Decay Scheme of Mg^{28}†

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The 21.3-h half-life β-ray emitter Mg^{28} was produced in the $Mg^{26}(t, p)Mg^{28}$ reaction and the subsequent γ radiations were studied with Ge(Li) and NaI(Tl) detectors. Strong γ rays with measured energies of 400.6±0.2, 941.7±0.4, and 1342.2±0.2 keV were found to be in coincidence with the 30.6-keV γ rays from the Al^{28} first excited state, in agreement with earlier results. Four additional weak γ rays detected in Mg^{28} decay include a γ-ray energy of 1589.4±0.4 keV in coincidence with the 30.6-keV γ ray, a γ-ray energy of 648.1±0.5 keV in coincidence with the 941.7-keV γ ray, and two ground-state γ-ray transitions having energies of 1620.0±0.4 and 1372.8±0.2 keV. In the proposed level scheme, Mg^{28} decays with a 95±1% β-ray branch ($\log ft=4.49\pm0.03$) to the $J^\pi=1^+$ 1372.8±0.2-keV level of Al^{28}, and with a 5±1% β-ray branch ($\log ft=4.59\pm0.08$) to a 1620.0±0.4-keV level which we assign a spin parity of $J^\pi=1^+$.

I. INTRODUCTION

THE only really useful radioactive isotope of magnesium for tracer applications is Mg^{28}. It is employed in biological and medical research studies of photosynthesis, cell reproduction, hibernation, protein synthesis, and neuromuscular function. The Mg^{28} half-life[1] of 21.3 h and the emission of several γ rays per decay makes Mg^{28} ideal for tracer usage in such studies.

Until the middle of 1968, Mg^{28} was produced at Brookhaven in the graphite reactor by irradiating a Li^6-Mg^{26} alloy with slow neutrons. Tritons from the $Li^6+n\rightarrow\alpha+t$ reaction produced Mg^{28} by the $Mg^{26}(t,p)Mg^{28}$ reaction. Other research centers were supplied with the Mg^{28} activity by the Brookhaven Nuclear Engineering Department (now the Applied Science Department).

With the installation of the triton-beam facility in the 3.5-MeV Van de Graaff accelerator at Brookhaven, which occurred at about the same time that the graphite reactor was shut down, it was decided to produce Mg^{28} directly in the accelerator by means of the $Mg^{26}(t,\mathrm{p})Mg^{28}$ reaction. Mori *et al.*[2] had measured the cross section versus triton energy for this reaction, and they found that the cross section reaches a broad plateau at $E_t\sim3.4$ MeV, where the total cross section is $\simeq100$ mb. Their measurements of the thick-target yield of Mg^{28} showed that quite favorable yields could be obtained with targets enriched in Mg^{26}. In subsequent developments at Brookhaven, ~1 mCi of Mg^{28} could be produced in less than 2 h of Van de Graaff bombardment, and the specific activity of the Mg^{28} samples was more than 10 times greater than that of the reactor-produced material.

The decay scheme of Mg^{28} apparently has not been studied since 1954, when Sheline *et al.*[3] measured the radiations with NaI(Tl) scintillation detectors. They observed γ rays with energies of 400, 949, and 1346 keV in coincidence with a 30.6-keV γ ray from the first excited state of Al^{28}. The equilibrium daughter activity Al^{28} decays with a 2.27-min half-life. Sheline *et al.* also observed the 1778.7-keV ground-state γ ray following the 100% Al^{28} decay to the first excited state of Si^{28}. The decay scheme of Mg^{28} proposed by Sheline *et al.* consisted of a single β-ray branch to a 1380-keV level in Al^{28} followed by γ-ray transitions to the 30.6-keV state either directly or via a cascade through a 980-keV level. Since Mg^{28} is even-even, and therefore its ground state has $J^\pi=0^+$, a spin parity of $J^\pi=0^+$ or 1^+ was assigned to the 1380-keV state of Al^{28}, because of the allowed $\log ft$ value of 4.4 for the β decay. The ground state and 30.6-keV first excited state of Al^{28} were assigned the values $J^\pi=3^+$ and 2^+, respectively, and various alternatives were suggested for the spin-parity assignment to the 980-keV state.

Lawergren's study[4] of the $Si^{30}(d,\alpha)Al^{28}$ reaction has cast some further light on the spin parity of the 980-keV state. His data indicate $J^\pi=0^\pm$ or 1^- with $J=0$ preferred.

In their most recent compilation, Endt and Van der Leun[1] have invoked the isobaric spin-selection rule, excluding $\Delta T=\pm1$, $0^+\rightarrow0^+$, β transitions to make the unique assignment of $J^\pi=1^+$ to the Al^{28} 1372-keV level.

The present investigation was undertaken in order to check the decay scheme proposed by Sheline *et al.* and to measure the γ-ray energies with the precision afforded by Ge(Li) detectors. It became apparent that there were additional features in the decay scheme not observed previously. We report these new results and present a revised decay scheme in this paper.

II. EXPERIMENTAL PROCEDURES AND RESULTS

The Mg^{28} production runs are currently being made at this laboratory by bombarding a Mg target with a beam of 3.4-MeV tritons from the Van de Graaff accelerator. The target consists of a $\frac{1}{4}$-in.-diam metallic

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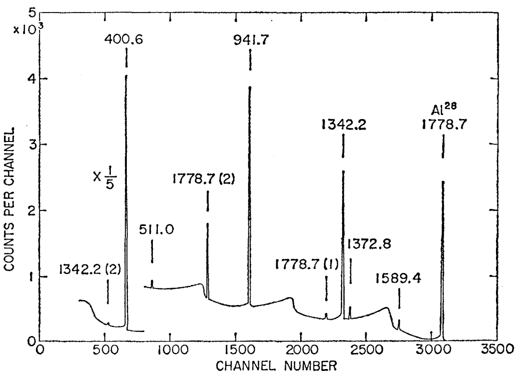

FIG. 1. γ-ray spectrum of Mg^{28}-Al^{28} using a 6-cm^3 Ge(Li) detector. Data have been omitted. Energies are in keV, and the peaks are full-energy-loss peaks unless designated as one-escape (1) or two-escape (2) peaks.

rod of magnesium, enriched to 99.77% in Mg^{26}, clamped in a water-cooled target holder. A beam current of 20–30 μA is normally used. After a bombardment of 20–40 μA h the activity is removed from the end of the rod by an etching process which takes off a layer about 0.001 in. thick containing most of the activity. The present studies of Mg^{28} were made with small portions of production samples. Several days were allowed to elapse before making any measurements in order to allow for the decay of 110-min F^{18} which is produced in the $O^{16}(t,n)F^{18}$ reaction from oxygen in the target.

Measurements on the γ rays were made with several Ge(Li) detectors ranging in volume from 6 to 30 cm^3. One of the spectra recorded with the 6-cm^3 detector is shown in Fig. 1. Of the γ-ray transitions indicated in the figure, those at 400.6, 941.7, 1342.2, and 1778.7 keV were reported previously[3] as discussed in Sec. I.

Two new weak γ rays are apparent in Fig. 1 at energies of 1372.8 and 1589.4 keV. Except for the 511.0-keV annihilation radiation, due to pair production by those γ rays with energies above 1 MeV no further γ-ray peaks could be clearly identified in the Ge(Li) singles spectrum of Fig. 1. Runs made at various times during a period of $\sim$4 half-lives of Mg^{28} showed that the intensities of all of the peaks in Fig. 1 were decaying at the same rate, indicating that they all originated from the decay of Mg^{28}.

Since the coincidence experiments described below indicated that a state at 1620.0 keV is populated in Mg^{28} decay, a further search for the corresponding ground-state transition was made with a 30-cm^3 Ge(Li) detector which had a resolution of 2.0 keV (full width at half-maximum) for Co^{60} γ rays. Figure 2 shows the results of a 16-h run. Identification of the weak 1620.0-keV peak as the ground-state transition of the 1620.0-keV level in Al^{28} follows from an accurate measurement of its energy and the fact that its intensity decays at the same rate as the 1589.4-keV line. The weak 1620.0-keV peak has an intensity of 0.06±0.01 relative to the 1589.4-keV line.

Accurate energy measurements were made on the various Mg^{28} γ rays by comparing them with lines from superposed calibration standards.[5] Energies of 400.6±0.2, 941.7±0.4, 1342.2±0.2, and 1589.4±0.4 keV were derived in a series of measurements on the pulse-height spectra. The energy of the 400.6-keV line was found by comparison with the 383.8±0.1-keV line of Ba^{133} and the 511.0-keV Na^{22} γ ray; for the 941.7-keV γ ray, the comparison standards were the 834.81±0.03-keV Mn^{54} γ ray and the 1063.44±0.09-keV Bi^{207} line; the Co^{60} γ rays of 1173.23±0.04 and 1332.49±0.04 keV were used to obtain the energy of the 1342.2-keV line; and, finally, the 1589.4-keV γ-ray energy was obtained by internal measurement against the 1342.2±0.2- and 1778.7±0.2-keV lines, where the energy of the latter has been taken from the literature.[1]

The γ-γ coincidence measurements were carried out by displaying the output of a 30-cm^3 Ge(Li) detector in coincidence with a NaI(Tl) detector when a Mg^{28} source of 1–2 μCi was placed between the two detectors. Several such experiments were performed, the first using a 6-mm-thick $\frac{3}{2}$-in.-diam NaI(Tl) crystal in order to detect the 30.6-keV γ-ray line occurring in Mg^{28} decay. When a pulse-height window was imposed on the 30.6-keV line from the NaI(Tl) detector, the Ge(Li) coincidence spectrum contained the 400.6-, 941.7-, 1342.2-, and 1589.4-keV lines, all with the same relative intensities as in the singles spectrum. On the other hand,

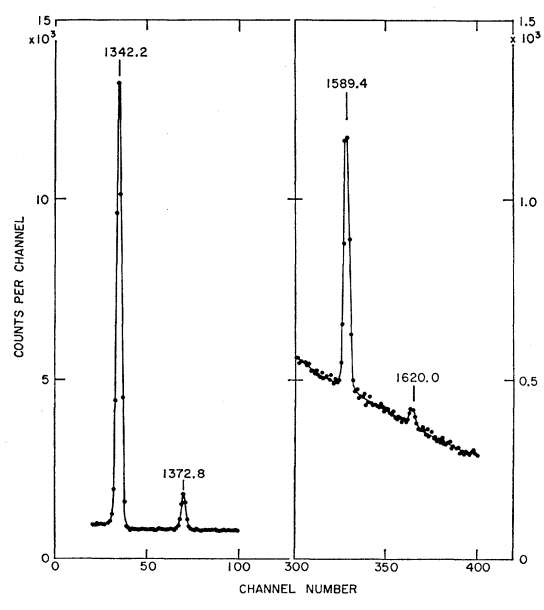

FIG. 2. Portions of the Mg^{28}-Al^{28} γ-ray singles spectrum studied with a 30-cm^3 Ge(Li) detector having high resolution. The very weak 1620.0-keV ground-state γ-ray transition is revealed in this experiment.

[5] Energies quoted for the standards used were taken from J. B. Marion, University of Maryland Technical Report No. 656, 1968 (unpublished), except for the Ba^{133} γ rays, which are from A. Schwarzschild (private communication).

the 1372.8-keV peak was not observed, and only a weak 1778.7-keV random-coincidence line was present.

For the second set of coincidence experiments a 3×3-in. NaI(Tl) scintillator was used, and a pulse-height channel was first placed around the 941.7-keV γ-ray peak. The Ge(Li) detector coincidence spectrum contained the 400.6-keV line and a weak peak corresponding to a γ ray of 648.1±0.5 keV. This coincidence spectrum is shown in Fig. 3.

Finally, the pulse-height channel on the NaI(Tl) detector output was moved up to the 1342-keV peak in order to search for a possible 1620.0→1372.8-keV transition which should have an energy of 247.2 keV. No such peak could be observed in the Ge(Li) spectrum.

Relative intensities of the various γ rays were obtained by correcting the peak areas in the Ge(Li)

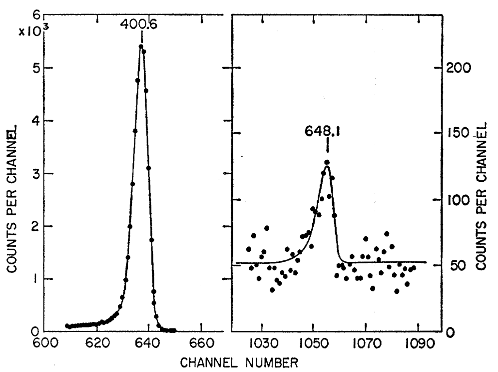

FIG. 3. Mg^{28} γ-ray spectrum in a 30-cm³ Ge(Li) detector in coincidence with a 3×3-in. NaI(Tl) detector when a pulse-height channel was imposed on the 941.7-keV peak in the spectrum of the NaI(Tl) detector.

spectra with the relative efficiencies taken from curves[6] appropriate to the two sizes of Ge(Li) detector used. Several such analyses gave results that agreed with each other within the estimated errors of 5–10%. The measured energies and relative intensities of the Mg^{28} γ rays are listed in Table I.

III. DISCUSSION

All of the experimental data discussed in Sec. II are consistent with the energy-level diagram shown in Fig. 4. This figure includes five of the first six known energy levels[1] of Al^{28}. The third excited state at 1017 keV has been deleted from Fig. 4, since we found no evidence in the present experiment for population of this level through either Mg^{28} β decay or cascade radiation from the 1620.0- or 1372.8-keV levels. The most significant change from the previously proposed[1,3] decay scheme of Mg^{28} is the new 5% β branch to a level in Al^{28} at 1620.0 keV. The energy of this state follows directly by summing the energies of the 1589.4- and 30.6-keV γ rays which are in coincidence. Based on the energy of the 1620.0-keV state, the cascade γ ray to the 972.2-keV level should have an energy of 647.8±0.5 keV. Thus, the transition assignment of the 648.1±0.5-keV γ ray which is in coincidence with the 941.7-keV γ rays is confirmed by the fact that its energy agrees with the expected value within the errors of 0.5 keV. As has already been seen in Fig. 2 and discussed above, the 1620.0-keV ground-state transition has been observed. The one remaining possible γ-ray transition involving the levels shown in Fig. 4, between the 1620.0- and 1372.8-keV levels, could not be found either in singles or coincidence experiments, and the upper limit on its intensity is given in Table I.

Another new feature of the decay scheme in Fig. 4 is the ground-state γ-ray branch from the 1372.8-keV level. This γ ray would have been almost impossible to detect in the earlier experiments[3] using NaI(Tl) detectors. The high resolution of the Ge(Li) detector

TABLE I. γ-ray energies and intensities in the decay of Mg^{28}.

Energy (keV)	Al^{28} transition assignment (levels in keV)	Intensity (% per β decay)
30.6	30.6→0	95.0
247.2	1620.0→1372.8	<0.05
400.6±0.2	1372.8→972.2	35.9
648.1±0.5	1620.0→972.2	0.085
941.7±0.4	972.2→30.6	35.9
972.2	972.2→0	<0.18
1342.2±0.2	1372.8→30.6	54.0
1372.8±0.2	1372.8→0	4.7
1589.4±0.4	1620.0→30.6	4.7
1620.0±0.4	1620.0→0	0.29

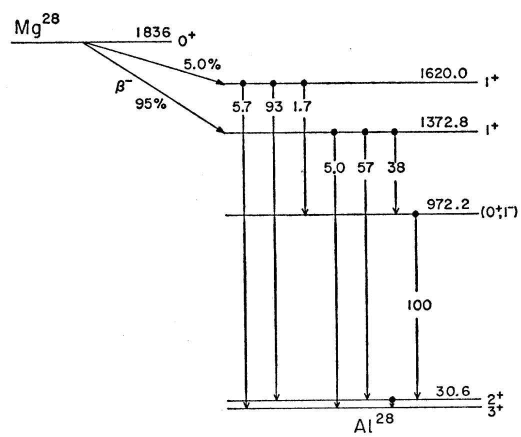

FIG. 4. Proposed decay scheme of Mg^{28}. Excitation energies are in keV and the γ-ray branching intensities from each level are in percent.

[6] S. Hechtl (private communication); T. K. Alexander (private communication).

also permits a sensitive search to be made for a possible ground-state γ-ray transition from the 972.2-keV state. An upper limit on the intensity of this peak, corrected for the relative detector efficiency, leads to the upper limit given in Table I. The corresponding upper limit on the fractional γ-ray branch 972.2→0 keV is <0.5%.

The Mg^{28} β-branching intensities shown in Fig. 4 were derived from the various γ-ray intensities. Based on the Mg^{28} total decay energy of 1836 keV and the half-life of 21.3 h [1] the logft values for the β-ray branches to the 1372.8- and 1620.0-keV levels of Al^{28} are 4.49±0.03 and 4.59±0.08, respectively. The logft values for the β transitions to the 1620.0- and 1372.8-keV levels establish that both β transitions must be allowed. Since the ground-state spin parity of Mg^{28} is $J^\pi=0^+$, the selection rule for allowed β transitions gives $J^\pi=0^+$ or 1^+ for both levels. If the 0^+ assignment is assumed, the β transition is pure Fermi and is, therefore, allowed only if $\Delta T=0$. If, therefore, the 1372.8- and 1620.0-keV levels are assumed to have $T=1$, the β decays are $\Delta T=1$ transitions which require them to be Gamow-Teller transitions. Since $0^+\to0^+$ Gamow-Teller transitions are not allowed, the assumption that $T=1$ for the 1620.0- and 1372.8-keV levels and the allowed nature of the β decay leads to unique $J^\pi=1^+$ assignments for both levels. This argument has been invoked previously for the 1372.8-keV level[1] and holds equally well for the 1620.0-keV level. The 0^+ assignment to either level would require some $T=2$ component in the wave function of that level. From the tabulated Coulomb displacement energy[7] we estimate that the first $T=2$ state in Al^{28}, the analog of the ground state of Mg^{28}, is at $E_x\simeq5.87$ MeV. If the 0^+ assignment to the 1372.8- and 1620.0-keV levels is assumed, and it is assumed that the Coulomb force mixes these levels with the analog of the Mg^{28} ground state, the measured ft values require 4 and 5% $T=2$ intensities for the 1620.0- and 1372.8-keV levels, respectively. Following the treatment of Bloom[8] and assuming the analog of the Mg^{28} ground state is at 5.87 MeV in Al^{28}, the 4 and 5% intensities require Coulomb matrix elements greater than 800 keV which, from Bloom's compilation, is much too large. Therefore, it appears quite unlikely that either the 1620.0- or 1372.8-keV levels could contain large enough $T=2$ components to make 0^+ assignments to these levels consistent with the observed ft values. The 1^+ assignment to both levels is, therefore, quite definite.

The observation in the present work of the ground-state γ-ray transitions from both the 1372.8- and 1620.0-keV levels further strengthens the $J^\pi=1^+$ assignments to these two states. Thus for $J^\pi=1^+$ the transitions to the $J^\pi=3^+$ ground state are $E2$ transitions which can compete with the predominant $M1$ or $M1+E2$ transitions to the $J^\pi=2^+$ 30.6-keV level. On the other hand, 0^+ assignments would require that the ground-state transitions be $M3$ which would not compete favorably with $E2$ transitions to the 30.6-keV state. We note that the fractional ground-state branches from the 1372.8- and 1620.0-keV states are approximately the same, but the fractional branch 1620.0→972.2 keV is ~20 times smaller than the 1372.8→972.2-keV branch in spite of the greater energy of the former transition.

Previous results on the Mg^{28} β endpoint energy[9] of 459±2 keV, as well as on the near equality[3] of the intensities of the 400.6- and 941.7-keV γ rays, show that there is no appreciable β-ray decay of Mg^{28} to the 972.2-keV level of Al^{28}. By allowing for a maximum possible difference of 10% in the intensities of the 400.6- and 941.7-keV γ rays an upper limit of <4% can be placed on the β decay of Mg^{28} to the 972.2-keV state. The corresponding lower limit on the logft value is >6.8. Since this value indicates a forbidden β-ray transition, it is unlikely that the 972.2-keV level can be $J^\pi=1^+$. If the spin parity were $J^\pi=2^\pm$ or $3^\pm$, the ground-state γ-ray transition would be $E1$ or $M1$ and should be observable. Thus our upper limit of <0.5% on the intensity of the 972.2→0 transition argues against the $2^\pm$ or $3^\pm$ possibilities, which is consistent with the results of Lawergren.[4] An assignment of $J^\pi=0^-$ to the 972.2-keV level would require an $M2$ transition to the 30.6-keV state. The upper limit[10] of 0.3 nsec on the mean lifetime of the 972.2-keV level would correspond to an $M2$ transition of at least 20 Weisskopf units. Therefore, we rule out a 0^- assignment. The two remaining possibilities, $J^\pi=0^+$ or 1^-, are consistent with all of the information available, although Lawergren's results[4] favor $J=0$. In either case, the β transition from Mg^{28} would be forbidden, the γ-ray transition to the 30.6-keV state would be consistent with lifetime information, and the ground-state γ-ray transition would be very weak.

The present observation that Mg^{28} decays to a state in Al^{28} at 1620.0±0.4 keV is not in very good agreement with the Al^{28} level at 1633±5 keV reported previously[1] from the $Al^{27}(d,p)Al^{28}$ reaction. Furthermore, the spin-parity assignment[1] of $J^\pi=2^+$ or 3^+ given previously to the 1633-keV level is inconsistent with our β-decay results. If it is the same state of Al^{28} that is involved in the present work, we conclude that the accuracy of the Q value measurements in the (d,p) reaction was overestimated and that the spin-parity assignment was incorrect.

The results of the present esperiment bear directly on Sheline's description[11] of the low-lying levels of Al^{28} in terms of four rotational bands. In particular, he assigns $J^\pi=4^+$ $K=3$ to the 1620.0-keV level. Since the ground state of Mg^{28} is 0^+, this predicts that the β transition to

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this level is forbidden and is inconsistent with the allowed logft measured in the present experiment.

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Study of Nuclear States in ^{60}Ni by Inelastic Electron Scattering

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Inelastic scattering of electrons from ^{60}Ni was studied with an over-all energy resolution of 0.1% by the use of 183- and 250-MeV electron beams from the Tohoku 300-MeV linear accelerator. Peaks were found at 1.33 (2^+), 2.16 (2^+), 2.50 (4^+), 3.13 (4^+), 3.67 (4^+), 4.04 (3^-), 4.85 (2^+, 4^+), 5.05 (4^+, 6^+), 6.20 (3^-), 6.85 (2^+, 5^-), and 7.05 MeV (3^-). The data were analyzed in Born approximation, using the Helm model to determine multipolarities and reduced transition probabilities. Distorted-wave calculations were also carried out for the 1.33-, 2.50-, and 4.04-MeV states. A striking feature of the present study is the discovery of the collective ($G=11$) 6^+ state at 5.05 MeV. The energies and transition probabilities of low-lying states were compared with the predictions of a recently developed theory based on the shell model.

I. INTRODUCTION

A NUMBER of studies to investigate the nuclear structure of ^{60}Ni have been performed by inelastic scattering, including (e, e'),[1,2] (p, p'),[3–6] and (α, α') [7–9] reactions. When the projectile is an electron, it is well known that the nuclear states are excited by the electromagnetic interaction, which is well understood, and which leads to precise determination of parameters,[10–12] i.e., the spin, parity, and reduced radiative transition probability. These parameters have been also extracted from inelastic scattering experiments using strongly interacting particles. Thus, it is interesting to compare the results of inelastic electron scattering with the results using other particles.

Inelastic electron scattering from ^{60}Ni was first studied at Stanford using incident electrons of 183 MeV. Recently, very precise measurements of the (e, e') reaction of ^{60}Ni were made at Yale,[2] where the energy resolution of the spectrum of the scattered electrons was 130 keV. Unfortunately, Duguay *et al.*[2] missed many excitations because of additional background due to instrumental scattering. The present paper reports the results of inelastic scattering of 183- and 250-MeV electrons from ^{60}Ni. The present experiments have better energy resolution than the previous (e, e') reaction at 183-MeV at Stanford.

Very recently, detailed shell-model calculations[13–16] were performed for the Ni isotopes, so comparison of the present experimental results with the theoretical predictions can be made for the energies and the

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