Rotating Superconductors*

NORMAN F. BRICKMAN†
Department of Physics, Yale University, New Haven, Connecticut 06520
(Received 13 February 1969)

Measurements are presented on the magnetic field generated within a superconductor when it is set into rotation—the so-called London moment. Solid rods and hollow cylinders have been investigated in a variety of materials in both single-crystal and polycrystalline specimens, and in both elemental and alloy superconductors. In the case of the hollow cylinders, wall thicknesses as thin as 27 Å have been employed. In all cases the results are in excellent agreement with the London formula.

I. INTRODUCTION

As first predicted by London,[1] a spatially uniform magnetic field should be generated within the body of any superconductor which is set into rotation. This effect has generally become known as the "London moment." For the case of a solid superconducting cylinder rotating about its longitudinal axis of symmetry, this field (H) is given by

$$\mathbf{H}=(2m^*c/|e^*|)\boldsymbol{\omega}, \tag{1}$$

where ω is the angular velocity of rotation, c is the speed of light, and m^* and e^* are the effective mass and charge, respectively, of the superconducting electron pairs. The units of Eq. (1) for H in gauss, assuming that m^* and e^* are each twice the free-electron values, are

$$H=1.137\times10^{-7}\omega,\quad \omega \text{ in rad/sec}$$
$$H=1.191\times10^{-8}\omega,\quad \omega \text{ in rev/min}.$$

The physical meaning behind the above equation is as follows: In the solid cylinder the superconducting electrons (BCS pairs) are rotating in synchronism with the lattice ions except for those within the penetration depth; here they rotate somewhat slower, and it is this lag which gives rise to a net surface current which, in turn, produces the London field. At the surface of the cylinder this lag amounts to $2\lambda\omega/a$, where λ is the penetration depth and a is the radius of the cylinder. It is due to the small magnitude of $\lambda(\approx5\times10^{-6}$ cm), in all superconductors, that the resulting field is so weak.

The situation with respect to a non-simply-connected superconductor (e.g., a hollow cylinder) is somewhat more complicated. There always exists in any experiment an external field, that of the earth, which is never completely compensated. The Meissner effect removes this field from the solid rod, but, in the tube, the field adjusts itself slightly so that the cavity contains an integral number of flux quanta and adds to any rotational field which may be present. A free-energy calculation for this geometry yields the result

$$n\left(\frac{\Phi_0}{\pi r^2}\right)=\frac{2m^*c}{|e^*|}\omega-H_h, \tag{2}$$

where n is an integer, Φ_0 the fluxoid quantum (2.07×10^{-7} G cm^2), r the radius of the hole, and H_h the field in the hole. Since the quantum number n is fixed by the external field, and thus remains constant, the above equation yields

$$\Delta H_h=(2m^*c/|e^*|)\Delta\omega, \tag{3}$$

where ΔH_h is the field change corresponding to a change in the rate of rotation, $\Delta\omega$. This same relation is obtained for the solid rod from Eq. (1).

The first reported measurements of the London moment are due to Hildebrandt,[2] who investigated hollow cylinders of lead and niobium using a flux-gate probe as the field sensing element. Speeds as high as 7500 rev/min were employed and formula (3) was found to hold to within an accuracy of about 7%. King, Hendricks, and Rorschach[3] investigated a hollow tin cylinder, also using a flux-gate probe, to speeds beyond 12 000 rev/min. They also agreed with formula (3) to within about an 8% error. Finally, Bol and Fairbank[4] investigated solid cylinders of tin and mercury at speeds up to about 6000 rev/min. Their method differed from those discussed above in that they measured the magnetic moment of the sample as produced by the rotational flux. There was considerable scatter in their measurements, but the results generally bore out the London formula.

In the meantime, a much more sensitive method for measuring small magnetic fields has been developed.[5]

* Assisted by the Army Research Office (Durham) and the National Science Foundation. Part of a dissertation submitted for the Ph.D. degree at Yale University.
† Present address: IBM Corporation, Components Division, Poughkeepsie, N. Y.

[1] F. London, *Superfluids* (John Wiley & Sons, Inc., New York, 1950), Vol. 1.

[2] A. F. Hildebrandt, Phys. Rev. Letters **12**, 190 (1964); A. F. Hildebrandt and M. Saffren, in *Proceedings of the Ninth International Conference on Low-Temperature Physics, Columbus, Ohio*, edited by J. G. Daunt *et al.* (Plenum Press, Inc., New York, 1965).

[3] C. A. King, J. B. Hendricks, and H. E. Rorschach, in *Proceedings of the Ninth International Conference on Low Temperature Physics, Columbus, Ohio*, edited by J. G. Daunt *et al.* (Plenum Press, Inc., New York, 1965).

[4] M. Bol and W. M. Fairbank, in *Proceedings of the Ninth International Conference on Low Temperature Physics, Columbus, Ohio*, edited by J. G. Daunt *et al.* (Plenum Press, Inc., New York, 1965).

[5] R. C. Jaklevic, J. Lambe, J. E. Mercereau, and A. H. Silver, Phys. Rev. **140**, A1628 (1965); J. E. Zimmerman and A. H. Silver, *ibid.* **141**, 367 (1966).

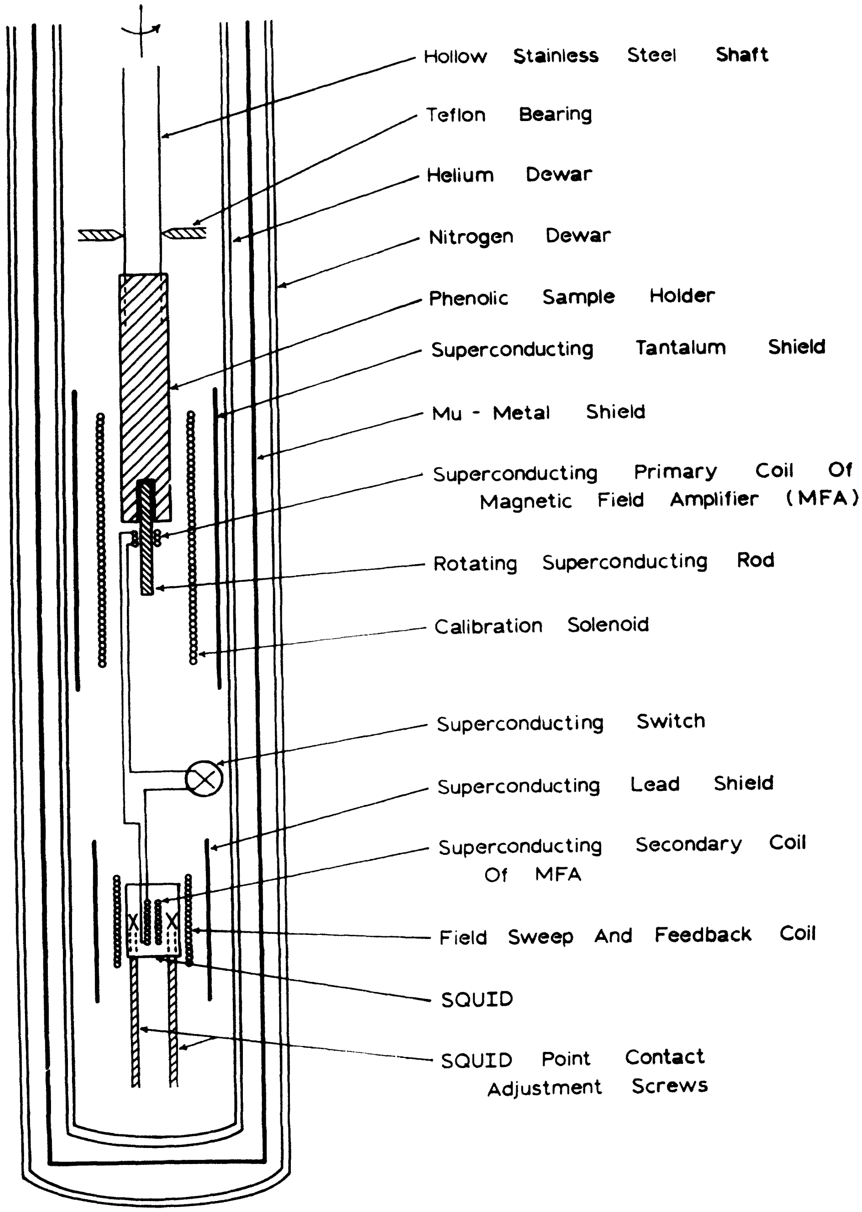

FIG. 1. Schematic drawing of apparatus.

This is based on the Josephson effect, and it has been adapted here for measurements of the London moment. As a result, the present accuracy is substantially better than was the case in the previous work and, in addition, a sufficiently wide range of materials was investigated to permit some comments as to the magnitude of the effective mass of the superconducting particles. Results will also be included of measurements of the London moment in thin-wall tubes. It has been predicted by Griffin[6] that the rotation of a thin-wall hollow cylinder should produce a field change given by

$$\Delta H_h = \frac{2m^*c}{|e^*|}\Delta\omega\left(1+\frac{2\lambda^2}{rd}\right)^{-1}, \tag{4}$$

where r is the radius of the hole and d is the wall thickness.

II. EXPERIMENTAL METHODS

The basic field-measuring device is a specially-designed superconducting quantum interference device (or SQUID),[5] which consists of a multiply connected

[6] A. Griffin, Solid State Commun. 4, 245 (1966).

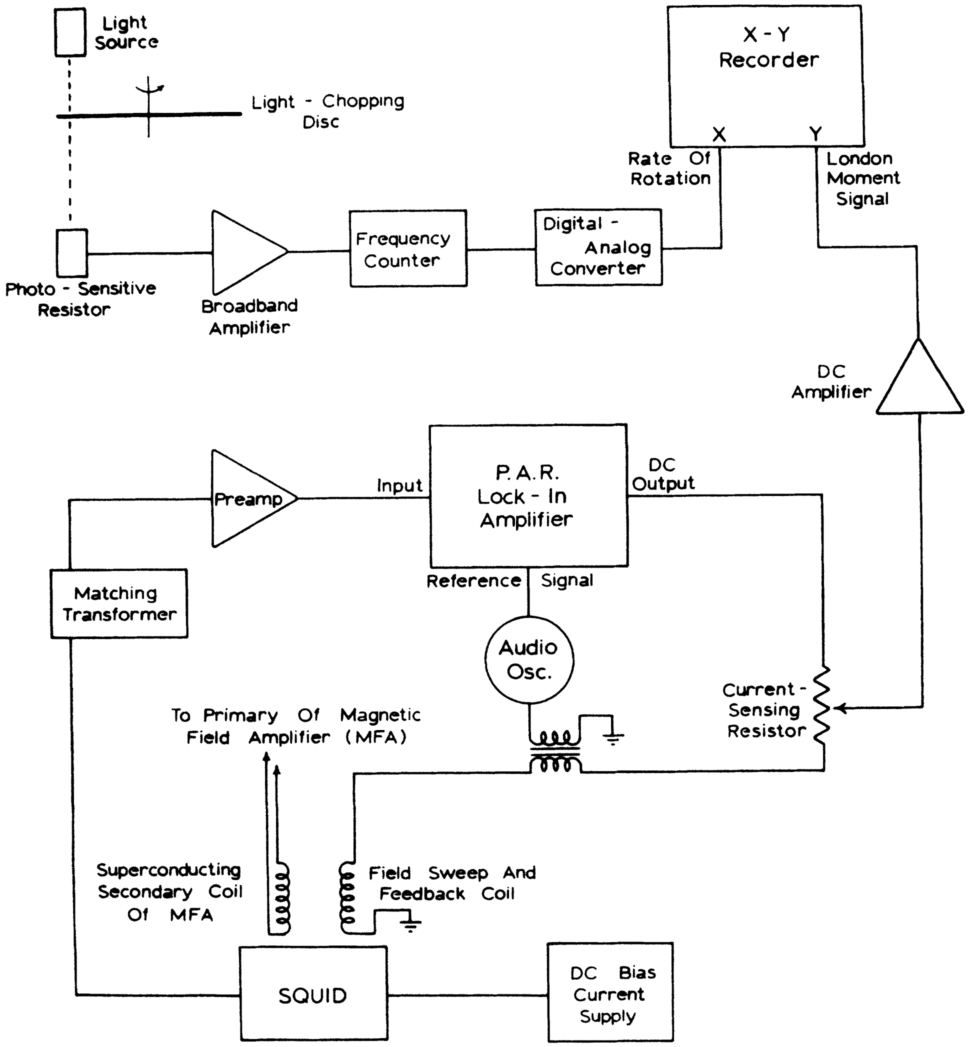

FIG. 2. Block diagram of electronics.

superconductor interrupted by two Josephson junctions. The present SQUID uses point contacts to approximate the Josephson junctions. Two pieces of niobium foil, 0.001 and 0.002 in. thick, are separated by a 0.002-in. sheet of Mylar. Two sharpened 0.020-in.-diam niobium wires are pressed into the sheets to make the two Josephson point contacts.

Figure 1 shows the entire experimental apparatus. The magnetic field amplifier (MFA) shown in the figure consists simply of two superconducting coils of niobium wire, in series, with a superconducting switch (operated from the top of the cryostat), provided to remove persistent currents from the coils prior to an experiment. The primary of the MFA surrounds the middle of the superconducting sample, while the secondary is located within the hole of the SQUID. Since the entire circuit will conserve flux, a field or flux change to be measured within the cylindrical sample results in a proportional flux change within the SQUID.

The tantalum and lead superconducting cylinders are for the purpose of providing time-invariant magnetic field levels in the experiment. Tantalum was chosen in order to be able to send it normal by over pressuring the helium Dewar, and so aid in calibrating the system. The μ-metal shield reduced the ambient magnetic field levels to about 8×10^{-4} G. A synchro receiver was used to rotate the shaft of the system to eliminate commutator-brush electrical noise, the synchro transmitter being located far from the rest of the apparatus. In making London moment plots, the rate of rotation was varied from 20 to 1050 rev/min or back down again, requiring 55 sec to go in either direction.

The electronics employed here is shown in Fig. 2, the circuits necessary for the operation of the SQUID having been described previously.[5] The feedback loop provides an output signal to the y axis of the xy recorder, which was shown to be accurately proportional to a field change within the sample that is being rotated, the system having a time constant of 0.4 sec. A light-

TABLE I. London-moment results for solid rods and a thick-wall hollow cylinder.

	Transition temperature T_c (°K)	Reduced temperature $t=T/T_c$	London-moment slope (% deviation from theory) Clockwise (%)	Counter-clockwise (%)	Average (%)
(A) Solid rod					
(1) 6-mm Sn polycrystalline	3.735	0.988	−7.8	+10.9	+1.6
		0.552	+1.6	− 1.4	+0.1
		0.32	−2.4	+ 1.9	−0.2
(2) 6-mm Sn single crystal	3.725	0.990	−3.8	0.0	−1.9
		0.55	−2.6	− 0.3	−1.5
		0.30	−0.9	− 2.9	−1.9
(3) 10.5-mm Sn single crystal	3.725	0.32	+1.1	− 1.2	−0.1
(4) 6-mm Al polycrystalline	1.165	0.94	+1.4	+ 0.2	+0.8
(5) 6-mm $In_{0.9850}Bi_{0.0150}$ polycrystalline	3.730	0.971	−3.7	+ 6.1	−0.5
		0.31	+0.1	− 0.4	−0.1
(6) 6-mm $In_{0.9750}Bi_{0.0250}$ polycrystalline	4.074	0.28	−1.4	+ 0.5	−0.5
(7) 6-mm $In_{0.1393}Pb_{0.8607}$ polycrystalline			−1.0	+ 1.2	+0.1
(B) Hollow cylinder					
2-μ Sn film on 6-mm Al polycrystalline rod					
(a) 1.65°K			+0.1	+ 1.5	+0.8
(b) 1.08°K			−0.6	− 0.1	−0.4

chopping disc mounted on the rotating shaft of the system provides for an input signal to a frequency counter and digital/analog converter, giving the x axis of the recorder a signal that is proportional to the rate of rotation.

The y axis of the xy recorder is calibrated in terms of the uniform London-moment field that is presumed to be generated within the cylinder during rotation. The calibration is based on the free-air calibration of the "calibration solenoid." The SQUID is placed within the solenoid to observe the change in the solenoid calibration due to the presence of the μ-metal shield, and the superconducting tantalum shield. Then, with the double superconducting-coil measuring system in operation and the superconducting sample present, a field change is applied with the calibration solenoid, taking into consideration the flux-conserving properties of both the sample that is present and the tantalum shield. When the system is now finally calibrated in terms of a field generated within a rotating sample, the flux-conservation nature of the tantalum shield and the sample that is rotating must again be taken into account. The demagnetizing factor of about 0.007 for the samples that have been tested here results in a negligible correction, relative to the over-all system calibration error of $\pm 3\%$ for the London-moment slopes ($\Delta H/\Delta\omega$).

All the samples that have been tested, except one, were 57.2 mm long, and had diameters ranging from 5.85 to 6.70 mm. One tin single-crystal rod was 10.39 mm in diameter and 96.4 mm long. The solid rods that have been tested were cut to length on a spark cutter in order to maintain their purity. High-purity aluminum and tin polycrystalline rods were purchased commercially, and were lightly etched to clean their surfaces and show the crystal faces. The single crystals were made by the Bridgman method from the same stock and are of random orientation. The polycrystalline-alloy rods were made of high-purity material but were not etched, as it was found that etching caused a different alloy to be formed on the surface of the rod.[7] Aluminum thin-film cylinders were made by vacuum evaporation from a tungsten boat onto a rotating Pyrex tube, with a starting pressure of 3×10^{-6} mm Hg. The film thicknesses were monitored to an accuracy of 10% with a Sloan deposit-thickness monitor that was calibrated with a Sloan sodium-light interferometer. Without breaking the vacuum, the films were often then coated with approximately 500 Å of silicon monoxide.

The critical temperatures of the solid rods (see Table I) agree well with the established values.[8] The aluminum thin films (see Table II) show the expected rise in critical temperature with decrease in film thickness, but only of less than half the magnitude of that found by Strongin and Kammerer.[9]

Whenever possible, the samples were kept stationary while cooling through T_c, and then rotated. Some of the solid rods and thin-film samples, however, tended to trap comparatively large amounts of flux in a direction not along the axis of symmetry, thus causing extra "noise" to appear in the London-moment plots. The Meissner effect in these cases was enhanced by rotating the sample at ≈ 600 rev/min while cooling through the transition temperature.[3] It was found, however, that there was never any difference in the London-moment slope for an individual sample that depended on whether or not it was rotating when cooled through its transition temperature.

[7] This possibility was suggested to me by Professor B. Serin, Rutgers University (private communication).

[8] T. Kinsel, E. A. Lynton, and B. Serin, Rev. Mod. Phys. 36, 105 (1964).

[9] M. Strongin and O. F. Kammerer, J. Appl. Phys. 39, 2509 (1968).

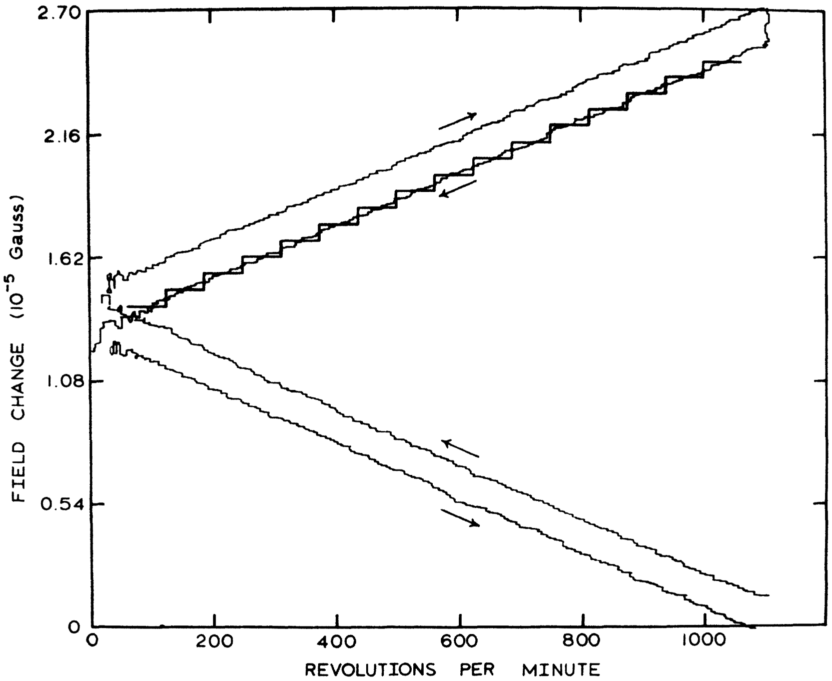

FIG. 3. Example of London-moment data. 6-mm tin single crystal rod, 1.20°K. The arrows indicate direction of speed change. The individual plots are spaced by changing the y-axis zero of reference. The steps that have been drawn in represent the postulated single-fluxoid steps (see text).

III. RESULTS

The results for the solid rods and a thick-wall hollow cylinder are tabulated in Table I. The theoretical London-moment slope is taken as that given by Eq. (3), where twice the free-electron values have been used for the effective mass and effective charge for the superconductor. Each value in the table represents an average of generally 25–30 xy-recorder plots, divided equally in each direction of rotation.

The average London-moment results listed in Table I provide a much closer verification of the theory than has previously been attained, and were made in the range of low rates of rotation (20–1050 rev/min) where no previous work has been reported. As the theory predicts, there is no consistent difference in the London-moment slope with either the direction of rotation or the size of the sample. Also, as expected, no temperature effects were observed.

It has been suggested[10] that the London moment might be produced as an array of vortex lines, each with a single quantum of magnetic flux. If this were the case, however, as opposed to the surface-current explanation that has been presented in this paper, the magnetic moment generated by rotation would not increase continuously with rate of rotation, but would go up in steps as additional flux quanta entered the material. The sensitivity and operating characteristics of the present system were such that any such steps would have been seen if they were present, but none were observed. The alloy rods listed in Table I were investigated in an effort to enhance any vortex-line

TABLE II. London-moment results for thin-wall hollow aluminum cylinders.

Thin-wall hollow Al cylinder (Å)	Transition temperature T_c	Reduced temperature $t=T/T_c$	Average experimental London-moment slope (G min/rev) ($\times 10^{-8}$)	Theoretical London-moment slope (G min/rev) ($\times 10^{-8}$)
1008 SiO coated	1.248	0.913	1.187	1.191
200	1.390	0.820	1.203	1.190
		0.953	1.264	1.188
		0.986	1.148	1.180
100	1.501	0.793	1.192	1.187
		0.962	1.150	1.175
		0.984	1.155	1.154
50, SiO coated	1.698	0.695	1.164	1.179
33, SiO coated	1.845	0.640	1.167	1.166
31,	1.876	0.613	1.111	1.163
27, SiO coated	1.865	0.638	1.091	1.154

[10] E. L. Andronikashvili, S. V. Odenov, and V. A. Shukhman, in Abstracts of the Tenth International Conference on Low-Temperature Physics, Moscow, 1966, Paper S-136 (unpublished).

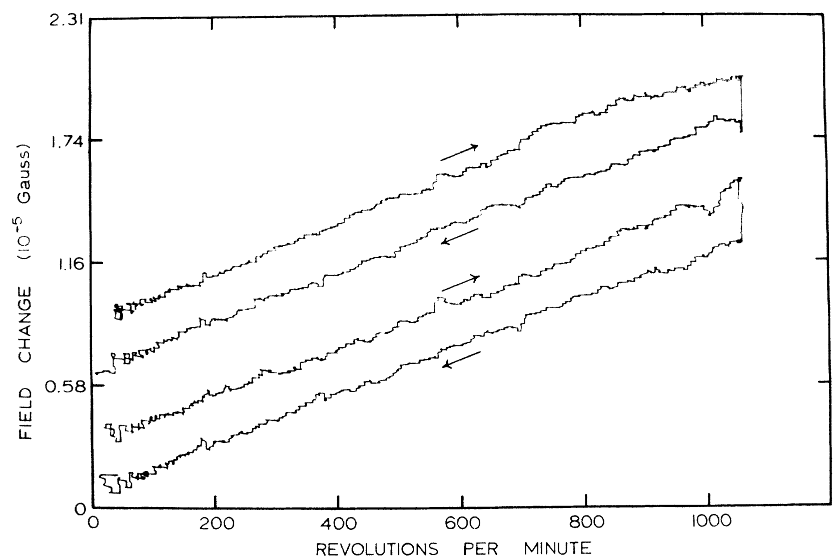

FIG. 4. Example of London-moment data. Thin-film hollow cylinder of aluminum with a 33 Å wall thickness, 1.18°K.

structure that might be present. The $In_{0.9850}Bi_{0.0150}$ rod was made both type I and type II by varying the temperature,[8] while the other two alloy rods were clearly type II. Since the materials were below H_{c1} (i.e., not in the mixed state) when rotated, no different results from the type-I superconductors were expected or obtained.

Figure 3 is an example of the London-moment data, with one plot for increasing speed and one for decreasing speed in each direction of rotation. The offset between plots is caused by simply moving the y-axis zero control of the xy recorder after making a plot. The y axis has been calibrated in terms of the uniform magnetic field that is generated in the rod upon rotation. The steps drawn over one of the plots in Fig. 3 represent the postulated single-fluxoid jumps discussed above.

Table II gives some results for thin-walled aluminum cylinders, the data being an average for the two directions of rotation, and where, as previously, no significant dependence on direction of rotation was observed. The theoretical slope is computed from Eq. (4), making use of the theoretical calculation of Miller[11] for the penetration depth in which we have taken the electron mean free path to be equal to the thickness of the film. It would clearly have been desirable from the standpoint of testing Eq. (4) to have used reduced temperatures closer to unity, especially for the thinnest films. Unfortunately, we were unable to obtain usable London-moment plots at temperatures above those given. Figure 4 shows a London-moment plot taken at 1.18°K for a tube with a 33 Å wall.

Finally, the results presented in Table I allow one to comment on the nature of the effective mass of the superconducting particles. The electron pairing basis of the BCS microscopic theory implies that m^* is twice the normal metal effective mass at the Fermi surface. The experimental work which tested the Josephson-junction frequency-voltage relation[12,13] has shown that the effective charge for the superconducting particles is at least very close to twice the free-electron charge. This leaves only the effective mass in the London-moment slope to be experimentally determined, and all the materials tested in the present experiment yield a value of close to twice the free-electron mass. This is to be expected in the present experiment, due to being able to equate the Hamiltonian for the system in the presence of a magnetic field with the zero-field Hamiltonian as transformed for a rotating system.[3,14] The result is Eq. (1), but with the effective mass and charge replaced by their free-electron values. In the only other type of experiment that gives a direct measurement for the effective mass of the superconducting particles, Zimmerman and Mercereau[15] have rotated a double-junction Josephson interferometer (vanadium metal) in a stationary magnetic field. The observed interference effects, as a function of the rate of rotation, yield a value for the effective mass of twice the free-electron mass, to within 4%.

ACKNOWLEDGMENTS

I am very indebted to my research advisor, Professor C. T. Lane, for his guidance and assistance throughout this project. I wish to acknowledge also many useful discussions with my colleagues in the low-temperature physics group.

[11] P. B. Miller, Phys. Rev. **113**, 1209 (1959).

[12] D. N. Langenberg, W. H. Parker, and B. N. Taylor, Phys. Rev. **150**, 186 (1966).

[13] J. Clarke, Phys. Rev. Letters **21**, 1566 (1968).

[14] Professor M. J. Stephen, Rutgers University (private communication).

[15] J. E. Zimmerman and J. E. Mercereau, Phys. Rev. Letters **14**, 887 (1965).