Effective N-N Interaction and Electromagnetic Operators Deduced from the Tabakin Potential: Application to Nickel Isotopes*

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The properties of low-lying states of even and odd nickel isotopes are derived by means of the quasiparticle second-Tamm-Dancoff theories in the frame of the mixed configurations $(2p_{3/2}, 1f_{5/2}, 2p_{1/2}, 1g_{9/2})^n$. The Tabakin potential renormalized for particle-hole excitations of the core is used. The agreement between theoretical and experimental spectra is good for Ni^{58-60} but becomes worse for Ni^{61-64}. Calculated effective electromagnetic operators are used to compute $E2$ and $M1$ transition rates, $E2$ and $M1$ static moments, and inelastic electron scattering form factors. The theory fails in the case of the $E2$ operator, while it is successful for the $M1$ operator. Spectroscopic factors for one-nucleon transfer reactions are calculated and found to be in general agreement with experiment.

I. INTRODUCTION

RECENTLY,[1–7] the nuclear shell model and its approximations have been extensively applied to the study of nickel isotopes within the mixed neutron configurations $(2p_{3/2}, 1f_{5/2}, 2p_{1/2})^n$. From the existing literature one can draw some definite conclusions which we can summarize as follows.

The shell-model description of the nickel isotopes in terms of the above configurations is able to reproduce in a semiquantitative way the experimental levels.[1–3,5] On the other hand, the first and second quasiparticle approximations reproduce well the results of the exact shell-model calculations.[4,6,7] Nevertheless, the influence of neglected configurations must somehow be taken into account. This fact manifests itself in two different ways. First, the effect of virtual transitions to neglected configurations must necessarily be taken into account when one tries to use within the valence shells a force derived from a realistic nucleon-nucleon interaction.[1,7] Second, effective $E2$ and $M1$ operators, quite different from the corresponding "bare" ones, must be used to explain the electromagnetic properties of nuclear states.[2]

In the present work we adopt the microscopic point of view, i.e., we try to avoid as far as we can the introduction of quantities to be determined in a phenomenological way. We use the one-three (QTD 13), two (QTD), and two-four (QSTD) quasiparticle Tamm-Dancoff approximations, and we extend the configuration mixing to include the $1g_{9/2}$ shell because this level may be of importance for the heaviest isotopes. The two-body force is that of Tabakin corrected for core polarization according to the method of Kuo and Brown.[8,9] The effective operators to be used in evaluating the electromagnetic properties of nuclear states are calculated, using the adopted two-body force, in a way consistent with the method used to "core-polarize" the two-body force itself.[10]

The greatest limitation of our calculation, as also of similar ones, is the lack of a self-consistent determination (in the Hartree sense) of the single-particle or single-quasiparticle parameters. The results of such calculations are not yet available in the literature. In such a situation we are forced to use a phenomenological approach. Our choice of the valence single-particle energy levels is essentially based on the experimental levels of Ni^{57}. The single-particle energy levels for shells outside the valence region are chosen according to a simple, reasonable rule. Actually, the results of the core-polarization procedure are insensitive to the details of such a choice. We use harmonic-oscillator radial wave functions.

In order to appreciate properly the quality of the agreement with the experimental data which has been

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obtained, it must be stressed that, in practice, no adjustable parameter has been used. In fact, the only parameter for which we have tried various values is the energy of the $1g_{9/2}$ level. It turned out that the results are practically insensitive to a rather large variation of this parameter, with the only obvious exception of the 3^- and $\frac{9}{2}^+$ levels of the even and odd isotopes, respectively.

II. SPECTRA AND ELECTROMAGNETIC PROPERTIES OF EVEN NICKEL ISOTOPES

We calculate the properties of low-lying states of nickel isotopes within the configurations $(2p_{3/2}, 1f_{5/2}, 2p_{1/2}, 1g_{9/2})^n$, n being the number of neutrons exceeding 28. Inclusion of the $1g_{9/2}$ among the valence shells allows us to construct 3^- states. In agreement with most authors,[1-7] we take for the unperturbed single-particle (sp) energies $E_0(2p_{3/2})$, $E_0(1f_{5/2})$, $E_0(2p_{/12})$, the values obtained from the experimental spectrum of Ni^{57}, i.e., 0, 0.78, 1.08 MeV, respectively.

For $E_0(1g_{9/2})$ we have tried several values ranging from 3 to 5 MeV.[2,5,11] We found that only the 3^- states are sensitive to this parameter in a relevant way. In fact, the quasiparticle (qp) energies and the occupation probabilities for the three lowest-valence shells change little when $E_0(1g_{9/2})$ is varied in the above range. We choose $E_0(1g_{9/2})=3.5$ MeV, which gives rather good results for the energies of the 3^- states.

The two-body interaction within the chosen configurations is obtained from Tabakin potential[12] by renormalizing for one-particle–one-hole excitations of the core.[8,9] We use harmonic-oscillator radial wave functions with size parameter $b=2.063$ F, this value being obtained by fitting the rms radius of Ni^{60}. Holes (h) run on the six core shells $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, and $1f_{7/2}$. Particles (p) run on the four valence shells, and on the four upper shells $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, and $3s_{1/2}$. Since the occupation probability of the valence shells is small, we do not correct for the partial inhibition of the excitations from core to valence shells due to the Pauli principle. For the same reason we do not take into account excitations from valence to upper shells. The energy denominators are "simplified",[9] i.e., they are taken equal to the difference $E_0(p)-E_0(h)$ between particle and hole sp energies. These are chosen to be

$$E_0(1p_{3/2})=E_0(1p_{1/2})=E_0(1d_{5/2})=E_0(2s_{1/2})=E_0(1d_{3/2})=-10 \text{ MeV},$$

$$E_0(1f_{7/2})=-5 \text{ MeV},$$

$$E_0(2p_{3/2})=0, \qquad E_0(1f_{5/2})=0.78 \text{ MeV},$$

$$E_0(2p_{1/2})=1.08 \text{ MeV}, \qquad E_0(1g_{9/2})=3.5 \text{ MeV},$$

$$E_0(1g_{7/2})=E_0(2d_{5/2})=E_0(2d_{3/2})=E_0(3s_{1/2})=10 \text{ MeV}.$$

The most delicate choice here is that of the position of the $1f_{7/2}$ level. A smaller value of the $(1f_{7/2})$-valence energy gap would be perhaps more realistic since the 2.6-MeV level of Ni^{57} is indicated as an excitation from the $1f_{7/2}$ shell,[2] but for a significantly smaller value it would be hard to believe in the core-polarization procedure. Essentially, we choose the smallest value of the energy gap for which the core-polarization procedure makes sense, and we test its consequences.

The Bogolyubov-Valatin canonical transformation is performed to define qp's. The parameters in the transformation are determined by the usual BCS equations.[13] Self-energies due to valence particles are not supposed to be included in the sp energies E_0, and are therefore added to the E_0's.

To calculate the QSTD eigenvectors and eigenvalues corresponding to the low-lying states of the even Ni isotopes, we construct the complete orthonormal bases involving all possible two-qp and four-qp excitations of the considered J^π. The dimensions of the QSTD secular matrices are for 0^+, 26×26; for 2^+, 59×59; for 3^+, 46×46; for 4^+, 57×57; and for 3^-, 47×47.

Before diagonalizing our secular matrices, we project out the spurious vectors due to nucleon number nonconservation entirely lying in our Hilbert space.[14] For $J^\pi=0^+$, we use both definitions of four-qp spurious vector, with ($\langle 0 \mid \psi_{sp4}\rangle \neq 0$) and without ($\langle 0 \mid \psi_{sp4}'\rangle = 0$) component on the qp vacuum. For $J^\pi \neq 0^+$, in which case all definitions give practically the same results, we use the definition "with blocking."[15]

The QTD and QSTD spectra for Ni^{58}, Ni^{60}, Ni^{62}, and Ni^{64} are given in Table I. The agreement with experimental values has a semiquantitative character. This can be called a good result, since no adjustment of parameters [except $E_0(1g_{9/2})$] was performed. The agreement is better for Ni^{58} and Ni^{60} than for Ni^{62} and Ni^{64}. This fact can be due to a possible inadequacy, for the heaviest isotopes, of our choice of the single-particle energies essentially based on the levels of Ni^{57}. It is seen that the QTD and QSTD spectra are similar and that, correspondingly, the four-quasiparticle percentages in the QSTD wave functions are small. This indicates that the residual interaction between quasiparticles is small when compared to quasiparticle energies. For $J^\pi=0^+$, we give in Table I the results obtained with both definitions of spurious vectors. As already known,[15] the definition with qp-vacuum component causes a rather strong depletion of the qp-vacuum component in the ground state.

To compute electromagnetic transition rates and inelastic electron scattering form factors, we use effective electromagnetic operators. We calculate such operators in a way consistent with the procedure

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TABLE I. QTD and QSTD spectra for the even Ni isotopes. Energies are in MeV. Four-qp percentages are given in parentheses beside each QSTD eigenvalue. For $J^\pi = 0^+$, the results obtained with spurious vectors with and without qp-vacuum component are labeled II and I, respectively. Experimental energies are taken from Lederer *et al.*[a]

	Ni^{58}				Ni^{60}			Ni^{62}			Ni^{64}		
J^π	QTD		QSTD	Expt	QTD	QSTD	Expt	QTD	QSTD	Expt	QTD	QSTD	Expt
0^+	0.000	I	−0.028(0.5)	0.	0.000	I −0.119(2.4)	0.	0.000	I −0.246(5.7)	0.	0.000	I −0.215(3.9)	0.
	2.542		2.348(7.9)	2.77	2.159	1.944(8)	2.29	1.663	1.410(8.2)	2.047	2.171	1.898(8.8)	(2.27)
	3.956	II	0.155(40)		3.253	II −0.096(39)		3.783	II −0.306(40)		4.588	II −0.280(38)	
			2.348(8)			1.942(8)			1.395(9)			1.896(9)	
2^+	1.432		1.355(3.5)	1.45	1.573	1.397(10.1)	1.3325	1.797	1.650(5)	1.172	1.823	1.621(7.3)	1.34
	2.362		2.189(11.9)	2.90	2.537	2.099(22.2)	2.158	2.507	2.167(15.9)	2.303	3.098	2.626(33.1)	(2.89)
	2.878		2.608(15.1)	(3.26)	2.600	2.453(7.5)	3.12	3.095	2.696(20.6)		3.271	3.008(10.7)	
3^+					2.954	2.640(18.2)	2.63	2.884	2.421(20.7)	(2.89)			
					3.488	3.122(15.7)		3.345	3.116(11.9)				
						3.819(94.6)			4.181(89.7)				
4^+	2.488		2.401(3.3)	2.45	2.631	2.533(4.1)	2.5057	3.000	2.893(4.4)	2.34	3.110	2.871(11.9)	(2.61)
	4.016		3.744(63.1)		3.909	3.485(72.6)		3.396	3.144(14.2)		3.596	3.339(42.6)	
	7.939		4.086(40.5)		6.923	3.876(34.4)		5.285	3.803(92.5)		4.030	3.408(10.4)	
3^-	4.487		4.402(4.4)	4.50	4.294	4.138(8.7)	(4.08)	3.964	3.695(12.7)	(3.77)	3.449	3.139(14.7)	(3.84)
	5.617		5.445(15.7)		5.085	4.800(16.7)		4.259	3.957(16.1)		3.984	3.682(13.9)	
			5.823(91.8)			5.425(95.8)			5.053(99.5)			4.499(98.8)	

[a] C. M. Lederer, J. M. Hollander, and I. Perlman, *Table of Isotopes* (John Wiley & Sons, Inc., New York, 1967), 6th ed.

TABLE II. Effective charge matrix for the $E2$ operators. The corresponding values obtained with "complete" denominators (see Ref. 10) are given in parenthesis.

n' \ n	$2p_{\frac{1}{2}}$	$2p_{\frac{3}{2}}$	$1f_{\frac{5}{2}}$	$1g_{\frac{9}{2}}$
$2p_{\frac{1}{2}}$	...	0.4694 (0.5034)	0.8091 (0.8179)	...
$2p_{\frac{3}{2}}$	0.4694 (0.5034)	0.4710	0.8552 (0.8609)	...
$1f_{\frac{5}{2}}$	0.8091 (0.8179)	0.8552 (0.8609)	0.8266	...
$1g_{\frac{9}{2}}$	...	...	...	0.5116

adopted in renormalizing the interaction for core polarization, i.e., we take into account the interaction of valence neutrons with photons through one-particle–one-hole excitations of the core. Details of the theory of effective electromagnetic operators are given in Refs. 10 and 16. The same particle-hole excitations considered in renormalizing the interaction are taken into account. The energy denominators are also the same. For electron scattering our effective operators are calculated at each value of the momentum transfer. Usually effective electromagnetic operators are simulated by assigning to valence particles a phenomenological effective charge different from the free-particle one. In order to compare our wave functions with those of other authors we calculate electromagnetic observable quantities with both types of effective operators.

We define an effective charge matrix by

$$e_\lambda(n', n) = \langle n' \,||\, \hat{O}_{E\lambda}^{(\mathrm{eff})} \,||\, n\rangle / \langle n' \,||\, \hat{O}_{E\lambda} \,||\, n\rangle_{\mathrm{ref}}, \qquad (1)$$

where $\langle n' \,||\, \hat{O}_{E\lambda} \,||\, n\rangle_{\mathrm{ref}}$ is the sp matrix element calculated with neutron effective charge $e_\lambda^{\mathrm{eff}}=1$. The matrix $e_2(n', n)$ for the $E2$ operator in the long-wavelength limit is given in Table II. For comparison, we give in parentheses below each off-diagonal matrix element the corresponding one calculated with "complete"[10] denominators. As we can see, these elements are systematically a little larger than those with "simplified" denominators. Nevertheless the difference is very little and gives only negligible differences in transition rates.

Reduced transition rates are defined by

$$B(E\lambda) = [1/(2J_i+1)]\,|\langle J_f \,||\, \hat{O}_{E\lambda}^{(\mathrm{eff})} \,||\, J_i\rangle|^2, \qquad (2)$$

where

$$\hat{O}_{E\lambda\mu} = \sum_{i=1}^{A} e_i r_i^\lambda Y_{\lambda\mu}(\Omega_i), \qquad (3)$$

and the reduced matrix element is defined as in Ref. 17. Our calculated $B(E2, 0_1^+ \to 2_1^+)$'s are given in Table III.

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TABLE III. Reduced transition probabilities $B(E2, 0_1^+ \to 2_1^+)$ for the even Ni isotopes (in e^2 F^4). Columns labeled $e(n', n)$ give the results obtained with the calculated effective operator. The states 0_1^+ are obtained with spurious vectors without qp-vacuum component. Experimental values are taken from Refs. a and b.

	Ni^{58}			Ni^{60}			Ni^{62}			Ni^{64}	
	$e^{\mathrm{eff}}=1$	$e(n', n)$	Expt	$e^{\mathrm{eff}}=1$	$e(n', n)$	Expt	$e^{\mathrm{eff}}=1$	$e(n', n)$	Expt	$e^{\mathrm{eff}}=1$	$e(n', n)$	Expt
$B(E2, 0_1^+ \to 2_1^+)$	176.95	51.33	730±70	202.59	58.26	970±80	265.95	142.37	840±170	397.97	199.03	870±170

[a] P. H. Stelson and L. Grodzins, Nucl. Data **1**, 21 (1965).

[b] R. K. Mohindra and D. M. Van Patter, Phys. Rev. **139**, B274 (1965).

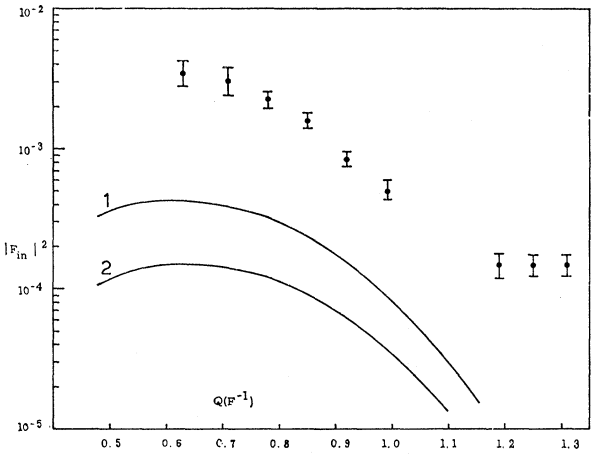

FIG. 1. Form factors $|F_{in}|^2$ for the reaction $Ni^{60}(e, e')Ni^{60}(2_1^+)$ at 183 MeV as a function of the momentum transfer Q. Curve 1 is obtained with $e^{eff}=1$, curve 2, with the calculated effective operators. Experimental bars are taken from Ref. 20.

Spurious vectors without qp-vacuum components are used for the 0_1^+ states. Consideration of the results obtained with $e_2^{eff}=1$ shows that a value $e_2^{eff}\approx 2$ would be necessary to get argeement with experiment. This is essentially consistent with what is obtained by other authors with different models and effective interactions.[2,5] On the other hand, our calculated effective charge matrix of Table II has elements equal to about 0.5 or 0.8. Correspondingly, the quantities $B(E2,\ 0_1^+\rightarrow 2_1^+)$ given by the calculated effective operator are too small by a factor of the order of 10. Better results could be obtained by modifying the value of $E_0(1f_{7/2})$, but it seems more reasonable to conclude that the particle-hole theory of the effective $E2$ operator fails in the frame of the configuration mixing considered here.

The $B(E2)$'s for transitions other than $0_1^+\rightarrow 2_1^+$ have revealed themselves highly unstable. For example, the quantity $B(E2,\ 2_2^+\rightarrow 0_1^+)$ for Ni^{62} is reduced by a factor 25 when the calculated effective operator is used instead of the operator corresponding to $e_2^{eff}=1$. Similarly, the quantity $B(E2,\ 2_3^+\rightarrow 0_1^+)$ for Ni^{60} is increased by a factor 2. Clearly (see Table II), this indicates that the result is due to the game of cancellations. Owing to the uncertainties which affect our (and other authors') wave functions and electromagnetic operators, no reliable predictions can be made

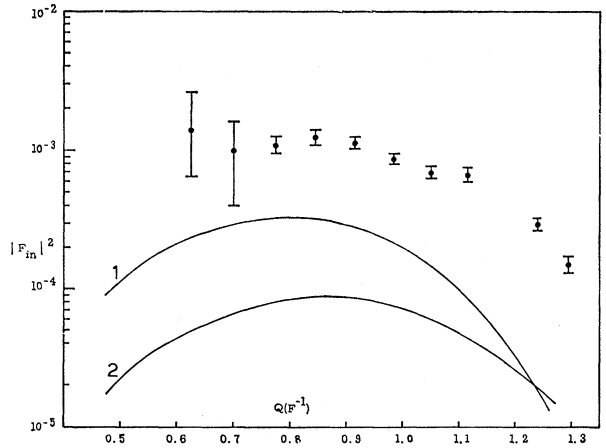

FIG. 2. The same as Fig. 1, but for the reaction $Ni^{60}(e, e')Ni^{60}(3_1^-)$.

TABLE IV. Form factors $|F_{in}|^2$ for the reactions Ni^{60} (e, e') $Ni^{60}(2_1^+)$ and $Ni^{60}(e, e')Ni^{60}(3_1^-)$. Column labeled e_{th} is obtained with the calculated effective operators. Experimental values are taken from Ref. 21.

E (MeV)	θ (deg)	q (F^{-1})	$10^4\,\lvert F_{in}\rvert^2$ $e_{eff}=1$	e_{th}	Expt	
45.12	70	0.259	1.124	0.334	2.55	
45.18	90	0.319	2.055	0.622	6.13	
45.19	110	0.370	2.910	0.897	7.94	
45.25	130	0.410	3.525	1.104	10.89	
59.95	90	0.425	3.414	1.066	11.12	2_1^+
54.80	110	0.449	3.819	1.209	12.69	
59.94	110	0.492	4.152	1.337	14.93	
54.38	150	0.526	4.359	1.436	15.84	
64.23	110	0.528	4.319	1.412	16.48	
59.95	90	0.415	0.872	0.168	1.39	
54.80	110	0.438	1.156	0.227	2.05	
59.94	110	0.481	1.532	0.307	2.74	3_1^-
63.72	110	0.512	1.817	0.371	3.56	
54.38	150	0.513	1.997	0.412	3.61	

TABLE V. The same as Table IV, but for Ni^{62}.

E (MeV)	θ (deg)	q (F^{-1})	$10^4\,\lvert F_{in}\rvert^2$ $e_{eff}=1$	e_{th}	Expt	
45.10	70	0.259	1.484	0.805	2.55	
45.18	90	0.320	2.713	1.481	5.91	
45.17	110	0.370	3.830	2.106	8.82	
45.28	130	0.411	4.632	2.562	10.82	2_1^+
54.88	110	0.451	5.008	2.781	13.28	
56.10	120	0.487	5.413	3.024	14.12	
64.24	110	0.529	5.596	3.143	16.68	
58.42	70	0.329	0.061	0.016	0.60	
56.24	70	0.316	0.052	0.014	0.46	
60.27	70	0.340	0.069	0.019	0.51	
56.25	90	0.390	0.139	0.038	0.68	
60.25	90	0.419	0.179	0.049	1.01	3_1^-
56.24	110	0.452	0.256	0.072	1.47	
56.10	120	0.476	0.313	0.089	2.09	
60.26	150	0.572	0.533	0.160	3.84	

TABLE VI. QTD 13 spectra for the odd Ni isotopes. Energies are in MeV. Three-qp percentages are given in parentheses beside each QTD 13 eigenvalue. Experimental values are taken from Refs. 23–25.

	Ni^{59}		Ni^{61}		Ni^{63}	
J^π	QTD 13	Expt	QTD 13	Expt	QTD 13	Expt
$\frac{1}{2}^-$	0.41 (8)	0.47	0.00 (10)	0.28	0.00 (5)	0.00
	2.59 (96)	(1.31)	2.12 (92)	(0.65)	2.64 (97)	(1.01)
	0.00 (2)	0.00	0.46 (2)	0.00	0.78 (3)	0.16
$\frac{3}{2}^-$	2.14 (99)	0.89	2.12 (100)	(1.11)	2.19 (98)	0.53
	2.28 (99)		2.44 (98)	1.17	2.44 (98)	
$\frac{5}{2}^-$	0.82 (5)	0.34	0.70 (3)	0.07	0.22 (4)	0.09
	1.52 (97)	(1.69)	1.94 (99)	0.91	2.03 (99)	
$\frac{7}{2}^-$	2.53 (100)	(1.97)	2.38 (100)	(1.46)	2.39 (100)	(1.71)
	2.68 (100)	(2.65)	3.02 (100)		2.72 (100)	(1.91)
$\frac{9}{2}^+$	2.63 (6)	3.07	1.99 (7)	(2.13)	0.72 (7)	(1.27)
	4.45 (99)		3.66 (99)		2.74 (99)	(2.52)

for such quantities. We only mention that large values can be obtained for the ratio $B(E2, 2_2^+ \to 2_1^+)/B(E2, 2_2^+ \to 0_1^+)$. For example, we get the value 247 for Ni^{62} when the calculated $E2$ operator is used.

We calculate inelastic electron scattering form factors for excitation of 2_1^+ and 3_1^- levels of Ni^{60} and Ni^{62}. The form factor is defined by

$$|F_{\text{in}}|^2 = \sigma(E_0, \theta)/Z^2\sigma_{\text{Mott}}(Z=1), \qquad (4)$$

where $\sigma(E_0, \theta)$ is the differential cross-section. We use essentially the Born approximation,[18] but we correct roughly for distortion effects by using a modified momentum transfer [see Eq. (8.13) of Ref. 18]. We include only the Coulomb part in $|F_{\text{in}}|^2$ [Eq. (3.64) of Ref. 18] because the transverse parts are negligible.[19] We checked numerically that this is also true in the present case. We compared our results with the experimental data of Crannell *et al.*,[20] referring to 183-MeV electrons, and with the data of Duguay *et al.*,[21] who used electrons of various energies near 50 MeV.

In Fig. 1, we give our results for the reaction $Ni^{60}(e, e')Ni^{60}(2_1^+)$ at 183 MeV. The angular distribution is good, but the absolute values are too small by a factor 10 for $e^{\text{eff}} = 1$ and about 20 for the calculated effective operator. In Fig. 2, the results for the reaction $Ni^{60}(e, e')Ni^{60}(3_1^-)$ at the same energy are given. Also in this case the angular distribution is good but the absolute values are too small by approximately the same factors. Our results at the energies and angles of Ref. 21 for Ni^{60} and Ni^{62} are given in Tables IV and V. The energy angle distribution is rather good. The Born approximation, roughly corrected for the distortion effects as mentioned above, seems to work even at these energies ($\sim$50 MeV), at least for $Z=28$. The absolute values are again too small. They are better for Ni^{62} than for Ni^{60}. This is what happens for transition rates. We can conclude that the comparison between theoretical and experimental values for inelastic electron scattering to 2_1^+ levels gives essentially the same results as for transition rates. Inelastic electron scattering to 3_1^- levels gives new information but the result is that, also in the $E3$ case, the calculated effective operator is too small.

TABLE VII. Effective magnetic reduction matrix defined by Eq. (6).

n \ n'	$2p_{\frac{1}{2}}$	$2p_{\frac{3}{2}}$	$1f_{\frac{5}{2}}$	$1g_{\frac{9}{2}}$
$2p_{\frac{1}{2}}$	0.8478	0.4220		
$2p_{\frac{3}{2}}$	0.4220	0.5262	∞	
$1f_{\frac{5}{2}}$		∞	0.4121	
$1g_{\frac{9}{2}}$				0.5216

TABLE VIII. Magnetic moments (in μ_N) for the $(\frac{3}{2}^-)_1$ and $(\frac{5}{2}^-)_1$ states of Ni^{61}. Columns labeled μ^{bare} and μ^{eff} give, respectively, the values obtained with the bare and effective operators. Experimental values are taken from Refs. 28 and 29.

J^π	μ^{bare}	μ^{eff}	Expt
$(\frac{3}{2}^-)_1$	-1.75	-0.93	-0.74868
$(\frac{5}{2}^-)_1$	1.30	0.49	± 0.3

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TABLE IX. Reduced transition probability $B(M1, (\frac{5}{2}^-)_1 \rightarrow (\frac{3}{2}^-)_1)$ for Ni^{61}. Rows labeled μ^{bare} and μ^{eff} give, respectively, the values obtained with the bare and effective operators. Columns labeled M_{11}, M_{13}, M_{31}, and M_{33} give the contributions to the transition amplitude coming from parts of the nuclear states with different number of quasiparticles. $B(M1)$ is in $(\mu_N)^2$. The experimental value is taken from Holland *et al.*[a]

$(\frac{5}{2}^-)_1 \rightarrow (\frac{3}{2}^-)_1$	M_{11}	M_{13}	M_{31}	M_{33}	$B(M1)$	Expt
μ^{bare}	0.	−0.012	−0.006	−0.009	7.7×10^{-4}	2.49×10^{-2}
μ^{eff}	−0.105	0.006	0.002	−0.004	1.02×10^{-2}	

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III. SPECTRA AND ELECTROMAGNETIC PROPERTIES OF ODD Ni ISOTOPES

For the odd Ni isotopes we use within QTD 13 the same sp parameters and the same two-body interaction used for the even isotopes. The dimensions of the secular matrices in the space of one and three qp are 9×9 for $J^\pi=\frac{1}{2}^-$, 16×16 for $\frac{3}{2}^-$, 17×17 for $\frac{5}{2}^-$, 12×12 for $\frac{7}{2}^-$, and 17×17 for $\frac{9}{2}^+$. Here also we have to project out the spurious kets before diagonalization. For each J^π for which there is a 1-qp state in the basis, there exists one spurious ket which is defined as in Ref. 22. This definition is perfectly analogous to what we used in the case of the even isotopes for $J^\pi\neq0^+$.

Our calculated spectra are given in Table VI, together with the experimental data.[23–25] The agreement is not good for Ni^{61} and Ni^{63}, and not bad for Ni^{59}. The trend to a better agreement for the lightest isotopes, already observed in the even isotopes, is confirmed in the odd ones. The first state of each J^π is an almost pure 1-qp state, except, of course, for $\frac{7}{2}^-$.

Similar to what we did for the even isotopes, we compute the electromagnetic observable quantities by means of calculated effective operators. To our knowledge, the only existing experimental data refer to Ni^{61} and are the quadrupole moment of the ground state $(\frac{3}{2}^-)_1$, the magnetic moments of $(\frac{3}{2}^-)_1$ and $(\frac{5}{2}^-)_1$, and the reduced rate for the transition $(\frac{5}{2}^-)_1 \rightarrow (\frac{3}{2}^-)_1$.

The quadrupole moment is defined by

$$Q=\tfrac{4}{5}\sqrt{\pi}\langle J\,J\,J\,-J\mid 2\,0\rangle\langle J\parallel \hat{O}_{E2}{}^{\rm eff}\parallel J\rangle. \qquad (5)$$

We get for Ni^{61} $Q[(\frac{3}{2}^-)_1]=0.0756$ b with $e^{\rm eff}=1$ and 0.0417 b with the calculated effective operator. The experimental value[26] is 0.162 b. Since the quadrupole moment depends linearly on charge, while transition rates depend quadratically, these results are analogous to the results for the $B(E2)$'s. We note that the sign is correct. We mention that the 1-qp–3-qp terms in the matrix element for Q are as important as the 1-qp–1-qp term, in spite of the small 3-qp percentage in the wave function. The sensitivity of the quadrupole moment to small admixtures in the wave function has already been pointed out.[27]

We calculate the static magnetic moments and the $M1$ transition rate with the bare operator due to the intrinsic magnetic moment of valence neutrons only and with the calculated effective operator including corrections for particle-hole excitations. In order to compare the two operators, we define an effective magnetic reduction matrix by

$$\mu(n',n)=\langle n'\parallel \hat{\mu}^{\rm eff}\parallel n\rangle/\langle n'\parallel \hat{\mu}\parallel n\rangle, \qquad (6)$$

where $\hat{\mu}=(e\hbar/2Mc)(g_s\mathbf{s}+g_l\mathbf{l})$ is the single-particle magnetic moment operator. The matrix element $\langle n'\parallel \hat{\mu}^{\rm eff}\parallel n\rangle$ is given by formula (6) of Ref. 10 (with neutrons and protons interchanged). We take into account the same transitions and use the same energy denominators as in the case of the renormalization of the interaction. The effective magnetic reduction matrix is given in Table VII. It is seen that the reduction effect is rather strong, being near to 50% for all the elements, except one which is reduced by 15%. The element $\mu(\frac{5}{2},\frac{3}{2})$ is infinite because the corresponding bare element is l-forbidden, while $\langle\frac{3}{2}\parallel \hat{\mu}^{\rm eff}\parallel\frac{3}{2}\rangle=0.7668\ \mu_N$. As we shall see, this breaking of a selection rule has an important effect on $B(M1, (\frac{5}{2}^-)_1\rightarrow(\frac{3}{2}^-)_1)$. At first sight, the effect to be expected from using $\mu(n',n)$ instead of 1 is a decrease of the calculated quantities.

The magnetic moment is defined by

$$\mu=[J/(J+1)(2J+1)]^{1/2}\langle J\parallel\hat{\mu}\parallel J\rangle. \qquad (7)$$

Our results are given in Table VIII, together with experimental values.[28,29] We see that the correction in passing from bare to effective operator is in the right direction and of approximately the right magnitude. We mention that the main contribution to the magnetic moment for both states comes from the 1-qp–1-qp term in the matrix elements.

The reduced $M1$ transition rate is defined by

$$B(M1)=(3/4\pi)(2J_i+1)^{-1}\,|\langle J_f\parallel\hat{\mu}\parallel J_i\rangle|^2.$$

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TABLE X. Experimental and calculated spectroscopic factors $(2J+1)S_J(0_1^+, J_1^\pi)$ from stripping reactions for the lowest-lying $(J)_1^\pi$ states in the odd-mass residual nickel isotopes Ni^{59} to Ni^{63}.

Residual nucleus	l, J^π	Experimental $(2J+1)S$		Calculated Present work $(2J+1)S$		a $(2J+1)S$	Energy (MeV) Expt	Energy (MeV) Present work
		b	c	I	II		b	
Ni^{59}	1, $(\frac{3}{2}^-)_1$	2.77	2.74	2.59	1.58	2.36	0.00	0.00
	3, $(\frac{5}{2}^-)_1$	5.19	4.05	5.37	3.25	5.24	0.34	0.82
	1, $(\frac{1}{2}^-)_1$	1.24	1.26	1.60	1.04	1.70	0.47	0.41
	4, $(\frac{9}{2}^+)_1$	7.50	10.60	9.28	5.62		3.07	2.63
		b	d				b	
Ni^{61}	1, $(\frac{3}{2}^-)_1$	1.67	1.49	1.20	0.66	1.73	0.00	0.46
	3, $(\frac{5}{2}^-)_1$	3.37	3.04	5.24	3.26	4.35	0.07	0.70
	1, $(\frac{1}{2}^-)_1$	1.21	1.23	1.18	0.92	1.38	0.28	0.00
	4, $(\frac{9}{2}^+)_1$	7.1	8.45	8.87	5.53		2.13	1.99
		e					e	
Ni^{63}	1, $(\frac{1}{2}^-)_1$	0.75		0.52	0.38		0.00	0.00
	3, $(\frac{5}{2}^-)_1$	2.39		4.85	3.01		0.09	0.22
	1, $(\frac{3}{2}^-)_1$	1.07		0.42	0.27		0.16	0.78
	4, $(\frac{9}{2}^+)_1$	~6.1		8.39	5.44		1.31	0.82

[a] Reference 2. [b] Reference 23. [c] Reference 25, part 1. [d] Reference 25, part 2. [e] Reference 30.

Our results with both bare and effective operators are given in Table IX. Rather surprisingly, a very strong enhancement of $B(M1)$ is found in passing from bare to effective operators. Inspection of the various contributions to the amplitude shows that this is due to the fact that the 1-qp–1-qp term is 0 for the bare operator while it is large for the effective operator. In turn, this is a consequence of breaking l forbiddenness. The result with the effective operator is in good agreement with the experimental value. Contrary to the $E2$

TABLE XI. Experimental and calculated spectroscopic factors $S_J(J_1^\pi, 0_1^+)$ from pickup reactions for the lowest-lying $(J^\pi)_1$ states in the odd-mass residual nickel isotopes Ni^{59} to Ni^{63}.

Residual nucleus	l, J_1^π	Experimental[a] S_{SE}	Experimental[a] S_{CB}	Calculated present work S I	Calculated present work S II	Energy (MeV) Expt[a]	Energy (MeV) Present work
Ni^{59}	1, $\frac{3}{2}^-$	2.22	2.36	2.66	1.79	0.00	0.00
	3, $\frac{5}{2}^-$	1.15	1.24	0.45	0.31	0.34	0.82
	1, $\frac{1}{2}^-$	0.41	0.52	0.60	0.27	0.47	0.41
Ni^{61}	1, $\frac{3}{2}^-$	2.77	3.72	3.25	2.07	0.00	0.46
	3, $\frac{5}{2}^-$	2.59	3.72	0.68	0.51	0.07	0.70
	1, $\frac{1}{2}^-$	0.88	1.40	1.29	0.82	0.28	0.00
	4, $\frac{9}{2}^+$	~0.72	1.32	0.28	0.14	2.14	1.99
Ni^{63}	1, $\frac{1}{2}^-$	0.47	1.16	1.33	0.85	0.00	0.00
	3, $\frac{5}{2}^-$	3.43	6.69	1.71	1.22	0.09	0.22
	1, $\frac{3}{2}^-$	2.42	4.24	3.28	2.13	0.16	0.78
	4, $\frac{9}{2}^+$	~0.82	~1.98	1.07	0.62	1.27	0.82

[a] Reference 24.

TABLE XII. Experimental and calculated spectroscopic factors from one-neutron transfer reactions on Ni^{61}. (a) Spectroscopic factors $S'=\frac{1}{4}(2J+1)S_l(\frac{3}{21}^-, J_n^+)$ from the reaction Ni^{61} $(d, p)Ni^{62}$, leading to some low-lying states (J_n^+) in the even mass residual nucleus Ni^{62}. (b) Spectroscopic factors $S=S_l(J_n^+, \frac{3}{21}^-)$ from the reaction $Ni^{61}(d, t)Ni^{60}$ leading to some low-lying states (J_n^+) in the even-mass residual nucleus Ni^{60}.

Residual nucleus	l, J_n^+	Experimental[a] E (MeV)	S'	Calculated present work S' I	S' II	E (MeV)
(a)	$1, 0_1^+$	0.00	0.45	0.81	0.52	0.00
Ni^{62}	$1, 2_1^+$	1.17	0.31	0.09		1.65
	$1, 0_2^+$	2.05	0.085	0.000	0.003	1.70
	$1, 2_2^+$	2.34	0.037	0.51		2.17
	$3, 4_1^+$	2.34	0.55	1.66		2.89
		b				
		E (MeV)	S	S		
(b)	$1, 0_1^+$	0.00	0.46	0.30	0.17	0.00
Ni^{60}	$1, 2_1^+$	1.33	0.62	0.72		1.40
	$3, 2_2^+$	2.17	0.22	0.02		2.10
	$1, 0_2^+$	2.29	0.11	0.14	0.13	2.06
	$3, 4_1^+$	2.52	0.44	0.21		2.53

[a] Reference 30.
[b] Reference 24.

case, some reasonable change (decreasing) in the energy denominators used in the renormalization procedure could probably give an even better agreement for all $M1$ data.

IV. SPECTROSCOPIC FACTORS FOR ONE-NEUTRON TRANSFER REACTIONS

Stripping and pick-up reactions both on even and odd isotopes of nickel, have been extensively analyzed in the last years by several experimental groups.[23–25,30] Theoretically they have been studied in the framework of the exact shell-model approach to nuclear structure.[2,5] They may provide important nuclear-structure information. In particular, the spectroscopic structure factors, extracted according to the distorted-wave Born-approximation (DWBA) theory from the results of a single-nucleon transfer reaction, yield direct information about the various single-particle strengths contained in the states excited in the reaction. They turn out to be quantities very sensitive to the details of the nuclear wave functions used to describe both the even and the odd nuclei involved.

We calculated these spectroscopic factors for stripping (d, p) and pickup (d, t) reactions on all the even- and the odd-mass nickel isotopes from Ni^{58} to Ni^{64}, using systematically the QTD 13 eigenfunctions for the states of the odd-mass isotopes, and the QSTD eigenfunctions for the states of those of even mass. The general definitions and expressions used in these calculations are those of Ref. 31. In Table X, we give our numerical results for the stripping (d, p) reactions on the even isotopes Ni^{58}, Ni^{60}, and Ni^{62} and compare them with the experimental data of Refs. 23, 25, and 30 and the predictions of Cohen *et al.*[2]

In Table XI, we give the results for pickup (d, t) reactions on the even isotopes Ni^{60}, Ni^{62}, and Ni^{64} and compare them with the two sets of spectroscopic factors of Ref. 24 extracted from the experimental cross sections via two different DWBA calculations. In Table XII, we show the calculated spectroscopic factors for the stripping and the pickup reactions on Ni^{61}, together with the corresponding experimental data.[24,30] Both variants of the QSTD theory for the eigenvectors 0^+ of the even isotopes discussed in Ref. 15 are used.

As for the case of tin isotopes,[32,33] the spectroscopic factors obtained with the variant $\langle 0 \mid \psi_{sp}' \rangle = 0$ are almost everywhere larger than those obtained with the variant $\langle 0 \mid \psi_{sp} \rangle \neq 0$. The numerical difference is larger here than for tin. In the present case, however, the comparison with experimental values does not favor any of the two sets of results, a somewhat intermediate value between the two sets being generally in fair agreement with the experiment.

V. CONCLUSIONS

The agreement we found between theoretical and experimental spectra is much better for the lightest than for the heaviest isotopes. A reason for this could be the inadequacy of the single-particle energies for the heaviest isotopes. We note that we do not compensate for such a possible inadequacy by adjusting the effective interaction. In fact, this is completely determined by the Tabakin potential and by the procedure adopted for taking core polarization into account. We think that a better determination of valence sp energies and of energy denominators in renormalization procedure can bring a really good agreement between theoretical and experimental spectra.

The calculated $E2$ effective operator was found too small, roughly by a factor 4. It is not possible to remedy this situation by varying the energy denominators. This failure seems to indicate that a larger configuration mixing, including at least the $1f_{7/2}$ proton shell, is necessary.

On the other hand, fairly good results have been obtained with the effective $M1$ operator. The reduction

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effect due to core polarization is sufficient to explain the small observed values of the static moments. The breaking of l forbiddenness accounts for the large observed value of $B(M1, (\frac{5}{2}^-)_1 \rightarrow (\frac{3}{2}^-)_1)$. These results, together with the similar results obtained for Sn^{116},[10] indicate the $M1$ observables as quantities sensitive to the degree of configuration mixing in a strong but regular way.

Essential agreement with experiment is obtained for one-nucleon transfer reaction spectroscopic factors. The ambiguity related to the two possible definitions of spurious vectors for $J^\pi = 0^+$ indicates that calculations in a number conserving approximation to exact shell model would be desirable.

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Decay of the 6.18-MeV $J^\pi = 0^+$ Level of Be^{10}†

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The reaction $Be^9(d, p)Be^{10}$ (at $E_d = 3.25$ MeV) has been used to investigate the decay of the 6.18-MeV $J^\pi = 0^+$ level of Be^{10}. Both of the γ-ray transitions $6.18 \rightarrow 5.96$ and $6.18 \rightarrow 3.37$ have been observed by means of γ-γ coincidence experiments using Ge(Li) and NaI(Tl) detectors. The mean life of the 6.18-MeV state is found to be $1.1_{-0.3}^{+0.4}$ psec from a Doppler-shift measurement on the $6.18 \rightarrow 5.96$ transition. It is concluded that the $6.18 \rightarrow 5.96$ transition takes place mainly to the $J^\pi = 1^-$ component of the $J^\pi = (1^-, 2^+)$ 5.96-MeV doublet and has an energy of 219.4 ± 0.3 keV. The intensities of proton groups leading to levels of Be^{10} in the (d, p) reaction at $E_d = 3.25$ MeV were determined with a Buechner spectrograph By combining the present results with previous measurements on the 6.18-MeV $E0$, ground-state transition, the branching ratios (in %) from the 6.18-MeV state to the states at 5.96 MeV ($J^\pi = 1^-$), 3.37 MeV ($J^\pi = 2^+$), and 0 MeV ($J^\pi = 0^+$) are found to be 4.6 ± 1.5, 95 ± 2, and 0.24 ± 0.08, respectively. The significance of these results is discussed. Incidental results are a value of 169.25 ± 0.04 keV for the C^{13} $3.85 \rightarrow 3.68$ transition and a meanlife of 2.0 ± 0.6 psec for the B^{10} 2.15-MeV level.

I. INTRODUCTION

THE $J^\pi = 0^+$ state at an excitation energy of 6.18 MeV in Be^{10} is thought to be an interloper in the $1p$ shell; that is, one of those states which, although of the "correct parity" $(-)^A$, does not belong to the $(1s)^4(1p)^{A-4}$ configuration. Other examples are the $J^\pi = 0^+$ states at 7.65 MeV in C^{12}, 6.59 MeV in C^{14}, and 6.05 MeV in O^{16}.[1] The Be^{10} state is thought[2] to be predominantly $(1p)^{A-6}(2s, 1d)^2$ as is the C^{14} state.[3] The interloper states in even-even nuclei are almost certainly considerably more complicated.[4,5] One reason for this is that 4-particle–4-hole states are energetically favored in even-even nuclei considerably more than in odd-odd nuclei.

Available evidence indicates that the well-known enhancement of $E2$ transition rates between states of $(1p)^{A-4}$ is present to about the same extent for transitions between interloper states in the $1p$ shell. An example is the $E2$ transition between the $J^\pi = 5^+$ and 3^+ states of N^{14} at 8.96 and 6.44 MeV, respectively. These states are identified as interlopers[3] involving

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