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Magnetic Moment and hfs Anomaly for He^3†

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The ratio of the nuclear magnetic moments of He^3 and H^1 has been measured by NMR techniques using a gaseous sample. The result is $-\mu_{He^3}(He)/\mu_p(H_2)=0.76178685$ (±0.1 ppm), uncorrected for magnetic shielding in He and H_2, and with these shielding corrections, $-\mu_{He^3}/\mu_p=0.76181237\pm0.6$ ppm. With this new value for the magnetic moment ratio the hfs anomaly is found to be $\delta_{He^3}=+(217\pm3)\times10^{-6}$. By comparison with the theory for δ_{He^3} the contribution of exchange currents to δ_{He^3} is calculated to be $+(15\pm5)\times10^{-6}$.

I. INTRODUCTION

A COMPLETE theory of the three-nucleon system must comprehend all the experimental information about the He^3 nucleus. Conversely, data about He^3 should aid in the development of the theory of the three-nucleon system and in the examination of the magnitude of a three-nucleon force. Recently,[1-3] studies have been made of He^3 and H^3 through the elastic scattering of high-energy electrons, and values for the rms charge and magnetic moment radii have been determined. Further relevant quantities are the nuclear masses, spins, magnetic dipole moments, and the hfs anomalies.

In this paper we report a new high-precision measurement of the magnetic moment ratio μ_{He^3}/μ_p by an NMR technique.[4] We achieve an accuracy of 0.1 ppm, which represents an improvement by a factor of 10 relative to an earlier determination.[5] With this new value and the value of the hfs interval for He^3 in the ground $1^2S_{1/2}$ state of He^+, a new value for the hfs anomaly for He^3 is determined.

Section II includes a description of the experimental apparatus. The measurements and results are discussed in Sec. III. The He^3 hfs anomaly is discussed in Sec. IV.

II. EXPERIMENTAL

The basic experimental technique used was that of NMR.[6] The dc magnetic field was produced by a Varian 12-in.-high resolution magnet equipped with field homogenizing coils. Field sweep was provided by driving the magnet power supply sweep input with a triangular voltage.

The measurements were made with gaseous samples at total pressures up to about 60 atm. A high-pressure bulb was built following the design of Garwin and Reich[7] and is shown schematically in Fig. 1. The spherical bulb, fabricated from Araldite 503 epoxy,[8] has a $\frac{1}{4}$-in. radius and 0.025 in. wall thickness. Two rf coils (4 turns of No. 30 copper wire) were wound on the bulb with their polar axes orthogonal. Brass capillary tubing was inserted in the diametrically opposed necks. Feather-edge disks (2 mil brass foil) were soldered to the tubing to prevent leakage around the tubular necks. The whole arrangement was cast in epoxy. Magnetic field modulation coils (1-in.-diam pancakes) were placed on the sample bulb. Destruction of bulbs fabricated in this manner occurred at about 75 atm. The gaseous samples contained high-purity He^3 and commerical-grade O_2 and H_2. The gas mixing was done in a Toepler pumping system and the mixtures were compressed into the sample bulb with a head of mercury backed by a high-pressure N_2 tank. The presence of the H_2 allows a comparison of the helium magnetic moment $\mu_{He^3}(He)$ to the proton moment $\mu_p(H_2)$. The nuclear spin-relaxation time for pure He^3 is long,[7,9] due to the weakness of the nuclear dipole-dipole relaxation mechanism. Hence, oxygen was introduced to reduce the He^3 relaxation time to about 1 sec.

A spectrometer was used which permitted simultaneous observations of the NMR signals from He^3 and protons in H_2 at the same magnetic field site, thus largely eliminating the effects of magnetic field drifts and gradient shifts. A schematic diagram of the spectrometer is shown in Fig. 2. Nuclear sideband detection systems[10] were used to observe the He^3 and proton resonances. The basic system is a simple tank circuit probe driven by an rf signal generator and uses conventional amplification and detection techniques.

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[9] G. K. Walters, L. D. Schearer, and F. D. Colegrove, Bull. Am. Phys. Soc. **9**, 11 (1964); R. L. Gamblin and T. R. Carver, *ibid.* **9**, 11 (1964); H. G. Robinson and T. Myint, Appl. Phys. Letters **5**, 116 (1964).

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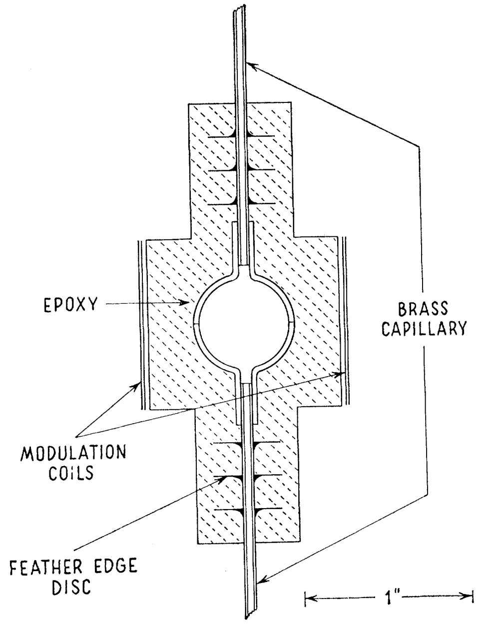

FIG. 1. High-pressure sample bulb.

The magnetic field is modulated at an audio angular frequency ω_m. Under slow passage conditions and with $\gamma H_1 \ll \omega_m$, a spectrum of resonances can be observed at magnetic field values

$$H = H_0 \pm k\omega_m/\gamma = (1/\gamma)(\omega_0 \pm k\omega_m), \quad (1)$$

in which γ is the gyromagnetic ratio of the sample nuclei, H_1 is the amplitude of the rf magnetic field, H_m is the amplitude of the modulation field, and H_0 is the value of the dc magnetic field at resonance when the rf frequency is ω_0. The sideband number is denoted by the integer k. We observe the $k=1$ sidebands. For stronger rf fields the $k=1$ resonances occur at magnetic field values

$$H = H_0 \pm \omega_m/\gamma \mp \gamma H_1^2/2\omega_m, \quad (2)$$

where the correction term of order $(\gamma H_1/\omega_m)^2$ is included. Hence the resonance condition depends on H_1.

The constant driving frequencies for the He^3 and H_2 probes were derived from a Hewlett-Packard 5100 A/ 5110 A digital-frequency synthesizer (dfs). The dfs has a 0–50-MHz output which can be varied in steps of 0.01 Hz. The rms frequency stability is approximately 1 in 10^{11} at 50 MHz. The H^1 frequency of 50 MHz was obtained with an harmonic generator and crystal filter driven by the dfs standard 1 MHz. The filter has a Q of 19 000 and gives 50-dB rejection of the 49- and 51-MHz harmonics. The He^3 frequency of about 38 MHz was taken directly from the dfs output.

The preamplifiers have noise figures of approximately 3 dB and gains of approximately 20 dB. National HRO-60 and Eddystone 770-R receivers were used as rf amplifiers and detectors. The am outputs were fed to lock-in detectors whose outputs were recorded on a two-pen strip chart recorder. The magnetic field modulation current and lock-in reference signals were obtained from an audio oscillator.

The operation of the spectrometer was checked by measuring the gyromagnetic moment ratio $\gamma_{Li^7}/\gamma_p(H_2O)$. The samples were saturated aqueous solutions of LiCl and $LiNO_3$ in distilled water. The measurement technique used is the same as that described in Sec. III. The results of these measurements were

$$\gamma_{Li^7}(LiNO_3)/\gamma_p(H_2O) = 0.388636251(20),$$

$$\gamma_{Li^7}(LiCl)/\gamma_p(H_2O) = 0.388636264(20),$$

in which 1-standard-deviation errors are indicated. The agreement of these values is taken as meaning that the Li^7 resonances arise from ionized lithium, as previously suggested.[11] The average is

$$\gamma_{Li^7}/\gamma_p(H_2O) = 0.388636258(14),$$

in agreement with previous determinations.[12]

III. EXPERIMENTAL PROCEDURE AND RESULTS

The amplitude of the sideband signals is dependent on H_m and γ. At the beginning of each measurement H_m was adjusted so that the amplitudes of the first sideband resonances from He^3 and protons in H_2 were approximately equal. The rf levels were adjusted to avoid saturation. The dc magnetic field was adjusted until one of the H^1 resonances, e.g., the one at $H_0 + \omega_m/\gamma_p(H_2)$, was in the center of the sweep trace. Here $H_0 = \omega_p/\gamma_p(H_2)$, where ω_p is the driving frequency for the proton resonance. The corresponding upper sideband resonance for He^3 will occur at the same dc magnetic field for the proper driving frequency ω_{He^3}. The separation of the centers of the He^3 and H^1 resonances was measured as a function of ω_{He^3} and the frequency for which the line centers coincide on the sweep trace was determined. A similar measurement was made on the other set of corresponding sidebands $[H = H_0 - \omega_m/\gamma_p(H_2)]$. The effects of resonance shifts due to H_1 [Eq. (2)] were eliminated by taking advantage of the fact that these shifts are symmetric around the center $(H = H_0)$ of the resonance spectrum. Let ω_1 and ω_2 denote, respectively, the He^3 angular frequencies at which corresponding H^1 and He^3 resonance line centers above and below H_0 coincide. From (2) the nuclear magnetic moment ratio is

$$\mu_{He^3}(He)/\mu_p(H_2) = (\omega_1 + \omega_2)/2\omega_p, \quad (3)$$

where the shift terms have cancelled. Further checks for systematic errors were made by measuring the magnetic moment ratio $\mu_{He^3}(He)/\mu_p(H_2)$, with both absorption and dispersion mode signals, with the two

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[12] G. Lindstrom, Arkiv Fysik **4**, 1 (1951); G. K. Yagola and E. E. Bogatyryov, Ukr. Fiz. Zh. **7**, 45 (1962).

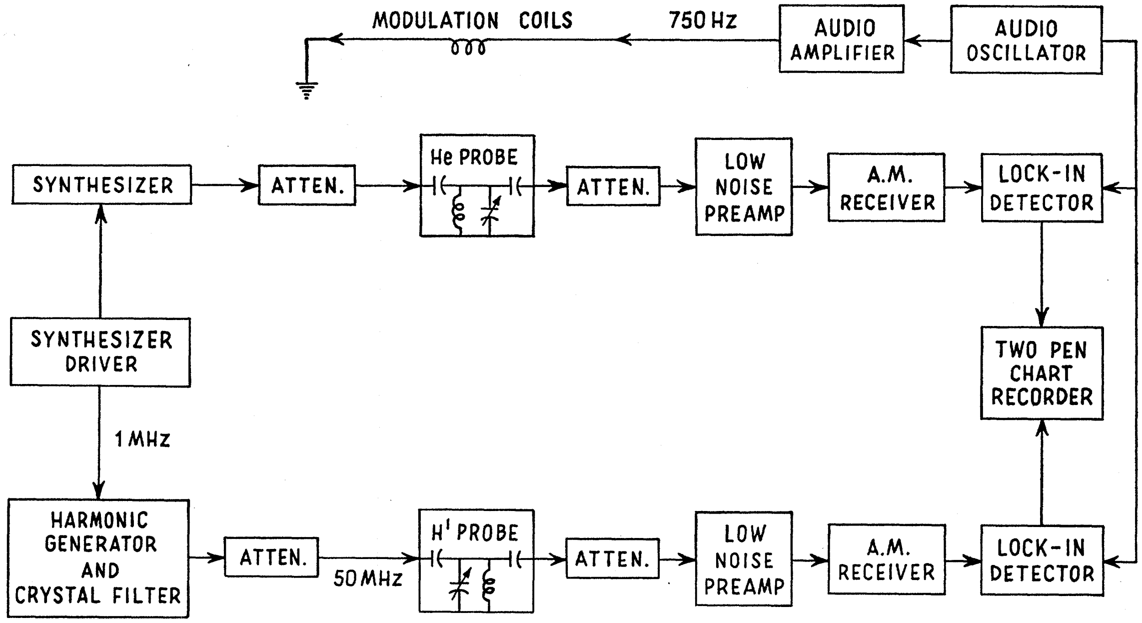

FIG. 2. Schematic diagram of the dual nuclear sideband spectrometer.

detection heads interchanged, with modulation frequencies $\omega_m/2\pi$ of 750 Hz and 1200 Hz, and for three values of the partial pressure ratio $P(H_2):P(O_2):P(\mathrm{He})$. Data were taken with increasing and decreasing magnetic field sweeps.

In a typical run the separation of the resonance centers was varied in 20-Hz steps over a range of approximately 200 Hz. Figure 3 shows a resonance trace. The signal-to-noise ratio is approximately 15. The He^3 and H^1 resonance linewidths are 49 and 76 Hz, respectively. The line centers were coincident for $\omega_{\mathrm{He}^3}/2\pi = 38.088770$ MHz and $\omega_p/2\pi = 50.000000$ MHz.

The observed helium and hydrogen linewidths are given in Table I. Values of the linewidth ratio multiplied by the inverse ratio of the nuclear moments,

$$[\Delta\nu_{1/2}(\mathrm{H}^1)/\Delta\nu_{1/2}(\mathrm{He}^3)][\mu_{\mathrm{He}^3}(\mathrm{He})/\mu_p(\mathrm{H}_2)]$$

are given in the last column of Table I. This ratio is equal to 1 within experimental error, i.e., the linewidths for hydrogen and helium are approximately the same (in magnetic field units). Hence we conclude that the broadening was primarily due to magnetic field inhomogeneity. The average linewidth is 17.5 ± 3.0 mG (1.5 ppm).

The values of the moment ratio $\mu_{\mathrm{He}^3}(\mathrm{He})/\mu_p(\mathrm{H}_2)$ are listed in Table II, where we have indicated the negative sign of μ_{He^3}.[13] They are plotted in Fig. 4 as a function of $P(\mathrm{He}^3)$. The value obtained by Anderson[5] is also shown in Fig. 4.

Analysis of these data for dependence of the measured moment ratio on the partial pressure of He, H_2, and O_2 shows that any pressure shifts are negligible within experimental error. A rough theoretical estimate indicates that no observable pressure shifts would be expected.

The final result is

$$-\mu_{\mathrm{He}^3}(\mathrm{He})/\mu_p(\mathrm{H}_2) = 0.76178685\ (\pm0.1\ \mathrm{ppm}), \quad (4)$$

where the quoted error is 1 standard deviation.

Magnetic shielding corrections must be made to determine the ratio of the magnetic moments of the bare nuclei, $\mu_{\mathrm{He}^3}/\mu_p$.

$$\mu_{\mathrm{He}^3}/\mu_p = [\mu_{\mathrm{He}^3}(\mathrm{He})/\mu_p(\mathrm{H}_2)][(1+\sigma_{\mathrm{He}})/(1+\sigma_{\mathrm{H}_2})]. \quad (5)$$

The calculated[14] magnetic shielding correction for He^3 is $\sigma_{\mathrm{He}} = 59.935\times10^{-6}$, where relativistic corrections of order α^2Z^2 have been neglected.[15] The shielding correction for H_2 is $\sigma_{\mathrm{H}_2} = (26.43\pm0.60)\times10^{-6}$.[16] Using these values with (4) and (5), we find

$$-\mu_{\mathrm{He}^3}/\mu_p = 0.76181237\pm0.6\ \mathrm{ppm}. \quad (6)$$

IV. DISCUSSION

The hfs anomaly δ is defined in general by the equation[17]

$$\Delta\nu = \Delta\nu_p(1-\delta), \quad (7)$$

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[14] R. L. Glick, J. Phys. Chem. 65, 1871 (1961).

[15] V. W. Hughes and W. B. Teutsch, Phys. Rev. 94, 761 (1954).

[16] T. Myint, D. Kleppner, N. F. Ramsey, and H. G. Robinson, Phys. Rev. Letters 17, 405 (1966).

[17] P. Kusch and V. W. Hughes, in *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1959), Vol. 37/1.

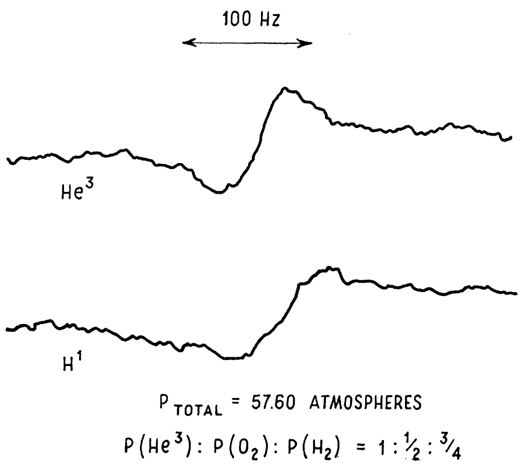

FIG. 3. Typical resonance pattern from He^3 and H^1 observed in the present experiment.

in which $\Delta\nu$ is the hfs interval and $\Delta\nu_p$ is the theoretical value of the hfs interval under the assumption that the nuclear magnetic moment is a point magnetic moment. The theoretical expression for $\Delta\nu$ in He^3 in the ground $1^2S_{1/2}$ state of the He^+ ion can be written as[18]

$$\Delta\nu_{\text{theor}}(\text{He}^+) = \left(\frac{128}{3}\alpha^2 c\,\text{Ry}\right)\frac{\mu_{\text{He}^3}}{\mu_0}\left(1-3\frac{m}{M_{\text{He}^3}}\right) \times(1+6\alpha^2+a_e+\epsilon_1+\epsilon_2+\epsilon_3)(1-\delta_{\text{He}}), \quad (8)$$

where

$$a_e=\alpha/2\pi-0.328(\alpha^2/\pi^2),$$

$$\epsilon_1=-2\alpha^2(\tfrac{5}{2}-\ln 2),$$

$$\epsilon_2=-(32\alpha^3/3\pi)(\ln 2\alpha)[\ln 2\alpha-\ln 4+(281/480)],$$

$$\epsilon_3=(4\alpha^3/\pi)(18.4\pm5).$$

Here α=fine-structure constant, c=velocity of light,

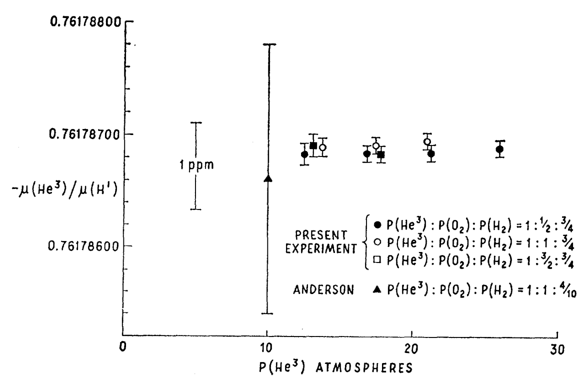

FIG. 4. The ratio $-\mu(He^3)/\mu(H^1)$ versus $P(He^3)$ for various values of the ratio $P(He^3):P(O_2):P(H_2)$. Also shown is Anderson's value Ref. 5.

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TABLE I. Linewidths ($\Delta\nu_{1/2}$).

P_{total} (atm)	$\Delta\nu_{1/2}(H^1)$ (Hz)	$\Delta\nu_{1/2}(He^3)$ (Hz)	$\frac{\Delta\nu_{1/2}(H^1)}{\Delta\nu_{1/2}(He^3)}\ \frac{\mu(He^3)}{\mu(H^1)}$
	$P(He^3):P(O_2):P(H_2)=1:\frac{1}{2}:\frac{3}{4}$		
27.4	75±7	51±8	1.1±0.2
37.6	76±6	52±7	1.1±0.2
47.3	73±6	50±6	1.1±0.2
57.6	76±7	49±7	1.2±0.2
	$P(He^3):P(O_2):P(H_2)=1:1:\frac{3}{4}$		
37.6	75±7	50±7	1.1±0.2
47.3	77±8	52±6	1.1±0.2
57.6	72±6	49±6	1.1±0.2
	$P(He^3):P(O_2):P(H_2)=1:\frac{3}{2}:\frac{3}{4}$		
47.3	73±7	54±7	1.1±0.2
57.6	75±6	48±8	1.2±0.2

Ry=rydberg, μ_0=Bohr magneton, m=electron mass, M_{He^3}=He^3 nuclear mass. The relativistic reduced mass correction of order $\alpha m/M_{\text{He}^3}$ is included in δ_{He}. For evaluation of $\Delta\nu_{\text{theor}}$, the factor μ_{He^3}/μ_0 is written in the form

$$\frac{\mu_{\text{He}^3}}{\mu_0}=\frac{\mu_{\text{He}^3}}{\mu_p}\frac{\mu_p}{\mu_e}\frac{\mu_e}{\mu_0}, \quad (9)$$

where μ_e is the spin magnetic moment of the electron. We use μ_{He^3}/μ_p from Eq. (6), and published values for μ_p/μ_e [16] and μ_e/μ_0,[19] to obtain

$$\mu_{\text{He}^3}/\mu_0=1.1587414\times10^{-3}\ (\pm0.9\text{ ppm}). \quad (10)$$

Using Eq. (10), the value of α from the ac Josephson measurements of e/h,[20] and the values of the other

TABLE II. Values of the moment ratio $\mu(He^3)/\mu(H^1)$ obtained at various pressures.

$P(He^3):P(O_2):P(H_2)$	$P(He^3)$ (atm)	$-\mu(He^3)/\mu(H^1)$[a]
$1:\frac{1}{2}:\frac{3}{4}$	25.6	0.76178688(9)
$1:\frac{1}{2}:\frac{3}{4}$	21.1	0.76178684(8)
$1:\frac{1}{2}:\frac{3}{4}$	16.7	0.76178684(9)
$1:\frac{1}{2}:\frac{3}{4}$	12.5	0.76178683(9)
$1:1:\frac{3}{4}$	20.9	0.76178694(9)
$1:1:\frac{3}{4}$	17.3	0.76178687(9)
$1:1:\frac{3}{4}$	13.7	0.76178689(10)
$1:\frac{3}{2}:\frac{3}{4}$	17.6	0.76178684(10)
$1:\frac{3}{2}:\frac{3}{4}$	13.0	0.76178690(10)

[a] Uncorrected for diamagnetic shielding.

[19] A. Rich, Phys. Rev. Letters **20**, 967 (1968).
[20] W. H. Parker, B. N. Taylor, and D. N. Langenberg, Phys. Rev. Letters **18**, 287 (1967).

constants as given by Cohen and Dumond,[21] we find from Eq. (8)

$$\Delta\nu_{\text{theor}}(\text{He}^+) = (8667.52\pm0.06)(1-\delta_{\text{He}})\,\text{MHz}, \quad (11)$$

in which the error is due principally to the uncertainty in the value of α. Using the experimental value[22]

$$\Delta\nu_{\text{expt}}(\text{He}^+) = 8665.649905\pm0.00005\ \text{MHz} \quad (12)$$

with Eq. (11), we find

$$\delta_{\text{He}} = (216\pm7)\times10^{-6}. \quad (13)$$

This value agrees with an earlier determination of δ_{He} based on $\Delta\nu_{\text{expt}}(\text{He}^+, 2^2S_{1/2})$,[3] in view of the different values of α that are used.

A relevant quantity which can be obtained independent of the uncertainty in α is

$$\frac{1-\delta_{\text{He}}}{1-P} = \frac{\Delta\nu_{\text{expt}}(\text{He}^+, 1^2S_{1/2})}{\Delta\nu_{\text{expt}}(\text{H}, 1^2S_{1/2})}\frac{\Delta\nu_p(\text{H}, 1^2S_{1/2})}{\Delta\nu_p(\text{He}^+, 1^2S_{1/2})}, \quad (14)$$

where

$$\frac{\Delta\nu_p(\text{H}, 1^2S_{1/2})}{\Delta\nu_p(\text{He}^+, 1^2S_{1/2})} = \frac{1}{8}\frac{\mu_p}{\mu_{\text{He}^3}}\left(1-3\frac{m}{M_{\text{H}^1}}+3\frac{m}{M_{\text{He}^3}}\right)$$
$$\times[(1-\tfrac{9}{2}\alpha^2-\alpha^2(\ln2-\tfrac{5}{2})+8\alpha^3/3\pi$$
$$\times\{4\ln^2 2\alpha-\ln^2\alpha-(\ln4-281/480)(4\ln2\alpha-\ln\alpha)\}$$
$$-3(\alpha^3/\pi)(18.4)]. \quad (15)$$

P includes the $\alpha(m/M_{\text{H}^1})$ and nucleon structure corrections for H. With the experimental values for the hfs splittings [2,24] and (6), we have

$$(1-\delta_{\text{He}})/(1-P) = 1-(182.3\pm0.6)\times10^{-6}. \quad (16)$$

Using the theoretical value[25] for P of $(35\pm3)\times10^{-6}$ with (16), we find

$$\delta_{\text{He}} = (217\pm3)\times10^{-6}. \quad (17)$$

The theoretical expression for δ_{He} is

$$\delta_{\text{He}}(\text{theor}) = (202\pm3)\times10^{-6}+\delta_{\text{exch}}, \quad (18)$$

for a singlet n-p effective range[4,27] $r_{0e}=2.7F$. Here δ_{exch} represents the contribution of exchange currents to the hfs anomaly. We find, using (17) and (18),

$$\delta_{\text{exch}} = (15\pm5)\times10^{-6}. \quad (19)$$

At present there seem to be no theoretical calculations of the contribution of exchange currents.

In view of the satisfactory agreement now of the experimental and theoretical values of P for hydrogen when the new value[20] of α is used and of the existence of accurate experimental values for the hfs anomalies of D, H^3 and He^3, it is important to reconsider the theory of the hfs anomalies for these light nuclei, including the contribution of exchange currents.

ACKNOWLEDGMENTS

The authors would like to thank R. J. Blume and S. A. Lewis for valuable experimental assistance.

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