 74.8 | 39.7 | 114.5 | 12.7 |
| 1400 | 10.8 | 98.5 | 6.30 | 11.5 | 72.5 | 42.5 | 115.0 | 12.8 |
| 1500 | 11.2 | 104 | 6.01 | 11.8 | 70.9 | 45.0 | 115.9 | 12.9 |

mass, $r$ is the ion-atom distance of separation, $V(r)$ is the potential energy, and $l$ is the angular momentum. If $r_X$ is the level-crossing radius,

$$v_x = \{(2/\mu)[E(1 - b^2/r_x{}^2) - V(r_x)]\}^{1/2} . \qquad (8)$$

$b_m$ is found by setting $v_X$ equal to zero in Eq. (8):

$$b_m = r_x[1 - V(r_x)/E]^{1/2} . \qquad (9)$$

Using Eqs. (4), (5), (8), and (9), we evaluate Eq. (3) and find

$$\sigma(E) = 4\pi r_x{}^2[1 - V(r_x)/E]\,[Q_3(y/\{\tfrac{2}{\mu}[E - V(r_x)]\}^{1/2}) - Q_3(2y/\{\tfrac{2}{\mu}[E - V(r_x)]\}^{1/2})] , \qquad (10)$$

where $Q_n(x) = \int_1^\infty e^{-xt} t^{-n} dt$.

Not all molecular states will involve level crossings which can give rise to the creation of an inner shell vacancy. We assume that only one molecular configuration is involved in the creation of the inner shell vacancy, and that $\alpha$ is the probability that this configuration is formed. (For a discussion of $\alpha$ see Ref. 4, pp. 138 and 139.) In this case, Eq. (10) becomes

$$\sigma(E) = 4\pi\alpha r_x{}^2[1 - V(r_x)/E]\,[Q_3(y/\{\tfrac{2}{\mu}[E - V(r_x)]\}^{1/2}) - Q_3(2y/\{\tfrac{2}{\mu}[E - V(r_x)]\}^{1/2})]. \qquad (11)$$

For $E \gg V(r_X)$, Eq. (11) reduces to

$$\sigma(E) = 4\pi\alpha r_x^2[Q_3(y/v) - Q_3(2y/v)], \tag{12}$$

where $v$ is the relative velocity at infinite distance of separation. The calculated cross section, which is plotted in Fig. 1, has a maximum of $1.41\alpha r_x^2$ occurring at a value of $v/y = 2.36$. The cross section includes two unspecified parameters which act as scale factors on Fig. 1: $(4\pi\alpha r_x^2)^{-1}$ is the scale factor for the cross section, and $y^{-1}$ is the scale factor for the velocity.

In Fig. 1 the theory and experiment are compared. The values used to normalize the experiment are $\alpha r_x^2 = 4.4\times10^{-18}$ cm$^2$ and $y = 5.75$ (keV/amu)$^{1/2} = 1.15$ a.u. The agreement is within experimental error.

According to the Landau-Zener theory (in a.u.)

$$y = 2\pi H^2/(d\epsilon/dR), \tag{13}$$

where $H$ is the off-diagonal matrix element coupling the crossing states, and $\epsilon$ is the energy splitting between unperturbed states. The error in the value of $y$, based on a 30% error in the x-ray cross section, is estimated to be ±15%.

A lower bound on the level-crossing radius can be found by setting $\alpha = 1$. This gives us $r_x = 2.1\times10^{-9}$ cm $= 0.40$ a.u. as a lower bound. The error in $r_x$ includes any error in the fluorescence yield $\omega K$.

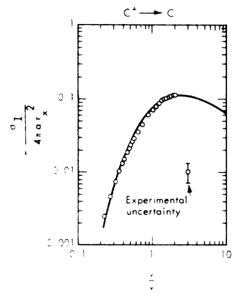


FIG. 1. Comparison between theory and experiment. The values $4\pi\alpha r_x^2 = 5.5\times10^{-17}$ cm$^2$ and $y = 5.75$ (keV/amu)$^{1/2}$ have been chosen to normalize the experimental data for comparison with Eq. (12). The experimental cross sections were also reduced by a factor of 2 to correct for the fact that x rays from both the projectile and the target would be detected.

## IV. CONCLUSION

The experimental cross section for carbon $K$-shell excitation is in good agreement with a theoretical model based on the Landau-Zener theory of level crossings. The model has made the simplifying assumptions that only one level crossing is involved in creating the inner shell vacancy and that the trajectories of the ion and the atom obey the classical equations of motion for a central potential. The quantity $y$ and a lower bound on the curve crossing radius $r_x$ are obtained directly by the comparison of the theory and the experiment.

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