

## Distorted-Wave Analysis of the $^{11}\text{B}(d, p)$ Reaction Leading to the Lowest Unbound Level in $^{12}\text{B}^\dagger$

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The 3.39-MeV state of  $^{12}\text{B}$  is 20 keV above the threshold for neutron decay, and has been seen both in  $^{11}\text{B}(d, p)^{12}\text{B}$  and in  $^{11}\text{B}(n, n)^{12}\text{B}$ . We present an analysis of  $^{11}\text{B}(d, p)^{12}\text{B}$  by a new technique and combine the stripping and elastic scattering data to obtain the spin and parity of the state in question.

IN an earlier study of the  $^{11}\text{B}(d, p)^{12}\text{B}$  reaction<sup>1</sup> at  $E_d=8.0$  MeV, in which plane-wave Born-approximation (PWBA) analysis was used, the 3.39-MeV state of  $^{12}\text{B}$  was assigned positive parity and a spin  $J_f \leq 3$ . The state was also seen in the  $^{10}\text{B}(t, p)^{12}\text{B}$  reaction<sup>2</sup> at  $E_t=10$  MeV. PWBA analysis there gave an angular momentum transfer  $l=0$ , from which came the prediction  $J_f^\pi=3^+$ . However, owing mainly to the uncertainties in PWBA analyses, neither assignment may be considered definite.

Recently, this state has been observed as a resonance in the  $^{11}\text{B}+n$  system.<sup>3</sup> That experiment yielded a resonance energy  $E_n(\text{lab})=20.8$  keV, and a width given by  $(2J_f+1)\Gamma_n=20-22$  eV.

The present work consists of a distorted-wave Born-approximation (DWBA) analysis of the unpublished  $^{11}\text{B}(d, p)^{12}\text{B}$  data of Schiffer *et al.*,<sup>4</sup> obtained at a deuteron energy of 12 MeV. This analysis uses a new method for calculating stripping to unbound states. The main difficulty in such calculations is that the radial integrals occurring in the expression for the reaction amplitude converge very slowly. The technique of Huby and Mines<sup>5</sup> was to insert a convergence factor  $\Phi(\alpha, r)$  [e.g.,  $\Phi(\alpha, r)=e^{-\alpha r}$ ] into the integrand, evaluate the integral for several values of  $\alpha$ , and then numerically extrapolate to  $\alpha=0$ . Even though that method has succeeded in at least one application,<sup>6</sup> it does not seem to remove the essential difficulty. When  $\alpha$  is small enough for the extrapolation to be reliable, one still has to integrate out to very large radii ( $\geq 100$  F).<sup>7,8</sup>

In the present method,<sup>9,10</sup> this difficulty is overcome by contour integration in the complex  $r$  plane. Within

the range of the nuclear force  $r \leq R$  the integration is done along the real axis. Outside this range, the integrand is split into two parts which are integrated along the contours  $r=R \pm i\rho$ ,  $0 \leq \rho < \infty$ . This transformation can be justified by using the analytic properties of Coulomb (or Hankel) functions as functions of  $r$ . The resulting external integrand tends exponentially to zero at large  $\rho$ . The external integral can therefore be evaluated using only a small number of points.

The present  $^{11}\text{B}(d, p)^{12}\text{B}$  data<sup>4</sup> were measured at  $E_d=12.0$  MeV. In this region of nuclei, the  $(d, p)$  reaction is certainly expected to proceed via  $l \leq 3$ . The shape of the angular distribution for the 3.39-MeV state is not at all consistent with  $l=0$  or  $l=3$ . Thus, the majority of the calculations were restricted to  $l=1, 2$ .

In the present calculation, the wave function for the neutron transferred to the 3.39-MeV state of  $^{12}\text{B}$  was taken to be an optical-model wave function corresponding to an energy of 20 keV in the  $^{11}\text{B}+n$  system. The code ABACUS<sup>11</sup> was used to generate the wave function from a real Woods-Saxon optical potential. The parameters used to reproduce the resonance were  $V_0=55.744$  MeV,  $V_{s0}=6.0$  MeV,  $r_0=1.26$  F, and  $a=0.6$  F.

From shell-model systematics, the  $d_{5/2}$  orbit is expected to lie several MeV below the  $d_{3/2}$  orbit. For this reason most of the  $l=2$  calculations were performed for  $j=\frac{5}{2}$ . The  $l=1$  calculations were done for both  $j=\frac{1}{2}$  and  $j=\frac{3}{2}$ . In addition to local zero-range calculations, calculations were also performed with nonlocal finite-range (FRNL) corrections within the local-energy approximation (LEA)—both performed with a specially modified<sup>7,10</sup> version of Tamura's Fortran code DWMAIN.<sup>12</sup> Most of the calculations were done with the average proton potential of Watson *et al.*,<sup>13</sup> who have carried out an extensive analysis of proton scattering from  $1p$ -shell nuclei. For the  $Z, A$ , and bombarding energy in the present case, the parameters of their potential are  $V_0=59.0$  MeV,  $W=0$ ,  $W_D=9.60$  MeV,  $V_{s0}=5.54$  MeV,  $r_0=r_0'=r_{s0}=r_{0c}=1.14$  F,  $a=a_{s0}=0.57$  F, and  $a'=0.50$  F. Several deuteron potentials were taken from

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TABLE I. Comparison of results of  $^{11}\text{B}(n, n)$  and  $^{11}\text{B}(d, p)$  analysis for the 3.39-MeV state of  $^{12}\text{B}$ .

$l$ (Assumed)	$\Gamma_{sp}^a$	$(2J+1)\Gamma_n^b$		$\Gamma_n/\Gamma_{sp}^d$		
		$\Gamma_{sp}$	$(2J+1)S^c$	$S$	$J^\pi$	$S$
1	3.3 keV	$7 \times 10^{-3}$	7.0	$10^{-3}$	$2^+$	1.40
2	8 eV	2.5	4.0	0.63	$3^-$	0.57

<sup>a</sup> Calculated from a Woods-Saxon potential whose parameters are given in text.

<sup>b</sup> The  $^{11}\text{B}+n$  analysis (Ref. 3) yields  $(2J+1)\Gamma_n=20-22$  eV.

<sup>c</sup> Present work.

<sup>d</sup> Consistency between stripping and elastic scattering requires that this quantity be approximately 1.

other analyses in this mass region.<sup>14</sup> The deuteron parameters used in calculating the curves plotted in Fig. 1 were Pot. 2:  $V_0=119.1$  MeV,  $W=0$ ,  $W_D=5.92$  MeV,  $V_{so}=6.0$  MeV,  $r_0=a=r_{so}=a_{so}=0.90$  F,  $r_0'=1.513$  F,  $a'=0.81$  F,  $r_{0c}=1.40$  F; and Pot. 3:  $V_0=100$  MeV,  $W=10$  MeV,  $W'=0$ ,  $V_{so}=0$ ,  $r_0=r_{0c}=1.40$  F,  $r_0'=1.74$  F,  $a=0.60$  F,  $a'=0.80$  F.

As may be seen from Fig. 1, the results at forward angles ( $\theta \lesssim 60^\circ$ ) were rather insensitive to the FRNL corrections and were also insensitive (within 10-30%) to changes in the distorting potentials. The best fits for  $l=1$  and  $l=2$  are shown in Fig. 2 along with the data.

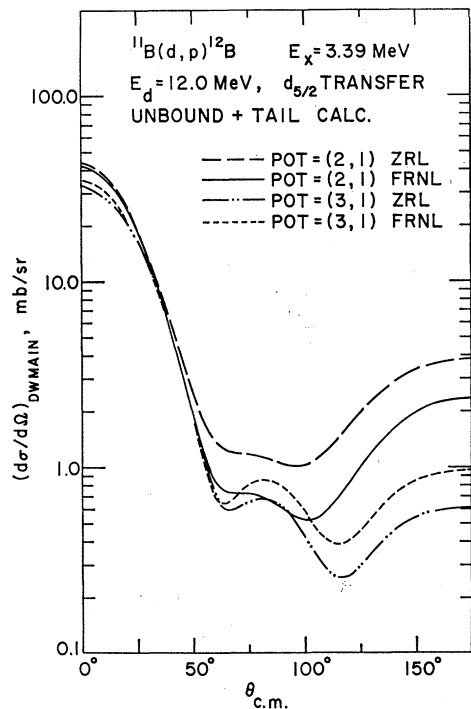


FIG. 1. DWBA calculations for  $d_{5/2}$  transfer to the 3.39-MeV state of  $^{12}\text{B}$ : zero-range local (ZRL) and finite-range nonlocal (FRNL). The potentials are given in the text.

The slope of the experimental angular distribution favors  $l=2$ , although by itself the difference in the shapes is perhaps not great enough to allow an unambiguous assignment.

However, the spectroscopic factors present a much stronger argument. Comparison of the data with the calculations yields  $(2J_f+1)S=7.0$  for  $l=1$  and  $(2J_f+1)S=4.0$  for  $l=2$ . In view of the large  $l=1$  strength at lower excitation energies in  $^{12}\text{B}$ , this  $l=1$  spectroscopic factor is much larger than one would expect. (In fact, since  $l=1$  implies  $J_f \leq 3$ , this spectroscopic factor for  $l=1$  is greater than the single-particle limit.) Comparison with the  $^{11}\text{B}+n$  data gives even stronger evidence for  $l=2$ . The  $^{11}\text{B}+n$  results yield a neutron width given by  $(2J_f+1)\Gamma_n=20-22$  eV. If  $\Gamma_{sp}$  is the single-particle width, then the quantity  $(2J_f+1)\Gamma_n/\Gamma_{sp} \approx (2J_f+1)\theta_n^2$  should be roughly equal to  $(2J_f+1)S$ . In the present analysis,  $\Gamma_{sp}$  was taken to be the width calculated from the real Woods-Saxon well used for the appropriate  $l$  values. (This estimate is expected to be in error by no more than a factor of 2.)

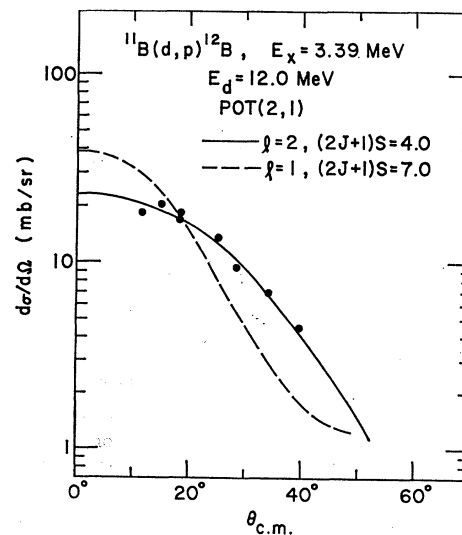


FIG. 2. Best fits for  $l=1$  and  $l=2$  transfer. The potentials are given in the text.

<sup>14</sup> Reference 8 and references therein.

The results of the analysis are shown in Table I. It can be seen that  $\theta_n^2$  and  $S$  differ by about a factor of  $10^3$  for  $l=1$ , but agree to within a factor of 2 for  $l=2$ .

Adding a  $d_{5/2}$  neutron to the ground state of  $^{11}\text{B}$  ( $J^\pi=3^-$ ) yields possible spins 1, 2, 3, and 4 and negative parity. (A  $d_{3/2}$  transfer would have allowed  $J=0$ , but the  $d_{3/2}$  strength is expected to lie several MeV above the  $d_{5/2}$  strength.  $J=0$  is also independently ruled out by the results of Mooring *et al.*) If the final spin were  $J=1$  or 2, the state could be reached via  $l=0$  as well as  $l=2$ . There is no indication of any  $l=0$  strength in either the  $^{11}\text{B}(d, p)$  or the  $^{11}\text{B}+n$  work. Therefore, the final spin should be either  $3^-$  or  $4^-$ . The  $\gamma$ -decay branching ratio of this state<sup>3</sup> seems to rule out  $J=4$ , leaving

$J^\pi=3^-$  as the most likely assignment consistent with all the data.

In summary, the comparison of reaction and elastic scattering data has yielded an unambiguous assignment of negative parity for the 3.39-MeV state of  $^{12}\text{B}$ , in contradiction to the results of earlier analyses.<sup>1,2</sup> The spin assignment is not unique, but combining all the available data for this state leads to  $J^\pi=3^-$  as the most likely assignment.

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## Comprehensive Formalism for Nuclear-Reaction Problems. III. Calculable Theories with Systematic "Discretization"\*

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The comprehensive formalism of earlier papers is specialized in the direction of calculable theories of nuclear reactions especially those based on the shell model such as that of Bloch and Fano. Unlike the basis states used in previous specialization, the shell-model states do not, in general, have the property that  $(H+\mathcal{L})$  is Hermitian. We show that this complication is no real obstacle, and that only a weaker Hermiticity condition is needed. The final result is a number of forms of calculable theories of the  $R$ -matrix kind. No continuum-continuum interaction problems exist, since continua do not occur. This and other merits of the results are discussed from the viewpoint of practical calculations.

### I. INTRODUCTION

IN the two earlier papers<sup>1,2</sup> (I and II), we have set up a comprehensive theory of reactions based on Bloch's  $\mathcal{L}$  operator,<sup>3</sup> and have specialized this to yield six existing theories (I), and also a theory of line broadening with fine structure (II). Each of the seven theories is characterized by the basis which is used to develop the Green's-function operator,  $\mathcal{G} \equiv (H+\mathcal{L}-E)^{-1}$ . The merit of the combination  $H+\mathcal{L}$  is that it facilitates the derivation of results for the scattering matrix, etc. If the set  $|p\rangle$  are eigenstates of a Hamiltonian  $H^0$  and

satisfy  $\mathcal{L}^0|p\rangle=0$ , and if  $(H+\mathcal{L})$  is Hermitian in the states  $|p\rangle$ ,

$$\mathcal{G} = \sum_{p,q} |p\rangle \langle p| \mathcal{G} |q\rangle \langle q|. \quad (1)$$

The term Hermitian for operators like  $(H+\mathcal{L})$  will be used even though  $(H+\mathcal{L})$  is "trivially" non-Hermitian, because of the use of complex boundary conditions, so that  $(H+\mathcal{L})^\dagger = H+\mathcal{L}^*$ . The term non-Hermitian will be used whenever  $(H+\mathcal{L})^\dagger \neq H+\mathcal{L}^*$ . On defining

$$\mathcal{G}^0 = (H^0 + \mathcal{L}^0 - E)^{-1}, \quad (2)$$

$$h = (H - H^0) + (\mathcal{L} - \mathcal{L}^0), \quad (3)$$

then

$$\mathcal{G} = (1 + \mathcal{G}^0 h)^{-1} \mathcal{G}^0 \quad (4)$$

and the matrix elements  $\langle p| \mathcal{G} |q\rangle$  that give the scattering matrix are obtained from the matrix version of Eq. (4), with operators  $\mathcal{G}$ ,  $\mathcal{G}^0$ ,  $h$  replaced by their

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